AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
MATHEMATICS
Paper 3

Please note that question 13 uses the original Large Data Set "Family Food". This was replaced by the data set "Transport Stock Vehicle Database" in A-level exams from June 2020.
If you'd like to see the original data set, please contact maths@aqa.org.uk.

Friday 15 June 2018 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 A circle has equation $(x-4)^2+(y+4)^2=9$

$r^2 = 9$

πr^2

What is the area of the circle?

Circle your answer.

[1 mark]

3π 9π 16π 81π

2 A curve has equation $y = x^5 + 4x^3 + 7x + q$ where q is a positive constant.

Find the gradient of the curve at the point where $x = 0$

Circle your answer.

[1 mark]

0 4 7 q

$\frac{dy}{dx} = 5x^4 + 12x^2 + 7$
$= 7$

3 The line L has equation $2x + 3y = 7$

Which one of the following is perpendicular to L?

Tick **one** box.

[1 mark]

$2x - 3y = 7$ ✗ ☐

$3x + 2y = -7$ ☐

$2x + 3y = -\frac{1}{7}$ ✗ ☐

$3x - 2y = 7$ ☑

$3y = 7 - 2x$

$y = \frac{7}{3} - \frac{2}{3}x$

$\frac{3}{2}$

$y = c + \frac{2}{3}x$

$3x - 7 = 2y$ $\frac{3}{2}x - \frac{7}{2} = y.$

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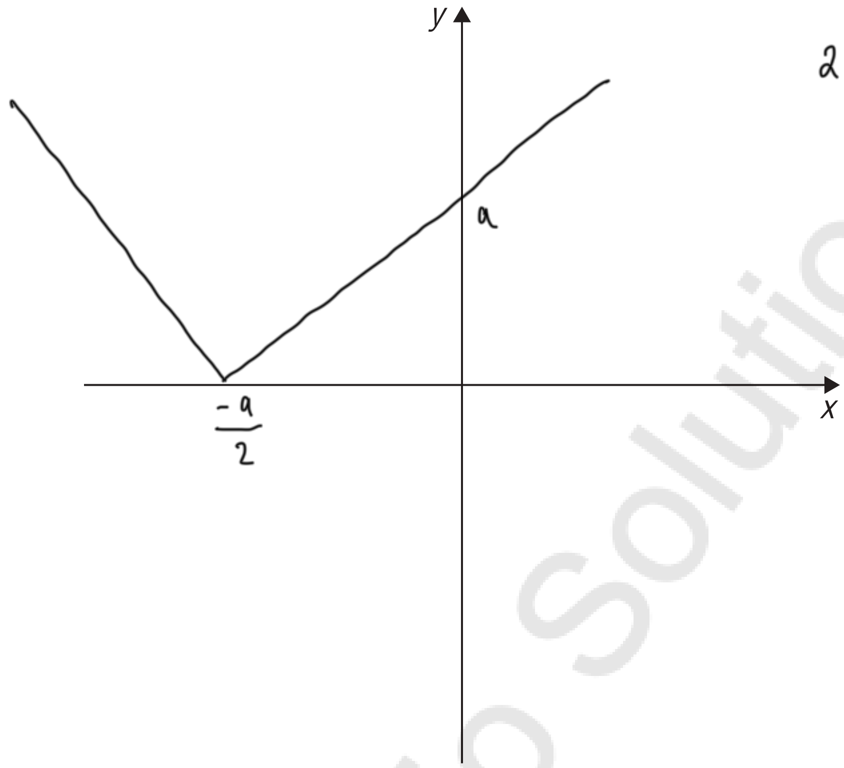
4 Sketch the graph of $y = |2x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

[3 marks]



$2x + a = 0$

$x = \frac{-a}{2}$

$y > 0$

5 Show that, for small values of x, the graph of $y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ can be approximated by a straight line.

[3 marks]

$\sin\theta \approx \theta \quad \tan\theta \approx \theta$

$\sin\frac{x}{2} \approx \frac{x}{2} \quad \tan\frac{x}{3} \approx \frac{x}{3}$

$y = 5 + \frac{4x}{2} + \frac{12x}{3}$

$y = 5 + 2x + 4x$

$y = 5 + 6x$ straight line $y = mx + c$

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6 A function f is defined by $f(x) = \frac{x}{\sqrt{2x-2}}$

6 (a) State the maximum possible domain of f.

[2 marks]

$\sqrt{2x-2} > 0$

$2x - 2 > 0$

$2x > 2$

$x > 1$ Domain : $\{x \in \mathbb{R} : x > 1\}$

6 (b) Use the quotient rule to show that $f'(x) = \frac{x-2}{(2x-2)^{\frac{3}{2}}}$

[3 marks]

$u = x$ $v = (2x-2)^{1/2}$

$u' = 1$ $v' = 2 \cdot \frac{1}{2} \cdot (2x-2)^{-1/2} = (2x-2)^{-1/2}$

$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(2x-2)^{1/2} - x(2x-2)^{-1/2}}{(2x-2)^1}$

$\times (2x-2)^{1/2}$ $= \frac{(2x-2) - x}{(2x-2)^{3/2}}$

$= \frac{x-2}{(2x-2)^{3/2}}$

Do not write outside the box

6 (c) Show that the graph of $y = \mathrm{f}(x)$ has exactly one point of inflection.

[7 marks]

$f''(x) = 0$

$f'(x) = \dfrac{x-2}{(2x-2)^{3/2}}$ $\quad u = x-2 \quad v = (2x-2)^{3/2}$

$u' = 1 \quad v' = \frac{3}{2}\cdot 2 \cdot (2x-2)^{1/2}$

$= 3\cdot(2x-2)^{1/2}$

$f''(x) = \dfrac{(2x-2)^{3/2} - 3(x-2)(2x-2)^{1/2}}{(2x-2)^3}$

$(2x-2)^{3/2} - 3(x-2)(2x-2)^{1/2} = 0$

$(2x-2)^{1/2}(2x-2-3(x-2)) = 0$

$(2x-2)^{1/2}(-x+4) = 0$

$x = 1$ or $x = 4$

Reject $x = 1$ as root in $f(x)$ domain

$x = 3 \quad f''(3) = \dfrac{1}{32} > 0$ (concave up)

$x = 5 \quad f''(5) = -\dfrac{\sqrt{2}}{256} < 0$ (concave down)

confirmed sign change → exactly one point of inflection at $x = 4$

6 (d) Write down the values of x for which the graph of $y = \mathrm{f}(x)$ is convex.

[1 mark]

convex when $f''(x) > 0$

$1 < x < 4$

Do not write outside the box

7 (a) Given that $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$, find y in terms of a.

[4 marks]

$\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$ $\quad 2\log_a 7 = \log_a 7^2$

$\log_a y = \log_a 49 + \log_a 4 + \frac{1}{2}$ $\quad \log_a a = 1$

$\log_a a^{1/2} = \frac{1}{2}$

$\log_a y = \log_a 49 + \log_a 4 + \log_a a^{1/2}$

$\log a + \log b = \log ab$

$\log_a y = \log_a 196\sqrt{a}$

$y = 196\sqrt{a}$

7 (b) When asked to solve the equation

$$2\log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2\log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2\log_a x = \log_a \frac{9}{4}$$ log a - log b = log $\frac{a}{b}$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$ n log a = log a^n

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \underline{\frac{3}{2}} \text{ or } \underline{-\frac{3}{2}}$$

Explain what is wrong with the student's solution.

[1 mark]

$x = -3/2$ $2\log_a(-3/2)$ which is undefined

log of a negative number does not exist

Turn over for the next question

8 (a) Prove the identity $\dfrac{\sin 2x}{1+\tan^2 x} \equiv 2\sin x\cos^3 x$

[3 marks]

$$\text{LHS} = \frac{\sin 2x}{1+\tan^2 x} = \frac{2\sin x\cos x}{1+\tan^2 x}$$

$\sin 2x = 2\sin x\cos x$

$\sec^2 x = 1+\tan^2 x$

$\sec^2 x = \dfrac{1}{\cos^2 x}$

$$= \frac{2\sin x\cos x}{\sec^2 x}$$

$$= 2\sin x\cos^3 x = \text{RHS}$$

Do not write outside the box

8 (b) Hence find $\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,d\theta$

[6 marks]

$x = 2\theta$

$4 \times \int \frac{\sin 4\theta}{1+\tan^2 2\theta}\,d\theta = 4 \times \int 2\sin 2\theta \cos^3 2\theta \,d\theta$

$= \int 8\sin 2\theta \cos^3 2\theta \,d\theta$

$f(\theta) = \cos^4 2\theta$

$f'(\theta) = -4\sin 2\theta \times 2 \qquad = -\cos^4 2\theta + C$

$-8\sin 2\theta \cos^3 2\theta$

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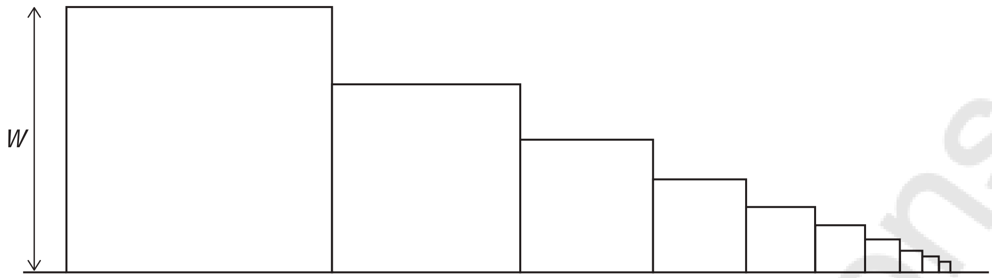
9 Helen is creating a mosaic pattern by placing square tiles next to each other along a straight line.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

9 (a) Find, in terms of w, the length of the sides of the second largest tile.

[1 mark]

Area of 2^{nd} tile $= \frac{w^2}{2}$

Side $= \sqrt{\left(\frac{w^2}{2}\right)} = \frac{w}{\sqrt{2}} = \frac{w\sqrt{2}}{2}$

9 (b) Assume the tiles are in contact with adjacent tiles, but do not overlap.

Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$.

[4 marks]

$a = w \quad r = \frac{1}{\sqrt{2}} \quad r < 1$

$S_\infty = \frac{w}{(1 - 1/\sqrt{2})} \approx 3.41w$

$3.41w < 3.5w$

$\therefore$ total length $< 3.5w$ for any number of tiles.

9 (c) Helen decides the pattern will look better if she leaves a 3 millimetre gap between adjacent tiles.

Explain how you could refine the model used in part **(b)** to account for the 3 millimetre gap, and state how the total length of the series of tiles will be affected.

[2 marks]

+3mm for each tile to the total length since there are infinitely many tiles, and each tile has +3mm to its length, the total length has no upper limit.

Turn over for the next question

Do not write outside the box

10 Prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

[7 marks]

Assume $\sqrt[3]{2}$ is rational

Then $\sqrt[3]{2} = \frac{a}{b}$ where a, b are integers with no common factors.

$$2 = \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

$2b^3 = a^3 \Rightarrow a^3$ is even so a is even

let $a = 2d$

$2b^3 = (2d)^3 = 8d^3$

$b^3 = 4d^3 \Rightarrow b^3$ is even $\Rightarrow b$ is even

So b and a must have common factor 2 $\lightning$

as we proved that a and b have no common factors.

Hence $\sqrt[3]{2}$ is irrational

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Section B

Answer **all** questions in the spaces provided.

11 The table below shows the probability distribution for a discrete random variable X.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	k	$2k$	$4k$	$2k$	k

Find the value of k.

Circle your answer.

[1 mark]

$\frac{1}{2}$ $\quad$ $\frac{1}{4}$ $\quad$ $\frac{1}{10}$ $\quad$ 1

$k + 2k + 4k + 2k + k = 10k$ $\quad$ $10k = 1$

$k = \frac{1}{10}$

Turn over for the next question

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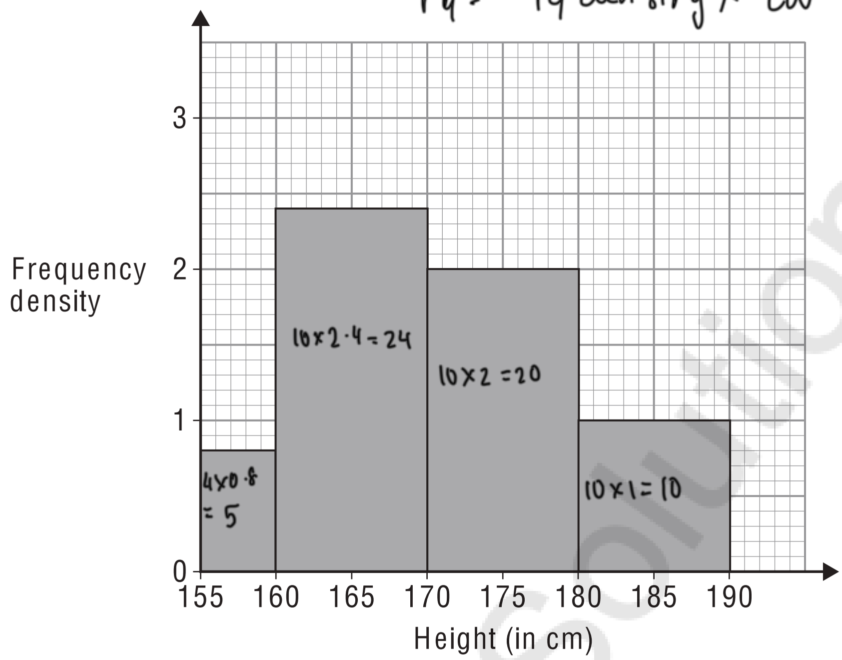
12 The histogram below shows the heights, in cm, of male A-level students at a particular school.

fq = fq density x cw



Which class interval contains the median height?

68 ÷ 2 = 34

Circle your answer.

4 + 24 + 20 + 10 = 68

[1 mark]

[155, 160) [160, 170) [170, 180) [180, 190]

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13 The table below shows an extract from the Large Data Set.

Year	**2011**	**2012**	**2013**	**2014**	**% change since 2011**
Other takeaway food brought home	0	0	0	0	–29

Sarah claims that the –29% change since 2011 is incorrect, as there is no change between 2011 and 2014.

Using your knowledge of the Large Data Set to justify your answer, explain whether Sarah's claim is correct.

[3 marks]

The values shown are rounded to the nearest whole number, the actual values in the Large data set are non zero. Using the unrounded values which are available to many decimal points would show the -29%. Hence Sarahs claim is incorrect.

Turn over for the next question

Do not write outside the box

14 A teacher in a college asks her mathematics students what other subjects they are studying.

She finds that, of her 24 students:

12 study physics
8 study geography
4 study geography and physics

14 (a) A student is chosen at random from the class.

Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent.

[2 marks]

$P(P) = \frac{12}{24} = \frac{1}{2}$ $\quad P(G) = \frac{8}{24} = \frac{1}{3}$

$P(A \cap B) = P(A) \times P(B)$

$\frac{4}{24} = \frac{1}{6}$ $\quad \frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$ ✓

since $P(G) \times P(P) = P(G \cap P)$ the events are independent

14 (b) It is known that for the whole college:

the probability of a student studying mathematics is $\frac{1}{5}$

the probability of a student studying biology is $\frac{1}{6}$

the probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

Calculate the probability that a student studies mathematics or biology or both.

[4 marks]

$P(M \cap B) = P(M) \times P(B \mid M)$

$= \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$= \frac{1}{5} + \frac{1}{6} - \frac{3}{40}$

$= \frac{35}{120} = \frac{7}{24}$

$P(A \cup B) = P(A) + P(B) - P(A \cup B)$

Turn over for the next question

Do not write outside the box

15 Abu visits his local hardware store to buy six light bulbs.

He knows that 15% of all bulbs at this store are faulty.

15 (a) State a distribution which can be used to model the number of faulty bulbs he buys.

[1 mark]

$X \sim B(6, 0.15)$

15 (b) Find the probability that all of the bulbs he buys are faulty.

[1 mark]

$P(X=6) = (0.15)^6 = 0.0000114$

15 (c) Find the probability that at least two of the bulbs he buys are faulty.

[2 marks]

$P(X \geq 2) = 1 - P(X \leq 1)$

$P(X \leq 1) = 0.7764$

$1 - 0.7764 = 0.224$

15 (d) Find the mean of the distribution stated in part **(a)**.

[1 mark]

$\text{Mean} = np = 6 \times 0.15 = 0.9$

15 (e) State two necessary assumptions in context so that the distribution stated in part **(a)** is valid.

[2 marks]

The probability of the lightbulb being faulty is fixed and whether one bulb is faulty is independent of another bulb being faulty.

Turn over for the next question

Do not write outside the box

16 A survey of 120 adults found that the volume, X litres per person, of carbonated drinks they consumed in a week had the following results:

$$\sum x = 165.6 \qquad \sum x^2 = 261.8$$

16 (a) (i) Calculate the mean of X.

[1 mark]

Mean $= \frac{\Sigma x}{n} = \frac{165.6}{120} = 1.38$

16 (a) (ii) Calculate the standard deviation of X.

[2 marks]

$SD = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{261.8}{120} - (1.38)^2}$

$= 0.5266$ to 4dp.

0.526 to 0.529

16 (b) Assuming that X can be modelled by a normal distribution find

16 (b) (i) $P(0.5 < X < 1.5)$

[2 marks]

$P(0.5 < X < 1.5) = 0.5428$ to 4dp

Do not write outside the box

16 (b) (ii) $P(X = 1)$

[1 mark]

$P(X = 1) = 0$

16 (c) Determine with a reason, whether a normal distribution is suitable to model this data.

[2 marks]

$\mu - 3 \times \delta = 1.38 - 3(0.526)$

$= -0.198$

$-0.198 < 0$

→ volume of drink cannot be negative

so normal distribution may not be appropriate

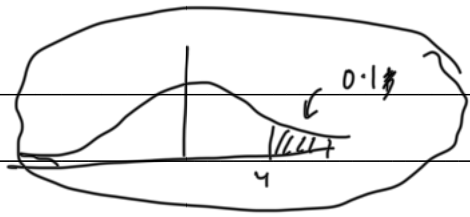
16 (d) It is known that the volume, Y litres per person, of energy drinks consumed in a week may be modelled by a normal distribution with standard deviation 0.21

Given that $P(Y > 0.75) = 0.10$, find the value of μ, correct to three significant figures.

[4 marks]

$Y \sim N(\mu, 0.21)$

$P(Y > 0.75) = 0.1$



$Z \sim N(0,1)$ $\quad Z = \dfrac{(Y - \mu)}{\delta} =$

$P\left(Z > \dfrac{(0.75 - \mu)}{0.21}\right) = 0.1 \qquad P(Z > z) = 0.1$

$\dfrac{(0.75 - \mu)}{0.21} = 1.2816$

$0.75 - \mu = 0.26914$

$\mu = 0.481$ (to 3 sf)

Turn over ▶

17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

X = # of marches won

$H_0: p = 0.5$

$H_1: p \neq 0.5$

under H_0: $X \sim B(10, 0.5)$

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.8281$

$= 0.172$

$0.172 > 0.05$ $\quad \frac{\alpha}{2}$ $\quad \alpha = 0.1$

Accept the null hypothesis, Suzannes new racket has not made a difference to X

Do not write outside the box

17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

$P(Y \geqslant y) \leq 0.1 \quad H_0: Y \sim B(20, 0.5)$

$P(Y \geqslant 13) = 1 - P(Y \leq 12) = 0.1316 > 0.1$ (not significant)

$P(Y \geqslant 14) = 1 - P(Y \leq 13) = 0.0577 < 0.1$ (significance)

minimum number of matches won

Turn over for the next question

Do not write outside the box

18 In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.

Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g

18 (a) After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.

18 (a) (i) State the sampling method used to collect the survey.

[1 mark]

opportunistic sampling

18 (a) (ii) Explain why this sample should not be used to conduct a hypothesis test.

[1 mark]

the sample is not random, so people who wanted and were available were surveyed.

Do not write outside the box

18 (b) A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g

Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.

Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation.

[6 marks]

$H_0: \mu = 66.5$

$H_1: \mu < 66.5$

test statistic

$$z = \frac{(65.4 - 66.5)}{21.2/\sqrt{750}}$$

$$= \frac{-1.1}{0.7743} = -1.42$$

critical value : -1.28

$-1.42 < -1.28 \rightarrow$ Reject H_0

Sufficient evidence that the advertising campaign has reduced chocolate consumption.

END OF QUESTIONS

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