AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
MATHEMATICS
Paper 2

Wednesday 13 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
TOTAL	

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

1 Which of these statements is correct?

Tick **one** box.

[1 mark]

$x = 2 \Rightarrow x^2 = 4$ ☑

$x^2 = 4 \Rightarrow x = 2$ ✗ ☐

$x^2 = 4 \Leftrightarrow x = 2$ ✗ ☐

$x^2 = 4 \Rightarrow x = -2$ ✗ ☐

2 Find the coefficient of x^2 in the expansion of $(1 + 2x)^7$

Circle your answer.

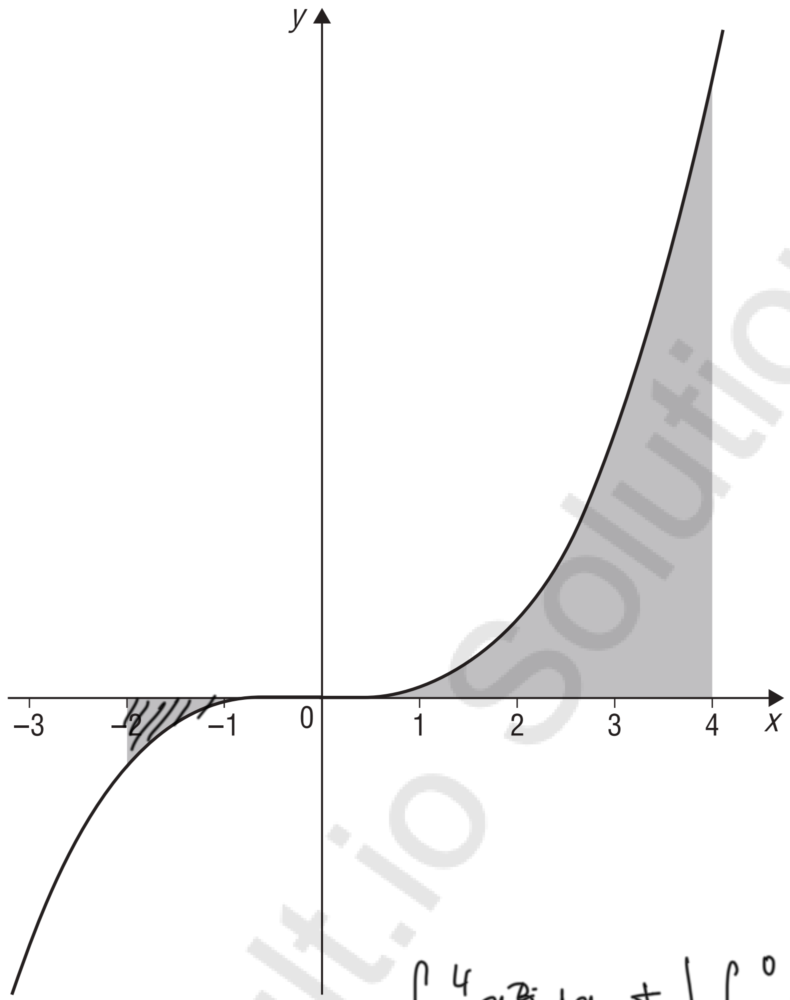
[1 mark]

42 4 21 84

$$\binom{7}{2} \times (2x)^2 = 21 \times 4x^2 = 84x^2$$

Do not write outside the box

3 The graph of $y = x^3$ is shown.



Find the total shaded area.

$\int_0^4 x^3\,dx + \left|\int_{-2}^0 x^3\,dx\right|$

Circle your answer.

[1 mark]

-68 60 68 128

$= \left[\frac{x^4}{4}\right]_0^4 + \left[\frac{x^4}{4}\right]_{-2}^0$

$= 64 + |-4| = 64 + 4 = 68$

Turn over ▶

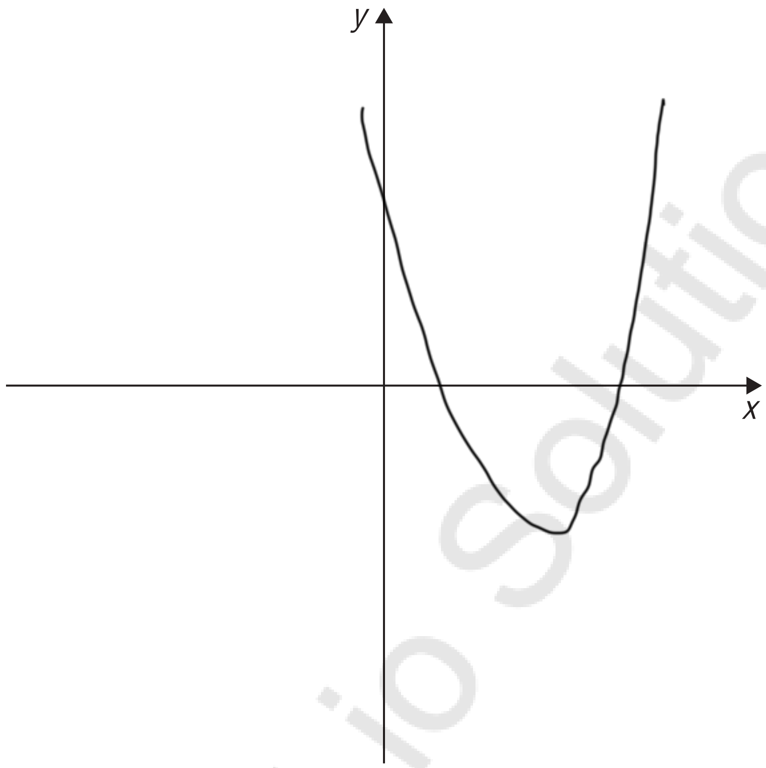
Do not write outside the box

4 A curve, C, has equation $y = x^2 - 6x + k$, where k is a constant.

The equation $x^2 - 6x + k = 0$ has two distinct positive roots.

4 (a) Sketch C on the axes below.

[2 marks]



Do not write outside the box

4 (b) Find the range of possible values for k.

Fully justify your answer.

[4 marks]

$b^2 - 4ac > 0$ 2 disinct roots

$(-6)^2 - 4(1)(k) > 0$

$36 - 4k > 0$

$36 > 4k$

$9 > k$

$9 > k > 0$ as we have +ve y intercept

Turn over for the next question

Do not write outside the box

5 Prove that 23 is a prime number.

[2 marks]

$\sqrt{23} \approx 4.8$

23 is odd so 2 is not a factor

$23 = 7 \times 3 + 2$, so 3 is not a factor

Since 23 has no prime factors less than or equal to $\sqrt{23}$, it is prime

Do not write outside the box

6 Find the coordinates of the stationary point of the curve with equation

$$(x + y - 2)^2 = e^y - 1$$

[7 marks]

$2\left(1+\frac{dy}{dx}\right)(x+y-2) = \frac{dy}{dx}e^y$

$\frac{dy}{dx} = 0$

$2(1+0)(x+y-2) = 0$

$2(x+y-2) = 0$

$x+y-2 = 0$

$(0)^2 = e^y - 1$

$e^y = 1$

$y = 0 \qquad 1-1=0$

$x + 0 - 2 = 0$

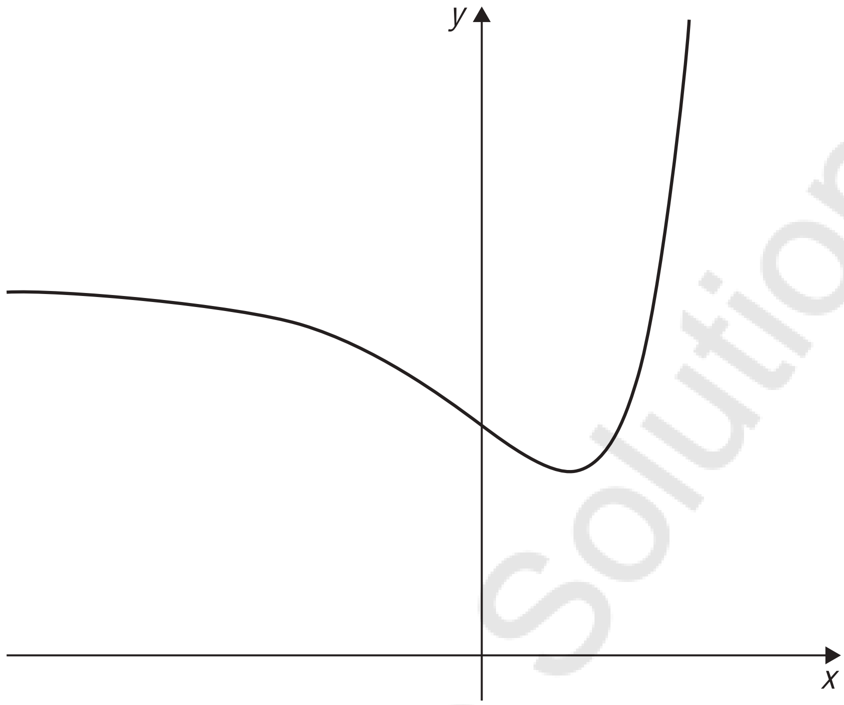
$x = 2$

stationary point $(2,0)$

Do not write outside the box

7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

$\int (x-1)e^x \, dx =$ $\quad u = x-1 \quad v = e^x$

$u' = 1 \quad v' = e^x$

$\int uv' = uv - \int vu' \, dx$

$= e^x(x-1) - \int e^x \, dx$

$= e^x(x-1) - e^x + c$

$= e^x(x-1-1) + c \Rightarrow e^x(x-2) + c = y$

$y \geq e \quad \frac{dy}{dx} = 0 \quad (x-1)e^x = 0 \quad x - 1 = 0 \quad x = 1$

$(1, e) \quad e = e^1(1-2) + c$

$e = -e + c \Rightarrow c = 2e$

$f(x) = e^x(x-2) + 2e$

Turn over for the next question

Do not write outside the box

8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

$R\sin(x-\alpha) = R\sin(x)\cos(\alpha) - R\cos(x)\sin(\alpha)$

$R\cos(\alpha) = \sqrt{3}$

$R\sin(\alpha) = 3$

$\frac{R\sin(\alpha)}{R\cos(\alpha)} = \frac{3}{\sqrt{3}} \Rightarrow \tan(\alpha) = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$R^2 = (R\cos(\alpha))^2 + (R\sin(\alpha))^2$

$R^2 = (\sqrt{3})^2 + (3)^2 = 3+9 = 12$

$R = \sqrt{12} = 2\sqrt{3}$

$y = 2\sqrt{3}\sin(x - \pi/3) + 4$

$y = \sin x \rightarrow y = \sin(x - \pi/3)$

translation by vector $\begin{pmatrix} \pi/3 \\ 0 \end{pmatrix}$

stretch in y axis scale factor $2\sqrt{3}$

translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

Do not write outside the box

8 (b) (i) Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

$$\frac{1}{\sqrt{3}\sin x - 3\cos x + 4} = \frac{1}{2\sqrt{3}\sin(x-\pi/3)+4}$$

$$-1 \le \sin(x - \pi/3) \le 1 \qquad \frac{1}{2\sqrt{3}(1)+4} \times \frac{(4-2\sqrt{3})}{(4-2\sqrt{3})}$$

$$= \frac{4-2\sqrt{3}}{16-12} = \frac{4-2\sqrt{3}}{4}$$

$$= \frac{2-\sqrt{3}}{2}$$

8 (b) (ii) Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

$$\frac{1}{(-1)(2\sqrt{3})+4} = \frac{1}{4-2\sqrt{3}} \times \frac{(4+2\sqrt{3})}{(4+2\sqrt{3})}$$

$$= \frac{4+2\sqrt{3}}{16-12} = \frac{4+2\sqrt{3}}{4} = \frac{2+\sqrt{3}}{2}$$

Turn over for the next question

Do not write outside the box

9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8 - t)$$

[3 marks]

$\frac{dx}{dt} = \frac{k(8-t)}{x}$

$x\frac{dx}{dt} = k(8-t)$ $t=2$ $\frac{dx}{dt} = 72$ $x = 336$

$336 \times 72 = 6k$

$\frac{336 \times 72}{6} = k = 4032$

$x\frac{dx}{dt} = 4032(8-t)$

Do not write outside the box

9 (b) Hence, show that

$$x^2 = 4032t(16 - t)$$

[3 marks]

$x \frac{dx}{dt} = 4032(8-t)$

$x \, dx = 4032(8-t) \, dt$

$\int x \, dx = 4032 \int (8-t) \, dt$

$\frac{x^2}{2} = 4032\left(8t - \frac{t^2}{2}\right) + C$ $x = 336$

$x^2 = 4032(16t - t^2) + C$ $t = 2$

$(336)^2 = \underline{4032(16(2) - (2)^2)} + C$

$112896 = 112896 + C$

$C = 0$ $x^2 = 4032t(16-t)$

Question 9 continues on the next page

Do not write outside the box

9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

$x\frac{dx}{dt} = 4032(8-t)$ $\quad$ $x^2 = 4032t(16-t)$

$\frac{dx}{dt} = 24$

$x \cdot 24 = 4032(8-t)$

$x = \frac{4032(8-t)}{24} = 168(8-t)$

$(168(8-t))^2 = 4032t(16-t)$

$28224(8-t)^2 = 4032t(16-t)$

$7(8-t)^2 = t(16-t)$

$7(64-16t+t^2) = 16t-t^2$

$448-112t+7t^2 = 16t-t^2$

$8t^2-128t+448 = 0$

$t^2-16t+56 = 0$

$t = \frac{16 \pm \sqrt{16^2-4(56}}{2} = \frac{16 \pm \sqrt{32}}{2}$

$= 8 \pm 2\sqrt{2}$

$8-2\sqrt{2} \approx 5.172$ hours

$0.172 \times 60 = 10.3 \approx 10$ mins

5 hours 10 minutes

14:40 $\rightarrow$ 2:40 pm

Do not write outside the box

9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

At stall opening $x = 0$ (no sales)

$x = 0$ $\frac{dx}{dt} = \frac{k(8 - t)}{x}$ we have

a denominator of 0 so the model

is undefined

Turn over for Section B

Do not write outside the box

Section B

Answer **all** questions in the spaces provided.

10 A garden snail moves in a straight line from rest to $1.28\,\text{cm s}^{-1}$, with a constant acceleration in 1.8 seconds.

Find the acceleration of the snail.

Circle your answer.

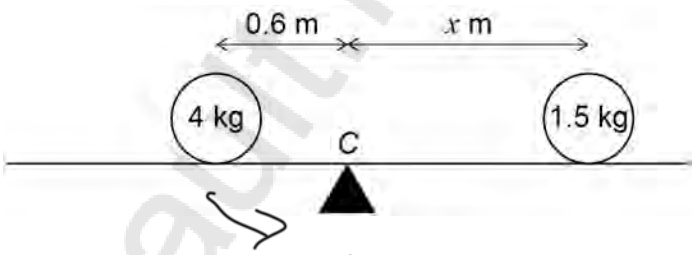
$u = 0\,\text{ms}^{-1}$
$v = 1.28\,\text{cms}^{-1} = 0.0128\,\text{ms}^{-1}$
$t = 1.8\,\text{s}$

[1 mark]

$2.30\,\text{m s}^{-2}$ $0.71\,\text{m s}^{-2}$ $0.0071\,\text{m s}^{-2}$ $0.023\,\text{m s}^{-2}$

$v = u + at$
$0.0128 = 0 + 1.8a$
$\frac{0.0128}{1.8} = a$

11 A uniform rod, *AB*, has length 4 metres.
The rod is resting on a support at its midpoint *C*.
A particle of mass 4 kg is placed 0.6 metres to the left of *C*.
Another particle of mass 1.5 kg is placed x metres to the right of *C*, as shown.



The rod is balanced in equilibrium at *C*.

Find x.

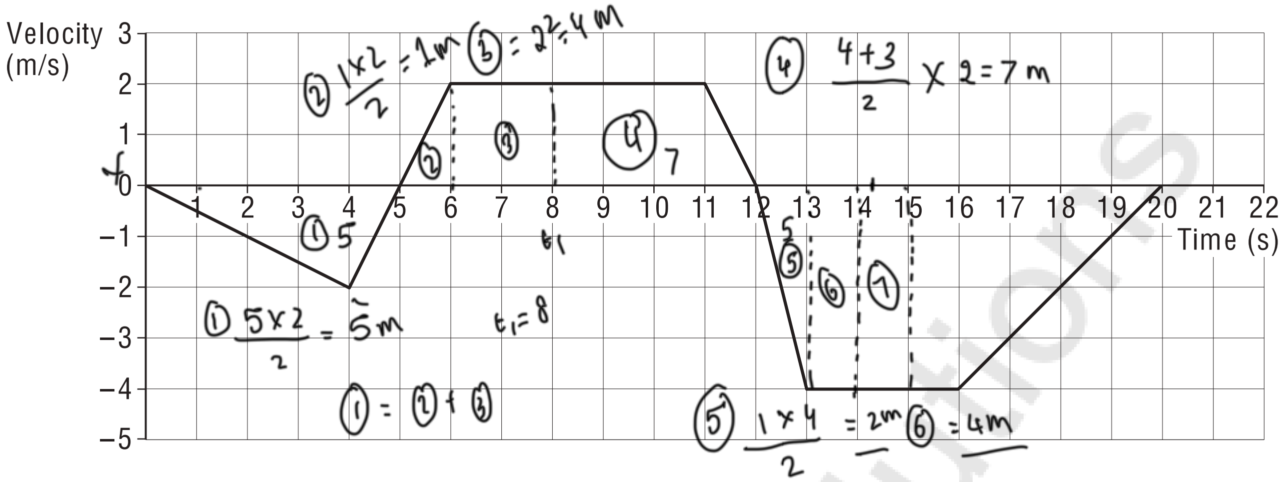
Circle your answer.

[1 mark]

1.8 m 1.5 m 1.75 m 1.6 m

$cw = acw \rightarrow 1.5x = 2.4$
$acw = 4 \times 0.6 = 2.4\,\text{Nm}$ $x = \frac{2.4}{1.5} = 1.6$
$cw = 1.5x$

12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

$m = \frac{\Delta v}{\Delta t} = \frac{(0 - -4)}{(12 - 13)} = \frac{4}{-1} = -4ms^{2}$

acceleration $-4ms^{-2}$

magnitude is $4ms^{-2}$

12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

S = Area under graph

0 − 5 = travelled -5m

5 − 8 = travelled back

8 − 12 = travelled 7m

12 − 14.25 = travelled back

$t_1 = 8s$ $t_2 = 14.25s$

Do not write outside the box

13 **In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.

[3 marks]

$F_{max} = \mu m g$

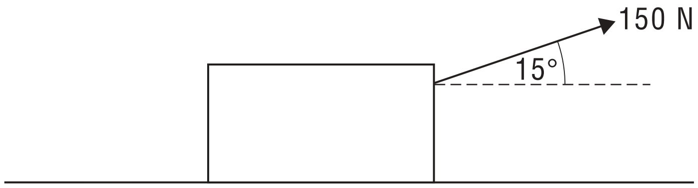
$= 0.85 \times 20 \times 9.8$

$= 166.6\,\text{N}$

166.6 > 150 so friction is not overcome

Do not write outside the box

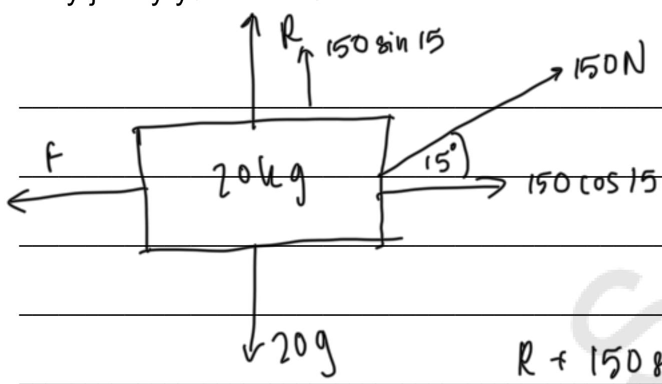
13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]



$R + 150\sin 15 = 20g$

$R = 20 \times 9.8 - 150\sin 15$

$= 157.2\text{N}$

$F_{max} = \mu R$

$= 0.85 \times 157.2 = 133.6\text{N}$

$150\cos 15 = 144.9\text{N}$

$144.9\text{N} > 133.6\text{N} = F_{max}$

Applied force is greater than max friction so the crate moves

Turn over ▶

Do not write outside the box

14 A quadrilateral has vertices A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$$

14 (a) Write down the vector $\overrightarrow{AB}$

[1 mark]

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= (-3, -5, -1) + (-1, 2, 7)$

$= (-4, -3, 6)$

14 (b) Show that $ABCD$ is a parallelogram, but not a rhombus.

[5 marks]

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = (1, -2, -7) + (0, 7, 6)$

$= (1, 5, -1)$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD} = (0, -7, -6) + (4, 10, 0)$

$= (4, 3, -6)$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = (-3, -5, -1) + (4, 10, 0) = (1, 5, -1)$

$\overrightarrow{AD}, \overrightarrow{BC}$ // and $\overrightarrow{AB}, \overrightarrow{DC}$ are parallel

$|AD| = \sqrt{(1)^2 + (5)^2 + (-1)^2} = 3\sqrt{3}$

$|AB| = \sqrt{(-4)^2 + (-3)^2 + (6)^2} = \sqrt{61}$

two pairs parallel ⇒ parallelogram

adjacent sides are not equal so ABCD is not a rhombus

Do not write outside the box

15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in $\mathrm{m\,s^{-2}}$ of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

The acceleration of A $= 0.138\,t^2$
The acceleration of B $= 0.024\,t^3$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

$V = \int 0.138t^2 \, dt = 0.046t^3 + c_1$

$t=0 \quad v=0 \quad 0+c_1=0 \quad c_1=0$

$s = \int 0.046t^3 \, dt = 0.0115t^4 + c_2$

$t=0 \quad s=0 \quad 0+c_2=0 \Rightarrow c_2=0$

$s = 0.0115t^4$

$100 = 0.0115t^4$

$t = \sqrt[4]{\frac{0.0115}{100}} = 9.657s \ (4 sf)$

Question 15 continues on the next page

Do not write outside the box

15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

$v = \int 0.024t^3 \, dt = 0.006t^4 \quad (c=0) \; (v=0, t=0)$

$s = \int 0.006t^4 \, dt = 0.0012t^5 \quad (c=0) \; (s=0, t=0)$

$s = 0.0012t^5$

$100 = 0.0012t^5$

$t = \sqrt[5]{\frac{0.0012}{100}} = 9.642 \text{ s} : B$

$9.642 < 9.657$ so minibus B reaches 100 ms first, so B should be bought

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

driver reaction time, driver B may have reacted to the flag being dropped quicker

16 **In this question use $g = 9.81\,\mathrm{m\,s^{-2}}$**

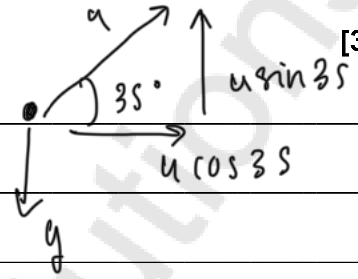
A particle is projected with an initial speed u, at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u.

[3 marks]



$v = u + at$ ↑

↑ $v = 0$, $u = u\sin 35$

$a = -9.81$ $t = 1.5$

$0 = u\sin 35 - (9.81 \times 1.5)$

$\dfrac{9.81 \times 1.5}{\sin 35} = u \approx 25.7\,\mathrm{ms^{-1}}$ (3 sf)

16 (b) Find the total time that the particle is in flight.

[3 marks]

$s = -10$ $u = 25.7\sin 35$ $a = -9.81$

$s = ut + \frac{1}{2}at^2$

$-10 = 25.7\sin 35\, t - \frac{1}{2}(9.81)t^2$

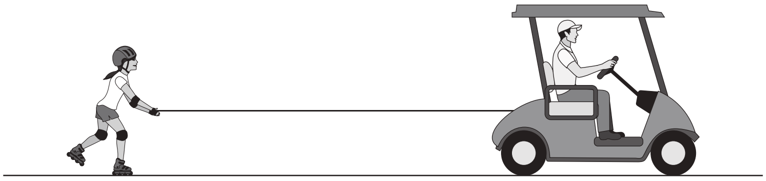
$4.905t^2 - 25\sin 35 t - 10 = 0$

$t = \dfrac{25\sin 35 \pm \sqrt{(25\sin 35)^2 - 4(4.905)(-10)}}{9.81}$

$t \approx 3.57\,\mathrm{s}$

Do not write outside the box

17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.



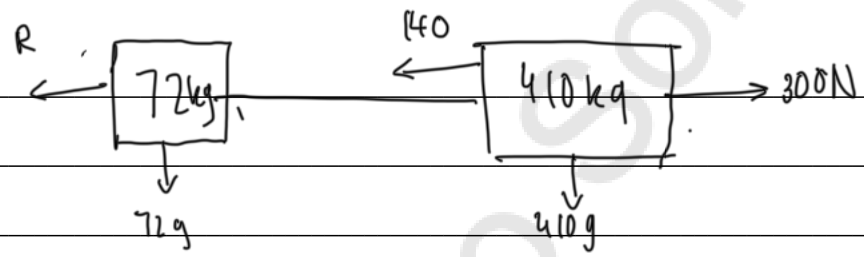
The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\text{m}\,\text{s}^{-2}$

17 (a) (i) Find R.

[3 marks]



$f = ma$

$\rightarrow \quad 300 - 140 - R = (410 + 72)\,0.2$

$160 - R = 482 \times 0.2$

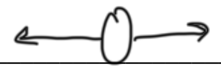
$160 - R = 96.2$

$R = 63.6\text{N}$

Do not write outside the box

17 (a) (ii) Find the tension in the rope.

[3 marks]



$\rightarrow \quad T - R = 72 \times 0.2$

$T - 63.6 = 14.4$

$T = 78\text{N}$

17 (b) State a necessary assumption that you have made.

[1 mark]

the rope is not elastic, so tension is constant.

Question 17 continues on the next page

Do not write outside the box

17 (c) The roller-skater releases the rope at a point *A*, when she reaches a speed of $6\,\text{m s}^{-1}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from *A*.

17 (c) (i) Determine whether the roller-skater will stop before reaching the stationary buggy.

Fully justify your answer.

[5 marks]

$-R = ma$ skate

$-63.6 = 72a$

$\frac{-63.6}{72} = a = -0.883\,\text{ms}^{-2}$

$v^2 = u^2 + 2as \quad v = 0 \quad u = 6 \quad a = -0.883 \quad s = ?$

$0 = (6)^2 + 2(-0.883)s$

$\frac{-(6)^2}{2(-0.883)} = s = 20.4\,\text{m} \qquad 20.4 > 20$

the skater does not stop before reaching the buggy

Do not write outside the box

17 (c) (ii) Explain the change in motion that the driver noticed.

[2 marks]

When the shaker releases the rope, tension is removed (which was acting backwards on the buggy) Hence F increases which leads to the buggy accelerating faster.

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED