AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# GCSE MATHEMATICS

**H**

Higher Tier Paper 3 Calculator

Monday 10 June 2024 Morning Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

## Advice

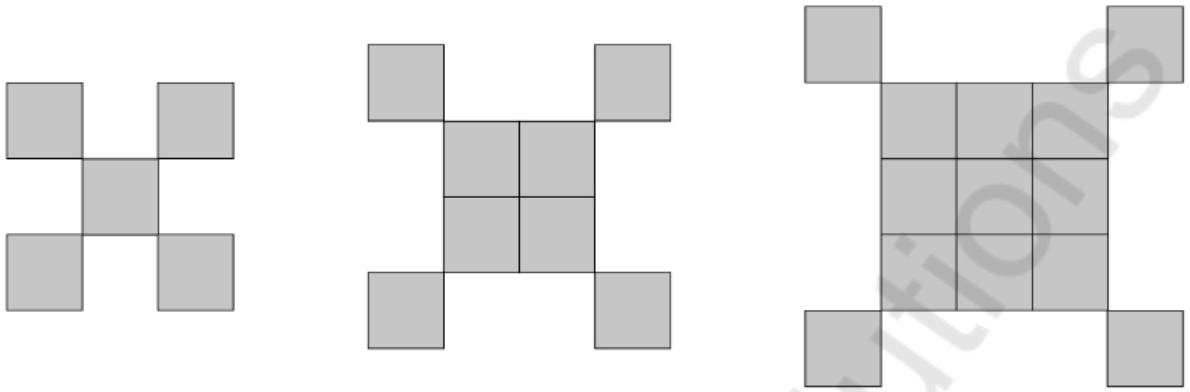
In all calculations, show clearly how you work out your answer.

| For Examiner's Use | |
|---|---|
| Pages | Mark |
| 2–3 | |
| 4–5 | |
| 6–7 | |
| 8–9 | |
| 10–11 | |
| 12–13 | |
| 14–15 | |
| 16–17 | |
| 18–19 | |
| 20–21 | |
| 22–23 | |
| 24–25 | |
| 26 | |
| **TOTAL** | |

Do not write outside the box

Answer **all** questions in the spaces provided.

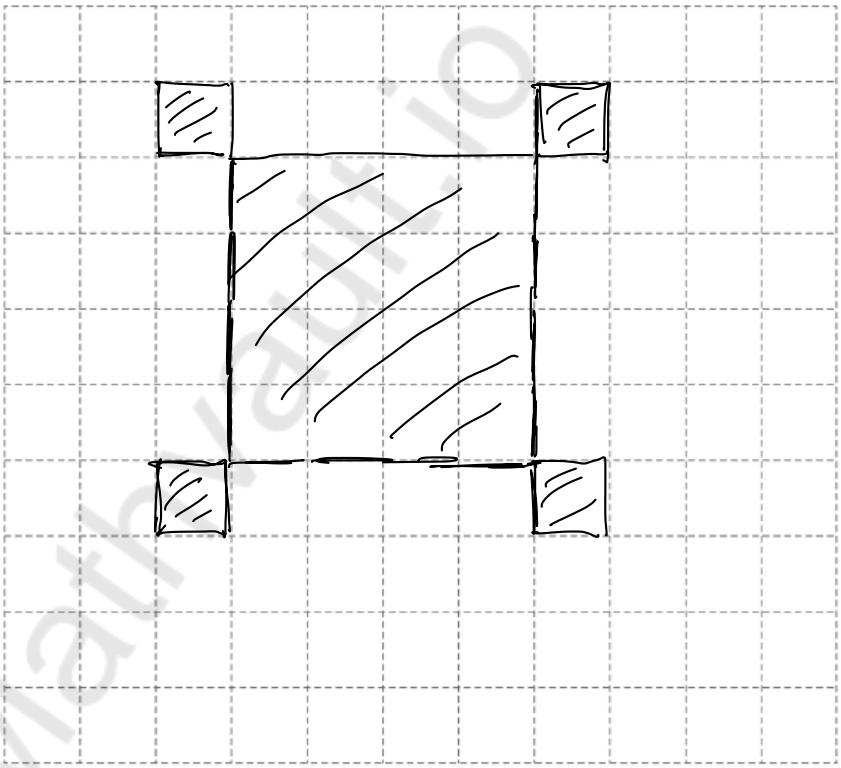
**1** Here are the first three Patterns in a sequence made up of small squares.



Pattern 1 Pattern 2 Pattern 3

**1 (a)** On the grid, draw Pattern 4

**[1 mark]**



Do not write outside the box

**1 (b)** The expression for the number of small squares in Pattern $n$ is $n^2 + 4$

Work out the least value of $n$ for which the number of small squares is greater than 500

**[1 mark]**

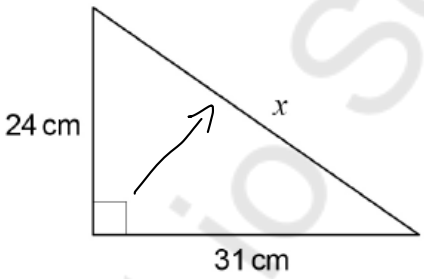
$n^2+4 > 500$

$n^2 > 496$

$n > \sqrt{496}$ $\quad n > 22.3$

$n =$ 23

**2**




Not drawn accurately

Use Pythagoras' theorem to work out the value of $x$.

Give your answer as a decimal.

**[3 marks]**

$a^2+6^2=c^2$

$24^2+31^2 = x^2$

$1537 = x^2$

$\sqrt{1537} = x$

$x \approx 39.2$ cm

Answer 39.2 cm

5

*Do not write outside the box*

**3** Rick claims most of the flats in his 8-floor building are energy efficient.

He samples 45 flats from floors 1 to 5

Give a reason why this sample may **not** be useful in testing Rick's claim.

**[1 mark]**

Total of 8 floors but sample only includes 1st floor to 5th floor. Floor 6, 7 and 8 not included in sample. Sample is not representative of the whole building.

**4** $3(x-1) \equiv 3x-3$ is an identity.

Tick **one** box.

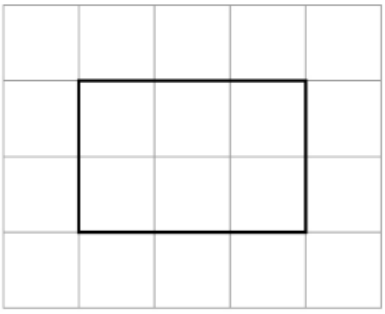
**[1 mark]**

- [x] It is true for **all** values of $x$
- [ ] It is true for **some** values of $x$
- [ ] It is true for **no** values of $x$

*Do not write outside the box*

**5** The front elevation of a cuboid is shown on this centimetre grid.

**Front elevation**

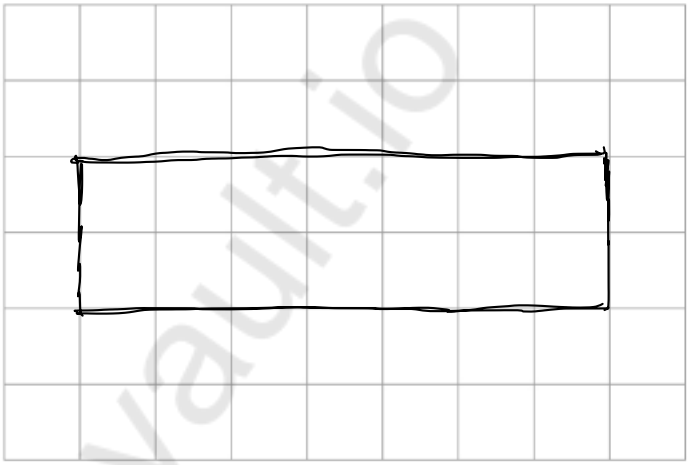


The volume of the cuboid is 42 $cm^3$

Draw the **side elevation** on this centimetre grid.

**[2 marks]**

**Side elevation**



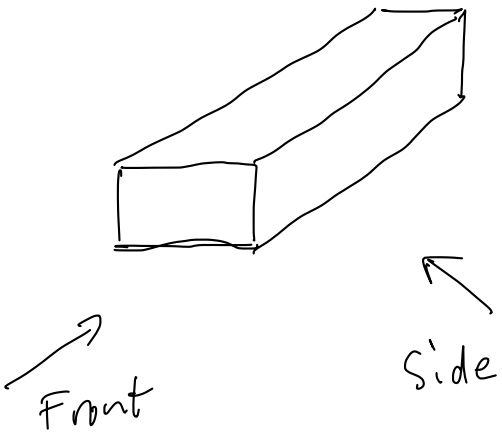
$V = l \times w \times h$

$42 = 3 \times w \times 2$

$42 = 6w$

$\div 6 \quad \div 6$

$7 = w$




4

**Turn over ►**

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**6 (a)** On Monday, Larrs swims 50 metres in 40 **seconds** at a constant speed.

On Tuesday, Larrs swims 1.5 kilometres.

Assume he swims at the same constant speed as on Monday.

How many **minutes** does he swim for on Tuesday?

**[5 marks]**

÷1000
1000m = 1km
50m = 0.05km
÷1000

Speed on monday = $\frac{0.05\text{km}}{\frac{2}{3}\text{ mins}}$

60s = 1min
÷6 ÷6
10s = $\frac{1}{6}$ min
×4 ×4
40s = $\frac{4}{6}$ mins
40s = $\frac{2}{3}$ mins

Speed on monday = $\frac{3}{40}$ km/minute

Speed on monday = 0.075 km/minute

$S = \frac{D}{T}$

$S \times T = D$

$T = \frac{D}{S}$

Time on Tuesday = $\frac{1.5\text{km}}{0.075\text{km/minute}}$

Time on Tuesday = 20 minutes

Answer 20 minutes

**6 (b)** In fact, on Tuesday Larrs swims at a slower constant speed than on Monday.

What does this mean about the number of minutes he swims for on Tuesday?

Tick the correct box.

**[1 mark]**

- ☐ It is less than the answer to part (a)
- ☐ It is the same as the answer to part (a)
- ☑ It is greater than the answer to part (a)
- ☐ It is not possible to say

*Do not write outside the box*

**7** Draw the graph of $y = 1 - \frac{1}{2}x$ for values of $x$ from –2 to 4

**[3 marks]**

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $y$ | $2$ | $1.5$ | $1$ | $0.5$ | $0$ | $-0.5$ | $-1$ |

$(-2, 2)$

$(-1, 1.5)$

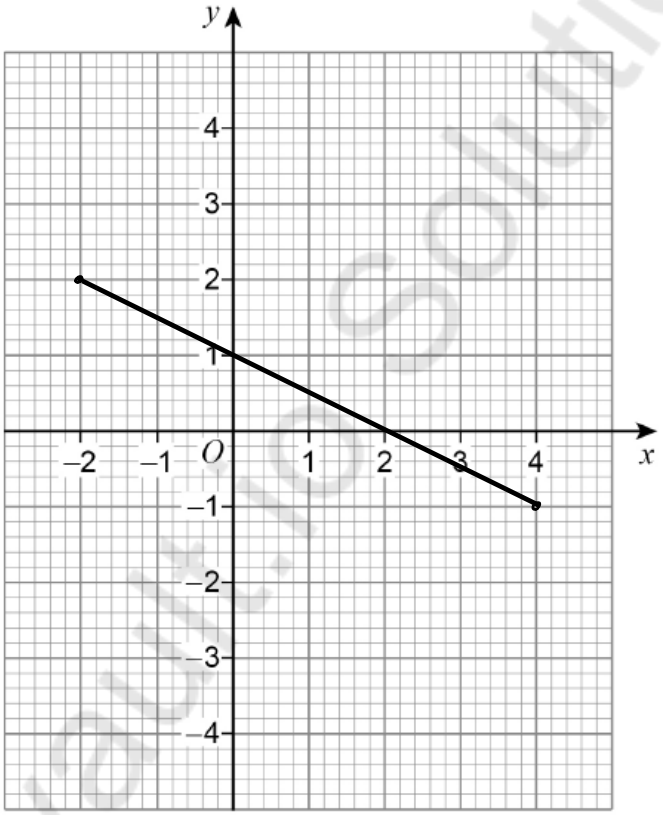
$(0, 1)$

$(1, 0.5)$

$(2, 0)$

$(3, -0.5)$

$(4, -1)$






9

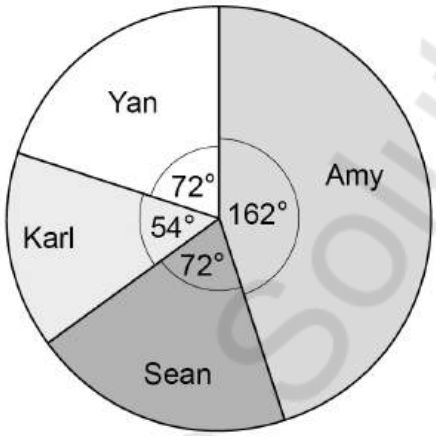
*Do not write outside the box*

**8** Four people are taking part in a television talent show.

Here are Amy's marks from the 6 judges.

| | | | | | |
|---|---|---|---|---|---|
| 8 | 9 | 9 | 6 | 9 | 10 |

The pie chart represents the phone vote.




Amy's total score is found by

| |
|---|
| 4 × the **mean** of her marks |
| + |
| her **percentage** of the phone vote |

Do not write outside the box

Work out Amy's total score.

**[4 marks]**

mean Judge's Scores = $\frac{8+9+9+6+9+10}{6}$

mean Score = 8.5

4 x mean = 4 x 8.5 = 34

% of phone votes = $\frac{162}{360} \times 100 = 45\%$

34 + 45 = 79

Answer 79



**Turn over for the next question**

4

*Do not write outside the box*

**9** Town A has

a population of 84 000

an area of 7 **square miles**.

Town B has a population density of 4695 people per **square kilometre**.

$$\text{Population density} = \frac{\text{population}}{\text{area}}$$

Which town has the greater population density?

Use 1 square mile = 2.6 square kilometres

Tick a box.

Town A ☐ Town B ☑

Show working to support your answer.

**[3 marks]**

7 square miles = $2.6 \times 7 = 18.2$ square km

P.D. (Town A) = $\frac{84\,000}{18.2} = 4615$ square km

$4615 < 4695$

*Do not write outside the box*

**10** On a biased dice,

P(lands on 6) = 0.38

This dice is rolled 150 times.

How many times would you expect the dice **not** to land on 6 ?

**[3 marks]**

P(not land on 6) = 1 − 0·38 = 0·62

0·62 × 150

Answer 93



**Turn over for the next question**

6

*Do not write outside the box*

**11** Write a number in each box to make the calculations correct.

**[2 marks]**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 10 | ÷ | –2 | × | $-1$ | = | 5 | | |
| $\frac{1}{3}$ | × | $4\pi$ | × | 6 | = | $8\pi$ | | |

$\frac{10}{-2} = -5$

$-5 \times x = 5$

$\div -5 \qquad \div -5$

$x = \frac{5}{-5} = -1$

$\frac{1}{3} \times y \times 6 = 8\pi$

$\frac{6y}{3} = 8\pi$

$2y = 8\pi$

$\div 2 \qquad \div 2$

$y = 4\pi$

Do not write outside the box

**12** Cards are either gold or not gold.

P(not Gold) = 1 − 0.05 = 0.95

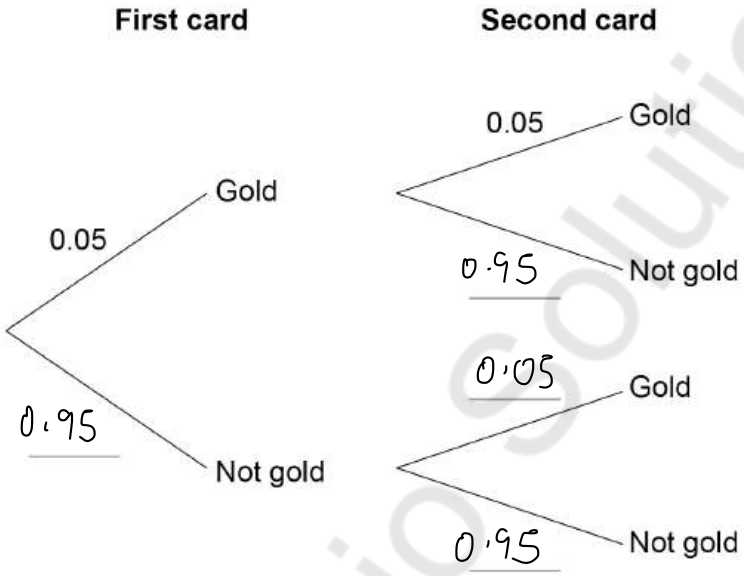
P(gold) = 0.05

Harim chooses a card at random and replaces it.

He then chooses a second card.

**12 (a)** Complete the tree diagram.

**[2 marks]**




**12 (b)** What is the probability that **at least one** of Harim's cards is gold?

**[3 marks]**

(G, G) + (G, NG) + (NG, G)

(0.05 × 0.05) + (0.05 × 0.95) + (0.95 × 0.05)

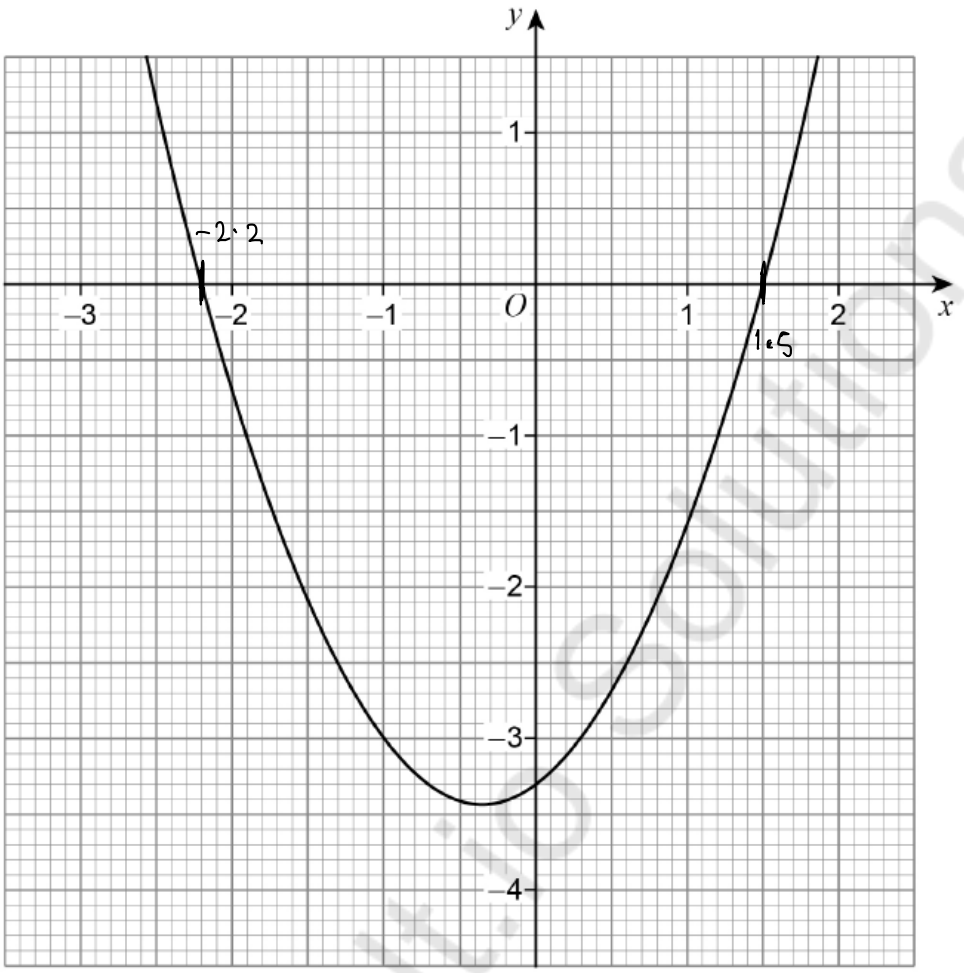
0.0975

Answer 0.0975

7

*Do not write outside the box*

**13** Here is a quadratic graph with equation $y = f(x)$




Write down the roots of the equation $f(x) = 0$

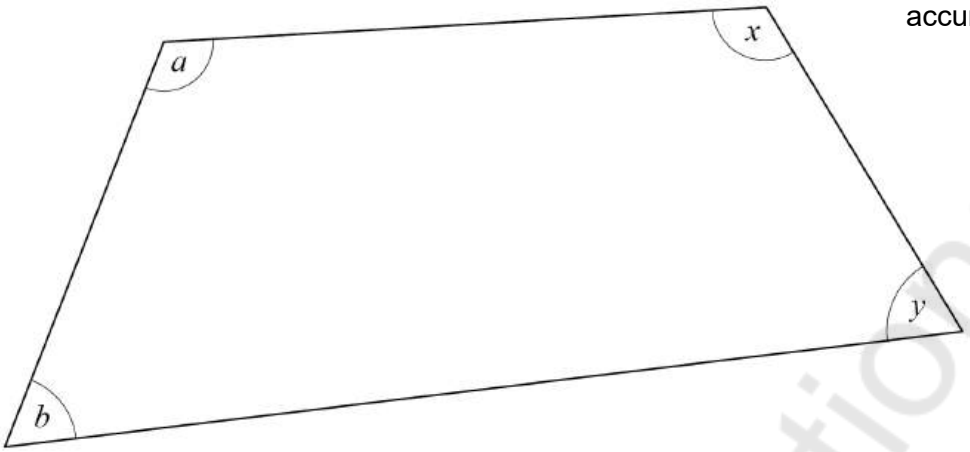
**[2 marks]**

Answer $x = 1{\cdot}5$, $x = -2{\cdot}2$

*Do not write outside the box*

**14**

Not drawn accurately




$b = 45°$ and $a : b = 7 : 3$ and $x : y = 4 : 1$

Show that $a : y = 5 : 2$

**[3 marks]**

$a + b + x + y = 360$ (angles in a quadrilateral sum to 360°)

$a \Rightarrow 7$
$b \Rightarrow 3$ ] 10

$x \Rightarrow 4$
$y \Rightarrow 1$ ] 5

÷3 ↓ 3 parts = 45° ↓ ÷3
1 part = 15°

$a = 7 \times 15 = 105°$

$a + b = 105 + 45 = 150°$

$150 + x + y = 360$

$x + y = 210$

5 parts = 210°
↓ ÷5 ↓ ÷5
1 part = 42°

$y = 42°$

$a : y = 105 : 42$
↓ ÷21 ↓ ÷21

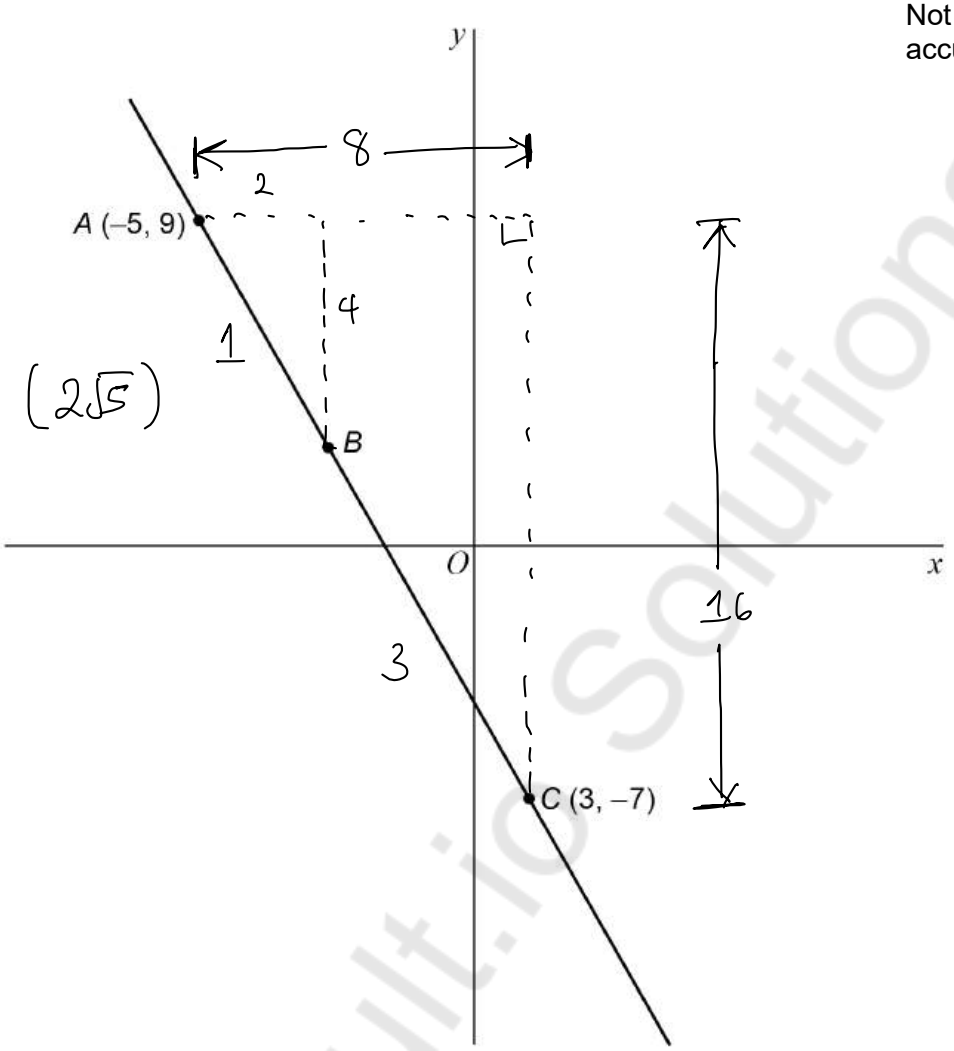
$a : y = 5 : 2$ shown.

**5**

*Do not write outside the box*

**15** A straight line passes through points $A$ (–5, 9), $B$ and $C$ (3, –7).

Not drawn accurately




**15 (a)** $AB : BC = 1 : 3$

Work out the coordinates of point $B$.

**[3 marks]**

$8^2 + 16^2 = AC^2$

$\sqrt{8^2+16^2} = AC$

$8\sqrt{5} = AC$

1 part $= \frac{8\sqrt{5}}{4} = 2\sqrt{5} = \sqrt{20}$

$x^2 + y^2 = 20$

$4 + 16 = 20$

$x = 2 \quad y = 4$

$-5 + 2 = -3 \quad 9 - 4 = 5$

Answer ( −3 , 5 )

Do not write outside the box

**15 (b)** Work out the equation of the line perpendicular to $AC$ that passes through $C$.

**[4 marks]**

$A(-5, 9)$

$C(3, -7)$

$m_{AC} = \frac{9-(-7)}{-5-3}$

$m_{AC} = \frac{9+7}{-8}$

$m_{AC} = \frac{16}{-8}$

$m_{AC} = -2$

$m_{AC_\perp} = \frac{-1}{-2}$

$m_{AC_\perp} = \frac{1}{2}$

$y = mx + c$

$-7 = \frac{1}{2}(3) + c$

$-14 = 3 + 2c$

$-3 \quad -3$

$-17 = 2c$

$-8.5 = c$

$y = \frac{1}{2}x - 8.5$

Answer $y = \frac{1}{2}x - 8.5$



**Turn over for the next question**

7

*Do not write outside the box*

**16** Jing rolls a fair six-sided dice 72 times.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Frequency** | 16 | 11 | 10 | 8 | 14 | 13 |

Is the relative frequency of rolling a 5 greater than the theoretical probability?

Tick a box.

Yes ☑ No ☐

Give a reason for your answer.

**[3 marks]**

Theoretical probability of 5 $= \frac{1}{6}$

Relative frequency $= \frac{14}{72} = \frac{7 \div 6}{36 \div 6} = \frac{7/6}{6}$

$\frac{7/6}{1} > \frac{1}{6}$

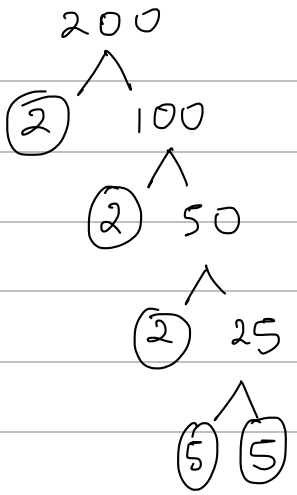
Do not write outside the box

**17 (a)** $a$ and $b$ are different prime numbers.

$a^3 \times b^2 = 200$

Work out the value of $a^4 \times b$

**[3 marks]**




$200 = 2 \times 2 \times 2 \times 5 \times 5$

$200 = 2^3 \times 5^2$

$200 = a^3 \times b^2$

$a = 2 \quad b = 5$

$a^4 \times b = 2^4 \times 5$

$a^4 \times b = 80$

Answer 80

**17 (b)** $c$ and $d$ are different prime numbers.

Circle the equation for which $c^4 \times d^2 \times e$ is a cube number.

**[1 mark]**

$e = cd$ $\quad$ (circled) $e = c^2d$ $\quad$ $e = c^2d^2$ $\quad$ $e = c^3d^3$

$c^4 \times d^2 \times cd$

$c^5 \times d^3$

Cube root ↓

$c^{5/3} d$

×

$c^4 \times d^2 \times c^2 d$

$c^6 \times d^3$

$c^2 d$ ✓

$c^4 \times d^2 \times c^2 d^2$

$c^6 \times d^4$

$c^2 d^{4/3}$

×

$c^4 \times d^2 \times c^3 d^3$

$c^7 d^5$

$c^{7/3} d^{5/3}$

×

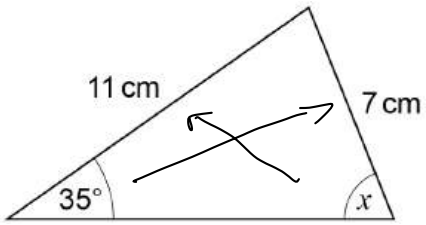
**Turn over for the next question**

7

*Do not write outside the box*

**18** Here is triangle A.

Not drawn accurately




**18 (a)** Use the sine rule to show that $x = 64°$ to the nearest degree.

**[3 marks]**

$\frac{\sin x}{11} = \frac{\sin 35}{7}$

$\sin x = \frac{11 \sin 35}{7}$

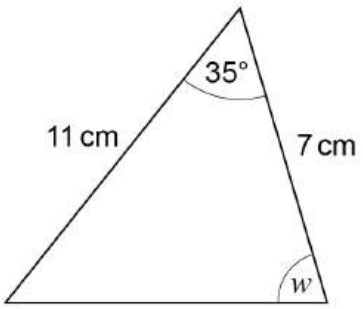
$x = \sin^{-1}\left(\frac{11 \sin 35}{7}\right) = 64.3340263$

$x \approx 64°$ to the nearest degree

Do not write outside the box

**18 (b)** Here is triangle B.

Not drawn accurately




Anna thinks that $w$ must be 64° to the nearest degree.

She says,

"This is because triangle B has two sides and one angle the same as triangle A."

**Without further calculation**, is she correct?

Tick a box.

Yes ☐ No ☑

Give a reason for your answer.

**[1 mark]**

In triangle B, 11cm side and 7cm side form the 35° angle but in triangle A, the 11cm side and 7cm side do not form the 35° angle thus SAS condition is not satisfied

**Turn over for the next question**

4

Do not write outside the box

**19** $f(x) = x - 3 \quad g(x) = 4x - 7$

**19 (a)** Work out the value of fg(6)

**[2 marks]**

$g(6) = 4(6) - 7$ $\quad f(17) = 17 - 3$

$g(6) = 24 - 7$ $\quad fg(6) = 14$

$g(6) = 17$

Answer 14

**19 (b)** Solve $(f(x))^2 = g(x)$

**[4 marks]**

$(x - 3)^2 = 4x - 7$

| | $x$ | $-3$ |
|---|---|---|
| $x$ | $x^2$ | $-3x$ |
| $-3$ | $-3x$ | $+9$ |

$x^2 - 3x - 3x + 9$

$x^2 - 6x + 9 = 4x - 7$

$-4x \quad -4x$

$x^2 - 10x + 9 = -7$

$+7 \quad +7$

$x^2 - 10x + 16 = 0$

$x^2 - 2x - 8x + 16 = 0$

$x(x - 2) - 8(x - 2) = 0$

$16$

$-1, -16$

$-2, -8$

$(x - 8)(x - 2) = 0$

$x - 8 = 0 \quad x - 2 = 0$

$+8 \quad +8 \quad +2 \quad +2$

$x = 8 \quad x = 2$

Answer $x = 8 \quad x = 2$

Do not write outside the box

**20** $P$, $Q$, and $R$ have positive values.

$P$ is directly proportional to $Q$

When $P = 8$, $Q = 2$

$R$ is inversely proportional to $Q^2$

When $R = 10$, $Q = 3$

Work out the value of $R$ when $P = 0.5$

**[5 marks]**

$P \propto Q$

$P = kQ$

$8 = k(2)$

$4 = k$

$P = 4Q$

$0.5 = 4Q$

$\frac{1}{8} = Q$

$R \propto \frac{1}{Q^2}$

$R = \frac{C}{Q^2}$

$10 = \frac{C}{3^2}$

$10 \times 3^2 = C$

$10 \times 9 = C$

$90 = C$

$R = \frac{90}{Q^2}$

$12 = \frac{90}{\left(\frac{1}{8}\right)^2} = \frac{90}{\frac{1}{64}} = 90 \div \frac{1}{64}$

$R = 90 \times \frac{64}{1}$

$R =$ 5760

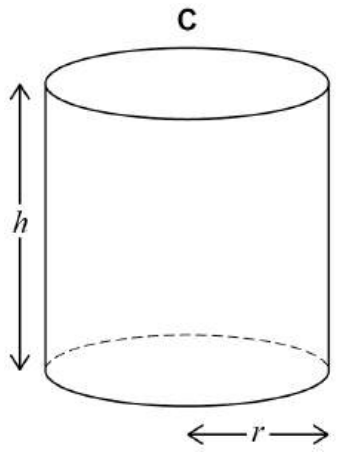
**Turn over for the next question**

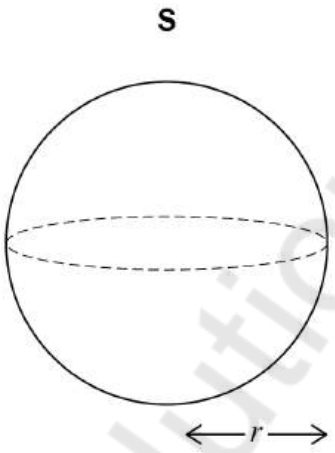
11

*Do not write outside the box*

**21** A cylinder, C, and a sphere, S, each have radius $r$

C has height $h$




Volume of Cylinder $= \pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

where $r$ is the radius

**21 (a)** volume of C = volume of S

Work out the ratio $r : h$

You **must** show your working.

**[3 marks]**

$\pi r^2 h = \frac{4}{3}\pi r^3$ $\div \pi$

$r^2 h = \frac{4}{3} r^3$ $\div r^2$

$h = \frac{4}{3} r$ $\div h$

$1 = \frac{4}{3} \times \frac{r}{h}$ $\times \frac{3}{4}$

$\frac{3}{4} = \frac{r}{h}$

$3 : 4 = r : h$

Answer 3 : 4

*Do not write outside the box*

**21 (b)** A **different cylinder** has radius $3r$ and height $2h$.

How many times bigger is the volume of this cylinder than the volume of C?

**[2 marks]**

$V_{cylinder} = \pi r^2 h$

$V = \pi \times (3r)^2 (2h)$

$V = \pi \times 9r^2 \times 2h$

$V = 18\pi r^2 h$

$V_{new} = 18 \times V_C$

Answer 18

**22** Fatima is choosing a 4-digit code.

Each digit is a whole number from 0 to 9

She decides

all her digits will be odd numbers

no digits will be repeated.

How many different codes can she make?

**[2 marks]**

0, (1), 2, (3), 4, (5), 6, (7), 8, (9)

$5 \times 4 \times 3 \times 2 = 120$

1 3 5 7 ✓

1 5 7 9 ✓

1 3 7 9 ✓

3 5 7 9 ✓

1 3 5 9 ✓

1 □ □ □

3 5 7

5 3 7

7 3 5

7 5 3

3 7 5

5 7 3

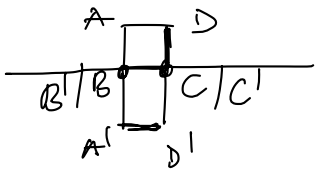
$6 \times 4 \times 5$

Answer 120

7

Do not write outside the box

**23** Quadrilateral *ABCD* is reflected in edge *BC*.




How many of the vertices are invariant?

Circle your answer. does not change

**[1 mark]**

1 (2) 0 4

**24** Write $2x^2 - 12x + 7$ in the form $d(x + e)^2 + f$ where $d$, $e$ and $f$ are integers.

**[3 marks]**

$2(x^2 - 6x) + 7$

$2[(x-3)^2 - 9] + 7$

$2(x-3)^2 - 18 + 7$

$2(x-3)^2 - 11$

$(x-3)^2 = x^2 - 6x + 9$

$-9 \quad -9$

$(x-3)^2 - 9 = x^2 - 6x$

Answer $d = 2$, $e = -3$, $f = -11$

**END OF QUESTIONS**

4

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