OCR B (MEI) A-level Maths Notes

James Chen

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This document provides my full notes for OCR B (MEI) A-level Mathematics (for the 2026 sitting).
This document does not distinguish between AS and A-level.
Exam technique strategies are covered throughout, written in blue.
The content covered is directly based on the specification. I have split these notes into topics in the same order as the specification, but since it has no official sub-topics, I have chosen the sub-topic structuring myself.

This document only focuses on the content you need to know. If you would like more motivation, intuition, derivations, and/or proofs of the content, refer to my OCR A notes document.

Warning: I am an OCR A student, so there may be some differences between exam boards which I have not accounted for.

Contents

1 General Notes

1.1 Exam Structure

The exam consists of three papers, each of which being mandatory: Pure Mathematics & Mechanics, Pure Mathematics & Statistics, Pure Mathematics & Comprehension.
The Pure Mathematics & Mechanics paper consists of 100 marks and lasts 2 hours.
The Pure Mathematics & Statistics paper consists of 100 marks and lasts 2 hours.
The Pure Mathematics & Comprehension paper consists of 75 marks and lasts 2 hours. There is a 60 mark section of pure questions and a 15 mark section of comprehension questions based on an insert.

The Pure Mathematics & Mechanics paper and Pure Mathematics & Statistics paper both consist of two sections: Section A, which consists of shorter questions with minimal interpretation, and Section B, which consists of longer questions and problem solving.

1.2 Command Words

- It is expected that numerical answers are simplified even if the question doesn't ask for simplification
- Exact: unrounded
- Prove: formal mathematical argument in detail, including a concise conclusion
- Show that: sufficiently detailed explanation to cover every step of working
- Determine: justification and working needed
- Verify: a clear substitution of the given value needed
- Find/Solve/Calculate: working may be necessary but no justification is required
- Give/State/Write down: no working nor justification is required
- In this question you must show detailed reasoning: detailed and complete analytical method which allows the argument to be followed (not restricting use of a calculator)
- Hence: next step should be based on what has gone before
- Hence or otherwise: using information from the previous parts may be useful but not necessary, alternate methods may be more time-consuming or complex

- You may use the result: a result that you don't need to know can be used
- Plot: mark points and potentially join with lines
- Sketch: a diagram showing the key features, like turning points and asymptotes
- Draw: draw to a sensible level of accuracy

1.3 What to do when stuck

- Could it be a certain key fact or detail you are missing?
- Scan the formula booklet to see if there any relevant formulae you have forgotten
- Look at other questions in the paper to see if there is anything related
- See if you can obtain the answer immediately using your calculator, and then let the answer guide you
- Scattergun to obtain as many marks as possible
- If you need to show that something is true, work backwards from the final statement to see what you could do to get there in the other direction
- Harvest as many marks as possible from the other parts of the question using what you know (especially if you're stuck on a show that)

1.4 Checking Procedures

Some checking procedures are specific to the content, so this is a list of generic ones:

- Significant figures
- +c
- Simplifying fractions
- Division by 0
- Correct reversal of inequality signs
- Substitute answer into original question
- Sanity checks (do the answers make sense?) (how many solutions should I have?)
- Information given in question (e.g. domain restrictions)
- Input into calculator correctly

2 Proof

There are several methods of proof (or disproof) in the specification:

- Proof by deduction: proceeding from known truths to the conclusion via logical steps
- Proof by exhaustion: checking every possible case The possible cases may be given as a range, e.g. prove something is true for all numbers between 2 and 10. Sometimes, the exhaustion may not be explicit, such as proving something is true for all natural numbers by proving it is true for odd numbers and even numbers separately.
- Disproof by counterexample: giving at least one counterexample to a claim in order to disprove it
- Proof by contradiction: assuming some statement is true and reaching a logical contradiction, implying the statement must have been false

The logical connective $\equiv$ refers to congruence, i.e. identities, and implies the relation is always true (not just for certain cases or values).
The logical connective $\Rightarrow$ refers to implication: $A \Rightarrow B$ means "if A is true then B is true". This does not necessarily mean that if B is true, then A is true.
The logical connective $\iff$, known as 'iff' or 'if and only if', refers to implication in both directions: $A \iff B$ means "if A is true then B is true" and "if B is true then A is true".

Integers are whole numbers and real numbers are any number on the number line.
Rational numbers can be written in the form $\frac{p}{q}$, where p, q are coprime integers (numbers with no common factors apart from 1), and $q \neq 0$. Irrational numbers cannot be written in this form.

There are two standard proofs by contradiction that you have to know about: the proof of the irrationality of $\sqrt{2}$, and the infinitude of the primes.

Assume that $\sqrt{2}$ is rational, so it can be written in the form $\frac{p}{q}$, where p, q are coprime integers (numbers no common factors apart from 1), and $q \neq 0$.
$\sqrt{2} = \frac{p}{q} \Rightarrow 2 = \frac{p^2}{q^2} \rightarrow 2q^2 = p^2$
Since p^2 is even, p is even, so p can be written as $2k$ for some integer k.
Therefore, $2q^2 = (2k)^2 = 4k^2 \Rightarrow 2k^2 = q^2$, so q is even, but p is also even, so p and q are not coprime, causing a contradiction, so our assumption was false, and therefore $\sqrt{2}$ is irrational.

Assume there are finitely many primes. Let's assume there are n primes, which are $p_1, p_2, p_3, ..., p_n$. Then, let $P = p_1p_2p_3...p_n + 1$. P is either a prime or a product of primes (as every number is a product of primes). Dividing P by each of the primes in our set gives a remainder of 1, so P is not perfectly divisible by any prime in our set. P is therefore either a prime larger than the primes in our set or a number with at least one prime factor which is not in our set. In both cases, we have found a prime not in our set, contradiction our assumption that we have found all the primes, so there must be infinitely many prime numbers.

Many people use the symbol # to represent a contradiction, but this is not widely recognised, and may risk marks lost if the examiner is not familiar with it. You should always explicitly say that a contradiction has been reached.

3 Algebra

3.1 Basics of algebra

Constants do not change, variables may change, the coefficient of something is whatever is multiplied to that something, an expression is a standalone thing like $x^2 + 2x$, an equation is an equality like $x^2 + 2x = 0$, a function assigns outputs to inputs, an identity is true in all cases, an index is an exponent or power, a term is a part of a bigger expression, and an unknown is a quantitiy to be solved for.

You should be able to solve linear equations in one unknown.
You should be able to change the subject.

3.2 Quadratics

You should be able to solve quadratics by the quadratic formula, factorising, and completing the square.
We may not necessarily have quadratic functions in x. We could replace x with anything! For example, x^4+x^2+1 is a quadratic in x^2, $e^{2x}+e^x+1$ is a quadratic in e^x, and $\sin^2 x + \sin x + 1$ is a quadratic in $\sin x$.
To make things easier, it may be worth using a different letter like y to represent the thing the quadratic is in, then substituting x back at the end.
For a quadratic $ax^2 + bx + c = 0$, the discriminant is $\Delta = b^2 - 4ac$. The notation D can also be used. If $b^2 - 4ac > 0$, there are two real distinct roots. If $b^2 - 4ac = 0$, there is one repeated real root. If $b^2 - 4ac < 0$, the roots are not real.

3.3 Simultaneous equations and inequalities

Variables can be eliminated from simultaneous equations by adding or subtracting them (typically after scaling one or both of the equations). Rearranging to obtain an expression for one of the variables can allow for substitution into the other equation.

You should know how to solve linear and quadratic inequalities. Reminder

that to solve quadratic inequalities, you find the critical values (the roots when the inequality sign is replaced with an equals sign), and sketch a graph of the quadratic to identify the region you need.
When plotting inequalities on graphs, you need to explicitly state whether your shaded region is the accepted or rejected region, and whether the boundaries are included.

There are two types of notation for intervals: interval notation and set notation.

The interval $a < x < b$ is represented in interval notation by (a, b). We use a round bracket for strict inequalities and a square bracket for non-strict inequalities. The interval $a \leq x < b$, for example, is represented by $[a, b)$. If either of the limits are infinity, we tend to use round brackets.

Sets are denoted with curly brackets: $\{\ \}$. We start a set with $\{x :$ which means "x such that" and then write our condition, for example $\{x : 2 \leq x < 3\}$ means the set of all values of x such that x is greater than or equal to 2 and less than 3. We use $\cup$ in between two sets to represent 'or' and $\cap$ to represent 'and'.

3.4 Surds and indices

Fractions of the form $\frac{a}{\sqrt{b}}$ can be rationalised by multiplying by $\frac{\sqrt{b}}{\sqrt{b}}$. Fractions of the form $\frac{a}{b+\sqrt{c}}$ can be rationalised by multiplying by $\frac{b-\sqrt{c}}{b-\sqrt{c}}$.
It is usually best to rationalise the denominator of any fractions with irrational denominators, though questions may not always penalise not doing so.

The laws of indices are:

- $x^a x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $(x^a)^b = x^{ab}$
- $x^{-a} = \frac{1}{x^a}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
- $x^0 = 1$

3.5 Partial fractions

We can decompose rational fractions into partial fractions.
$\frac{1}{(ax+b)(cx+d)(ex+f)}$ can be written as $\frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{ex+f}$ for constants A, B, C.
$\frac{1}{(ax+b)(cx+d)^2}$ can be written as $\frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$
To determine the values of A, B, C, we multiply everything by the denominator

of the left hand side, and then substitute any values of x we want as the identity has to hold for all values of x. We tend to choose convenient values of x which makes some of the terms on the right hand side go away. We can obtain simultaneous equations for the constants.

4 Functions

4.1 Polynomials and the factor theorem

You should know how to expand brackets, collect like terms, and factorise.
You should know what quadratics, cubics, and parabola are.

The factor theorem states that $f(a) = 0$ if and only if $(x - a)$ is a factor of $f(x)$. Also, $f(\frac{b}{a}) = 0$ if and only if $(ax - b)$ is a factor of $f(x)$.
Sometimes, a factor of a polynomial has to be determined by inspection before you can use polynomial division.

You should be able to simplify rational expressions and perform polynomial division.

4.2 Definition, domain, and range of functions

The definition of a function is a mapping from the domain to the range such that for each x in the domain, there is a unique y in the range with $f(x) = y$.
A many-to-one function assigns multiple inputs to the same output while a one-to-one function only assigns one input to each output.
The exam may sometimes write $f : x \to y$ to represent a function f from the domain x to the range y.
The domain is the set of all possible inputs of the function, and the range is the set of all possible outputs of the function.
The range must be given in terms of $f(x)$ (or f), not x. For example, the range of $f(x) = x^2$ is $f(x) \geq 0$, not $x \geq 0$.

4.3 Inverse and composite functions

The inverse function of a function $f(x)$ is the function $f^{-1}(x)$ such that $f^{-1}(f(x)) = x$. The inverse function is the reflection of the function in the line $y = x$.
We write fg to refer to $f(g(x))$, i.e. putting $g(x)$ into $f(x)$ to form a composite function.

4.4 The modulus function

The modulus function $|x|$ is equal to x if $x \geq 0$ and is equal to $-x$ if $x < 0$, i.e. it turns x positive no matter its sign (gives the magnitude of x).

5 Graphs

You should be able to sketch curves of polynomials up to quartics and curves of the form $y = \frac{a}{x}$ or $y = \frac{a}{x^2}$.
To sketch the graph of a function which includes modulus functions, you have to reflect any portions of the graph under the x axis about the x axis.

If $y = f(x)$ then $y = f(x + a)$ translates by a units to the left, $y = f(x) + a$ translates by a units up, $y = f(ax)$ stretches parallel to the x axis by scale factor $\frac{1}{a}$, and $y = af(x)$ stretches parallel to the y axis by scale factor a.
When transformations are combined, remember to apply these rules to every instance of x in the function.

Completing the square on a quadratic can be used to find the line of symmetry and turning point. The curve $y = a(x+p)^2+q$ has a minimum at $(-p, q)$ when $a > 0$ or a maximum at $(-p, q)$ when $a < 0$, and a line of symmetry at $x = -p$.

Stationary points of inflection, which are covered more in calculus, may be required when sketching. They are points where the gradient changes from increasing to decreasing or vice versa, and there is a point with gradient zero in between.

6 Coordinate Geometry

6.1 Straight lines

The equation of a line can be written in the forms $y = mx + c, y - y_1 = m(x - x_1), ax + by + c = 0, \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$.
The midpoint of the line segment between the points (x_1, y_1) and (x_2, y_2) is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. The distance between these points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
For two lines with gradients m_1, m_1, they are parallel if $m_1 = m_2$ and they are perpendicular if $m_1 m_2 = -1$.

6.2 Circles

The equation of a circle is $(x - a)^2 + (y - b)^2 = r^2$ where the center is (a, b) and the radius is r.
Some questions give the equation of a circle in the form $ax^2+by^2+cx+dy+e = 0$. This can be converted into the standard equation by completing the square on both the x terms and the y terms.

The following circle theorems should be known:

- The angle in a semicircle is a right angle
- The perpendicular from the centre of a circle to a chord bisects the chord

- The radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at the point

6.3 Parametric equations

We may define a curve by writing x and y in terms of some other parameter, usually t or θ, to obtain parametric equations for the curve. To convert from parametric to cartesian, we need to find a way to eliminate the parameter, sometimes using known identities (e.g. trig identities).
A circle with radius r can be written as the parametric equations $x = r\cos\theta, y = r\sin\theta$.

The gradient of a parametric curve with parameter t is $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

7 Sequences and Series

7.1 Binomial expansion

If n is a natural number, then $(a + bx)^n = a^n + \binom{n}{1}bx + \binom{n}{2}(bx)^2 + ... + \binom{n}{r}a^r(bx)^{n-r} + ... + b^n$, which is in the formula booklet.
$n!$, or n factorial, is the product of every natural number from 1 to n, with 0! defined as 1.
${}^nC_r$ or $\binom{n}{r}$ is the number of ways to choose r objects from n objects (without order mattering), and ${}^nC_r = \frac{n!}{r!(n-r)!}$, which is in the formula booklet.
The value of $\binom{n}{r}$ is the rth entry in the nth row of Pascal's triangle.

We can extend the binomial expansion to any rational n using the formula $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$, which is in the formula booklet. If the term which isn't x is not 1, we can divide the entire expression by some factor to make it 1, e.g. $(2+x)^{0.5} = 2^{0.5}(1 + \frac{x}{2})^{0.5}$ and we can proceed using $0.5x$ instead of x.
The range of validity is $|x| < 1$, with x replaced with whatever you used in the expansion.

These expansions can be used for approximations. If x is small, the terms with high powers can be neglected as they are close to 0. We can substitute particular values of x into these expansions to obtain these approximations.

7.2 Properties of sequences

Sequences can be given as a formula for the nth term or a recursion relation of the form $x_{n+1} = f(x_n)$.
Increasing sequences are where every term is greater than or equal to the last.
Decreasing sequences are where every term is less than or equal to the last.
If the terms of a sequence repeat after certain intervals, it is periodic, and the

number of terms between each repeat is the period.
A series represents the sum of a certain number of terms of a sequence.
A finite sequence has a finite number of terms and an infinite sequence has an infinite number of terms.
The notation $\sum_{r=a}^{b} f(r)$ represents the sum of the values of $f(r)$ from a to b inclusive (every integer in this range).

7.3 Arithmetic and geometric sequences

An arithmetic sequence is where the terms increase with a constant difference. If we denote the first term as a and the common difference as d, then the nth term is $a + d(n-1)$, and the sum of the first n terms is $\frac{n}{2}(a+l)$ or $\frac{n}{2}(2a + d(n-1))$, and these sum formulae are in the formula booklet. We have the special case of the sum of the first n natural numbers $S_n = \frac{n(n+1)}{2}$ which is not in the formula booklet.

A geometric sequence is where the terms increase with a common ratio. If we denote the first term as a and the common ratio as r, then the nth term is ar^{n-1} and the sum of the first n terms is $S_n = \frac{a(1-r^n)}{1-r}$, and this sum formula is in the formula booklet. As long as $|r| < 1$, the sum to infinity is $S_\infty = \frac{a}{1-r}$, which is also in the formula booklet. $|r| < 1$ guarantees that the series converges (ends up on a specific value), whereas $|r| > 1$ means the sequence diverges (does not end up on a specific value).

8 Trigonometry

8.1 Definitions and graphs of sin, cos, and tan

You should know the definitions of sin, cos, and tan. On a unit circle, $\cos\theta = x, \sin\theta = y, \tan\theta = \frac{y}{x}$.

You should the know the graphs, symmetries, and periodicities of sin, cos, and tan. You should know the necessary exact values, but a new thing is recognising the exact values for angles given in radians, which may take some practice.

8.2 Sine and cosine rules, area of a triangle

You should know the sine and cosine rules.
The area of a triangle is given by $A = \frac{1}{2}ab\sin C$, where C is the included angle between sides a, b.

8.3 Trigonometric equations

You should be able to solve simple trigonometric equations. The question will give a range for the solution. If this needs to be changed via a transformation of the variable, it is worth explicitly changing the required range.

8.4 Radians and small angle approximations

Multiplying an angle in degrees by $\frac{\pi}{180}$ converts it to radians (and vice versa). If θ is the angle subtended by a sector of a circle with radius r, the arc length is $r\theta$ and the area is $\frac{1}{2}r^2\theta$. The radian is defined as the angle subtended by a sector whose arc length is equal to its radius.

When x is small and measured in radians, $\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$. These are given in the formula booklet.

8.5 Reciprocal and inverse trigonometric functions

We define $\csc\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \cot\theta = \frac{1}{\tan\theta}$. We also define $\arcsin\theta, \arccos\theta, \arctan\theta$ as the inverse functions of $\sin\theta, \cos\theta, \tan\theta$ respectively, giving principal values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. The domains of these inverse functions are the ranges of their respective trig functions. Their graphs are the reflections of their respective trig functions in the line $y = x$ (for the appropriate domain).
The reciprocal trig functions have the same domain as their respective trig functions except for where the denominator is 0. The ranges of $\csc x$ and $\sec x$ is all reals apart from between -1 and 1. The range of $\cot x$ is all reals.

8.6 Trigonometric identities and compound angle formulae

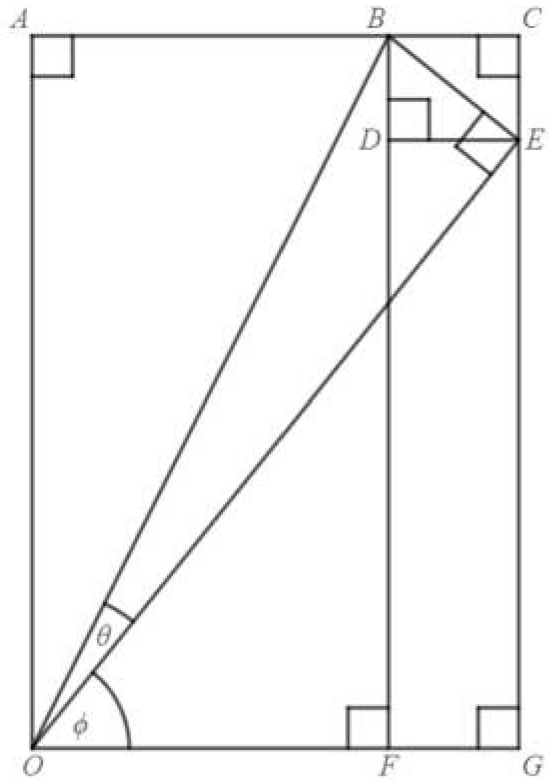
We have the trigonometric identities:
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

The following compound angle formulae are given in the formula booklet: $\sin(x+y) = \sin x\cos y + \sin y\cos x$
$\cos(x+y) = \cos x\cos y - \sin x\sin y$
$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$

Replacing y with x gives important double angle formulae (which aren't in the formula booklet and you need to know):
$\sin 2x = 2\sin x\cos x$
$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

You do not need to know but need to understand the geometric proofs of the compound angle formulae:

Consider the following diagram.



Why should we draw such a diagram? We want to consider $\cos(\theta+\phi)$, and this could be heavily related to right-angled triangles. We could therefore consider the separate right-angled triangles with angles $\theta, \phi, \theta+\phi$.
$\cos(\theta+\phi) = OF/OB$
$\cos\theta = OE/OB$
$\cos\phi = OG/OE$
$OG = OB\cos\theta\cos\phi =$
$OF = OG - FG = OG - BC = OG - BE\sin\phi = OG - OB\sin\theta\sin\phi$
$\cos(\theta+\phi) = \frac{OG-BC}{OB} = \frac{OB\cos\theta\cos\phi - OB\sin\theta\sin\phi}{OB} = \cos\theta\cos\phi - \sin\theta\sin\phi$

Now consider $\sin(\theta+\phi) = \sqrt{1-\cos^2(\theta+\phi)} = \sqrt{1-(\cos\theta\cos\phi-\sin\theta\sin\phi)^2}$
$= \sqrt{\sin^2\theta + \cos^2\theta - \sin^2\theta\sin^2\phi - \cos^2\theta\cos^2\phi + 2\sin\theta\sin\phi\cos\theta\cos\phi}$
$= \sqrt{\sin^2\theta(1-\sin^2\phi) + \cos^2\theta(1-\cos^2\phi) + 2\sin\theta\sin\phi\cos\theta\cos\phi}$
$= \sqrt{\sin^2\theta\cos^2\phi + \sin^2\phi\cos^2\theta + 2\sin\theta\sin\phi\cos\theta\cos\phi} = \sin\theta\cos\phi + \sin\phi\cos\theta$

We can divide sin by cos to get tan:

$\tan(\theta + \phi) = \frac{\sin\theta\cos\phi + \sin\phi\cos\theta}{\cos\theta\cos\phi - \sin\theta\sin\phi}$
Let's divide through by $\sin\theta\cos\phi$.
$\tan(\theta + \phi) = \frac{1+\tan\phi\cot\theta}{\cot\theta - \tan\phi}$
And let's now multiply by $\tan\theta$.
$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$

8.7 Harmonic form

We can write $a\sin x + b\cos x$ in any of the forms $R\sin(x+\alpha)$, $R\sin(x-\alpha)$, $R\cos(x+\alpha)$, $R\cos(x-\alpha)$. We have, in all cases, $R^2 = a^2 + b^2$. We can determine α by expanding the required form using a compound angle formula and by comparing coefficients of $\sin x, \cos x$ on both sides to determine the value fo $\tan\alpha$. By convention, $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$.
This makes graphs of $a\sin x + b\cos x$ easier to sketch, max and min values are easier to find, and equations can be solved easier.

9 Exponentials and Logarithms

9.1 Exponential graphs

You should be familiar with the graph of a^x when a is positive, including the example e^x.
By the chain rule, the gradient of e^{kx} is ke^{kx}, so exponential curves are proportional to their gradient, which is a common thing in the real world and it makes the exponential model suitable for many applications.

9.2 Logarithms and reduction to linear form

$\log_a x$ is the inverse function of a^x (where x and a are positive).
$a = b^c \Rightarrow c = \log_b a$
$\log_a a = 1, \log_a 1 = 0$
$\ln x$, the natural logarithm, is the inverse function of e^x and its graph is the reflection of e^x in the line $y = x$.

The following laws of logarithms need to be known:
$\log_a x + \log_a y = \log_a xy$
$\log_a x - \log_a y = \log_a \frac{x}{y}$
$\log_a(b^c) = c\log_a b$

Equations of the form $a^x = b$ can be solved by taking logarithms of both sides. Any base may be used, but the base a may be most useful here.

For relationships of the form $y = ax^n$. We may take the logarithm of both sides (with any base) to obtain a linear form. If we use the natural logarithm, we have $\ln y = \ln(ax^n) = \ln(a) + n\ln x$ (using the laws of logarithms). This means that a graph of $\ln y$ against $\ln x$ has gradient n and y-intercept $\ln(a)$.

10 Calculus

10.1 Derivatives and gradients

The derivative of $f(x)$ represents the gradient of the tangent to $y = f(x)$ at the point $(x, f(x))$. This is the rate of change of y with respect to x. If the derivative is at $x = a$, this is the limit of the gradient of a chord as x tends to a. We tend to denote the derivative of y as $\frac{dy}{dx}$ and the derivative of $f(x)$ as $f'(x)$. We represent second derivatives as $f''(x)$ and $\frac{d^2y}{dx^2}$. The second derivative is the rate of change of the gradient. If it is positive over a certain interval, the function is concave upward (also known as convex or convex downward) in that interval. If it is negative, the function is concave downward (also known as concave or convex upward). If it is zero and the curve changes from concave to convex or vice versa, there is a point of inflection.

10.2 Differentiation from first principles

The derivative is defined as $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ (which is in the formula booklet). Using this formula to differentiate functions is known as differentiation by first principles. We substitute in our function of x into the formula, expand, simplify, divide by h, and let h tend to 0.

The derivatives of $\sin x$ and $\cos x$ by first principles need to be known:
$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0}\frac{\sin x\cos h+\sin h\cos x-\sin x}{h}$
$= \lim_{h\to 0}(\frac{\cos h-1}{h}\sin x + \frac{\sin h}{h}\cos x)$.
We now need to use some standard limits (which I shall leave unproven): $\lim_{h\to 0}\frac{\sin h}{h} = 1, \lim_{h\to 0}\frac{\cos h-1}{h} = 0$. This gives us $\frac{d}{dx}\sin x = \cos x$.

$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$
$= \lim_{h\to 0}(\frac{\cos h-1}{h}\cos x - \frac{\sin h}{h}\sin x) = -\sin x$, using the same standard limits as before.

10.3 Differentiating standard functions

In general, $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$, $\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$, $\frac{d}{dx}(af(x)) = a\frac{d}{dx}f(x)$ for a constant a.

$\frac{d}{dx}x^n = nx^{n-1}$
$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}a^x = a^x\ln a$
$\frac{d}{dx}\sin x = \cos x$
$\frac{d}{dx}\cos x = -\sin x$
$\frac{d}{dx}\tan x = \sec^2 x$ (this is in the formula booklet)
$\frac{d}{dx}\ln x = \frac{1}{x}$

10.4 Tangents, normals, stationary points and their classifications

Differentiating the equation of a curve and plugging in a to the gradient function gives the gradient of the tangent at the point $x = a$. Taking the negative reciprocal gives the gradient of the normal at this point. The point can be used to find the equation of the tangent or normal.
Stationary points occur when $\frac{dy}{dx} = 0$. A stationary point is a local maximum when $\frac{d^2y}{dx^2} < 0$ and a local minimum when $\frac{d^2y}{dx^2} > 0$. If $\frac{d^2y}{dx^2} = 0$ and there is a sign change in the second derivative either side close to this point, there is a point of inflection. If $\frac{dy}{dx} = 0$ is also true at that point, it is a stationary point of inflection.

A function is increasing when $\frac{dy}{dx} \geq 0$ and decreasing when $\frac{dy}{dx} \leq 0$.

10.5 The chain, product, and quotient rules

The product rule states that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
The quotient rule states that $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$, which is in the formula booklet.
The chain rule states that $\frac{d}{dx}f(g(x)) = \left(\frac{d}{dx}g(x)\right)\frac{d}{dg(x)}f(x)$.
We also have the following results: $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

10.6 Implicit differentiation

We can differentiate implicit relations of x and y. We can differentiate both sides with respect to x by differentiating any term in x as normal, and by differentiating any term in y with respect to y, and multiplying these by $\frac{dy}{dx}$.

10.7 The fundamental theorem of calculus

The fundamental theorem of calculus states that differentiation and integration are essentially the reverse of each other. If $\frac{d}{dx}f(x) = f'(x)$ then $\int f'(x)dx = f(x) + c$ for some arbitrary constant c.

10.8 Integrating standard functions

Similar to differentiation, the following results apply: $\int f(x)+g(x)dx = \int f(x)dx + \int g(x)dx$, $\int f(x) - g(x)dx = \int f(x)dx - \int g(x)dx$, $\int af(x)dx = a\int f(x)dx$ for a constant a.
$\int x^n dx = \frac{1}{n}x^{n+1} + c$ as long as $n \neq 1$
$\int e^x dx = e^x + c$
$\int \frac{1}{x}dx = \ln x + c$
$\int \sin x dx = -\cos x + c$
$\int \cos x dx = \sin x + c$

These are all indefinite integrals as the integral signs have no limits, and we need to add an arbitrary constant. NEVER forget to add an arbitrary constant for indefinite integrals.

10.9 Definite integrals

The integral $\int_a^b f(x)dx$ represents the area between the curve $y = f(x)$ and the lines $x = a, x = b$. This integral is definite as there are limits. We do not need an arbitrary constant. The integral returns the signed area (positive if the curve is above the x axis, negative if the curve is below the x axis). If a question asks for the area, not the value of an integral, the area must be positive. We may therefore have to split up the integral into several integrals based on where the curve is above or below the x axis.

The area between the two curves $y = f(x), y = g(x)$ from $x = a$ to $x = b$ is $|\int_a^b f(x) - g(x)dx|$.

We can also integrate to the y axis. The area between a curve $f(x)$ and the y axis between $y = a$ and $y = b$ is $\int_a^b xdy$. We could write x in terms of y to help us evaluate this.

Integration can be viewed as the approximation of the area under a curve with a bunch of rectangles, where the limit of the sum of the areas of the rectangles is taken as the width of the rectangles tends to 0.

10.10 Integration by substitution and integration by parts

We may wish to transform an integral in the variable x to an integral in a different variable, such as u. If $u = f(x)$ and we have $I = \int_{x=a}^{x=b} g(x)dx$, then $I = \int_{u=f(a)}^{u=f(b)} g(f^{-1}(u))\frac{dx}{du}du$. This is essentially a reverse form of the chain rule, and there is a particularly important result to recognise (which is in the formula booklet): $\int \frac{f'(x)}{f(x)}dx = \ln(f(x)) + c$.
Integrals of the form $\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$ can typically be done by inspection or by an explicit substitution if you wish. This result is in the formula booklet.

We may integrate products of functions using integration by parts: $\int u'vdx = [uv] - \int uv'dx$. This is the reverse version of the product rule. Sometimes, the method has to be used more than once.
To integrate $\ln x$, you can use parts with $1 \times \ln x$ to obtain $\int 1 \times \ln xdx = (\int 1dx)\ln x - \int(\int 1dx)\frac{d}{dx}\ln xdx = x\ln x - \int \frac{x}{x}dx = x\ln x - x + c$.

Some rational functions such as $\frac{1}{(ax+b)(cx+d)}$ are hard to integrate. We can use partial fractions to decompose them into several fractions, which are easier to integrate as they are scalar multiples and translations of $\frac{1}{x}$, which integrates to $\ln x + c$.

10.11 Differential equations

For differential equations relating $\frac{dy}{dx}, y, x$ we have to use the method of the separation of variables. We need all y terms on the same side as $\frac{dy}{dx}$ and all x terms on the other side. We can then integrate both sides with respect to x, so we get rid of the $\frac{dy}{dx}$ and integrate its remaining side with respect to y, and integrate the other side with respect to x. We add an arbitrary constant, which could be evaluated using any given boundary conditions.
The arbitrary constant must be added at the first integration. Every time it changes into a different arbitrary constant, it is worth using a different variable to represent it.
A common trick is for the question to describe an object at rest at a certain time, which is a boundary condition as the derivative of its displacement with respect to time is 0.

11 Numerical Methods

11.1 Change of sign

As long as $f(x)$ is continuous on a certain interval, if $f(a) > 0, f(b) < 0$ or $f(a) < 0, f(b) > 0$, then $f(x)$ has a root between a and b. For change of sign questions, you should always explicitly say that there is a change of sign after plugging in the values.
This method can be used to determine roots to a certain accuracy by considering upper and lower bounds for what rounds to the root for a given number of significant figures.

Change of sign methods can fail when the function touches the x axis or has a vertical asymptote.

11.2 Iterative formulae and Newton-Raphson

If we can rearrange an equation $f(x) = 0$ into the form $x = g(x)$, we can use the iterative formula $x_{n+1} = g(x_n)$ with a chosen value of x_1 to look for roots. We can draw cobweb and staircase diagrams to represent this method. We draw the function curve and the line $y = x$ on a graph. We take an x_n, draw a line up to the function curve, draw a horizontal line to $y = x$, and repeat. This can in result in what looks a staircase if the function is convex (its second derivative is greater than 0) and it looks like a cobweb if the function is concave (its second derivative is less than 0).

The Newton-Raphson is a special iterative method where to estimate roots of $f(x) = 0$, we use $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for a given value of x_0, which is in the formula booklet.

11.3 The trapezium rule

We may wish to estimate the area under a curve $y = f(x)$ from $x = a$ to $x = b$ using trapezia. If we use n trapezia of equal width then the width of each one, h, is $\frac{b-a}{n}$. Let's let the x values $x_0, x_1, x_2, ..., x_n$ denote $a, a+h, a+2h, ..., b$ respectively. We can then use the trapezium rule (which is given in the formula booklet): $A \approx \frac{h}{2}(f(x_0) + f(x_n) + 2(f(x_1) + f(x_2) + ... + f(x_{n-1})))$. This gives an underestimate of the area when the curve is concave and an overestimate when the curve is convex.
We could also use rectangles to approximate integrals, and give upper and lower bounds for areas.

12 Vectors

A scalar has a magnitude and no direction while a vector has a magnitude and a direction. A two dimensional vector can be represented as $x\mathbf{i} + y\mathbf{j}$ or $\binom{x}{y}$. The vector notations $\vec{AB}$ or $\mathbf{a}$ or $\underline{a}$ could also be used.

Vectors in three dimensions can be represented as $x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

The magnitude of the vector $\mathbf{a}$ is denoted as $|\mathbf{a}|$. If $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2}$. Similarly, if $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$.

By convention, we define the direction of a vector as the angle it makes with the positive x axis, which should lie in the interval $[0, 360)$.
The direction can be calculated using $\tan^{-1} \frac{y}{x}$, but may not always be exactly equal to this, as this does not always return the angle desired. It may be the case that this value has to be subtracted from 180. It is worth drawing a quick diagram to make sure you have the correct angle.

We can add vectors by adding the components, or add them diagrammatically by drawing them tip to tail and drawing the resultant vector from the start to the end (to form a triangle of vectors).
We can multiply vectors by scalars by multiplying each component by the scalar (this has the geometric effect of enlarging the vector).

A position vector is a vector which starts at the origin and points outwards to a point. We tend to use lowercase letters, such as $\mathbf{a}$ to denote position vec-

tors.
A resultant vector can be split into its horizontal and vertical component vectors.
Two vectors are parallels if and only if they are scalar multiples of each other, i.e. you can multiply one by a constant to get the other.
Two vectors are equal if and only if their components are equal.
A unit vector has magnitude 1.

The distance between the points represented by position vectors $a\mathbf{i} + b\mathbf{j}$ and $c\mathbf{i} + d\mathbf{j}$ is $\sqrt{(c-a)^2 + (d-b)^2}$.

13 Sampling

The population refers to the entire set of people or objects for some context.
A sample is a subset of this population. Different samples can lead to different conclusions about the population.

A census involves recording data from every single member of the population,

Simple random sampling refers to giving a unique number to each member of the population, choosing a set of these numbers using a random number generator, and only recording data for the people represented by these numbers.
Advantages of simple random sampling include that it is less time consuming and easier than a census, and gives each member an equal chance of being considered, but disadvantages include it not necessarily representing the entire population.

Opportunity sampling is only using the first members you come across. Advantages include the convenience, whereas disadvantages include the potential bias.

Systematic sampling is essentially sampling with intervals (e.g. taking every 3rd person).

Stratified sampling is where we divide the population into strata based on some category (e.g. age or gender) and then use a different sampling method to sample within these strata. This may represent differences between different demographics if necessary, but is reliant on the advantages and disadvantages of other sampling methods.

Cluster sampling is where the population is split into clusters, such as by location, where each cluster represents the entire population. A sample of clusters is chosen and every member of these clusters contribute (sometimes sampling can be done within these clusters). It may be hard to ensure each cluster reflects the entire population, and there may be bias caused by the choice of particular clusters.

Quota sampling is where members of the population are chosen such that there are enough to reach pre-decided minimum numbers for certain quota (such as age or location).

Self-selected sampling (or volunteer sampling) is where the participants choose to take part themselves, such as by responding to an advertisement. Advantages include the ease of recruitment, targeting particular interests, and motivation. Disadvantages include overrepresentation of certain beliefs, lack of randomisation, and ethical concerns.

14 Data Presentation and Interpretation

14.1 Data presentation methods

Categorical data splits data into categories. Discrete data increases in steps while continuous data increases 'smoothly'. Ranked data is where data is given in a particular order, and the actual values are irrelevant - only which comes after the other is relevant.

You should be familiar with vertical line charts, dot plots, pie charts, and bar charts.

Reminder that stem and leaf diagrams consist of a stem (usually representing the first digit) with leafs representing all the data. There must be a key.

Reminder that box-and-whisker-plots consist of vertical lines at the minimum and maximum values, and the median, lower quartile, and upper quartile, with the two quartiles forming a box, and the min and max values connected to this box via whiskers.

Reminder that cumulative frequency diagrams show the total of a variable as some other variable changes, and must be strictly increasing.

Reminder that a histogram has bars of unequal width, with frequency density on the y axis and the classes on the x axis. The area is proportional to frequency.

Each data presentation method has advantages and disadvantages, such as whether or not they show all the data, how clearly they present data, whether or not they present the key features of the data (e.g. median), etc.

Data could be symmetrical, unimodal (having one mode), or bimodal (having two modes). Data could also be skewed in a certain direction (postively or negatively).

Diagrams representing unbiased samples become more representative of theoretical probability distributions with increasing sample size.

14.2 Bivariate data

You should be able to interpret scatter diagrams. The regression line is essentially the line of best fit. Scatter diagrams may include distinct sections of the population.
Association is where two variables influence each other, and correlation is a special case where the association is linear. They may either change in the same direction (positive correlation) or change in opposite directions (negative correlation) or may not impact each other at all (zero correlation). No matter what, correlation does not necessarily imply causation.
Data could be interpolated or extrapolated. Interpolation is predicting values between known data points and extrapolation is predicting values beyond the range on known data points, such as by using the regression line. Extrapolation may not always be valid.
There may be outliers in a scatter diagram, which are inconsistent with the rest of the data. The presence of any has to be judged by eye.

14.3 Measures of central tendency and spread

You should know how to determine the mean, median, and mode of a dataset. Sometimes, it may be appropriate to use a weighted mean, where each value doesn't contribute to the mean equally.
The midrange is the mean of the minimum and maximum values.
The ath percentile is the item which lies at the a% position when the elements are in order.
Reminder that the interquartile range is the upper quartile minus the lower quartile.

14.4 Variance and standard deviation

The sample variance is $s^2 = \frac{S_{xx}}{n-1}$, where $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$. The sample standard deviation is the square root of the sample variance. These results are in the formula booklet.

14.5 Outliers

An outlier is an item inconsistent with the rest of the data. There are two definitions for outliers in the spec: more than 1.5 times the interquartile range away from the nearer quartile, or more than 2 times the standard deviation from the mean.

Sometimes, these two definitions may disagree on whether an element is an outlier or not. In that case, it is inconclusive.

15 Probability

15.1 Basics of probability

Reminder that mutually exclusive events cannot occur at the same time, and independent events do not affect the probabilities of each other occurring.
Reminder that $P(A)$ is the probability of the event A occurring, and $P(A') = 1 - P(A)$ is the probability that A does not occur (the complement of A).
$P(X = x)$ is the probability that the random variable X takes the value x.

The expected frequency of an event A in n total trials is $nP(A)$.

You should be familiar with tree diagrams, sample space diagrams, and Venn diagrams.
We have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (probability of A given B), and the special case $P(A \cap B) = P(A)P(B)$ when A and B are independent.
When A and B are independent, $P(A|B) = P(A)$.

15.2 Modelling with probability

What it says on the tin.

16 Probability Distributions

16.1 General discrete distributions

Discrete probability distributions can be defined in a table or by a formula, which must specify which values the random variable can take.
The discrete uniform distribution has an equal probability of every value in its range occurring.

16.2 The binomial distribution

The conditions for a binomial distribution are: independent repetitions of a trial, two mutually exclusive and exhaustive events, and constant probabilities of success and failure.
Conditions must be satisfied for a model to hold, and assumptions are where we assume the conditions are met so that the model can be seen as suitable.
Assumptions must be made in context, e.g. for the number of MCQ questions answered correctly, you have to say 'MCQ questions are answered independently

of one another' and not 'they are independent' or 'MCQ questions are independent'.

If $X \sim B(n,p)$ then there are n trials with probability of success p, and $P(X=x) = \binom{n}{x}p^x(1-p)^{n-x}$. This is in the formula booklet.
The mean is np. This result is also in the formula booklet.

16.3 The normal distribution

The normal distribution is denoted as $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance. Probabilities can only be evaluated with your calculator. The area under the curve represents probability.
A normal distribution resembles a roughly bell-shaped unimodal symmetric histogram which tapers at the extremes. For example, if data is asymmetrical, it may not be well-modelled by a normal distribution. Histograms from increasingly large samples form a normal distribution. A binomial distribution may be modelled by a normal distribution based on these conditions.

Continuity corrections have to be used when using a normal distribution to model discrete data. This is because, for example, any values of the normal distribution between 10.5 and 11 are represented as 11 in the discrete distribution, so for example $P(X \leq 15)$ for the discrete distribution is equal to $P(X < 15.5)$ for the normal distribution.

A linear transformation of a normal variable is also a normal variable. If $X \sim N(\mu, \sigma^2)$ then $aX + b \sim N(a\mu + b, a^2\sigma^2)$.

The standard normal distribution Z is distributed $Z \sim N(0,1)$ and $Z = \frac{X-\mu}{\sigma}$.
If we need to find values of μ and σ given known probabilities, we have to transform to the standard normal.

About two thirds of the data lie within one standard deviation of the mean. About 95% lie within two standard deviations. Almost all lie within three standard deviations. The points of inflection of a normal curve lie at approximately one standard deviation from the mean.

17 Statistical Hypothesis Testing

17.1 The language of hypothesis testing

The null hypothesis, H_0, tends to be what would happen if our hypothesis were not true, and nothing has changed. The alternative hypothesis, H_1 tends to be what would happen if our hypothesis were true, and things have changed.
The significance level is the pre-decided maximum probability which would lead to rejection of the null hypothesis.

The test statistic is the distribution we use for the test (for our spec we only cover normal, binomial, correlation).
A 1-tail test is where we only test either if something is lower than it should be, or higher than it should be.
A 2-tail test is where we test for both. We tend to split the significance level in half in this case.
The critical value is the value such that anything more extreme leads to rejection of the null hypothesis.
The critical region is the range of values which leads to rejection of the null hypothesis.
The acceptance region is the range of values which leads to acceptance of the null hypothesis.
A p value (or the significance level) is the probability the null hypothesis is rejected if it is true.

This is the general recipe for a hypothesis test:

H_0: [insert null hypothesis]
H_1: [insert alternative hypothesis]
Define the parameter in context (you typically have to say the parameter represents the whole population)
Divide the significance level by 2 if the test is two-tailed.
State the test statistic / distribution
Determine the critical region and determine whether the data given is in it, OR determine the probability of the data occurring and compare to the significance level.
Conclude: "there is sufficient/insufficient evidence to reject H_0 at the [significance level]: [insert thing] is likely to [alternative hypothesis]".

17.2 Hypothesis test for the proportion in a binomial distribution

We can use the above recipe to perform hypothesis tests for the proportion in a binomial distribution.
For a given significance level, the probability of rejecting the null hypothesis will be less than or equal to this level.

17.3 Hypothesis test for the mean of a normal distribution

We can consider a sample mean $\bar{X}$ as a random variable, and we have that if $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, where n is the sample size. This is in the formula booklet.
Critical values can be calculated using calculator functions or the percentage point table in the formula booklet.

If the population variance is unknown but the sample size is large, the population variance is approximately equal to the sample variance.

17.4 Hypothesis test for correlation

Correlation is a measure of how close data points lie to a straight line. A rank correlation coefficient measures the correlation between the data ranks instead of the actual data values.
The null hypothesis is often either there is correlation or there is no correlation.

18 The Large Data Set

This document will only refer to the Large Data Set for the 2026 sitting.
Some of the data may be used in statistics questions, but you are never expected to memorise any of its data, however familiarity with the data set tends to be an advantage. The following information is useful to be aware of, but you do not necessarily need to memorise it.

Most of the data is based on boroughs in London, with some comparison to the rest of the UK.
The dataset covers employment rate, median house price, mean and median taxpayer income, pupils at the end of KS4 achieving 5+ A*-C including English and Maths, and household waste recycling rate.

19 Models And Quantities

Sometimes, we may wish to simplify real-world scenarios in our models, such as by using the following assumptions: light, smooth (no friction), uniform (weight acts at center), particle (body concentrated at a single point), inextensible, thin, rigid, long term.

Length (m), time (s), and mass (kg), are the three base quantities in the S.I. system and are mutually independent. Other quantities, such as velocity and force, are derived from these base quantities.
The unit for moment is Nm.

20 Kinematics

20.1 Representing kinematics

The displacement of an object is a vector quantity representing the magnitude and direction of its position from a given origin. (Distance is a scalar, the magnitude of displacement, and overall distance travelled has to be determined from the overall path of the object, and cannot be determined from the displacement

alone).
Velocity is the vector version of speed, representing the magnitude and direction of the speed.
Acceleration is the rate of change of velocity, also a vector. The magnitude of acceleration is a positive scalar.

The gradient of a displacement-time graph is the velocity. The gradient of a velocity-time graph is acceleration. The area between an acceleration-time graph and the x axis is velocity. The area between a velocity-time graph and the x axis is displacement.

For non-constant acceleration, we have $v = \frac{ds}{dt}, a = \frac{dv}{dt} = \frac{d^2s}{dt^2}, v = \int adt, s = \int vdt$.
We can extend this to vectors in two dimensions by differentiating or integrating the horizontal and vertical components separately.

20.2 Constant acceleration

The following SUVAT equations are given in the formula booklet (and only apply for constant acceleration):
$v = u + at, s = ut + \frac{1}{2}at^2, s = \frac{1}{2}(u+v)t, v^2 = u^2 + 2as, s = vt - \frac{1}{2}at^2$
These equations can be extended to 2D by replacing u, v, a, s with their respective vectors (these are also in the formula booklet). We exclude $v^2 = u^2 + 2as$ from this.

There are a variety of ways to derive these formulae, for example:

- $v = \int adt = at + c, t = 0 \Rightarrow v = u$ which gives $v = u + at$
- $s = \int vdt = \int u + atdt = ut + \frac{1}{2}at^2 + c, t = 0 \Rightarrow s = 0$ which gives $s = ut + \frac{1}{2}at^2$
- Finding formulae for areas under graphs using triangles
- Finding formulae for gradients
- Substituting $v = u+at$ into $s = \frac{1}{2}(u+v)t$ to get $s = \frac{1}{2}(2u+at)t = ut+\frac{1}{2}at^2$

21 Projectiles

A projectile moves only under the influence of gravity, i.e. $\mathbf{a} = \binom{0}{-g} = -g\mathbf{j}$, where $g = 9.8\text{ms}^{-2}$ for this spec. The standard modelling assumptions for projectile motion are: no air resistance, projectile is a particle, horizontal distance is small enough to assume gravity acts in the same direction, vertical distance is small enough to assume gravity is constant.

Given initial velocity u at angle θ to the horizontal, the horizontal component

of the velocity is $u\cos\theta$ and the vertical component is $u\sin\theta$.
The total duration of the motion is given by $s = ut + \frac{1}{2}at^2 \Rightarrow 0 = ut\sin\theta - \frac{1}{2}gt^2 \Rightarrow t = \frac{2u\sin\theta}{g}$.
The horizontal distance travelled for this duration is $\frac{2u\sin\theta}{g}u\cos\theta$.
The maximum height is given by $v^2 = u^2 + 2as \Rightarrow 0 = u^2\sin^2\theta - 2gh \Rightarrow h = \frac{u^2\sin^2\theta}{2g}$.
We can derive a cartesian equation for the projectile's motion. The vertical height at a certain time is given by $y = ut\sin\theta - \frac{g}{2}t^2$ and the horizontal displacement at a certain time is $x = ut\cos\theta \Rightarrow t = \frac{x}{u\cos\theta}$, so substituting gives $y = \frac{x}{u\cos\theta}u\sin\theta - \frac{g}{2}(\frac{x}{u\cos\theta})^2 = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta = \tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$.

22 Forces

22.1 Examples of forces and force diagrams

Weight is a downward force acting on all objects with mass. Tension is force in ropes, strings, etc which opposes motion. Thrust drives objects forwards and compression compresses objects. The normal contact force (or normal reaction force) is in the opposite direction to weight and perpendicular to the surface (its value depends on the other forces acting). There may be a frictional force if the surface is rough.

Our spec states $g = 9.8\text{ms}^{-2}$ for all questions, but you need to be aware that g is not a constant, and depends on location in the universe.

Force diagrams can be drawn to represent the magnitudes and directions of all forces acting on an object. External forces act between the object and the surroundings. Internal forces act within the object.

22.2 Vector treatment of forces and equilibrium

Vector addition can be used to find resultant forces of two or more forces acting on an object.
Forces may need to be resolved, in which case we can multiply by $\cos\theta$ or $\sin\theta$ to get the horizontal or vertical components of the force (depending on what θ is).
The velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force.

When the resultant force is 0, an object is in equilibrium, and all the forces on it form a closed shape.
A particle is in equilibrium if and only if the sum of the resolved parts of all the forces in any given direction is 0. Forces may be resolved horizontally and vertically, parallel and perpendicular to a surface, or in any chosen direction.

22.3 Friction

Frictional forces act in the opposite direction to a driving force.
The components of the contact force between two rough surfaces are the normal force and the friction force.
We use the model $F \leq \mu R$, where F is the frictional force, R is the normal contact force, and μ is the coefficient of friction.
Limiting friction describes the maximum friction before slipping occurs (before the system no longer remains in equilibrium) and occurs when $F = \mu R$.
We use the term smooth to describe contact without friction and rough to describe contact with friction.

23 Newton's Laws Of Motion

23.1 Newton's first law

A force is a vector and changes the velocity of an object with mass.
Newton's first law states that an object at rest or moving with a constant velocity remains at this constant velocity until acted upon by an external force.

23.2 Newton's second law

Newton's second law states that $F = ma$ for motion in a straight line of bodies with a constant mass moving under the action of a constant force. Force and acceleration may be considered as a two-dimensional vector here.

23.3 Newton's third law

Newton's third law states that every action has an equal and opposite reaction.

23.4 Connected particles

A system in which none of its components has any relative motion may be modelled as a single particle.
When an object is resting on a horizontal surface, the normal reaction force is equal and opposite to the weight.

24 Rigid Bodies

The moment of a force about a pivot is the product of the magnitude of the force and the perpendicular distance from the line of action of the force to the pivot. A system of forces may cause a turning effect on the object.
A rigid is body is in equilibrium if and only if the resultant force and resultant moment are 0.
For a uniform rod, the weight acts at the midpoint of the rod.
For a non-uniform rod, the weight acts at a given specific point or is to be

determined by moments.
For a rectangular lamina, the weight acts at the point of symmetry.