AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# Level 2 Certificate
# FURTHER MATHEMATICS

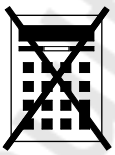
## Paper 1 Non-Calculator

Thursday 12 June 2025 Afternoon Time allowed: 1 hour 45 minutes

**Materials**

For this paper you must have:
- mathematical instruments
- the Formulae Sheet (enclosed).

You must **not** use a calculator.



**Instructions**
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.

| For Examiner's Use | |
|---|---|
| Pages | Mark |
| 2–3 | |
| 4–5 | |
| 6–7 | |
| 8–9 | |
| 10–11 | |
| 12–13 | |
| 14–15 | |
| 16–17 | |
| 18–19 | |
| 20–21 | |
| 22 | |
| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1 (a)** The function f is given by $f(x) = 5x + 3$

The range is $f(x) < 23$

Work out the domain of the function, giving your answer as an inequality.

**[1 mark]**

$23 = 5x + 3$

$20 = 5x$

$4 = x$

Answer $x < 4$

**1 (b)** The function g is given by $g(x) = x^3 - 4$

The domain is $-3 < x < 2$

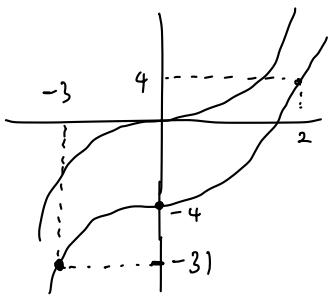
Work out the range of the function, giving your answer as an inequality.

**[2 marks]**

$g(-3) = (-3)^3 - 4 = -27 - 4 = -31$

$g(2) = (2)^3 - 4 = 8 - 4 = 4$

Answer $-31 < g(x) < 4$



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**1 (c)** The function h is given by $h(x) = \sqrt{2x+1}$

Work out $h^{-1}(x)$

**[2 marks]**

$x = \sqrt{2y+1}$

$x^2 = 2y+1$

$x^2 - 1 = 2y$

$\frac{x^2-1}{2} = y$

$h^{-1}(x) = \frac{x^2-1}{2}$

**2** $\begin{pmatrix} 5 & 0 \\ 1 & -6 \end{pmatrix}\begin{pmatrix} 12 \\ 7 \end{pmatrix} = 15\begin{pmatrix} a \\ b \end{pmatrix}$

Work out the values of $a$ and $b$.

**[2 marks]**

42
-12
30

$\begin{pmatrix} 5(12) + 0(7) \\ 1(12) + -6(7) \end{pmatrix} = \begin{pmatrix} 15a \\ 15b \end{pmatrix}$

$\begin{pmatrix} 60 \\ 12-42 \end{pmatrix} = \begin{pmatrix} 15a \\ 15b \end{pmatrix}$

$\begin{pmatrix} 60 \\ -30 \end{pmatrix} = \begin{pmatrix} 15a \\ 15b \end{pmatrix}$

$a = \frac{60}{15}$ $b = \frac{-30}{15}$

$a = 4$ $b = -2$

$a =$ 4 $b =$ -2

7

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**3** A linear sequence starts 2 $\overset{+3}{\rightarrow}$ 5 $\overset{+3}{\rightarrow}$ 8 $\overset{+3}{\rightarrow}$ 11

Work out the **position** of the term that is the first to have a value greater than 500

**[3 marks]**

$3n-1$ 2 5 8 11

$-1$ ↑

$3n$ 3 6 9 12

$3n - 1 > 500$

$3n > 501$

$n > 167$

At $n = 168$ value is greater than 500

Answer 168th

**4** *AC* is a straight line.

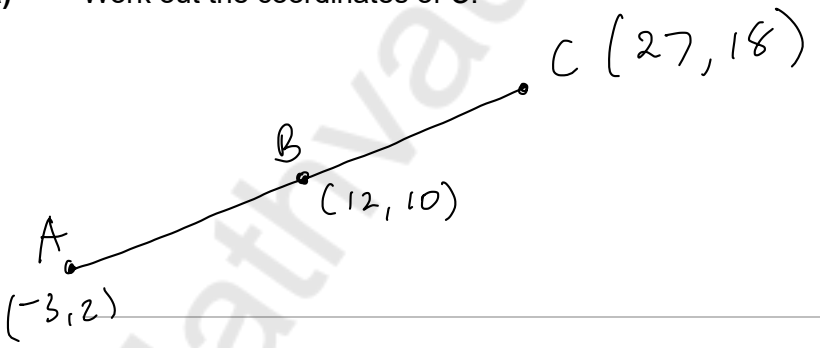
*A* is the point (–3, 2)

*B* is the midpoint of the line *AC*.

*B* is the point (12, 10)

**4 (a)** Work out the coordinates of *C*.

**[2 marks]**




$\Delta x = 12 - (-3) = 15$ $\Delta y = 10 - 2 = 8$

$C_x = 12 + 15 = 27$ $C_y = 10 + 8 = 18$

Answer ( 27 , 18 )

Do not write outside the box

**4 (b)** Work out the distance *AB*.

**[2 marks]**

$AB = \sqrt{[12-(-3)]^2 + [10-2]^2}$

$AB = \sqrt{225 + 64}$

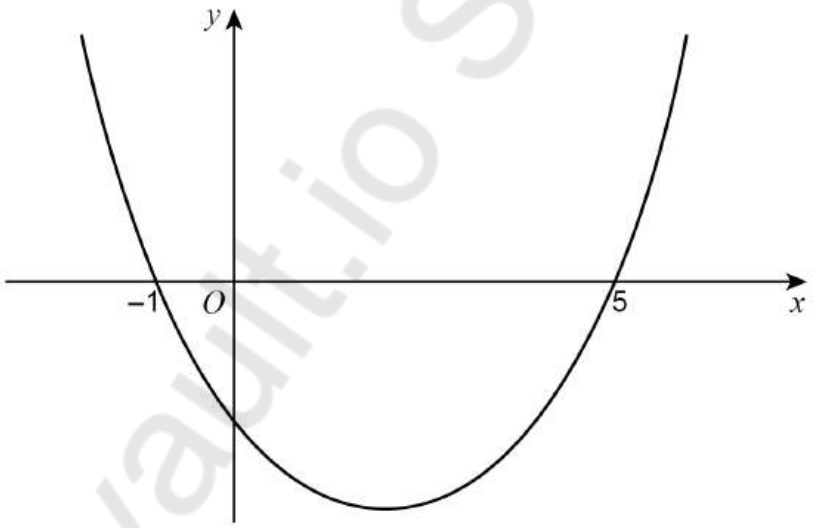
$AB = \sqrt{289}$

225
64
289

Answer $\sqrt{289}$ units

**5** Here is a sketch of $y = x^2 + bx + c$

The graph passes through (–1, 0) and (5, 0)




Work out the values of $b$ and $c$.

**[3 marks]**

$y = (x+1)(x-5)$

$y = x^2 + x - 5x - 5$

$y = x^2 - 4x - 5$

$b =$ $-4$ $c =$ $-5$

10

*Do not write outside the box*

**6** Simplify fully $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{48}}$

**[3 marks]**

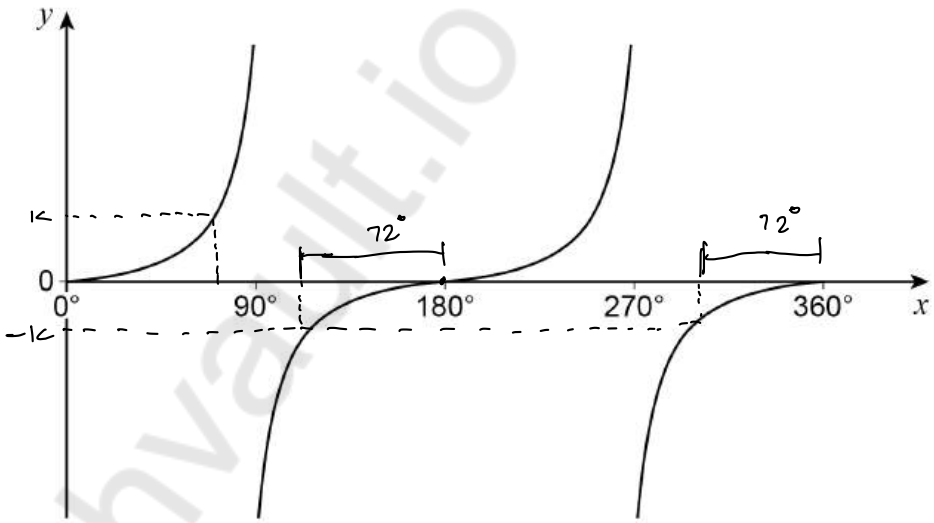
$\dfrac{\sqrt{25\times3}-\sqrt{9\times3}}{\sqrt{16\times3}}$

$\dfrac{5\sqrt{3}-3\sqrt{3}}{4\sqrt{3}}$

$\dfrac{2\sqrt{3}}{4\sqrt{3}}$

Answer $1/2$

**7** Here is a sketch of $y = \tan x$ for $0° \leqslant x \leqslant 360°$




You are given that $\tan 72° = k$

Work out the **two** values of $x$ between 0° and 360° for which $\tan x = -k$

**[2 marks]**

$180 - 72$ $360 - 72$

$108$ $288$

Answer $108$ and $288$

*Do not write outside the box*

**8** Work out $\left(\dfrac{x^{\frac{9}{2}}}{x^{\frac{1}{2}}}\right)^3$ as a power of $x$.

Circle your answer.

**[1 mark]**

$x^7$ $x^{12}$ $x^{15}$ $x^{27}$

$\left(x^{\frac{a}{2}-\frac{1}{2}}\right)^3 = \left(x^{\frac{8}{2}}\right)^3 = \left(x^4\right)^3 = x^{12}$

**9** 3-digit integers are made using three **different** digits.

The digits are chosen from 1, 2, 3, 4, 5, 6, 7, 8, 9

How many **more** odd integers than even integers can be made?

**[3 marks]**

ODD □ □ □ EVEN □ □ □

7 X 8 X 5 7 x 8 x 4

ODD ⇒ 280 Even ⇒ 224

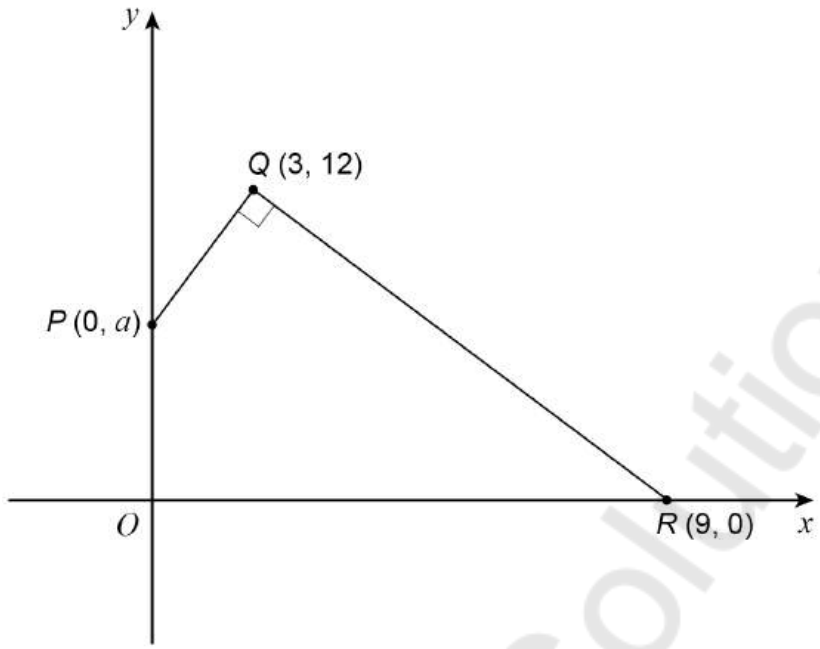
280
−224
056

Answer 56

**Turn over for the next question**

9

**10** $PQ$ is perpendicular to $QR$.

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Work out the value of $a$.

**[4 marks]**

$M_{QR} = \frac{0-12}{9-3} = \frac{-12}{6} = -2$

$M_{QP} = \frac{1}{2}$

$M_{QP} = \frac{12-a}{3-0} = \frac{1}{2}$

$\frac{12-a}{3} = \frac{1}{2}$

$12-a = 1.5$

$12-1.5 = a$

$a = 10.5$

$a =$ 10.5

*Do not write outside the box*

**11** Solve $x^3 + 10x^2 + 8x = 0$

Where appropriate give your answers in surd form.

Simplify your answers where possible.

**[4 marks]**

$x(x^2 + 10x + 8) = 0$

$x = 0 \qquad x^2 + 10x + 8 = 0$

$(x + 5)^2 - 25 + 8 = 0$

$(x+5)^2 - 17 = 0$

$(x+5)^2 = 17$

$x + 5 = \sqrt{17}$

$x = -5 + \sqrt{17} \qquad x = -5 - \sqrt{17}$

Answer $x = 0 \qquad x = -5+\sqrt{17} \qquad x = -5-\sqrt{17}$

**Turn over for the next question**

8

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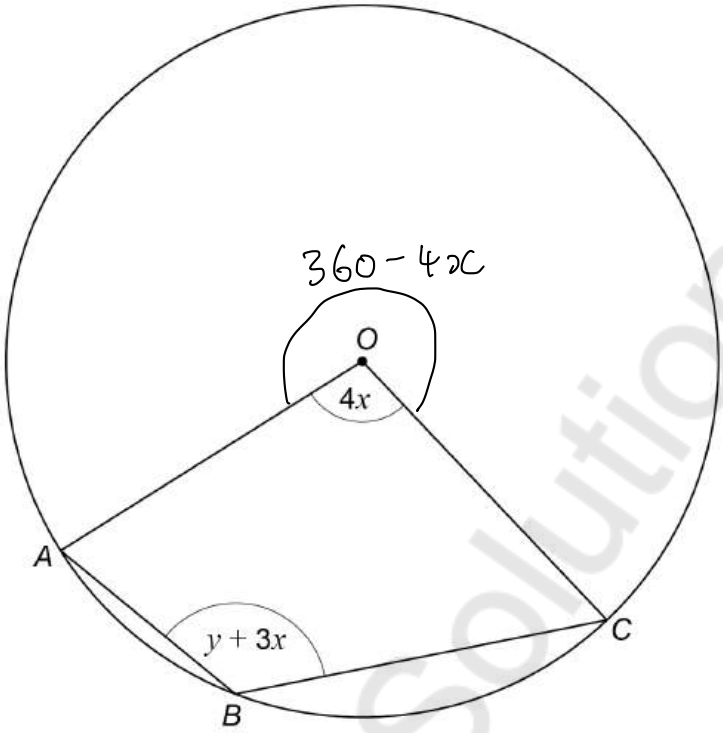
**12** *A*, *B* and *C* are points on a circle, centre *O*.




Prove that $y = 180 - 5x$

**[4 marks]**

$(y + 3x) \times 2 = 360 - 4x$

$2y + 6x = 360 - 4x$

$2y = 360 - 4x - 6x$

$2y = 360 - 10x$

$y = 180 - 5x$ shown.

Angle at centre is twice angle at circumference

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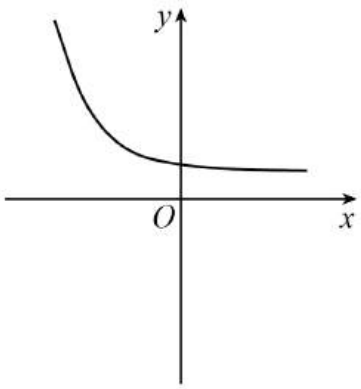
**13** Here are four sketch graphs.

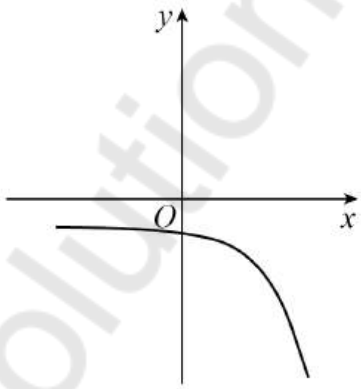
Circle the letter of the sketch graph that represents $y = 5^x$

**[1 mark]**

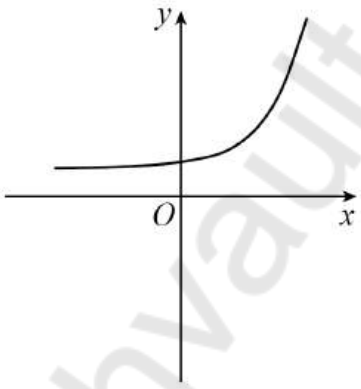
**A**



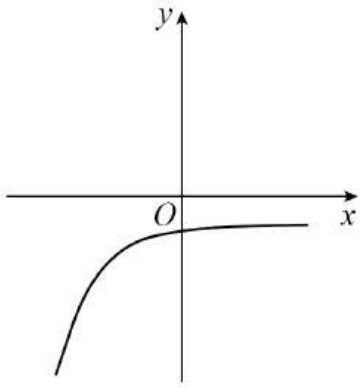

**B**




**C**




**D**






**Turn over for the next question**

5

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**14** Simplify fully $\dfrac{2x^2-4x-6}{x^2-3x} \div \dfrac{4x+4}{7x^2}$

**[4 marks]**

$\dfrac{2(x^2-2x-3)}{x(x-3)} \times \dfrac{7x^2}{4(x+1)}$

$\dfrac{-3}{-3,+1}$

$\dfrac{2(x-3)(x+1)}{x(x-3)} \times \dfrac{7x^2}{4(x+1)}$

$\dfrac{2(7x^2)}{x \times 4}$

$\dfrac{7x^2}{2x}$

$\dfrac{7x}{2}$

Answer $\dfrac{7x}{2}$

Do not write outside the box

**15** $(x-3)(x+4)(x+p) \equiv x^3 + qx^2 + rx + 24$

Work out the values of $p$, $q$ and $r$.

**[3 marks]**

$(x^2 - 3x + 4x - 12)(x + p)$

$(x^2 + x - 12)(x + p)$

$x^3 + px^2 + x^2 + px - 12x - 12p$

$x^3 + (p+1)x^2 + (p-12)x - 12p \equiv x^3 + qx^2 + rx + 24$

$-12p = 24$ $\quad p+1 = q$ $\quad p - 12 = r$

$p = -2$ $\quad -2+1 = q$ $\quad -2 - 12 = r$

$-1 = q$ $\quad -14 = r$

$p =$ $-2$ $\quad q =$ $-1$ $\quad r =$ $-14$

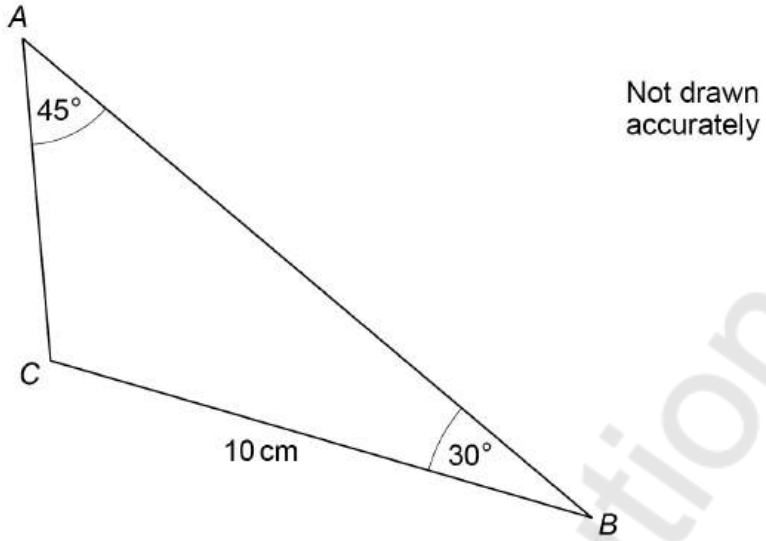


**Turn over for the next question**

7

Do not write outside the box

**16**




Work out the length *AC*.

Give your answer in the form $a\sqrt{2}$ where $a$ is an integer.

**[3 marks]**

$\frac{\sin 45}{10} = \frac{\sin 30}{AC}$

$\frac{\frac{\sqrt{2}}{2}}{10} = \frac{\frac{1}{2}}{AC}$

$AC \times \frac{\sqrt{2}}{2} = 10 \times \frac{1}{2}$

$AC \times \frac{\sqrt{2}}{2} = 5$

$AC = 5 \times \frac{2}{\sqrt{2}}$

$AC = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$AC = \frac{10\sqrt{2}}{2}$

$AC = 5\sqrt{2}$

Answer $5\sqrt{2}$ cm

Do not write outside the box

**17** The expansions of $(4a + x)^4$ and $(5 - 2x)^3$ have the same coefficient of $x^3$

Work out the value of $a$.

**[3 marks]**

$^4C_3 (4a)^1 (x)^3$

$4 \times 4a \times x^3$

$16ax^3$

$^3C_3 (5)^0 (-2x)^3$

$1 \times 1 \times -8x^3$

$-8x^3$

$16a = -8$

$a = -\frac{1}{2}$

$\frac{4!}{(4-3)!3!}$

$\frac{4 \times 3!}{1! \times 3!} = 4$

$\frac{3!}{(3-3)!3!} = 1$

$a = -\frac{1}{2}$

**Turn over for the next question**

6

*Do not write outside the box*

**18** A 60 cm piece of wire is bent to form three sides of a rectangle.
It is then attached to a fixed rod as shown in the diagram.

Not drawn accurately


**18 (a)** The area of the rectangle is $A$ cm$^2$

Show that $A = x(60 - 2x)$

**[2 marks]**

$A = xy$ $\quad x + y + x = 60$

$A = x(60 - 2x)$ $\quad 2x + y = 60$

$y = 60 - 2x$

Do not write outside the box

**18 (b)** Work out the maximum value of $A$ as $x$ varies.

**[4 marks]**

$A = x(60 - 2x)$

$A = 60x - 2x^2$

$\frac{dA}{dx} = 60 - 4x$

$\frac{dA}{dx} = 0$

$0 = 60 - 4x$

$4x = 60$

$x = 15$

$A = 15(60 - 2(15))$

$A = 15 \times 30 = 450 \text{ cm}^2$

Answer 450 $\text{cm}^2$

**Turn over for the next question**

6

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**19 (a)** $(x - 1)$ is a factor of $3x^3 + 2x^2 - 3x - 2$

Factorise fully $3x^3 + 2x^2 - 3x - 2$

**[3 marks]**

$3x^2 + 5x + 2$

$x - 1 \;\big|\; 3x^3 + 2x^2 - 3x - 2$

$-(3x^3 - 3x^2)$

$5x^2 - 3x$

$-(5x^2 - 5x)$

$2x - 2$

$2x - 2$

$0 \quad 0$

$(x-1)(3x^2+5x+2)$

$3 \times 2 = 6$

$+3, +2$

$(x-1)(3x^2+3x+2x+2)$

$(x-1)(3x(x+1)+2(x+1))$

$(x-1)(3x+2)(x+1)$

Answer $(x-1)(3x+2)(x+1)$

**19 (b)** $f(x) = (x - 2)(5x + 4)(3x + 2)$

$c$ is a positive integer.

The value of $f(c)$ can be written as the product of **two** prime factors.

Deduce the value of $f(c)$

**[2 marks]**

At $x = 3$

$1 \times 19 \times 11 \rightarrow$ product of two prime factors

$\therefore c = 3$

$f(3) = 209$

Answer 209

Do not write outside the box

**20** $y = \frac{x^3}{3} - 5x^2 + 28x + 11$

Show that $y$ is an increasing function for all values of $x$.

**[4 marks]**

$\frac{dy}{dx} = x^2 - 10x + 25$

$\frac{dy}{dx} = (x-5)^2 - 25 + 28$

$\frac{dy}{dx} = (x-5)^2 + 3$

Gradient function is always positive, $(x-5)^2 \geq 0$

$(x-5)^2 + 3 \geq 3$

$y$ is thus an increasing function for all values of $x$

**Turn over for the next question**

9

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**21** Show that $\dfrac{20\sin^2 x}{3+\cos^2 x-3\sin^2 x}$ can be simplified to $k\tan^2 x$ where $k$ is an integer.

**[2 marks]**

$\sin^2\theta + \cos^2\theta \equiv 1$

$3\sin^2\theta + 3\cos^2\theta \equiv 3$

$$\frac{20\sin^2\theta}{3\sin^2\theta + 3\cos^2\theta + \cos^2\theta - 3\sin^2\theta}$$

$$\frac{20\sin^2\theta}{4\cos^2\theta}$$

$$5\tan^2\theta$$

Do not write outside the box

**22** A circle has centre (2, 3) and radius 5

A straight line passes through (3, 7) and has a gradient of 2

Work out the coordinates of the **two** points where the circle and the straight line intersect.

**[6 marks]**

$(x-2)^2 + (y-3)^2 = 5^2$

$x^2 - 4x + 4 + [2x+1-3]^2 = 25$

$x^2 - 4x + 4 + (2x-2)^2 = 25$

$x^2 - 4x + 4 + [2(x-1)]^2 = 25$

$x^2 - 4x + 4 + 4[x-1]^2 = 25$

$x^2 - 4x + 4 + 4[x^2 - 2x + 1] = 25$

$x^2 - 4x + 4 + 4x^2 - 8x + 4 = 25$

$5x^2 - 12x + 8 - 25 = 0$

$5x^2 - 12x - 17 = 0$

$5x^2 + 5x - 17x - 17 = 0$

$5x(x+1) - 17(x+1) = 0$

$(5x-17)(x+1) = 0$

$5x - 17 = 0 \qquad x + 1 = 0$

$x = \frac{17}{5} \qquad x = -1$

$y - 7 = 2(x-3)$

$y = 2x - 6 + 7$

$y = 2x + 1$

$y = 2\left(\frac{17}{5}\right) + 1 \qquad y = 2(-1) + 1$

$y = \frac{34}{5} + 1 \qquad y = -2 + 1$

$y = \frac{39}{5} \qquad y = -1$

Answer ( $\frac{17}{5}$ , $\frac{39}{5}$ ) and ( $-1$ , $-1$ )

8

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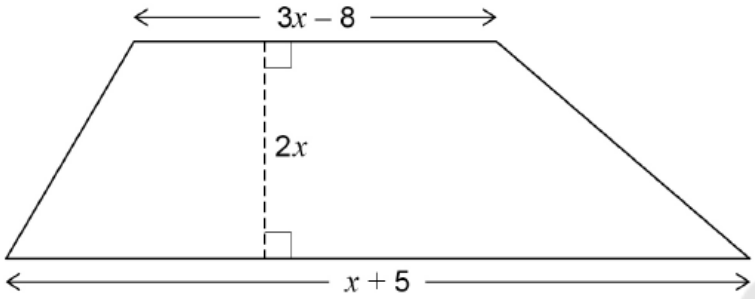
**23** The diagram shows a trapezium.

All lengths are in centimetres.




Not drawn accurately

The area of the trapezium is less than 22 cm$^2$

Work out the greatest possible range of values of $x$.

Give your answer in the form $m < x < n$ where $m$ and $n$ are both positive numbers.

**[5 marks]**

$A_T = \frac{(a+b) \times h}{2}$

$22 > \frac{(3x - 8 + x + 5)(2x)}{2}$

$44 > (4x - 3)(2x)$

$44 > 8x^2 - 6x$

$0 > 8x^2 - 6x - 44$

$0 > 4x^2 - 3x - 22$

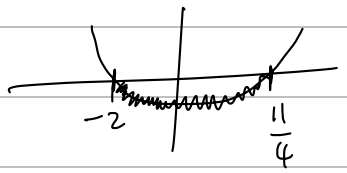
$0 > 4x^2 + 8x - 11x - 22$

$0 > 4x(x + 2) - 11(x + 2)$

$0 > (4x - 11)(x + 2)$

$-88$

$-11, +8$




Critical values: $4x - 11 = 0$ $x + 2 = 0$

$x = \frac{11}{4}$ $x = -2$

Answer $-2 < x < \frac{11}{4}$

**END OF QUESTIONS**

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