AQA

Please write clearly in block capitals.

Centre number | | | | | |　　Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

Level 2 Certificate
FURTHER MATHEMATICS
Paper 2 Calculator

Wednesday 18 June 2025 Afternoon Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked. •
In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1

Sequence	nth term
A	$\dfrac{6n}{7n+25}$
B	$\dfrac{20}{n+4}$

The 5th term in sequence A is also a term in sequence B.

Work out the **position** of the term in sequence B.

[3 marks]

At $n=5$, A: $\dfrac{6(5)}{7(5)+25} = \dfrac{1}{2}$

$\dfrac{1}{2} = \dfrac{20}{n+4}$

$n+4 = 40$

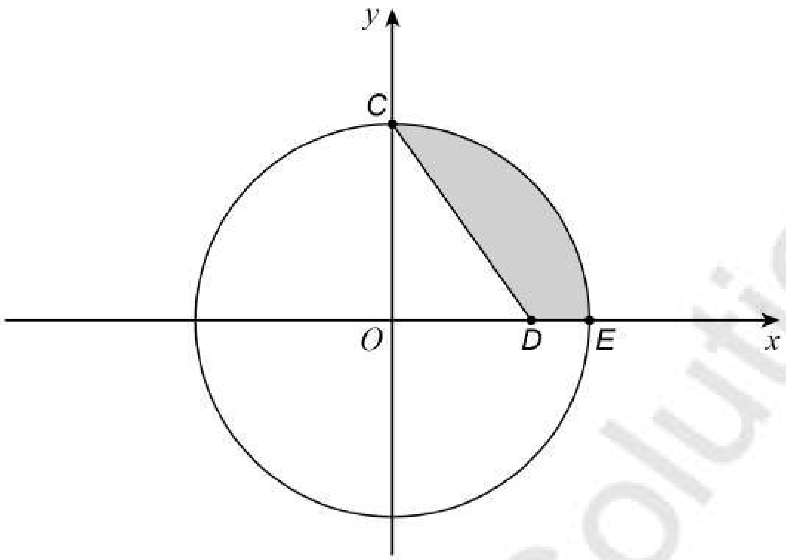
$n = 36$

Answer 36th term in sequence B

2 C and E are points on the circle with equation $x^2 + y^2 = 144$

CD is a straight line.

$OD : DE = 5 : 1$



Not drawn accurately

Work out the shaded area.

Give your answer as a decimal.

[4 marks]

$x^2 + y^2 = r^2$

$r^2 = 144$, $r = 12$

$OE = 12$

$\underbrace{5 : 1}_{6}$

$12 : 6$

$2 : 1$

$10 : 5$

$OD = 10$

$OC = 12$

Area of $\Delta OCD = \frac{10 \times 12}{2} = 60$

Area of quadrant $OCE = \frac{\pi \times 12^2}{4} = 36\pi$

Area of Shaded $= 36\pi - 60 \approx 53.1$

Answer 53.1 units^2

7

Turn over ▶

3 The equation of a straight line is $2ax + y = 25$ where a is a constant.

The line passes through the point (3, 1)

Work out the value of the gradient of the line.

[3 marks]

$x = 3$
$y = 1$

$2a(3) + 1 = 25$
$6a + 1 = 25$
$6a = 24$
$a = 4$

$2(4)x + y = 25$
$8x + y = 25$
$y = -8x + 25$

Answer -8

4 $-5 \leqslant n \leqslant 5$

Work out the range of possible values of n^2

[2 marks]

At $n=0$, $n^2=0$
At $n=5$ or $n=-5$, $n^2 = 25$

Answer $25 \geqslant n^2 \geqslant 0$

Do not write outside the box

5 The equation of a curve is $y = x^5 + px^4$ where p is a constant.

When $x = -1$ the gradient of the curve is -3

Work out the value of p.

[3 marks]

$\frac{dy}{dx} = 5x^4 + 4px^3$

$-3 = 5(-1)^4 + 4p(-1)^3$

$-3 = 5 - 4p$

$4p = 8$

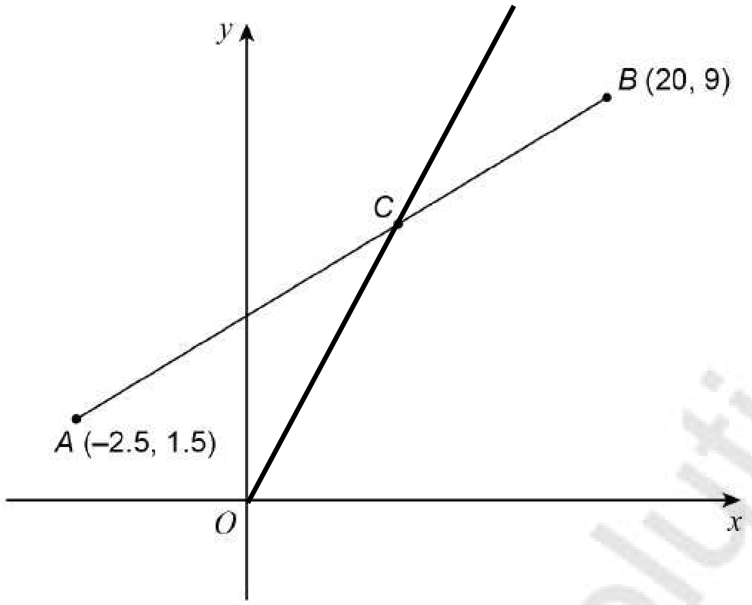
$p = 2$

$p =$ 2

Turn over for the next question

8

6 C is a point on the straight line AB.



$AC : CB = 3 : 2$

Work out the equation of the straight line passing through O and C.

[3 marks]

$C\left[\left(\frac{20-(-2.5)}{5}\times 3\right)+-2.5 \;,\; \left(\frac{9-1.5}{5}\right)\times 3+1.5\right]$

$C(11, 6)$

$m_{OC}=\frac{6-0}{11-0}=\frac{6}{11}$

$y-0=\frac{6}{11}(x-0)$

$y=\frac{6}{11}x$

Answer $y=\frac{6}{11}x$

Do not write outside the box

7 (a) $2^{5a+1} = 2^a$

Work out the value of a.

[2 marks]

$\ln 2^{5a+1} = \ln 2^a$

$5a+1(\ln 2) = a \ln 2$

$5a+1 = a$

$4a = -1$, $a = -0.25$

$a =$ -0.25

7 (b) $3^{-7} \div 3^c = \left(3^{-3}\right)^{2c}$

Work out the value of c.

[3 marks]

$3^{-7-c} = 3^{-3 \times 2c}$

$-7-c = -6c$

$5c = 7$

$c = 7/5$

$c =$ 1.4

8

8 k is a positive integer.

Prove that $4k(k+8) - (4k+9)(k-1)$ is a multiple of 9

[3 marks]

$4k^2 + 32k - [4k^2 + 5k - 9]$

$4k^2 + 32k - 4k^2 - 5k + 9$

$27k + 9$

$9(3k + 1)$

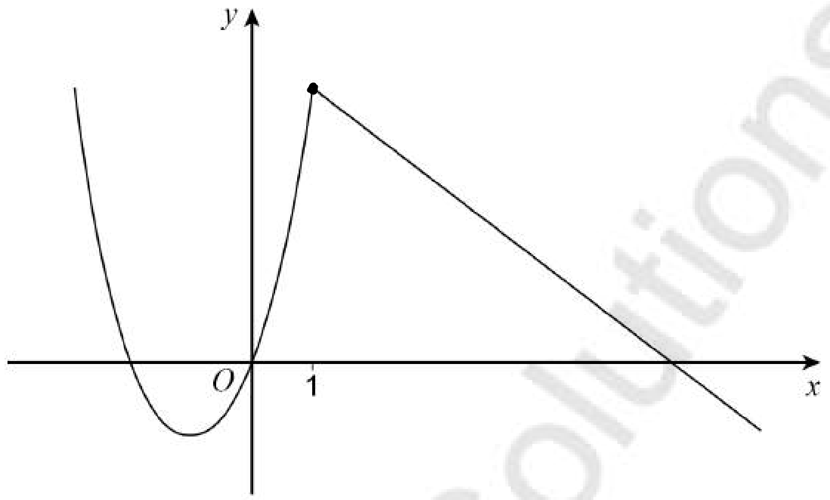
hence always a multiple of 9 for values of k that are a positive integer.

Do not write outside the box

9 $\quad \mathrm{f}(x) = x^2 + px \qquad x \leqslant 1$

$\qquad = 2p - \frac{1}{2}x \qquad x > 1$

A sketch of $y = \mathrm{f}(x)$ is shown.



Work out the value of p.

[2 marks]

$x^2 + px = 2p - \frac{1}{2}x$

$(1)^2 + p(1) = 2p - \frac{1}{2}(1)$

$1 + p = 2p - 0.5$

$1.5 = p$

$p =$ 1.5

Turn over for the next question

5

10 The first four terms of a quadratic sequence are

–1 11 31 59

Work out an expression for the nth term.

Give your answer in the form $an^2 + b$ where a and b are constants.

[2 marks]

$-1 \quad 11 \quad 31 \quad 59$

$+12 \quad +20 \quad +28$

$+8 \quad +8$

$2a = 8$

$a = 4$

$an^2 + bn + c$

$4n^2 + bn + c$

$4n^2 - 5$

At $n = 1$

$4 + b + c = -1$

$b + c = -5$

$b + c = 2b + c$

$b = 0$

$c = -5$

At $n = 2$

$16 + 2b + c = 11$

$2b + c = -5$

Answer $4n^2 - 5$

Do not write outside the box

11 (a) Simplify fully $\frac{3}{4w} + \frac{1}{6w}$

[2 marks]

$\frac{3}{4\omega}{}^{\times 3}_{\times 3} + \frac{1}{6\omega}{}^{\times 2}_{\times 2}$

$\frac{9}{12\omega} + \frac{2}{12\omega} = \frac{11}{12\omega}$

Answer $\frac{11}{12w}$

11 (b) Factorise fully $(x - 1)^9 + (x - 1)^8(x + 4)$

[2 marks]

$(x-1)^8 [x-1 + x+4]$

$(x-1)^8 (2x+3)$

Answer $(x-1)^8(2x+3)$

11 (c) Factorise fully $8y^2 - 50$

[2 marks]

$2(4y^2 - 25)$

$2(2y+5)(2y-5)$

Answer $2(2y+5)(2y-5)$

8

Turn over ►

12 (a) Circle the matrix that represents an enlargement, scale factor 3, centre O.

[1 mark]

$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ $\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$ $\begin{pmatrix} 3 & 0 \\ 3 & 0 \end{pmatrix}$

12 (b) *OABC* is the unit square.

O (0, 0) *A* (1, 0) *B* (1, 1) *C* (0, 1)

K is the transformation represented by $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

L is the transformation represented by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

OABC is mapped to *OA′B′C′* under a **combined** transformation of **K** followed by **L**.

Work out the coordinates of *A′*, *B′* and *C′*.

[3 marks]

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad A'(0,-1)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad B'(-1,-1)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad C'(-1,0)$$

A′ (0 , −1) *B′* (−1 , −1) *C′* (−1 , 0)

Do not write outside the box

13 Rearrange $x = \dfrac{3k^2}{5k^2 - 2}$ to make k the subject.

[4 marks]

$5xk^2 - 2x = 3k^2$

$5xk^2 - 3k^2 = 2x$

$k^2(5x - 3) = 2x$

$k^2 = \dfrac{2x}{5x-3}$

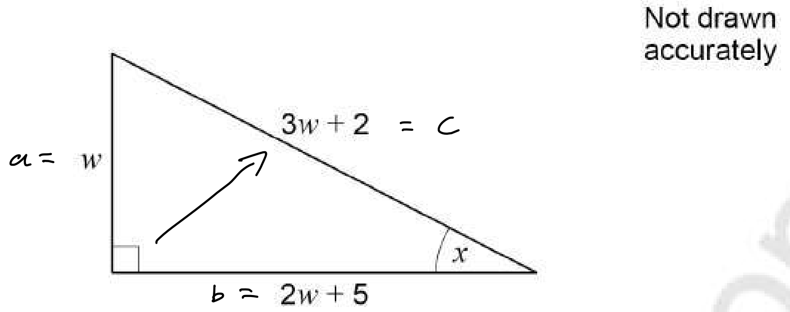
$k = \left(\dfrac{2x}{5x-3}\right)^{1/2}$

Answer $\left(\dfrac{2x}{5x-3}\right)^{1/2}$

Turn over for the next question

8

14 All lengths shown are in centimetres.



14 (a) Use Pythagoras' theorem to show that $4w^2 - 8w - 21 = 0$

[2 marks]

$a^2 + b^2 = c^2$

$w^2 + (2w+5)^2 = (3w+2)^2$

$w^2 + 4w^2 + 20w + 25 = 9w^2 + 12w + 4$

$0 = 9w^2 - w^2 - 4w^2 + 12w - 20w + 4 - 25 = 0$

$0 = 4w^2 - 8w - 21 = 0$ shown.

Do not write outside the box

14 (b) Work out the size of angle x.

-84

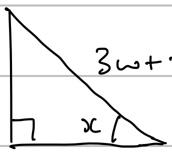
[3 marks]

$0 = 4w^2 - 8w - 21$

$\dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-21)}}{2(4)} = w$

$w = \dfrac{7}{2}$ $\qquad w = -\dfrac{3}{2}$ reject

$0 = 3.5 = w$



$2(3.5)+5$

$A = 12$

SOH CAH TOA

$\tan x = \dfrac{3.5}{12}$

$x = \tan^{-1}\left(\dfrac{3.5}{12}\right)$

$x \simeq 16.3^{\circ}$ (3 s.f.)

$x =$ 16.3 °

Turn over for the next question

5

15 For a curve $y = f(x)$

$$\frac{dy}{dx} = 1 - \frac{9}{x^2}$$

15 (a) Show that the curve has a turning point where $x = 3$

[1 mark]

$1 - \frac{9}{3^2}$

$1 - \frac{9}{9}$

$1 - 1$

$\frac{dy}{dx} = 0$ at $x = 3$ $\therefore$ Turning point occurs at $x = 3$

15 (b) Use $\frac{d^2y}{dx^2}$ to work out the nature of the turning point.

You **must** show your working.

[2 marks]

$\frac{dy}{dx} = 1 - 9x^{-2}$

$\frac{d^2y}{dx^2} = 18x^{-3}$

At $x = 3$, $\frac{d^2y}{dx^2} = \frac{18}{3^3} = \frac{2}{3}$

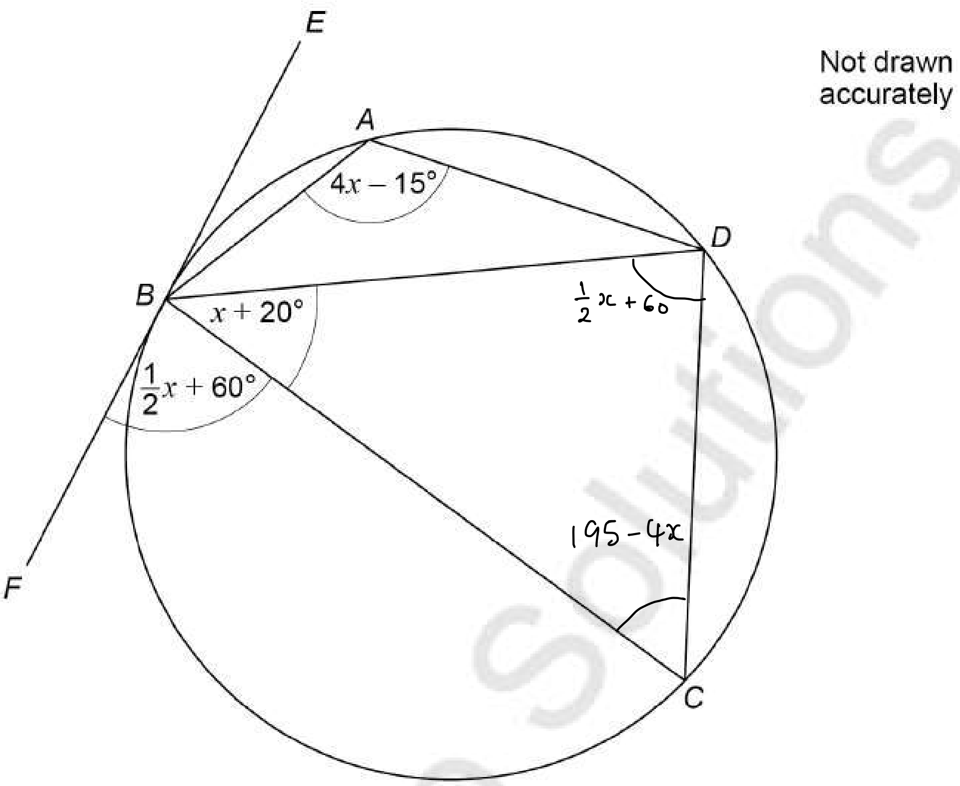
$\frac{2}{3} > 0$ Thus $x = 3$ is a local minimum.

Answer local minimum at $x = 3$

Do not write outside the box

16 A, B, C and D are points on a circle.

EBF is a tangent.



Work out the value of x.

[4 marks]

$F\hat{B}C = B\hat{D}C$ (Alternate Segment Theorem)

$D\hat{C}B = 180 - (4x - 15) = 180 - 4x + 15 = 195 - 4x$ (opposite angles in cyclic quadrilateral sum to 180)

$x + 20 + \frac{1}{2}x + 60 + 195 - 4x = 180$ (Sum of angles in a $\Delta = 180°$)

$-1.5x + 275 = 180$

$95 = 1.5x$

$x = 63.\dot{3}$

$x =$ $63.\dot{3}$ °

7

Turn over ►

Do not write outside the box

17 Solve

$$2a + 4b + c = 12 \quad \text{—— ①}$$
$$3a - b + 2c = -2 \quad \text{—— ②}$$
$$a - 5b - 3c = 6 \quad \text{—— ③}$$

Do **not** use trial and improvement.

You **must** show your working.

[5 marks]

$c = 12 - 2a - 4b$

$3a - b + 2(12 - 2a - 4b) = -2$

$3a - b + 24 - 4a - 8b = -2$

$-a - 9b = -26$

$a + 9b = 26$

$7a + 63b = 182$

$a - 5b - 3(12 - 2a - 4b) = 6$

$a - 5b - 36 + 6a + 12b = 6$

$7a + 7b = 42$

$7a + 63b = 182$

$56b = 140$

$b = 2.5$

$a = 26 - 9b$

$a = 26 - 9(2.5)$

$a = 3.5$

$c = 12 - 2(3.5) - 4(2.5)$

$c = -5$

$a =$ 3.5 $\quad b =$ 2.5 $\quad c =$ −5

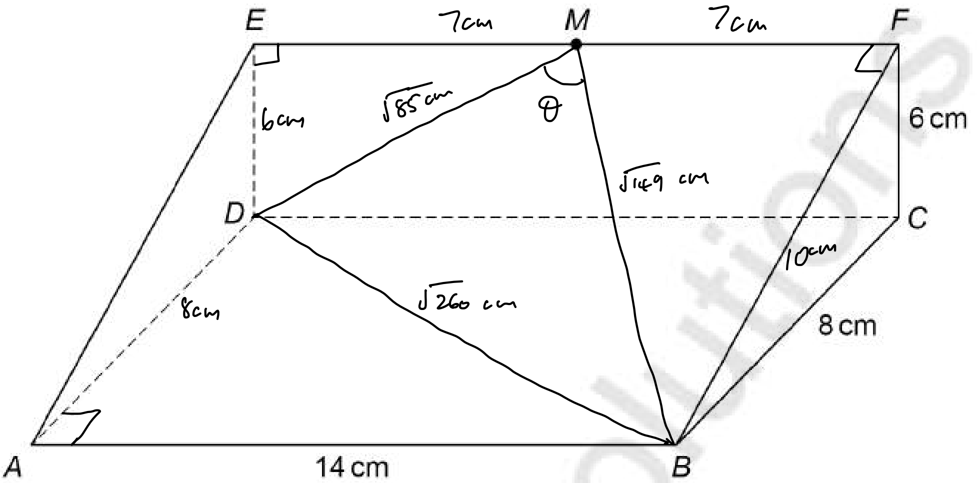
Do not write outside the box

18 *ABCDEF* is a triangular prism.

ABCD is a rectangle and angle *BCF* is 90°

AB = 14 cm *BC* = 8 cm *CF* = 6 cm

M is the midpoint of *EF*.



Work out the size of angle *BMD*.

[5 marks]

$BF = \sqrt{8^2+6^2} = 10\text{cm}$

$BM = \sqrt{7^2+10^2} = \sqrt{149}\text{ cm}$

$DM = \sqrt{7^2+6^2} = \sqrt{85}\text{ cm}$

$BD = \sqrt{8^2+14^2} = \sqrt{260}\text{ cm}$

$260 = 85 + 149 - 2\sqrt{12665}\cos B\hat{M}D$

$\cos B\hat{M}D = \dfrac{85+149-260}{2\sqrt{12665}}$

$B\hat{M}D = \cos^{-1}\left[\dfrac{-26}{2\sqrt{12665}}\right]$

$B\hat{M}D \simeq 96.6^\circ$ (3 s.f.)

Answer 96.6 °

10

Turn over ►

19 $f(x) = 4x^3 - 7$ $g(x) = 6x + 8$

Work out the value of x for which $f(2x) = gf(x)$

[4 marks]

$f(2x) = 4(2x)^3 - 7$ $gf(x) = 6[4x^3 - 7] + 8$

$f(2x) = 4(8x^3) - 7$ $gf(x) = 24x^3 - 42 + 8$

$f(2x) = 32x^3 - 7$ $gf(x) = 24x^3 - 34$

$32x^3 - 7 = 24x^3 - 34$

$8x^3 = -27$

$x^3 = \frac{-27}{8}$

$x = \left(\frac{-27}{8}\right)^{1/3}$

$x = \frac{-3}{2}$

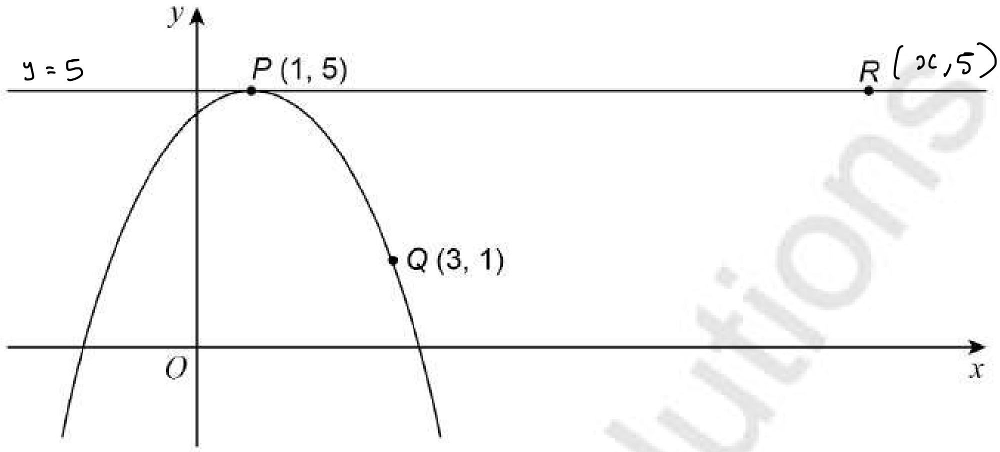
$x = -\frac{3}{2}$

Do not write outside the box

20 Here is a sketch of the curve with equation $y = 4 + 2x - x^2$

The tangent to the curve at the turning point P is shown.

The normal to the curve at Q meets this tangent at R.



Work out the x-coordinate of R.

[5 marks]

$y = 4 + 2x - x^2$

$\frac{dy}{dx} = 2 - 2x$

At $x = 3$ $\quad M_T = 2 - 2(3) = 2 - 6 = -4$

$M_N = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x - 3)$ – Equation of normal at Q

$5 - 1 = \frac{1}{4}(x - 3)$

$16 = x - 3$

$x = 19$

Answer 19

9

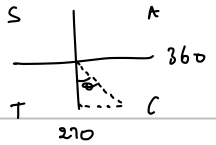
Turn over ►

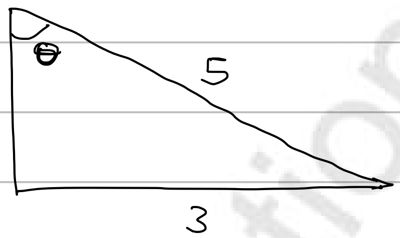
21 (a) θ is an angle between 270° and 360°

$$\cos\theta = \frac{1}{5k} \quad \text{and} \quad \tan\theta = -3k \qquad \text{where } k \text{ is a positive constant.}$$

Work out the value of θ.

[2 marks]





$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$-3k = \dfrac{\sin\theta}{1/5k}$

$\sin\theta = \dfrac{-3k}{5k} = -\dfrac{3}{5}$

$\theta = \sin^{-1}\left(\dfrac{3}{5}\right) \approx 36.9°$

$270° + 36.9° = 306.9°$

Answer 306.9 °

Do not write outside the box

21 (b) Work out the values of x between 0° and 360° for which

$$8\cos^2 x - \cos x = 0$$

[3 marks]

$\cos x(8\cos x - 1) = 0$

$\cos x = 0$ $\quad$ $8\cos x - 1 = 0$

$x = 90°, 270°$ $\quad$ $\cos x = \frac{1}{8}$

$x = \cos^{-1}\left(\frac{1}{8}\right)$

$x \simeq 82.8°, 360 - 82.8 = 277.2°$

Answer $x = 90°, 270°, 82.8°, 277.2°$

END OF QUESTIONS

5