AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________________________

Forename(s) ____________________________________

Candidate signature ____________________________________

I declare this is my own work.

A-level
MATHEMATICS
Paper 1

Tuesday 6 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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12	
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16	
TOTAL	

Answer **all** questions in the spaces provided.

1 Find the coefficient of x^7 in the expansion of $(2x - 3)^7$

Circle your answer.

[1 mark]

-2187 $\quad$ -128 $\quad$ 2 $\quad$ 128

$\binom{7}{7} = 1 \quad (2x)^7(-3)^0 = 128x^7$

2 Given that $y = 2x^3$ find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = 5x^2$ $\quad$ $\frac{dy}{dx} = 6x^2$ $\quad$ $\frac{dy}{dx} = \frac{x^4}{2}$ $\quad$ $\frac{dy}{dx} = 6x^3$

$6x^2$

3 The curve with equation $y = \ln x$ is transformed by a stretch parallel to the x-axis with scale factor 2

Find the equation of the transformed curve.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\ln x$ $\quad$ $y = 2\ln x$ $\quad$ $y = \ln\frac{x}{2}$ $\quad$ $y = \ln 2x$

4 Given that θ is a small angle, find an approximation for $\cos 2\theta$

Circle your answer.

[1 mark]

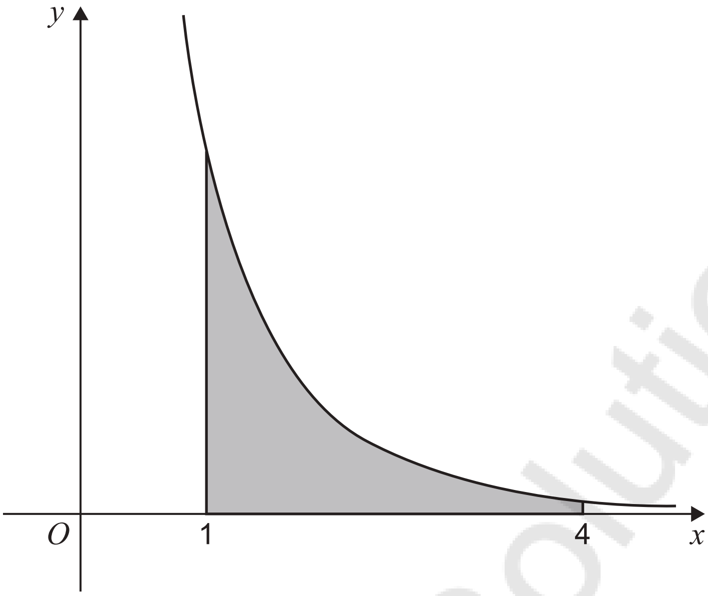
$1 - \frac{\theta^2}{2}$ $\quad$ $2 - 2\theta^2$ $\quad$ $1 - 2\theta^2$ $\quad$ $1 - \theta^2$

$\cos\theta \approx 1 - \frac{\theta^2}{2}$

$\cos 2\theta \approx 1 - \frac{(2\theta)^2}{2} = 1 - 2\theta^2$

Turn over for the next question

5 The graph of $y = \dfrac{5}{e^x - 1}$ is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

x	1	1.6	2.2	2.8	3.4	4
y	2.90988	1.26485	0.62305	0.32374	0.17263	0.09329

5 (a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

Give your answer to four decimal places.

[3 marks]

$h = \dfrac{(4-1)}{5} = \dfrac{3}{5} = 0.6$

Area $\approx \dfrac{h}{2} \times$ [first + last + 2(middle)]

$\approx \dfrac{0.6}{2} \times [2.90988 + 0.09329 + 2(1.26485 + 0.62305 + 0.32374 + 0.17263)]$

$\approx 0.3 \times [3.00317 + 2(2.38427)]$

$\approx 0.3 \times 7.77171$

$= 2.3315$ (4dp)

5 (b) Using your answer to part **(a)** deduce an estimate for $\int_1^4 \frac{20}{e^x - 1}\,dx$

[1 mark]

$\int_1^4 \frac{20}{e^x-1}\,dx = 4 \times \int_1^4 \frac{5}{e^x-1}\,dx$

$4 \times 2.3315 = 9.3$ (to 2sf)

Turn over for the next question

6 Show that the equation

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x+8)$$

has exactly one solution.

Fully justify your answer.

[5 marks]

$2\log a = \log a^2$ $\log_{10} x^2 = \log_{10} 4 + \log_{10}(x+8)$

$\log a + \log b$
$= \log(ab)$ $\log_{10} x^2 = \log_{10}[4(x+8)]$

$x^2 = 4(x+8)$

$x^2 = 4x + 32$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) = 0$

$x = 8$ or $\underline{-4}$

$x = -4$ is not valid as $\log_{10}(-4)$ has no real value. Therefor the only solution is $x = 8$

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7 (a) Given that n is a positive integer, express

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

as a single fraction not involving surds.

[3 marks]

$$\frac{7(5\sqrt{n}-3) - 7(3+5\sqrt{n})}{(3+5\sqrt{n})(5\sqrt{n}-3)}$$

$$= \frac{35\sqrt{n} - 21 - 21 - 35\sqrt{n}}{25n-9}$$

$$= \frac{-42}{25n-9}$$

7 (b) Hence, deduce that

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

is a rational number for all positive integer values of n

[1 mark]

since n is an integer and 42 and 25n - 9 are integers, the expression is a ratio of two integers, therefore it is rational for all positive integer values of n

Do not write outside the box

8 Show that

$$\int_0^{\frac{\pi}{2}} (x \sin 4x)\,\mathrm{d}x = -\frac{\pi}{8}$$

[6 marks]

$\int uv'\,dx = uv - \int u'v\,dx \qquad u = x \qquad v = -\frac{1}{4}\cos 4x$

$u' = 1 \qquad v' = \sin 4x$

$\left[-\frac{x}{4}\cos 4x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{4}\cos 4x\,dx$

$\left[-\frac{x}{4}\cos 4x\right]_0^{\pi/2} - \left[\frac{1}{16}\sin 4x\right]_0^{\pi/2}$

0

$-\frac{\pi}{8}\cos 2\pi - \left(\frac{1}{16}\sin 2\pi - \frac{1}{16}\sin 0\right)$

$= \frac{-\pi}{8}$

Turn over for the next question

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9 The points P and Q have coordinates $(-6, 15)$ and $(12, 19)$ respectively.

9 (a) (i) Find the coordinates of the midpoint of PQ

[1 mark]

$\frac{-6 + 12}{2} = 3$ $\quad \frac{15 + 19}{2} = 17$

$M = (3, 17)$

9 (a) (ii) Find the equation of the perpendicular bisector of PQ

Give your answer in the form $ax + by = c$ where a, b and c are integers.

[4 marks]

m of PQ $\rightarrow m = \frac{(19 - 15)}{(12 - -6)} = \frac{4}{18} = \frac{2}{9}.$

m of perpendicular bisector $= \frac{-9}{2}$

$(3, 17)$ $\quad y - 17 = \frac{-9}{2} \times (x - 3)$

$y - y_1 = m \times (x - x_1)$

$2y - 34 = -9 \times (x - 3)$

$2y - 34 = -9x + 27$

$9x + 2y = 61$

9 (b) (i) A circle passes through the points P and Q

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle.

[3 marks]

$9x + 2y = 61$ ①
$2x - 5y = -30$ ②

① $\div 2 \rightarrow y = \frac{(61 - 9x)}{2}$

$2x - \frac{5(61 - 9x)}{2} = -30$

$4x - 5(61 - 9x) = -60$
$4x - 305 + 45x = -60$
$49x = 245$
$x = 5$
$y = \frac{61 - 45}{2} = 8$

centre $= (5, 8)$

Q $(12, 19)$ $r^2 = (12 - 5)^2 + (19 - 8)^2 = 49 + 121 = 170$
$(x - 5)^2 + (y - 8)^2 = 170$

9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n

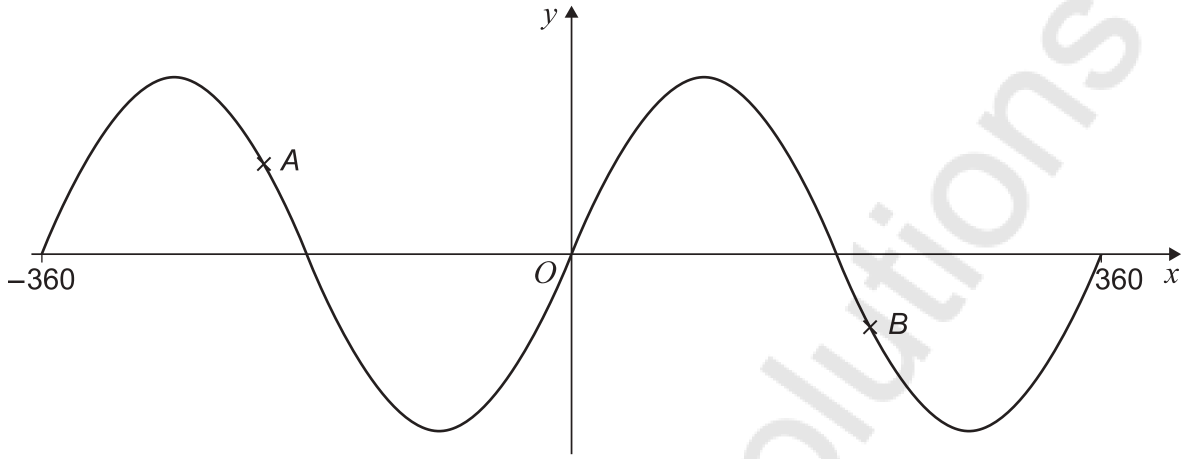
[1 mark]

x axis $y = 0$ $(x - 5)^2 + 64 = 170 \rightarrow (x - 5)^2 = 106$
$\rightarrow$ two sols.
y axis $x = 0$ $25 + (y - 8)^2 = 170$
$n = 4$ $(y - 8)^2 = 145 \rightarrow$ two solutions.

10 The curve with equation

$$y = \sin x°$$

for $-360 \leq x \leq 360$ is shown below.



10 (a) Point *A* on the curve has coordinates (a, 0.5)

10 (a) (i) Find the value of a

[2 marks]

$\sin(-\alpha) = -\sin(\alpha)$

$\sin(210) = \frac{-1}{2}$

$\sin(-210) = \frac{1}{2}$

$a = 210°$

10 (a) (ii) State the value of $\sin(180° - a°)$ $= \sin(e)$

[1 mark]

$\sin(180 - (-210)) = \sin(390) = \sin(30)$

$= 0.5.$

10 (b) Point *B* on the curve has coordinates $\left(b,\ -\dfrac{3}{7}\right)$

10 (b) (i) Find the exact value of $\sin(b° - 180°)$

[2 marks]

$\sin(\theta - 180) = -\sin(\theta)$

$\sin(b - 180) = -\sin b = -\left(\frac{-3}{7}\right) = 3/7$

10 (b) (ii) Find the exact value of $\cos b°$

[3 marks]

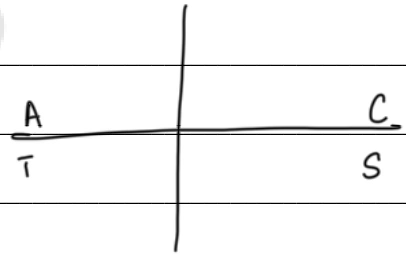
$\sin^2 b + \cos^2 b = 1$

$(-3/7)^2 + \cos^2 b = 1$

$9/49 + \cos^2 b = 1$

$\cos^2 b = 40/49$

$\cos b = \pm\sqrt{40/49} = \dfrac{\pm 2\sqrt{10}}{7}$



$\cos b = \dfrac{-2\sqrt{10}}{7}$ as $\sin b < 0$.
and b between 180 and 360

11 The nth term of a sequence is u_n

The sequence is defined by

$$u_{n+1} = pu_n + 70$$

where $u_1 = 400$ and p is a constant.

11 (a) Find an expression, in terms of p, for u_2

[1 mark]

$u_2 = p \times 400 + 70$

$= 400p + 70$

11 (b) It is given that $u_3 = 382$

11 (b) (i) Show that p satisfies the equation

$$200p^2 + 35p - 156 = 0$$

[3 marks]

$u_3 = pu_2 + 70 = p(400p + 70) + 70$

$= 400p^2 + 70p + 70$

$382 = 400p^2 + 70p + 70$

$191 = 200p^2 + 35p + 35$

$200p^2 + 35p - 156 = 0$

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11 (b) (ii) It is given that the sequence is a decreasing sequence.

Find the value of u_4 and the value of u_5

[3 marks]

$200p^2 + 35p - 156 = 0$

$\frac{-35 \pm \sqrt{35^2 - 4(200)(-156)}}{400} = 0.8$ or -0.975

$u_4 = 0.8 \times 382 + 70 = 305.6 + 70 = 375.6$

$u_5 = 0.8 \times 375.6 + 70 = 370.48$

11 (c) The limit of u_n as n tends to infinity is L

11 (c) (i) Write down an equation for L

[1 mark]

$L = 0.8L + 70$

11 (c) (ii) Find the value of L

[1 mark]

$L - 0.8L = 70$

$0.2L = 70$

$L = 350$

12 One of the rides at a theme park is a room where the floor and ceiling both move up and down for 10π seconds.

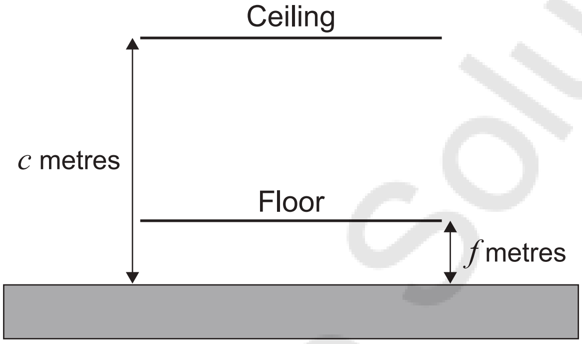
At time t seconds after the ride begins, the distance f metres of the floor above the ground is

$$f = 1 - \cos t$$

At time t seconds after the ride begins, the distance c metres of the ceiling above the ground is

$$c = 8 - 4\sin t$$

The ride is shown in the diagram below.



12 (a) Show that the initial distance between the floor and ceiling is 8 metres.

[1 mark]

$t = 0 \quad f = 1 - \cos(0) = 1 - 1 = 0$

$c = 8 - 4\sin(0) = 8 - 0 = 8$

$\cancel{\text{dist}}$ $c - f = 8 - 0 = 8$ m

12 (b) Show that the distance d metres between the floor and ceiling at time t is given by

$$d = 7 + R\cos(t + \alpha)$$

where R and α are positive constants to be found.

[5 marks]

$d = c - f = (8 - 4\sin t) - (1 - \cos t)$

$= 7 - 4\sin t + \cos t$

$R\cos(t+\alpha) = R\cos t\cos\alpha - R\sin t\sin\alpha$

$d = 7 + \cos t - 4\sin t$

$R\cos\alpha = 1$

$R\sin\alpha = 4$

$R = \sqrt{1^2 + 4^2} = \sqrt{17}$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{4}{1}$ $\tan^{-1} 4 = 1.33$ rad

$d = 7 + \sqrt{17}\cos(t + 1.33)$ ✓

12 (c) Hence, find the minimum distance between the ceiling and the floor.

Give your answer to the nearest centimetre.

[2 marks]

$d = 7 + \sqrt{17}\cos(t + 1.33)$ $\cos(t + 1.33) = -1$

$d_{min} = 7 - \sqrt{17} = 2.876..$

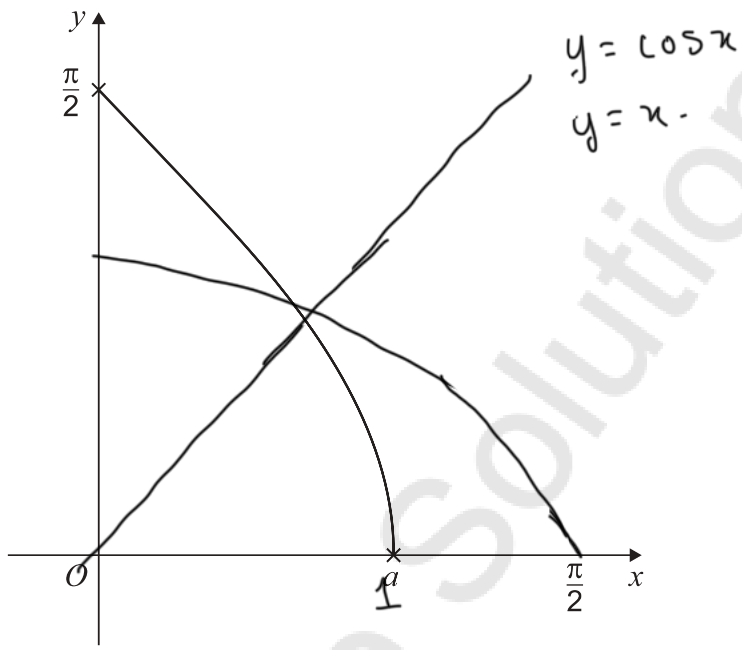
2.88 m (nearest cm)

Do not write outside the box

13 The function f is defined by

$$\mathrm{f}(x) = \arccos x \quad \text{for} \quad 0 \le x \le a$$

The curve with equation $y = \mathrm{f}(x)$ is shown below.



13 (a) State the value of a

[1 mark]

arccos domain $[-1, 1]$

$a = 1$ $\quad 0 \le x \le 1$

13 (b) (i) On the diagram above, sketch the curve with equation

$$y = \cos x \quad \text{for} \quad 0 \le x \le \frac{\pi}{2}$$

and

sketch the line with equation

$$y = x \quad \text{for} \quad 0 \le x \le \frac{\pi}{2}$$

[4 marks]

13 (b) (ii) Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$$\cos x = \arccos x$$

[1 mark]

if $x = \cos x$ then applying arc cos to both sides gives $\arccos x = x$ which means $\cos x = \arccos x$.

Question 13 continues on the next page

13 (c) Use the Newton-Raphson method with $x_0 = 0$ to find an approximate solution, x_3, to the equation

$$x - \cos x = 0$$

Give your answer to four decimal places.

[3 marks]

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ where $f(x) = 0$.

$f(x) = x - \cos x$

$f'(x) = 1 + \sin x$.

$x_1 = 0 - \frac{0 - \cos 0}{1 + \sin 0} = 0 - \frac{(-1)}{1} = 1$

$x_2 = 1 - \frac{1 - \cos 1}{1 + \sin 1} = 0.7504$

$x_3 = 0.7504\ldots - \frac{0.7504 - \cos x_2}{1 + \sin x_2}$

$= 0.7391$ (to 4dp)

Turn over for the next question

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14 (a) (i) Given that

$$y = 2^x$$

write down $\frac{dy}{dx}$

[1 mark]

$\frac{dy}{dx} = 2^x \cdot \ln 2$

14 (a) (ii) Hence find

$$\int 2^x \, dx$$

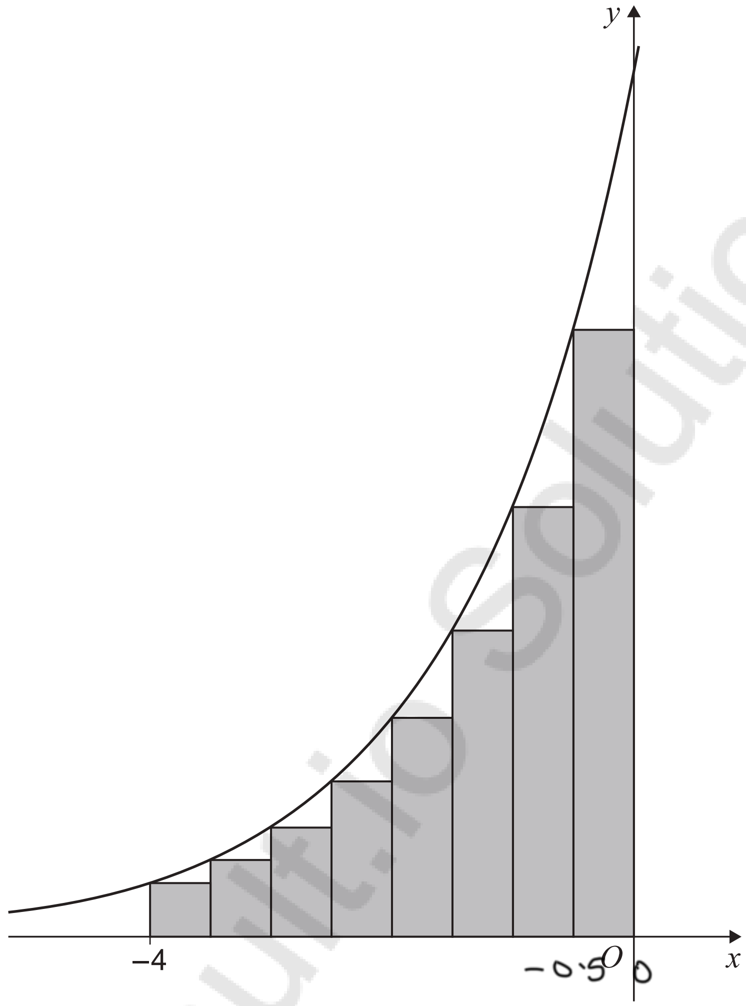
[2 marks]

$\int 2^x \, dx = \frac{2^x}{\ln 2} + C$

$\frac{dy}{dx} = 2^x \cdot \ln 2$

$y = \frac{2^x}{\ln 2}$ $\frac{dy}{dx} = \frac{2^x \ln 2}{\ln 2} = 2^x$

14 (b) The area, A, bounded by the curve with equation $y = 2^x$, the x-axis, the y-axis and the line $x = -4$ is approximated using eight rectangles of equal width as shown in the diagram below.



14 (b) (i) Show that the exact area of the largest rectangle is $\frac{\sqrt{2}}{4}$

[2 marks]

$-4 \div 8 = -0.5$

$0.5 \times 2^{-0.5} = 0.5 \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$

$\frac{1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$

Question 14 continues on the next page

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14 (b) (ii) The areas of these rectangles form a geometric sequence with common ratio $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form $k(1+\sqrt{2})$ where k is a rational number.

[3 marks]

$r = \frac{\sqrt{2}}{2}$ $a = \frac{\sqrt{2}}{4}$ $S_8 = \frac{a(1-r^8)}{1-r}$ $\left(\frac{\sqrt{2}}{2}\right)^8 = \frac{2^4}{2^8} = \frac{1}{2^4} = \frac{1}{16}$

$\frac{\frac{\sqrt{2}}{4}\left(1-\left(\frac{\sqrt{2}}{2}\right)^8\right)}{1-\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{4}\left(1-\frac{1}{16}\right)}{1-\frac{\sqrt{2}}{2}}$

$\frac{\frac{\sqrt{2}}{4}\left(\frac{15}{16}\right)}{\frac{2-\sqrt{2}}{2}} = \frac{\frac{15\sqrt{2}}{64}}{\frac{2-\sqrt{2}}{2}}$

$\frac{15\sqrt{2}}{64} \times \frac{2}{2-\sqrt{2}} = \frac{15\sqrt{2}}{32(2-\sqrt{2})}$

$\frac{15\sqrt{2}}{32(2-\sqrt{2})} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{30\sqrt{2}+30}{32(4-2)}$

$= \frac{30\sqrt{2}+30}{64} = \frac{15\sqrt{2}+15}{32} = \frac{15}{32}(\sqrt{2}+1)$

$k = \frac{15}{32}$

14 (b) (iii) More accurate approximations for A can be found by increasing the number, n, of rectangles used.

Find the exact value of the limit of the approximations for A as $n \to \infty$

[3 marks]

$$\int_{-4}^{0} 2^x \, dx = \frac{1}{\ln 2}\left[2^x\right]_{-4}^{0}$$

$$= \frac{1}{\ln 2}\left[2^0 - 2^{-4}\right]$$

$$= \frac{1}{\ln 2}\left[1 - \frac{1}{16}\right)$$

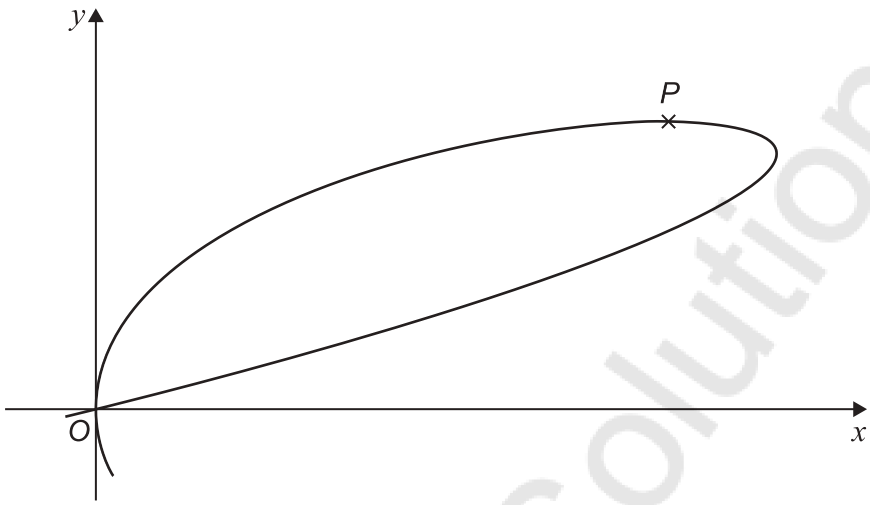
$$= \frac{15}{16\ln 2}$$

Turn over for the next question

15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at P as shown in the diagram below.



15 (a) Show that the y-coordinate of P satisfies the equation

$$y^2(y - 2) = 0$$

[7 marks]

$2x + 6y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

$2x - 4y = 0$

$x = 2y$

$(2y)^2 + 2y^3 - 4(2y)y = 0$

$4y^2 + 2y^3 - 8y^2 = 0$

$2y^3 - 4y^2 = 0$

$2y^2(y - 2) = 0$

$y^2(y-2) = 0$

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15 (b) Hence, find the coordinates of P

[2 marks]

$y^2(y-2)=0 \quad y=0$ or 2

$\times \quad y=0$ gives $x=0$ which is the Origin

$y=2 \quad x=2y \Rightarrow x=4$

$P(4,2)$

Turn over for the next question

16 (a) Given that

$$\frac{1}{16-9x^2} \equiv \frac{A}{4-3x} + \frac{B}{4+3x}$$

find the values of A and B

[3 marks]

$1 = A(4+3x) + B(4-3x)$

$x = -\frac{4}{3}$

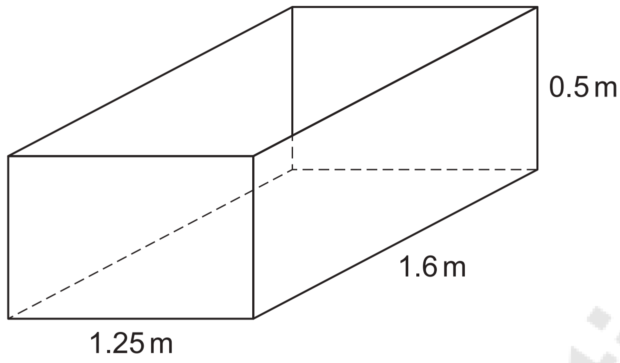
$1 = B\left(4-3\left(\frac{-4}{3}\right)\right)$

$1 = 8B \Rightarrow B = 1/8$

$x = \frac{4}{3}$ $1 = A\left(4+3\left(\frac{4}{3}\right)\right)$

$1 = 8A \Rightarrow A = 1/8$

16 (b) An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time t minutes after the container starts to be filled, the depth of water is d metres and water leaks out at a rate of $0.36d^2$ cubic metres per minute.

At time t minutes after the container starts to be filled, the volume of water in the container is V cubic metres.

16 (b) (i) Show that

$$\frac{dV}{dt} = \frac{16 - 9V^2}{100}$$

[4 marks]

$1.6 \times 1.25 = 2\text{m}^2$

so $V = 2d \qquad d = \frac{V}{2}$

Rate in $= 0.16\ \text{m}^3/\text{min}$

Rate out $= 0.36\,d^2 = 0.36\left(\frac{V}{2}\right)^2 = \frac{0.36V^2}{4}$

$= 0.09V^2$

$\frac{dV}{dt} = 0.16 - 0.09V^2 = \frac{(16 - 9V^2)}{100}$

16 (b) (ii) Hence, find t in terms of V

[5 marks]

$$\frac{dV}{dt} = \frac{16-9V^2}{100}$$

$$\int \frac{1}{16-9V^2}\,dV = \int \frac{1}{100}\,dt$$

$$\int \frac{1}{8(4-3V)} + \frac{1}{8(4+3V)}\,dV = \int \frac{1}{100}\,dt$$

$$\frac{1}{8}\int \frac{1}{4-3V} + \frac{1}{4+3V}\,dV = \int \frac{1}{100}\,dt$$

$$\frac{1}{8}\left[\frac{-1}{3}\ln|4-3v| + \frac{1}{3}\ln|4+3v|\right] = \frac{t}{100} + c$$

$$\frac{1}{24}\left[\ln|4+3v| - \ln|4-3v|\right] = \frac{t}{100} + c$$

At $t=0$, $V=0$

$$\frac{1}{24}\left[\ln|4| - \ln|4|\right] = 0 \quad c = 0$$

so $t = \frac{100}{24}\left(\ln(4+3V) - \ln(4-3V)\right)$

16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute.

[2 marks]

Full volume $= 1.6 \times 1.25 \times 0.5 = 1$

$$t = \frac{100}{24} \times \left(\ln(7) - \ln(1)\right)$$

$$t = \frac{100\ln 7}{24} = 8.1 \text{ minutes}$$

to the nearest minute 8 minutes

END OF QUESTIONS

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