

Y1S7 XMQs and MS

(Total: 43 marks)

1. P31(AS)_2018 Q3 . 7 marks - Y1S7 Hypothesis testing
2. P31(AS)_2019 Q5 . 6 marks - Y1S7 Hypothesis testing
3. P31(AS)_2020 Q4 . 7 marks - Y1S1 Data collection
4. P31(AS)_2021 Q4 . 10 marks - Y1S6 Statistical distributions
5. P31(AS)_2022 Q2 . 7 marks - Y1S6 Statistical distributions
6. P31_2022 Q4 . 6 marks - Y1S6 Statistical distributions

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Qu	Scheme	Marks	AO
3	(a) Let $N =$ the number of games Naasir wins $N \sim B(15, \frac{1}{3})$	M1	3.3
	(i) $P(N = 2) = 0.059946\dots$ awrt 0.0599	A1	1.1b
	(ii) $P(N > 5) = 1 - P(N \leq 5) = 0.38162\dots$ awrt 0.382	A1	1.1b
	(b) $H_0 : p = \frac{1}{3}$ $H_1 : p > \frac{1}{3}$	(3) B1	2.5
	Let $X =$ the number of games Naasir wins $X \sim B(32, \frac{1}{3})$	M1	3.3
	$P(X \geq 16) = 1 - P(X \leq 15) = 0.03765$ (< 0.05)	A1	3.4
	[Significant result so reject H_0 (the null model) and conclude:]	A1	3.5a
	There is evidence to support Naasir's claim (o.e.)	(4)	
		(7 marks)	
	Notes		
(a)	M1 for selecting a binomial model with correct n or p Award for sight of $B(15, \frac{1}{3})$ (o.e. e.g. in words) or implied by 1 correct answer 1 st A1 for awrt 0.0599 (from a calculator). Allow 0.05995 2 nd A1 for awrt 0.382 (from a calculator)		
(b)	B1 for correctly stating both hypotheses in terms of p or π Accept $p = 0.\dot{3}$ or any exact equivalent. $H_1 : p \geq \frac{1}{3}$ is B0 M1 for selecting a suitable model to use for the test. Award for sight of $B(32, \frac{1}{3})$ (o.e. e.g. in words) or implied by 0.03765 1 st A1 for use of the model to calculate an appropriate probability using calc. Sight of $P(X \geq 16)$ and answer awrt 0.0377		
ALT	CR May use CR so award 1 st A1 for CR of $X \geq 16$ must have seen some probabilities though: 1 of $P(X \leq 15) = 0.9623$ or $P(X \leq 14) = 0.9224$ or 0.9223 2 nd A1 for conclusion in context that there is support for Naasir's claim Must mention " <u>Naasir</u> " or " <u>his</u> " and " <u>claim</u> " or " <u>method</u> " (o.e.) <u>or</u> e.g. <u>probability</u> of <u>winning</u> a game is $> \frac{1}{3}$ or has <u>increased</u> Dependent on M1 and 1 st A1 but can ignore hypotheses.		
SC	Use of 0.3 for $\frac{1}{3}$ If used 0.3 instead of $\frac{1}{3}$ in (a) and score M0A0A0 can condone use of 0.3 in (b) 1 st A1 ft needs $P(X \geq 16) = 0.0138$ <u>or</u> CR of $X \geq 15$ and sight of 1 of $P(X \geq 15) = 0.0327$ or $P(X \geq 14) = 0.0694$ 2 nd A1 as before with 0.3 instead $\frac{1}{3}$ (if appropriate)		

5. Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$$H_0 : p = 0.15$$

$$H_1 : p \geq 0.15$$

Let X = the number of customers who buy chocolate.

$$X \sim B(30, 0.15)$$

$$P(X = 8) = 0.0420$$

$$0.0420 < 0.05 \text{ so reject } H_0$$

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

- (a) Identify the first two errors that Julie has made in her hypothesis test. (2)
- (b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer. (1)
- (c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05 (2)
- (d) Find the actual level of significance of this test. (1)



Question	Scheme	Marks	AOs
5(a)	The alternative hypothesis should be $H_1 : p > 0.15$	B1	2.5
	The calculation of the test statistic should be $P(X \geq 8)$ [= 0.0698]	B1	2.3
		(2)	
(b)	These will affect the conclusion (as the null hypothesis should not be rejected) since $P(X \geq 8)$ [= 0.0698] is greater than 0.05	B1	2.4
		(1)	
(c)	$P(X \leq 8) = 0.9722... > 0.95$ or $P(X \geq 9) = 0.0277... < 0.05$	M1	2.1
	CR: $\{X \geq 9\}$	A1	1.1b
		(2)	
(d)	awrt <u>0.0278</u>	B1ft	1.1b
		(1)	
(6 marks)			
Notes			
(a)	B1: Identifying that \geq should be $>$ in the alternative hypothesis B1: Identifying that $P(X = 8)$ should be $P(X \geq 8)$ Stating $P(X = 8)$ is incorrect on its own is insufficient Check for errors identified and corrected next to the question		
(b)	B1: Will affect conclusion and correct supporting reason		
(c)	M1: For use of tables to find probability associated with critical value [$P(X \leq 8)$ or $P(X \geq 9)$ with $B(30, 0.15)$ (may be implied by either correct probability awrt 0.97 or awrt 0.03) or by the correct CR] A1: $[30 \geq] X \geq 9$ o.e. e.g. $X > 8$ Allow '9 or more' or 'CR ≥ 9 '		
(d)	B1ft: awrt 0.0278 (allow awrt 2.78%) or correct ft their one-tailed upper CR from $B(30, 0.15)$ to 3s.f.		

4. A lake contains three different types of carp.

There are an estimated 450 mirror carp, 300 leather carp and 850 common carp.

Tim wishes to investigate the health of the fish in the lake.

He decides to take a sample of 160 fish.

- (a) Give a reason why stratified random sampling cannot be used. (1)
- (b) Explain how a sample of size 160 could be taken to ensure that the estimated populations of each type of carp are fairly represented.

You should state the name of the sampling method used. (2)

As part of the health check, Tim weighed the fish.

His results are given in the table below.

Weight (w kg)	Frequency (f)	Midpoint (m kg)
$2 \leq w < 3.5$	8	2.75
$3.5 \leq w < 4$	32	3.75
$4 \leq w < 4.5$	64	4.25
$4.5 \leq w < 5$	40	4.75
$5 \leq w < 6$	16	5.5

(You may use $\sum fm = 692$ and $\sum fm^2 = 3053$)

- (c) Calculate an estimate for the standard deviation of the weight of the carp. (2)

Tim realised that he had transposed the figures for 2 of the weights of the fish.

He had recorded in the table 2.3 instead of 3.2 and 4.6 instead of 6.4

- (d) Without calculating a new estimate for the standard deviation, state what effect
- (i) using the correct figure of 3.2 instead of 2.3
- (ii) using the correct figure of 6.4 instead of 4.6

would have on your estimated standard deviation.

Give a reason for each of your answers. (2)



Question	Scheme	Marks	AOs
4(a)	It is not possible to have a sampling frame	B1	2.3
		(1)	
(b)	Quota sampling and (catch 85 common carp, 45 mirror carp and 30 leather carp) or (ignore any fish caught of a type where the quota is full)	M1	1.1a
	Quota sampling and catch 85 common carp, 45 mirror carp and 30 leather carp and ignore any fish caught of a type where the quota is full	A1	1.1b
		(2)	
(c)	$\sigma = \sqrt{\frac{3053}{160} - \left(\frac{692}{160}\right)^2}$	M1	1.1b
	= 0.6129... awrt 0.613	A1	1.1b
		(2)	
(d)(i)	This would have no effect as the piece of data would remain in the same class	B1	2.2a
(ii)	This would increase the standard deviation as change in mean is small and $6.4 - 4.6 \approx 3\sigma$ therefore estimate of standard deviation will increase	B1	2.2a
		(2)	

(7 marks)

Notes

(a)	B1:	For the idea there cannot be a sampling frame/list
(b)	M1:	Quota sampling and either for the correct numbers of each type or for the idea that if quota full ignore the fish.
	A1:	Quota sampling and both the correct numbers of each type and for the idea that if quota full ignore the fish or sample until all quotas are full
(c)	M1:	A correct expression for σ
	A1:	Awrt 0.613 allow $s = \text{awrt } 0.615$
(d)	B1:	Correct deduction with suitable explanation Allow range for class. Do not allow there is no differences
	B1:	Correct deduction with suitable explanation. so would increase the standard deviation and a suitable reason. Allow the value is bigger than any others in the table oe

Qu	Scheme	Marks	AO
4. (a)	[$R =$ no. of red beads in Aliya's bracelet] $R \sim B(18, 0.14)$	B1 (1)	3.3
(b)(i)	$P(R = 1) = 0.19403\dots$ awrt 0.194	B1	1.1b
(ii)	$P(R \dots 4) = 1 - P(R \dots 3) = 1 - [0.76184\dots]$ $= 0.2381588\dots$ awrt 0.238	M1 A1 (3)	3.4 1.1b
(c)	Requires $p = 0.14$ to be constant so need a large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable.	B1 (1)	3.5b
(d)	$H_0 : p = 0.14$ $H_1 : p \neq 0.14$ [$X =$ number of red beads in the sample] $X \sim B(75, 0.14)$ $P(X \dots 4) = 0.01506\dots$ or if $B(75, 0.14)$ seen awrt 0.02 { $0.02 < 0.025$ so significant <u>or</u> reject H_0 } There is evidence that the proportion of red beads has changed	B1 M1 A1 A1 (4)	2.5 3.3 3.4 2.2b
(e)	p -value is $2 \times "0.01506\dots" = 0.030123\dots =$ awrt 0.03	B1ft (1)	1.1b
(10 marks)			
Notes			
(a)	B1 for $B(18, 0.14)$ accept in words e.g. <u>binomial</u> with $n = 18$ and $p = 0.14$		
(b)(i)	B1 for awrt 0.194		
(ii)	M1 for interpreting "at least 4" Need $1 - P(R \dots 3)$ <u>and</u> $1 - p$ [$0 < p < 1$] $P(R = 3) = 0.233\dots$ OK A1 for awrt 0.238		
(c)	B1 for mention of <u>large number of beads</u> and need for <u>$p = 0.14$ to be constant</u> for it to be suitable. Do NOT accept e.g. "events are independent"		
(d)	B1 for both hypotheses correct with use of p or π M1 for selecting a suitable model: sight or correct use of $B(75, 0.14)$ May be implied by sight of 0.015 or better <u>or</u> [$P(X > 4) =$] 0.9849... i.e. 0.985 or better 1 st A1 for use of the correct model awrt 0.015 (accept awrt 0.02 following a correct expression) Allow 1 st A1 for awrt 0.985 <u>only if</u> correct comparison with 0.975 is seen. Sight of $B(75, 0.14)$ and $P(X \dots 4) =$ awrt 0.02 scores M1A1 <u>No sight</u> of $B(75, 0.14)$ <u>but</u> sight of awrt 0.015 scores M1(\Rightarrow)A1[Condone $P(X = 4) = \dots$] 2 nd A1 (dep on M1A1) for a correct conclusion in context mentioning "proportion", "red" and "changed"		
NB	If there is a statement about H_0 or significance it must be compatible. May see CR i.e. $X \dots 4$ (mark when prob seen) and $X \dots 18$ (prob = 0.01406...) Ignore upper limit NB for information $P(X = 4) = 0.0104\dots$ and can only score M1A0A0 if $B(75, 0.14)$ seen		
(e)	B1ft for awrt 0.03 Allow ft of their probability in (d) provided at least 3sf used NB an answer of 0.02 in (d) leading to 0.04 in (e) is B0		
SC	Use of CR will give significance level of $0.01506\dots + 0.01406\dots = 0.029\dots$ score B1 no ft		

2. A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier A will be damp.

A random sample of 35 bags of sugar is taken from supplier A .

- (a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

- (i) exactly 2
(ii) more than 3

(3)

Supplier B claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier B and finds that only 2 of the bags are damp.

- (b) Carry out a suitable test to assess supplier B 's claim.

You should state your hypotheses clearly and use a 10% level of significance.

(4)



Qu	Scheme	Mark	AO
2. (a)	[$D =$ number of bags that are damp] $D \sim B(35, 0.08)$ NB $0.08 = \frac{2}{25}$	M1	3.3
(i)	$P(D = 2) = 0.2430497\dots$ awrt 0.243	A1	3.4
(ii)	$P(D > 3) = [1 - P(D \leq 3) = 1 - 0.69397\dots] = 0.30602\dots$ awrt 0.306	A1	1.1b
		(3)	
(b)	$H_0 : p = 0.08$ $H_1 : p < 0.08$	B1	2.5
	[$X \sim$] $B(70, 0.08)$	M1	2.1
	$[P(X \leq 2)] = 0.0739756\dots$ awrt 0.074	A1	1.1b
	[$0.074 < 0.10$ so significant, reject H_0 so...]		
	there <u>is</u> evidence to <u>support</u> supplier <u>B's claim</u> (o.e.)	A1	2.2b
		(4)	
		(7 marks)	
Notes			
(a)	M1 for selecting a correct model: sight of or use of $B(35, 0.08)$ [Condone $B(0.08, 35)$ May be implied by one correct answer or sight of $P(D \leq 3) =$ awrt 0.694 (or allow 0.693) or seeing $\binom{35}{2} 0.08^2 \times (1 - 0.08)^{35-2}$ Saying $B(35, 8\%)$ without a correct calculation would score M0		
(i)	1 st A1 for awrt 0.243		
(ii)	2 nd A1 for awrt 0.306 (Condone poor use of notation e.g. $P(D = 3) = 0.306\dots$ i.e. just mark ans)		
NB	$P(D \leq 3) = 0.539$ scores 2 nd A0 but would of course score M1		
(b)	B1 for both hypotheses correct in terms of p or π [Condone 8% for 0.08] M1 for sight or correct use of $B(70, 0.08)$ [Condone $B(0.08, 70)$ May be implied by prob of 0.074 or better 1 st A1 for final answer awrt 0.074 can condone poor notation e.g. $P(X = 2) =$ awrt 0.074 Can allow this mark for CR of $X \leq 2$ provided $[P(X \leq 2)] = 0.074$ (or better) is seen [Can allow 0.07 if $X \sim B(70, 0.08)$ and $P(X \leq 2)$ are both seen] 2 nd A1 (dep on M1A1 but independent of hypotheses) for a correct inference in context Must mention <u>claim</u> or <u>B and</u> idea of <u>support for</u> ... or <u>proportion/probability</u> (of damp bags) and idea of <u>less</u> than 8% or A 2 nd A0 for contradictory statements e.g. "accept H_0 so evidence to support B 's claim" 2 nd A0 if you see $0.0739\dots < 0.08$ so significant/ reject H_0 etc		
MR	0.8 for 0.08 In (a) allow M1 for $B(35, 0.8)$ then A0A0 In (b) allow B1 for Hypotheses and M1 for $B(70, 0.8)$ seen, then A0A0		

Question	Scheme	Marks	AOs
4(a)	$H_0: p = 0.1 \quad H_1: p \neq 0.1$	B1	2.5
		(1)	
(b)	Use of $X \sim B(50, 0.1)$ implied by sight of one of awrt 0.0052 or awrt 0.9755 or awrt 0.0245	M1	3.4
	Critical regions $X = 0$ or $X \geq 10$	A1	1.1b
	$X = 0$ and $X \geq 10$ plus $P(X = 0) = \text{awrt } 0.0052$ and $P(X \geq 10) = \text{awrt } 0.0245$	A1	1.1b
	SC: Both CR correct with no probabilities and no distribution seen scores M0A1A0		
		(3)	
(c)	0.0297	B1ft	1.1b
		(1)	
(d)	15 is <u>in the critical region</u> therefore there is evidence to support the <u>manager's</u> belief	B1ft	2.2b
		(1)	
(6 marks)			
Notes			
(a)	B1	For both hypotheses in terms of p or π . Connected to H_0 and H_1 correctly Condone 10% but not 10	
(b)	M1	Using correct distribution to find the probability associated with one tail of the CR If the correct distribution is <u>stated</u> (may be seen in part(a)) allow for one tail of the correct CR or one of (awrt 0.025 or awrt 0.005 or awrt 0.975) seen connected to a correct probability statement	
	A1	Lower CR $X = 0 / X < 1 / X \leq 0 /$ [condone eg $P(X = 0)$ labelled as CR] Or Upper CR $X \geq 10$ or $X > 9$ [condone $P(X \geq 10)$ oe labelled as CR]	
	A1	Both CR's correct with the relevant probabilities Allow \cup for "and" and $X > 9, X < 1, X \leq 0$ [do not allow $P(X = 0)$ or $P(X \geq 10)$ oe] Allow CR in different form eg $(9, \infty), [10, \infty)$	
(c)	B1ft	awrt 0.0297 or 2.97% or ft for the sum of the probabilities in (b) for "their 2 critical regions" if seen. If none seen it must be awrt 0.0297 SC M0 in (b) for a one tail test Allow B1ft for their one tail CR in (b) eg 0.0338 or 0.0245 or 0.0579	
(d)	B1ft	A correct statement about 15 and "their CR" or sight $P(X \geq 15) = 0.0000738\dots$ and comparison with "their 0.0245" and a compatible correct statement in context. eg There is evidence that there has been a change in the <u>proportion/probability</u> arriving <u>late</u> Condone increase rather than change Do not allow contradicting statements. NB No CR given in (b) then B0	