AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

A-level
MATHEMATICS
Paper 3

Tuesday 20 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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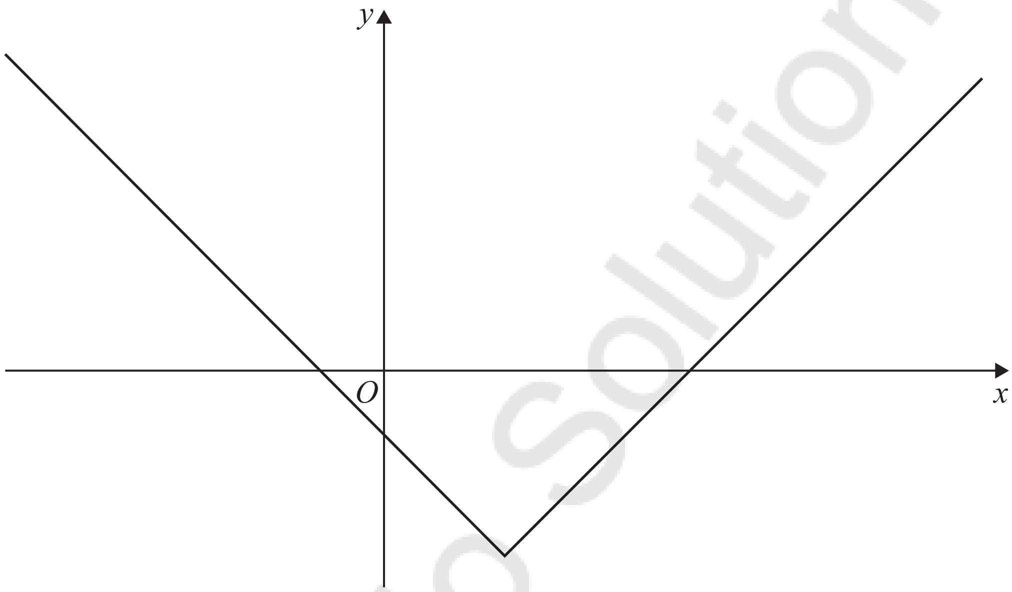
TOTAL	

Section A

Answer **all** questions in the spaces provided.

1 The graph of $y = \mathrm{f}(x)$ is shown below.



One of the four equations listed below is the equation of the graph $y = \mathrm{f}(x)$

Identify which one is the correct equation of the graph.

Tick (✓) **one** box.

[1 mark]

$y = |x + 2| + 3$ ✗ ☐

$y = |x + 2| - 3$ ☐

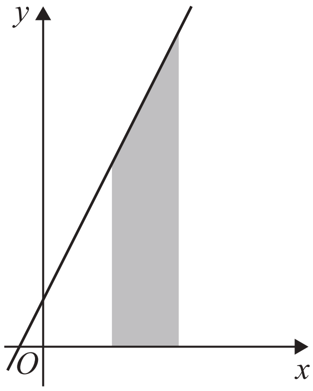
$y = |x - 2| + 3$ ✗ ☐

$y = |x - 2| - 3$ ☑ ✓

2 The trapezium rule is used to estimate the area of the shaded region in each of the graphs below.
Identify the graph for which the trapezium rule produces an overestimate.

Tick (✓) **one** box.

[1 mark]



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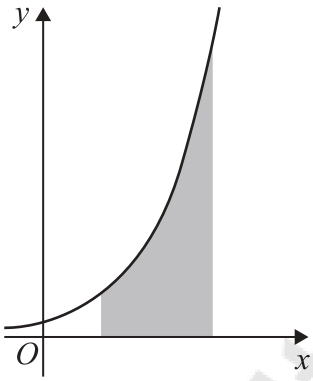

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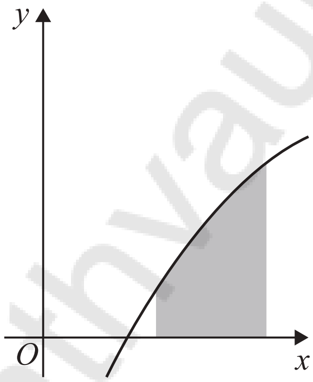

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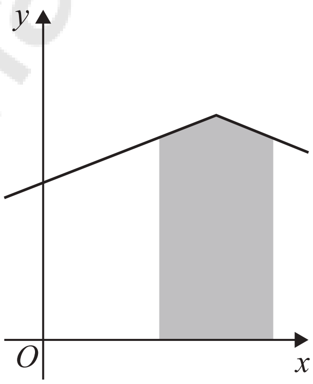

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3 A curve with equation $y = \mathrm{f}(x)$ passes through the point (3, 7)

Given that $\mathrm{f}'(3) = 0$ find the equation of the normal to the curve at (3, 7)

Circle your answer.

[1 mark]

$y = \frac{7}{3}x$ $\quad$ $y = 0$ $\quad$ $x = 3$ $\quad$ $x = 7$

4 Express

$$\frac{5 - \sqrt[3]{x}}{x^2}$$

in the form

$$5x^p - x^q$$

where p and q are constants.

[2 marks]

$\frac{5}{x^2} - \frac{x^{1/3}}{2} = 5x^{-2} - x^{1/3 - 2} = 5x^{-2} - x^{(-5/3)}$

5 A curve has equation $y = 3e^{2x}$

Find the gradient of the curve at the point where $y = 10$

[3 marks]

$\frac{dy}{dx} = 6e^{2x}$ $\frac{dy}{dx} = 6\exp\left(\frac{\ln 10/3}{2}\right)$

$10 = 3e^{2x}$ $= 20$

$\frac{10}{3} = e^{2x}$

$\ln\frac{10}{3} = 2x$

$\frac{\ln\frac{10}{3}}{2} = x$

Turn over for the next question

Do not write outside the box

6 (a) Sketch the curve with equation

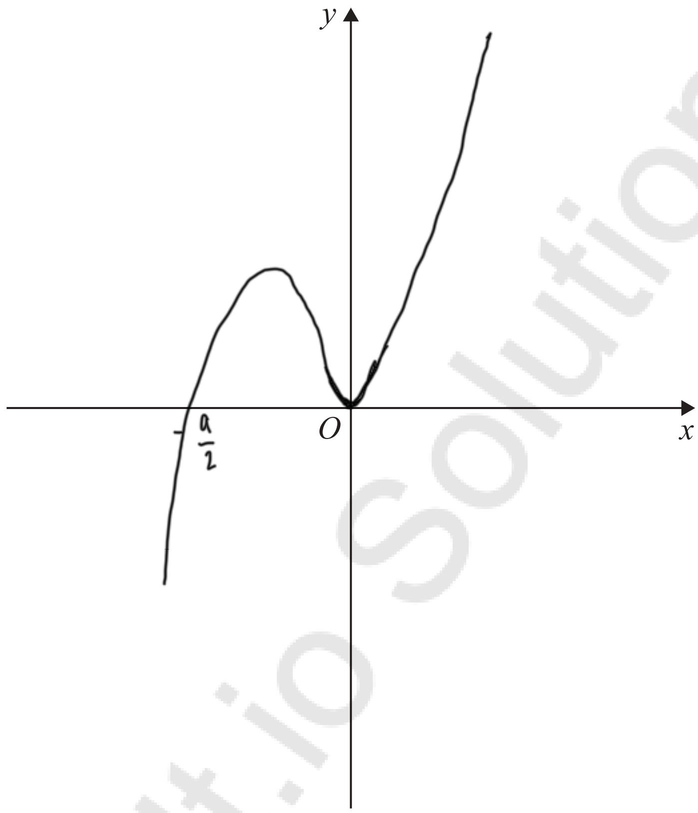
$$y = x^2(2x + a)$$

where $a > 0$

$x = 0$

$x = \frac{-a}{2}$

[3 marks]



6 (b) The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^2(2x + a) + 36$$

6 (b) (i) It is given that $x + 3$ is a factor of $\mathrm{p}(x)$

Use the factor theorem to show $a = 2$

[2 marks]

$(-3)^2(2(-3) + a) + 36 = 0$

$9(-6 + a) + 36 = 0$

$-54 + 9a + 36 = 0$

$9a - 18 = 0$

$9a = 18$

$a = 2$

6 (b) (ii) State the transformation which maps the curve with equation

$$y = x^2(2x + 2)$$

onto the curve with equation

$$y = x^2(2x + 2) + 36$$

[2 marks]

tranSformation of vector $\begin{pmatrix} 0 \\ 36 \end{pmatrix}$

6 (b) (iii) The polynomial $x^2(2x + 2) + 36$ can be written as $(x + 3)(2x^2 + bx + c)$

Without finding the values of b and c, use your answers to parts **(a)** and **(b)(ii)** to explain why

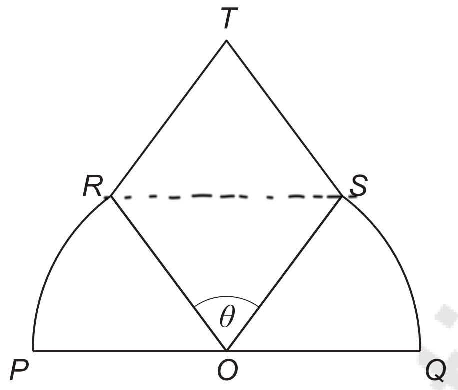
$$b^2 < 8c$$

[2 marks]

the translated graph only has one real root at $x = -3$
so the other quadratic factor has no real roots.
and for there to be no real roots $b^2 - 4ac$
$b^2 - 4 \times 2 \times c < 0 \quad b^2 < 8c$

Turn over for the next question

7 A new design for a company logo is to be made from two sectors of a circle, *ORP* and *OQS*, and a rhombus *OSTR*, as shown in the diagram below.



The points *P*, *O* and *Q* lie on a straight line and the angle *ROS* is θ radians.

A large copy of the logo, with $PQ = 5$ metres, is to be put on a wall.

7 (a) Show that the area of the logo, A square metres, is given by

$$A = \frac{25}{8}(\pi - \theta + 2\sin\theta)$$

[4 marks]

$A_S = \frac{1}{2} r^2 \theta$

$2 \times \frac{1}{2} \times r^2 \frac{(\pi - \theta)}{2} = r^2 \left(\frac{\pi - \theta}{2}\right)$

$A_\Delta = \frac{1}{2} ab \sin C$ $\quad PQ = 5m$

$A_\Diamond = r^2 \sin\theta$ $\quad PQ = 2r$

$r = 2.5$

$A_\Diamond = (2.5)^2 \sin\theta$ $\quad A_S = (2.5)^2 \left(\frac{\pi - \theta}{2}\right)$

$(2.5)^2 \left(\sin\theta + \frac{\pi - \theta}{2}\right)$

$2(2.5)^2 \ (2\sin\theta + \pi - \theta)$

$\frac{25}{8} \ (2\sin\theta + \pi - \theta)$

7 (b) (i) Show that the maximum value of A occurs when $\theta = \frac{\pi}{3}$

Fully justify your answer.

[6 marks]

$\frac{dA}{d\theta} = \left(\frac{25}{8}\right)(-1 + 2\cos\theta)$

$\frac{dA}{d\theta} = 0 \qquad -1 + 2\cos\theta = 0$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\frac{d^2A}{d\theta^2} = -\frac{25}{8} \times 2\sin\theta \quad \frac{25}{8} \times -2\sin\frac{\pi}{3} = -\frac{25}{8} \times \frac{\sqrt{3}}{2} < 0$

$\frac{dA}{d\theta} = 0$ and $\frac{d^2A}{d\theta^2} < 0$

Hence $\frac{\pi}{3}$ is a maximum

Question 7 continues on the next page

7 (b) (ii) Find the exact maximum value of A

[2 marks]

$$A = \frac{25}{8}\left(2\sin\frac{\pi}{3} + \frac{2\pi}{3}\right)$$

$$= \frac{25}{8}\left(\sqrt{3} + \frac{2\pi}{3}\right)$$

7 (c) Without further calculation, state how your answers to parts **(b)(i)** and **(b)(ii)** would change if *PQ* were increased to 10 metres.

[2 marks]

The angle giving the maximum area would not change as this was not dependent on r. the maximum area would be quadrupled as the radius has doubled and area is proportional to r^2

Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

8 Use the substitution $u = x^5 + 2$ to show that

$$\int_0^1 \frac{x^9}{(x^5+2)^3}\,dx = \frac{1}{180}$$

[7 marks]

$\frac{du}{dx} = 5x^4$ $\qquad \int \frac{x^9}{u^3} \frac{du}{5x^4}$

$\frac{du}{5x^4} = dx$ $\qquad \int \frac{x^5}{u^3} \frac{du}{5}$

$u - 2 = x^5$ $\qquad \int_2^3 \frac{u-2}{5u^3}\,du$

$x = 1 \quad u = 1 + 2 = 3$

$x = 0 \quad u = 0 + 2 = 2$ $\qquad \int_2^3 \frac{1}{5u^2} - \frac{2}{5u^3}\,du$

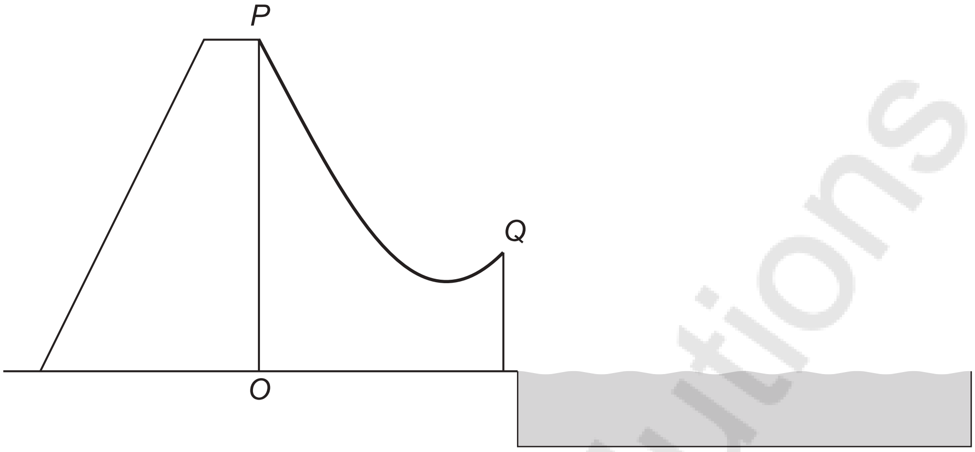
$$\left[\frac{-1}{5u} + \frac{1}{5u^2}\right]_2^3$$

$$\frac{-1}{15} + \frac{1}{45} + \frac{1}{10} - \frac{1}{20} = \frac{1}{180}$$

Turn over for the next question

9 A water slide is the shape of a curve *PQ* as shown in **Figure 1** below.

Figure 1



The curve can be modelled by the parametric equations

$$x = t - \frac{1}{t} + 4.8$$

$$y = t + \frac{2}{t}$$

where $0.2 \le t \le 3$

The horizontal distance from *O* is x metres.

The vertical distance above the point *O* at ground level is y metres.

P is the point where $t = 0.2$ and *Q* is the point where $t = 3$

9 (a) To make sure speeds are safe at *Q*, the difference in height between *P* and *Q* must be less than 7 metres.

Show that the slide meets this safety requirement.

[3 marks]

$t = 0.2 \qquad y = 0.2 + \frac{2}{0.2} = 10.2$

$t = 3 \qquad y = 3 + \frac{2}{3} = \frac{11}{3} = 3.67$

$10.2 - 3.67 = 6.53$

$6.53 < 7$

so the safety requirements are met

9 (b) (i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t

[3 marks]

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dt} = 1 - \frac{2}{t^2}$

$\frac{dx}{dt} = 1 + \frac{1}{t^2}$

$\frac{dy}{dx} = \frac{1 - \frac{2}{t^2}}{1 + \frac{1}{t^2}}$

Question 9 continues on the next page

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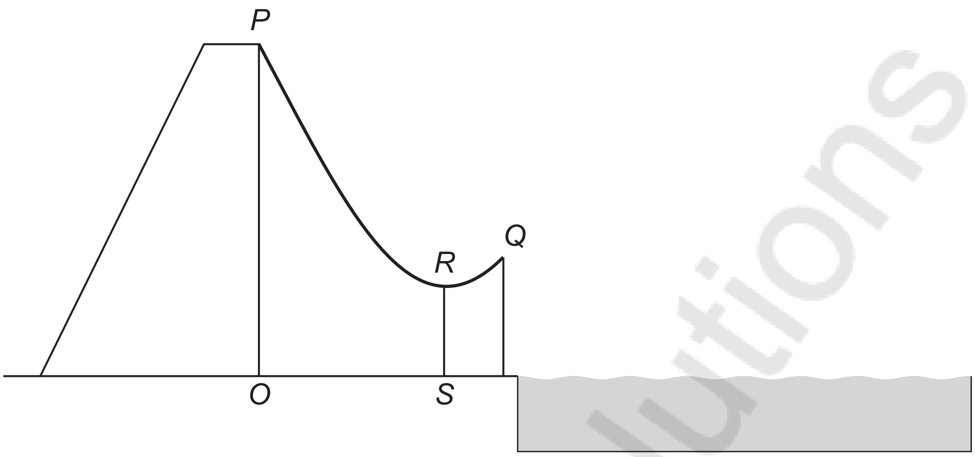
9 (b) (ii) A vertical support, *RS*, is to be added between the ground and the lowest point on the slide as shown in **Figure 2** below.

Figure 2



Find the length of *RS*

[4 marks]

$\frac{dy}{dx} = 0$ $\quad 1 - \frac{2}{t^2} = 0$

$t^2 = 2$

$t = \sqrt{2}$

$y = \sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

$RS = 2\sqrt{2}$ m $= 2.83$ m (3sf)

9 (b) (iii) Find the acute angle the slide makes with the horizontal at *Q*

Give your answer to the nearest degree.

[2 marks]

$t = 3 \qquad \frac{dy}{dx} = \frac{1 - \frac{2}{9}}{1 + \frac{1}{9}} = 0.7$

$\tan^{-1}(0.7) = 35^{\circ}$

END OF SECTION A

TURN OVER FOR SECTION B

Section B

Answer **all** questions in the spaces provided.

10 Which of the following is **not** a possible value for a product moment correlation coefficient?

Circle your answer.

[1 mark]

$-\frac{6}{5}$ $\qquad$ $-\frac{3}{5}$ $\qquad$ 0 $\qquad$ 1

$-1 \leq r \leq 1$

11 A and B are mutually exclusive events.

Which one of the following statements must be correct?

Tick (✓) **one** box.

[1 mark]

$P(A \cup B) = P(A) \times P(B)$ ☐

$P(A \cup B) = P(A) - P(B)$ ☐

$P(A \cap B) = 0$ ☑

$P(A \cap B) = 1$ ☐

Turn over for the next question

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12 It is known that, on average, 40% of the drivers who take their driving test at a local test centre pass their driving test.

Each day 32 drivers take their driving test at this centre.

The number of drivers who pass their test on a particular day can be modelled by the distribution B (32, 0.4)

12 (a) State one assumption, in context, required for this distribution to be used.

[1 mark]

the probability of passing the text is constant at 0.4.

12 (b) Find the probability that exactly 7 of the drivers on a particular day pass their test.

[1 mark]

$P(X=7) = \binom{32}{7} \times (0.4)^7 \times (0.6)^{25} = 0.0157$

12 (c) Find the probability that, at most, 16 of the drivers on a particular day pass their test.

[1 mark]

$P(X \leq 16) = 0.908$

12 (d) Find the probability that more than 12 of the drivers on a particular day pass their test.

[2 marks]

$P(X \geqslant 13) = 1 - P(X \leq 12)$

$= 1 - 0.4618$

$= 0.538$

12 (e) Find the mean number of drivers per day who pass their test.

[1 mark]

Mean $= np = 32 \times 0.4 = 12.8$

12 (f) Find the standard deviation of the number of drivers per day who pass their test.

[2 marks]

Variance $= np(1-p) = 32 \times 0.4 \times 0.6 = 7.68$

S.D $= \sqrt{7.68} = 2.77$

Turn over for the next question

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13 There are two types of coins in a money box:

- 20% are bronze coins
- 80% are silver coins

Craig takes out a coin at random and places it back in the money box.

Craig then takes out a second coin at random.

13 (a) Find the probability that both coins were of the same type.

[2 marks]

P(both bronze) = $0.2 \times 0.2 = 0.04$

P(both silver) = $0.8 \times 0.8 = 0.64$

P(both are same type) = $0.04 + 0.64 = 0.68$

13 (b) Find the probability that both coins are bronze, given that at least one of the coins is bronze.

[2 marks]

P(at least one bronze) = 1 − P(both silver)

$= 1 - 0.64 = 0.36$

P(both bronze | at least one) $= \dfrac{\text{P(both bronze)}}{\text{P(At least one)}}$

$= \dfrac{0.04}{0.36} = \dfrac{1}{9}$

$P(A|B) = \dfrac{P(A \cap B)}{P(A)}$

Turn over for the next question

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14 The mass of aluminium cans recycled each day in a city may be modelled by a normal distribution with mean 24 500 kg and standard deviation 5 200 kg.

14 (a) State the probability that the mass of aluminium cans recycled on any given day is **not** equal to 24 500 kg.

[1 mark]

$P(X = a) = 0$ for all a

$P(X \neq 24500) = 1$

14 (b) To reduce costs, the city's council decides to collect aluminium cans for recycling less frequently.

Following the decision, it was found that over a 24-day period a total mass of 641 520 kg of aluminium cans was recycled.

It can be assumed that the distribution of the mass of aluminium cans recycled is still normal with standard deviation 5 200 kg, and that the 24-day period can be regarded as a random sample.

Investigate, at the 5% level of significance, whether the mean daily mass of aluminium cans recycled has **changed**.

[7 marks]

$H_0 : \mu = 24500$ $\qquad \bar{X} = \dfrac{641520}{24} = 26730$

$H_1 : \mu \neq 24500$

$\bar{X} \sim N\left(24500, \dfrac{5200^2}{24}\right)$

$P(\bar{X} > 26730) = 1 - P(\bar{X} \leq 26730)$

$= 0.018$

$0.018 < 0.025$ so reject the H_0

there is sufficient evidence to suggest the mean daily mass of aluminium cans recycled has changed.

14 (c) A member of the council claims that if a different sample of 24 days had been used the hypothesis test in part **(b)** would have given the same result.

Comment on the validity of this claim.

[2 marks]

A different sample would have a different sample mean, which is what is being tested. The result of the hypothesis test could be different, so the claim is not valid.

Turn over for the next question

15 (a) A random sample of eight cars was selected from the Large Data Set.

The masses of these cars, in kilograms, were as follows.

950 989 1247 1415 1506 1680 1833 2040

It is given that, for the population of cars in the Large Data Set:

lower quartile $= 1167$

median $= 1393$

upper quartile $= 1570$

15 (a) (i) It was decided to remove any of the masses which fall outside the following interval.

median $-$ 1.5$\times$ interquartile range $\leq$ mass $\leq$ median $+$ 1.5$\times$ interquartile range

Show that only one of the eight masses in the sample should be removed.

[3 marks]

IQR = 1570 − 1167 = 403

lower limit 1393 − 1.5 × 403 = 788.5

upper limit 1393 + 1.5 × 403 = 1997.5

2040 > 1997.5 Hence it is an outlier,

there is only one outlier

15 (a) (ii) Write down the statistical name for the mass that should be removed in part **(a)(i)**.

[1 mark]

outlier

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15 (b) The table shows the probability distribution of the number of previous owners, N, for a sample of cars taken from the Large Data Set.

n	0	1	2	3	4	5	6 or more
$\mathbf{P}(N = n)$	0.14	0.37	0.9k	0.25	0.4k	1.7k	0

Find the value of $\mathrm{P}(1 \leq N < 5)$

[4 marks]

$0.14 + 0.37 + 0.9k + 0.25 + 0.4k + 1.7k = 1$

$0.76 + 3k = 1$

$3k = 0.24$

$k = 0.08$

$P(1 \leq N < 5) =$

$0.37 + 0.9 \times 0.08 + 0.25 + 0.4 \times 0.08 = 0.724$

Question 15 continues on the next page

15 (c) An expert team is investigating whether there have been any changes in CO_2 emissions from all cars taken from the Large Data Set.

The team decided to collect a quota sample of 200 cars to reflect the different years and the different makes of cars in the Large Data Set.

15 (c) (i) Using your knowledge of the Large Data Set, explain how the team can collect this sample.

[2 marks]

LDS has 2 years and 5 makes of cars, so we need to choose proportional quotas, 100 from each year or 40 from each make. They should select 20 cars from each of the 5 makes of car for each of the two years. $(20 \times 5 \times 2 = 200)$

15 (c) (ii) Describe **one disadvantage** of quota sampling.

[1 mark]

Quota sampling can produce a biased sample.

Turn over for the next question

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16 A farm supplies apples to a supermarket.

The diameters of the apples, D centimetres, are normally distributed with mean 6.5 and standard deviation 0.73

$N \sim (6.5, 0.73^2)$

16 (a) (i) Find $P(D < 5.2)$

[1 mark]

$P(D < 5.2) = 0.0375$

16 (a) (ii) Find $P(D > 7)$

[1 mark]

$P(D > 7) = 0.2467$

16 (a) (iii) The supermarket only accepts apples with diameters between 5 cm and 8 cm.

Find the proportion of apples that the supermarket accepts.

[1 mark]

$P(5 \leq D \leq 8) = 0.9601$

16 (b) The farm also supplies plums to the supermarket.

These plums have diameters that are normally distributed.

It is found that 60% of these plums have a diameter less than 5.9 cm.

It is found that 20% of these plums have a diameter greater than 6.1 cm.

Find the mean and standard deviation of the diameter, in centimetres, of the plums supplied by the farm.

[6 marks]

$P(D < 5.9) = 0.6$

$P(D > 6.1) = 0.2$

For $P(D < 5.9) = 0.6$

$P(Z < z) = 0.6 \quad z_1 = 0.2533$

For $P(D > 6.1) = 0.2$

$P(Z > z) = 0.2$

$1 - 0.2 = 0.8 = P(Z < z) \quad z_2 = 0.8416$

$5.9 < \mu < 6.1$

$\frac{5.9 - \mu}{\delta} = 0.2533 \Rightarrow 5.9 - \mu = 0.2533\delta \ (1)$

$\frac{6.1 - \mu}{\delta} = 0.8416 \Rightarrow 6.1 - \mu = 0.8416\delta \ (2)$

$(1) - (2) \quad (6.1 - \mu) - (5.9 - \mu) = 0.8416\delta - 0.2533\delta$

$0.2 = 0.5883\delta$

$\mu = 5.81 \text{ cm} \qquad \delta = \frac{0.2}{0.5883} = 0.34$

$\delta = 0.34 \text{ cm}$

$5.9 - \mu = 0.2533 \times 0.34 = 0.0861$

$\mu = 5.9 - 0.0861 = 5.814$

Turn over ▶

17 A council found that 70% of its new local businesses made a profit in their first year.

The council introduced an incentive scheme for its residents to encourage the use of new local businesses.

At the end of the scheme, a random sample of 25 new local businesses was selected and it was found that 21 of them had made a profit in their first year.

Using a binomial distribution, investigate, at the 2.5% level of significance, whether there is evidence of an increase in the proportion of new local businesses making a profit in their first year.

[6 marks]

$H_0: p = 0.7$ $X \sim B(25, 0.7)$

$H_1: p > 0.7$

$P(X \geq 21) = 1 - P(X \leq 20)$

$= 1 - 0.9095 = 0.0905$

$0.0905 > 0.025$

we do not reject H_0

There is insufficient evidence of an increase in the proportion of locals businesses that made a profit in the first year.

END OF QUESTIONS

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