Y1P5 XMQs and MS

(Total: 33 marks)

1. P2_Sample Q8 . 7 marks - Y1P5 Straight line graphs
2. P2_2019 Q7 . 7 marks - Y1P5 Straight line graphs
3. P1(AS)_2018 Q4 . 4 marks - Y1P5 Straight line graphs
4. P1(AS)_2019 Q1 . 4 marks - Y1P5 Straight line graphs
5. P1(AS)_2019 Q4 . 5 marks - Y1P5 Straight line graphs
6. P1(AS)_2020 Q4 . 6 marks - Y1P5 Straight line graphs

8.

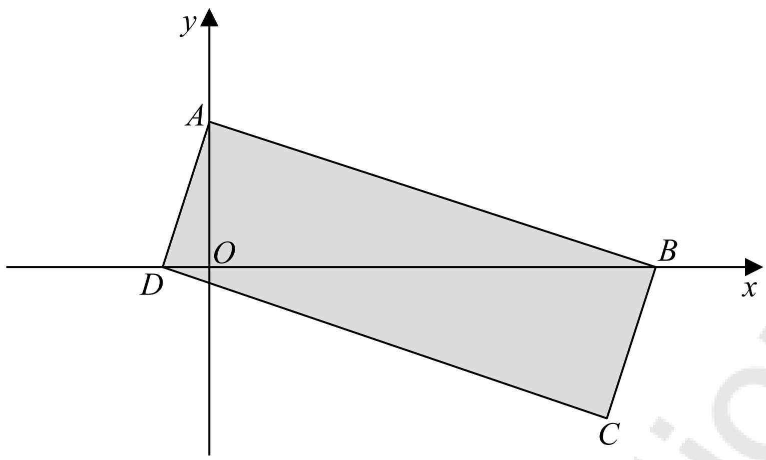

Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\frac{2}{5}$

B1: States that y coordinate of $A = 2$

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}$ '5.8'$\times$'2'

A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) **Way 1**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 2**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \frac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 3**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} **(**Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) **Way 1**	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) **Way 2**	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss = 1.12} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit = 0.04} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points (800, their 1100) and (300, their 680)
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c$, $80 = 300k + c \Rightarrow k = \frac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	**Special Case:** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

4. The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

Question	Scheme	Marks	AOs
4	States gradient of $4y-3x=10$ is $\frac{3}{4}$ oe or rewrites as $y=\frac{3}{4}x+\ldots$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
			(4 marks)

Notes

B1: States that the gradient of line l_1 is $\frac{3}{4}$ or writes l_1 in the form $y=\frac{3}{4}x+\ldots$

M1: Attempts to find the gradient of line l_2 using $\frac{\Delta y}{\Delta x}$ Condone one sign error Eg allow $\frac{9}{6}$

A1: For the gradient of l_2 $=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$ or the equation of l_2 $y=-\frac{3}{2}x+\ldots$

Allow for any equivalent such as $-\frac{9}{6}$ or -1.5

A1: CSO (on gradients)

Explains that they are neither parallel as the gradients not equal nor perpendicular as $\frac{3}{4}\times-\frac{3}{2}\neq-1$ oe

Allow a statement in words "they are not negative reciprocals " for a reason for not perpendicular and "they are not equal" for a reason for not being parallel

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

3.4	Using a model
3.5	Evaluating the outcome/ refining a model

Question	Scheme	Marks	AOs
1(a)	$2x+4y-3=0 \Rightarrow y=\mp\frac{2}{4}x+...$ Gradient of perpendicular = $\pm\frac{4}{2}$	M1	1.1b
	Either $m=2$ or $y=2x+7$	A1	1.1b
		(2)	
(b)	Combines 'their' $y=2x+7$ with $2x+4y-3=0 \Rightarrow 2x+4(2x+7)-3=0 \Rightarrow x=...$	M1	1.1b
	$x=-2.5$ oe	A1	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts to set given equation in the form $y=ax+b$ with $a=\mp\frac{2}{4}$ oe such as $\mp\frac{1}{2}$ **AND** deduces that $m=-\frac{1}{a}$ Condone errors on the "$+b$"

An alternative method is to find both intercepts to get gradient $l_1=\pm\frac{0.75}{1.5}$ and use the perpendicular gradient rule.

A1: Correct answer. Accept **either** $m=2$ **or** $y=2x+7$

This must be simplified and not left as $m=\frac{4}{2}$ or $m=2x$ unless you see $y=2x+7$.

Watch: There may be candidates who look at $\mathbf{2}x+4y-3=0$ and incorrectly state that the gradient is 2 and use the perpendicular rule to get $m=-\frac{1}{2}$ They will score M0 A0 in (a) and also no marks in (b) as the lines would be parallel. In a case like this don't allow an equation to be "altered"

Candidates who state $m=2$ or $y=2x+7$ **with no incorrect working** can score both marks

(b)

M1: Substitutes their $y=mx+7$ into $2x+4y-3=0$, condoning slips, in an attempt to form and solve an equation in *x*. Alternatively equates their $y=-\frac{1}{2}x+\frac{3}{4}$ with their $y=mx+7$ in an attempt to form and solve, condoning slips, an equation in *x*. Don't be too concerned by the mechanics of the candidates attempt to solve. (E.g. allow solutions from their calculators).

You may see $2x+4y-3=2x-y+7$ with y being found before the value of *x* appears

It cannot be awarded from "unsolvable" equations (e.g. lines that are parallel).

A1: $x=-2.5$

The answer alone can score both marks as long as both equations are correct and no incorrect working is seen.

Remember to isw after the correct answer and ignore any *y* coordinate

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

Question	Scheme	Marks	AOs
4 (a)	Attempts $H = mt + c$ with both $(3, 2.35)$ and $(6, 3.28)$	M1	3.3
	Method to find both *m* and *c*	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their '*b*'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	
			(5 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For creating a linear model with both pieces of information given.
Eg. Accept sight of $2.35 = 3m + c$ and $3.28 = 6m + c$ Condone slips on the 2.35 and 3.28.
Allow for an attempt at the "gradient" $m = \frac{3.28 - 2.35}{6 - 3} (= 0.31)$ or the intercept.
Allow for a pair of simultaneous in any variable even *x* and *y*

dM1: A full method to find both constants. For simultaneous equations award if they arrive at values for *m* and *c*.
If they attempted the gradient it would be for attempting to find "*c*" using $y = mx + c$ with their *m* and one of the points $(3, 2.35)$ or $(6, 3.28)$

A1: A correct model using allowable/correct variables. $H = 0.31t + 1.42$ Condone $h \leftrightarrow H, t \leftrightarrow T$
Allow equivalents such as $H = \frac{31}{100}t + \frac{142}{100}$, $t = \frac{H - 1.42}{0.31}$ but not $H = \frac{0.93}{3}t + 1.42$
Do not allow $H = 0.31t + 1.42$ m (with the units)

(b) To score any marks in (b) the model must be of the form $H = mt + b$ where $m > 0, b > 0$

B1ft: States or implies that 1.42 (with or without units) or 142 cm (including the units) is the original height or the height when $t = 0$
You should allow statements such as *c* = 1.42 or original height = 1.42 (m)
Follow through on their value of '*c*', so for $H = 0.25t + 1.60$ it is scored for stating the initial height is 1.60 (m) or 160 cm. Do not follow through if $c \leqslant 0$

B1ft: Compares 140 cm with their 1.42 (m) **and** makes a valid comment.
In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close or approximately the same. Allow $1.42\text{m} \approx 1.4\text{m}$ or similar
In the case of $H = 0.25t + 1.60$ it would be for stating that the fact that it does not support the use of the model as the values are too different. If they state $1.60 > 1.40$ this is insufficient. They cannot just state that they are not the same. It must be implied that there is a significant difference.
As a rule of thumb use "good model" for between 135cm and 145 cm.

This requires a correct calculation for their H, a correct statement with an appreciation shown for the units and a correct conclusion.
Notes on Question 4 continue
SC B1 B0 Award SC for incomplete answers which suggest the candidate knows what to do. Eg. In (b) $H = 0.31t + 1.42$ followed by in (c) It supports the model as when $t = 0$ it is approximately 140 cm

4. In 1997 the average CO_2 emissions of new cars in the UK was 190 g/km.

In 2005 the average CO_2 emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO_2 emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n.

(3)

In 2016 the average CO_2 emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

Question	Scheme	Marks	AOs
4 (a)	Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ Or attempts gradient eg $m = \pm\frac{190-169}{8}(=-2.625)$	M1	3.3
	Full method to find a linear equation linking *A* with *n* E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625\times19+190 = ...$	M1	3.4
	$A = 140.125$ g km^{-1}	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ considered.
Eg Accept sight of $190 = 0n + c$ or $169 = 8m + c$ or $A - 169 = m(n-8)$ or $A = 190 + mn$ where *m* could be a value.
Also accept an attempt to find the gradient $\pm\frac{190-169}{8}$ or sight of ±2.625 or $\pm\frac{21}{8}$ oe

dM1: A full method to find both constants of a linear equation
Method 1: Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously
Method 2: Uses gradient and a point Eg $m = \pm\frac{190-169}{8}(=-2.625)$ and $c = 190$
Condone different variables for this mark. Eg. *y* in terms of *x*.

A1: $A = -2.625n + 190$ or $A = -\frac{21}{8}n + 190$ oe

(b)

M1: Attempts to substitute $"n" = 19$ into their linear model to find *A*. They may call it *x* = 19
Alternatively substitutes $A = 120$ into their linear model to find *n*.

A1: $A = 140.125$ from $n = 19$ Allow $A = 140$
or $n = 26/27$ following $A = 120$

B1ft: Requires a correct calculation for their model, a correct statement and a conclusion
E.g For correct (a) $A = 140$ is (much) higher than 120 so the model is not suitable/appropriate.
Follow through on a correct statement for their equation. As a guide allow anything within [114,126] to be regarded as suitable. Anything less than 108 or more than 132 should be justified as unsuitable.

Note B0 Recorded value is not the same as/does not equal/does not match the value predicted