AQA

Please write clearly in block capitals.

Centre number | | | | | Candidate number | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS
Paper 1

Wednesday 6 June 2018   Morning   Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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11	
12	
13	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 $y = \dfrac{1}{x^2}$

$y = x^{-2}$

Find an expression for $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = -2x^{-3} = \dfrac{-2}{x^3}$

Circle your answer.

[1 mark]

$\dfrac{dy}{dx} = \dfrac{0}{2x}$ $\quad$ $\dfrac{dy}{dx} = x^{-2}$ $\quad$ $\dfrac{dy}{dx} = -\dfrac{2}{x}$ $\quad$ $\dfrac{dy}{dx} = -\dfrac{2}{x^3}$

2 The graph of $y = 5^x$ is transformed by a stretch in the y-direction, scale factor 5

State the equation of the transformed graph.

Circle your answer.

[1 mark]

$y = 5 \times 5^x$ $\quad$ $y = 5^{\frac{x}{5}}$ $\quad$ $y = \dfrac{1}{5} \times 5^x$ $\quad$ $y = 5^{5x}$

$f(x) \rightarrow a \times f(x$

$5^x \rightarrow 5 \times 5^x$

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3 A periodic sequence is defined by $U_n = \sin\left(\frac{n\pi}{2}\right)$

State the period of this sequence.

Circle your answer.

$\frac{2\pi}{k} = \frac{2\pi}{(\pi/2)}$

$= \frac{2}{1/2} = 4$

[1 mark]

8 $\quad$ 2π $\quad$ 4 $\quad$ π

4 The function f is defined by $f(x) = e^{x-4}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

[3 marks]

$y = e^{x-4}$ $\quad$ $\ln e^{x-4} = x-4 \ln e = x-4$

$\ln y = x-4$

$\ln y + 4 = x$ $\quad$ Range of $f(x)$ = domain $f^{-1}(x)$

$f^{-1}(x) = \ln y + 4$ $\quad$ $e^{x-4} > 0$,

$x > 0$

Turn over for the next question

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5 A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^{t} - 5$$

5 (a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{4} \times 2^{2t}$

[3 marks]

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\qquad y = a^x$

$\frac{dx}{dt} = 4 \times -\ln 2 \times 2^{-t}$ $\qquad \frac{dy}{dx} = \ln a \times a^x$

$\frac{dy}{dt} = 3 \times \ln 2 \times 2^t$

$\frac{dy}{dx} = \frac{3 \times \cancel{\ln} 2 \times 2^t}{4 \times -\cancel{\ln} 2 \times 2^{-t}} = \frac{-3}{4} \times 2^t$

5 (b) Find the Cartesian equation of the curve in the form $xy + ax + by = c$, where a, b and c are integers.

[3 marks]

$x = 4 \times 2^{-t} + 3$ $\qquad y = 3 \times 2^t - 5$

$x - 3 = 4 \times 2^{-t}$ $\qquad y + 5 = 3 \times 2^t$

$\frac{x-3}{4} = 2^{-t}$ $\qquad \frac{y+5}{3}$

$\frac{(x-3)(y+5)}{12} = 2^{-t} \times 2^t \Rightarrow 2^0 = 1$

$\frac{xy + 5x - 3y - 15}{12} = 1$

$xy + 5x - 3y - 15 = 12$

$xy + 5x - 3y = 27$ ✓

6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt{4+x}}$

[3 marks]

$$\frac{1}{\sqrt{4+x}} = (4+x)^{-1/2} = \left(4\left(1+\frac{x}{4}\right)\right)^{-1/2} = \frac{1}{2}\left(1+\frac{x}{4}\right)^{-1/2}$$

$$\frac{1}{2}\left[1+\frac{x}{4}\right]^{-1/2} \approx \frac{1}{2}\left[1+\left(-\tfrac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)}{2}\cdot\left(\frac{x}{4}\right)^2\right]$$

$$\approx \frac{1}{2}\left[1-\frac{x}{8}+\frac{3x^2}{128}+\cdots\right]$$

$$\approx \frac{1}{2}-\frac{x}{16}+\frac{3x^2}{256}$$

6 (b) Hence, find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^3}}$

[2 marks]

$$\frac{1}{2}-\frac{x}{16}+\frac{3x^2}{256}$$

$$\frac{1}{2}-\frac{-x^3}{16}+\frac{3(-x^3)^2}{256}$$

$$\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}$$

Question 6 continues on the next page

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6 (c) Using your answer to part **(b)**, find an approximation for $\int_0^1 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$, giving your answer to seven decimal places.

[3 marks]

$$\int_0^1 \frac{1}{\sqrt{4-x^3}}\,dx = \int_0^1 \frac{1}{2} + \frac{1}{16}x^3 + \frac{3}{256}x^6\,dx$$

$$= \left[\frac{1}{2}x + \frac{1}{64}x^4 + \frac{3}{1792}x^7\right]_0^1$$

$$= \frac{1}{2} + \frac{1}{64} + \frac{3}{1792} - 0$$

$$= 0.5172991$$

6 (d) (i) Edward, a student, decides to use this method to find a more accurate value for the integral by increasing the number of terms of the binomial expansion used.

Explain clearly whether Edward's approximation will be an overestimate, an underestimate, or if it is impossible to tell.

[2 marks]

underestimate because each term of the expansion is positive so there will always be more positive values to add.

6 (d) (ii) Edward goes on to use the expansion from part **(b)** to find an approximation for $\int_{-2}^{0} \frac{1}{\sqrt{4 - x^3}}\, dx$

Explain why Edward's approximation is invalid.

[2 marks]

$\left|\frac{1}{4}x^3\right| < 1$

$|x^3| < 4$

$|x| < \sqrt[3]{4}$

$-\sqrt[3]{4} < x < \sqrt[3]{4}$

-2 x

Turn over for the next question

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7 Three points A, B and C have coordinates A (8, 17), B (15, 10) and C (−2, −7)

7 (a) Show that angle ABC is a right angle.

[3 marks]

$AB^2 = (8-15)^2 + (17-10)^2 = (-7)^2 + (7)^2 = 98$

$AC^2 = (8--2)^2 + (17--7)^2 = (10)^2 + (24)^2 = 676$

$BC^2 = (15--2)^2 + (10--7)^2 = (17)^2 + (17)^2 = 578$

ΔABC $AB^2 + BC^2 = AC^2$

$98 + 578 = 676$

AC ABC must be a right angle as we have hypotenuse AC

7 (b) A, B and C lie on a circle.

7 (b) (i) Explain why AC is a diameter of the circle.

[1 mark]

Angles extended from the diameter are always right angles

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7 (b) (ii) Determine whether the point *D* $(-8, -2)$ lies inside the circle, on the circle or outside the circle.

Fully justify your answer.

[4 marks]

radius $= \frac{\sqrt{676}}{2} = 13$

$A = (8, 17)$ $C = (-2, -7)$

centre $= \left(\frac{8-2}{2}, \frac{17-7}{2}\right) = (3, 5)$

distance between D and centre to be ≤ 13

$(-8-3)^2 + (-2-5)^2 = (-11)^2 + (-7)^2$

$= 121 + 49 = 170 > 13^2$

so D lies outside the circle

Turn over for the next question

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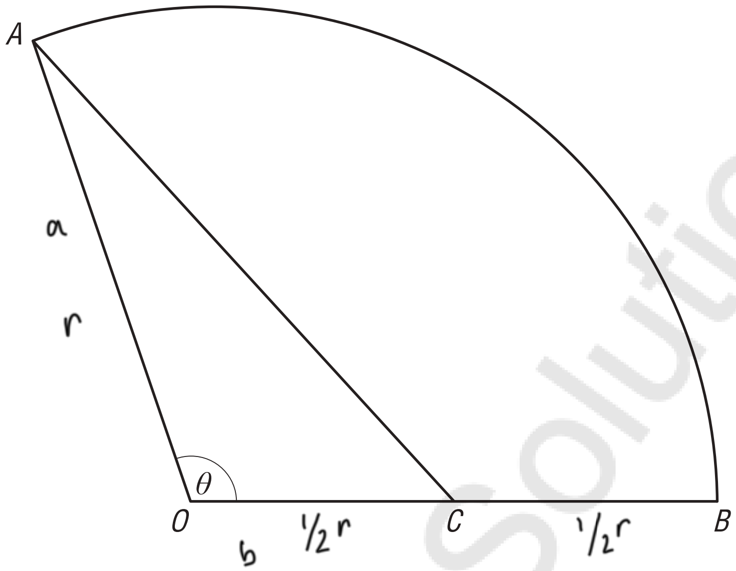
8 The diagram shows a sector of a circle *OAB*.

C is the midpoint of *OB*.

Angle *AOB* is θ radians.



8 (a) Given that the area of the triangle *OAC* is equal to one quarter of the area of the sector *OAB*, show that $\theta = 2\sin\theta$

[4 marks]

$A_\Delta = \frac{1}{2} ab \sin C$ $\quad A_s = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} r \frac{r}{2} \sin\theta$

$= \frac{r^2}{4} \sin\theta$

$r^2 \sin\theta = \frac{1}{2} r^2 \theta$

$2r^2 \sin\theta = r^2 \theta$

$2\sin\theta = \theta$

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8 (b) Use the Newton-Raphson method with $\theta_1 = \pi$, to find θ_3 as an approximation for θ. Give your answer correct to five decimal places.

[3 marks]

$\theta = 2\sin\theta$

$f(\theta) = 2\sin\theta - \theta$

$f'(\theta) = 2\cos\theta - 1$

$\theta_{n+1} = \theta_n - \frac{2\sin\theta - \theta}{2\cos\theta - 1}$

$\theta_2 = \pi - \frac{2\sin\pi - \pi}{2\cos\pi - 1} = \pi - \frac{-\pi}{-2-1} = \frac{2}{3}\pi = 2.0944..$

$\theta_3 = 2.0944.. - \frac{2\sin\theta_2 - \theta_2}{2\cos\theta_2 - 1} = 1.91322$ (5dp)

8 (c) Given that $\theta = 1.89549$ to five decimal places, find an estimate for the percentage error in the approximation found in part **(b)**.

[1 mark]

$\frac{1.91322 - 1.89549}{1.89549} \times 100 = 0.935\%$

Turn over for the next question

9 An arithmetic sequence has first term a and common difference d.

The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms.

9 (a) Show that $4a + 70d = 4a^2 + 20ad + 25d^2$

[4 marks]

$S_n = \frac{1}{2}n(2a + (n-1)d)$

$S_6 = 3(2a + 5d) = 6a + 15d$

$S_{36} = 18(2a + 35d) = 36a + 630d$

$S_6^2 = S_{36}$

$(6a + 15d)^2 = 36a + 630d$

$36a^2 + 180ad + 225d^2 = 36a + 630d$

$4a^2 + 20ad + 25d^2 = 4a + 70d$

9 (b) Given that the sixth term of the sequence is 25, find the smallest possible value of a.

[5 marks]

$a + 5d = 25 \Rightarrow a = 25 - 5d$

$4a + 70d = 4a^2 + 20ad + 25d^2$

$4(25-5d) + 70d = 4(25-5d)^2 + 20d(25-5d) + 25d^2$

$100 - 20d + 70d = 4(625 - 250d + 25d^2) + 500d - 100d^2 + 25d^2$

$100 - 20d + 70d = 2500 - 1000d + 100d^2 + 500d - 100d^2 + 25d^2$

$100 + 50d = 2500 - 500d + 25d^2$

$0 = 2400 - 550d + 25d^2$

$0 = 96 - 22d + d^2$

$0 = (d-6)(d-16)$

$d = 6$ or $d = 16$

$a = 25 - 5(16)$

$= 25 - 80 = -55$

Turn over for the next question

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10 A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using

$$m = m_0 e^{-kt}$$

where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

On average, it takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

10 (a) The scientist drinks two strong cups of coffee at 8 am. Use the model to estimate the mass of caffeine in the scientist's body at midday.

[4 marks]

$t = 5.7 \quad m = \frac{m_0}{2}$

$\frac{m_0}{2} = m_0 e^{-5.7k}$

$\frac{1}{2} = e^{-5.7k}$

$\ln(1/2) = -5.7k$

$-\ln(2) = -5.7k$

$k = \frac{\ln(2)}{5.7}$

$m_0 = 2 \times 200 = 400\text{mg}$

$m = 400e^{-kt} = 400e^{\frac{-\ln(2)}{5.7} \times 4}$

$m = 246$

10 (b) The scientist wants the mass of caffeine in her body to stay below 480 mg

Use the model to find the earliest time that she could drink another cup of strong coffee.

Give your answer to the nearest minute.

[3 marks]

$m(t)+200$

$m(t)+200<480 \Rightarrow m(t)<280$

$400e^{-kt}\leq 280$

$e^{-kt}\leq\frac{280}{400}=0.7$

$-kt\leq\ln(0.7)$

$t\leq\frac{\ln(0.7)}{-\ln 2/5.7}=\frac{-5.7\ln(0.7)}{\ln 2}$

≈ 2.93 hours

2 hours 56 minutes

10 (c) State a reason why the mass of caffeine remaining in the scientist's body predicted by the model may not be accurate.

[1 mark]

the model is based on the average person

Turn over for the next question

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11 The daily world production of oil can be modelled using

$$V = 10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4$$

where V is volume of oil in millions of barrels, and t is time in years since 1 January 1980.

11 (a) (i) The model is used to predict the time, T, when oil production will fall to zero.

Show that T satisfies the equation

$$T = \sqrt[3]{60T^2 + \frac{162\,000}{T}}$$

[3 marks]

$V=0$ $\quad 0 = 10 + 100\left(\frac{T}{30}\right)^3 - 50\left(\frac{T}{30}\right)^4$

$0 = 10 + 100\left(\frac{T^3}{27000}\right) - 50\left(\frac{T^4}{810\,000}\right)$

$0 = 10 + \frac{T^3}{270} - \frac{T^4}{16200}$

$0 = 162000 + 60T^3 - T^4$

$T^4 = 162000 + 60T^3$ $\quad T > 0$

$T^3 = \frac{162000}{T} + 60T^2$

$T = \sqrt{60T^2 + \frac{162000}{T}}$

11 (a) (ii) Use the iterative formula $T_{n+1} = \sqrt[3]{60T_n{}^2 + \frac{162\,000}{T_n}}$, with $T_0 = 38$, to find the values of T_1, T_2, and T_3, giving your answers to three decimal places.

[2 marks]

$\sqrt[3]{60(38)^2 + \frac{162000}{38}} = 44.963\ldots$ T_1

$\sqrt[3]{60(44.96\ldots)^2 + \frac{162000}{(44.96\ldots}} = 49.987$ T_2

$\sqrt[3]{60(49.987)^2 + \frac{162000}{(49.987\ldots)}} = 53.504$

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11 (a) (iii) Explain the relevance of using $T_0 = 38$

[1 mark]

38 years after 1980 is 2018
which is present day

11 (b) From 1 January 1980 the daily use of oil by one technologically developing country can be modelled as

$$V = 4.5 \times 1.063^t$$

Use the models to show that the country's use of oil and the world production of oil will be equal during the year 2029.

[4 marks]

$2029 - 1980 = 49 \quad t = 49$

$V = 10 + 100\left(\frac{49}{30}\right)^3 - 50\left(\frac{49}{30}\right)^4 = 89.885$ million

$V = 4.5 \times 1.063^{49} \approx 89.814$ million

Both values are approximately equal
and $t = 49$

Turn over for the next question

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12 $p(x) = 30x^3 - 7x^2 - 7x + 2$

12 (a) Prove that $(2x+1)$ is a factor of $p(x)$

[2 marks]

$x = -\frac{1}{2}$ $\quad 30\left(-\frac{1}{2}\right)^3 - 7\left(\frac{-1}{2}\right)^2 - 7\left(\frac{-1}{2}\right) + 2$

$\frac{-30}{8} - \frac{7}{4} + \frac{7}{2} + 2 = 0$

12 (b) Factorise $p(x)$ completely.

[3 marks]

$$\begin{array}{r}
15x^2 - 11x + 2 \\
2x+1 \,\big)\, \overline{30x^3 - 7x^2 - 7x + 2} \\
\underline{30x^3 + 15x^2} \\
-22x^2 - 7x \\
\underline{-22x^2 - 11x} \\
4x + 2 \\
\underline{4x + 2 -} \\
0
\end{array}$$

$(2x+1)(15x^2 - 11x + 2)$

$(2x+1)(5x-2)(3x-1)$

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12 (c) Prove that there are no real solutions to the equation

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

$30\sec^2 x + 2\cos x = 7\sec x + 7$

$\frac{30\sec^2 x}{\cos x} + 2 = \frac{7\sec x}{\cos x} + \frac{7}{\cos x}$

$30\sec^3 x + 2 = 7\sec^2 x + 7\sec x$

$30\sec^3 x - 7\sec^2 x - 7\sec x + 2 = 0$

$(2\sec x + 1)(5\sec x - 2)(3\sec x - 1) = 0$

$\sec x = -\frac{1}{2}, \frac{2}{5}$ or $\frac{1}{3}$

none of these are valid as $\sec x \in [-1, 1]$

Hence there are no real solutions

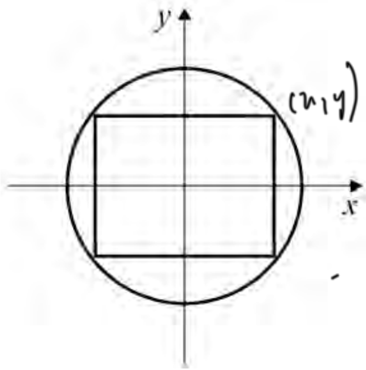
Turn over for the next question

Turn over ▶

Do not write outside the box

13 A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

[10 marks]

$x^2+y^2=16 \Rightarrow y=\sqrt{16-x^2}$

width $=2x$

height $=2y$

$A=(2x)(2y)=4xy$

$=4x\sqrt{16-x^2}$ $\quad 0<x<4$

$\frac{dA}{dx}=-4x^2(16-x^2)^{-1/2}+4(16-x^2)^{1/2}$

$u=4x \qquad v=(16-x^2)^{1/2}$

$u'=4 \qquad v'=\frac{1}{2}\cdot -2x\cdot(16-x^2)^{-1/2}$

$=-x(16-x^2)^{-1/2}$

$\frac{dA}{dx}=\frac{-4x^2}{\sqrt{16-x^2}}+4\sqrt{16-x^2}$

$=\frac{4(16-x^2)-4x^2}{\sqrt{16-x^2}}$

$64-4x^2-4x^2=64-8x^2=0$

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$x^2 = 8 \Rightarrow \quad x = 2\sqrt{2}$

$y = \sqrt{16 - 8} = \sqrt{8} = 2\sqrt{2}$

$x = y = 2\sqrt{2}$

$A = 4 \times 2\sqrt{2} \times 2\sqrt{2} = 32$ sq inches

$\frac{dA}{dx} = \frac{64 - 8x^2}{\sqrt{16 - x^2}} = (64 - 8x^2)(16 - x^2)^{-1/2}$

$u = (64 - 8x^2) \quad v = (16 - x^2)^{-1/2}$

$u' = -16x \quad v' = \frac{-1}{2} \times -2x (16 - x^2)^{-3/2}$

$= x(16 - x^2)^{-3/2}$

$\frac{d^2A}{dx^2} = x(16 - x^2)^{-3/2}(64 - 8x) - 16x(16 - x^2)^{-1/2}$

$x = 2\sqrt{2} \quad = -16 \quad -16 < 0$ hence it is a maximum, so the maximum area is 32

Turn over for the next question

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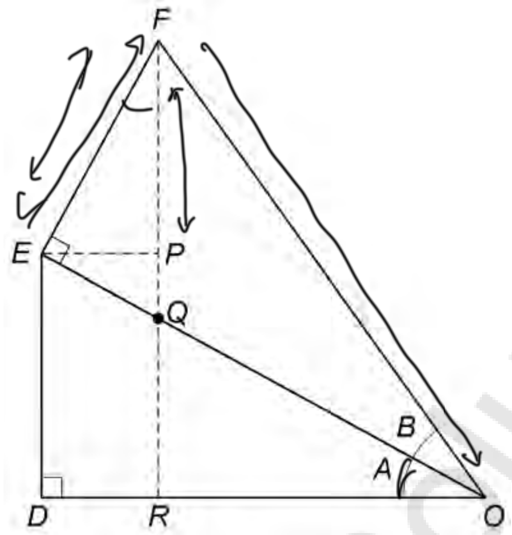
14 Some students are trying to prove an identity for $\sin(A + B)$.

They start by drawing two right-angled triangles *ODE* and *OEF*, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle *OFR*,

Line 1 $\sin(A+B) = \dfrac{RF}{OF}$

Line 2 $= \dfrac{RP + PF}{OF}$

Line 3 $= \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $= \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $=$ .... $+ \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

$\frac{EF}{OF} = \sin B$ by inspection

$\angle EQF$ and $\angle OQR$ are vertically opposite so

equal ✓ $\angle FEQ = \angle ROQ = A$ $\frac{PF}{EF} = \cos A$

Do not write outside the box

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $= \dfrac{DE}{OE} \times \dfrac{OE}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $=$ sin A cos B $+ \cos A \sin B$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when *A* and *B* are acute angles.

[1 mark]

In right angled triangles the angles are acute, so the proof only holds for acute angles

14 (d) Another student claims that by replacing *B* with $-B$ in the identity for $\sin(A + B)$ it is possible to find an identity for $\sin(A - B)$.

Assuming the identity for $\sin(A + B)$ is correct for all values of *A* and *B*, prove a similar result for $\sin(A - B)$.

[3 marks]

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(A-B) = \sin A \cos(-B) + \sin(-B) \cos A$

$\cos(-a) = \cos(a) \qquad \sin(-a) = -\sin(a)$

$\sin(A-B) = \sin A \cos B - \sin B \cos A$

Turn over ▶

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15 A curve has equation $y = x^3 - 48x$

The point *A* on the curve has x coordinate -4

The point *B* on the curve has x coordinate $-4 + h$

15 (a) Show that the gradient of the line *AB* is $h^2 - 12h$

[4 marks]

$x = -4 \qquad y = (-4)^3 - 48(-4)$

$= -64 + 192$

$= 128$

$x = -4+h \qquad y = (-4+h)^3 - 48(-4+h)$

$= -64 + 48h - 12h^2 + h^3 + 192 - 48h$

$= h^3 - 12h^2 + 128$

$m = \dfrac{h^3 - 12h^2 + 128 - 128}{-4+h - -4} = \dfrac{h^3 - 12h^2}{h}$

$= h^2 - 12h$

15 (b) Explain how the result of part **(a)** can be used to show that *A* is a stationary point on the curve.

[2 marks]

the gradient at $x = -4$ is $\lim_{h \to 0} h^2 - 12h = 0$, so the gradient is 0, which means its a stationary point.

END OF QUESTIONS

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