

Y2M4 XMQs and MS

(Total: 45 marks)

1. P3_2018 Q9 . 13 marks - Y2M4 Moments
2. P32_2019 Q4 . 11 marks - Y2M4 Moments
3. P32_2020 Q4 . 10 marks - Y2M4 Moments
4. P32_2022 Q4 . 11 marks - Y2M4 Moments

Mathvault.io Solutions

9.

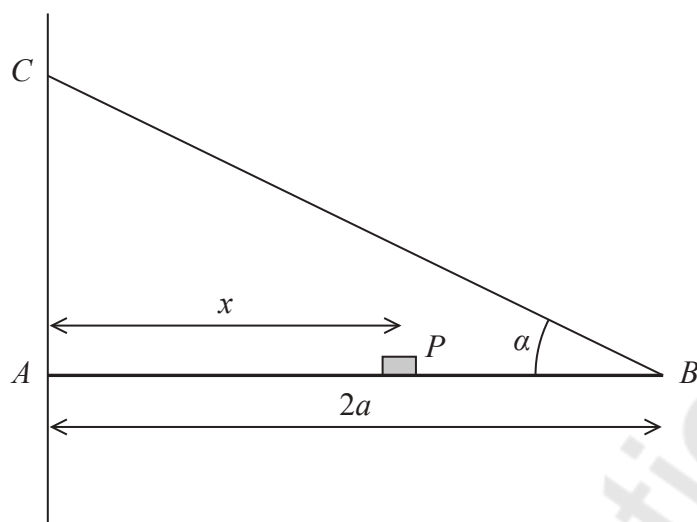


Figure 3

A plank, AB , of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C , which is vertically above A .

A small block of mass $3M$ is placed on the plank at the point P , where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$ (3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a . (2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan \beta$ (5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P . You must justify your answer carefully. (3)



Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a \sin \alpha = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a + 3x)}{2a \cdot \frac{3}{5}} = \frac{5Mg(3x + a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x + a)}{6a} \cos \alpha = 2Mg$ OR $2Mg \cdot 2a \tan \alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically OR Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3 \cdot \frac{2a}{3} + a)}{6a} \sin \alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ Or: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos \alpha}\right) \sin \alpha$	A1ft	1.1b
	$Y = \frac{5Mg}{2}$ N.B. May use $R \sin \beta$ for Y and/or $R \cos \beta$ for X throughout	A1	1.1b
	$\tan \beta = \frac{Y}{X}$ or $\frac{R \sin \beta}{R \cos \beta} = \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x + a)}{6a} \leq 5Mg$ and solve for x	M1	2.4
	$x \leq \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \leq \frac{5a}{3}$, if no incorrect statement seen. N.B. If the correct inequality is not found, their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	
(13 marks)			

Notes:

(a)

M1: Using $M(A)$, with usual rules, or any other complete method to obtain an equation in a , M , x and T only.

A1: Correct equation

A1*: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.

(b)

M1: Using an appropriate strategy to find x . e.g. Resolve horizontally with usual rules applying OR Moments about C . Must use the given expression for T .

A1: Accept $0.67a$ or better

(c)

M1: Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B , with usual rules

A1 ft: Correct equation with their x substituted in T expression or using $T = \frac{2Mg}{\cos\alpha}$

A1: Y (or $R\sin\beta$) = $\frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$

M1: For finding an equation **in $\tan\beta$ only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$

This is independent but must have found a Y .

A1: Accept $\frac{-5}{4}$ if it follows from their working.

(d)

M1: Allow $T = 5Mg$ or $T < 5Mg$ and solves for x , showing all necessary steps (M0 for $T > 5Mg$)

A1: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better.

B1: Treat as A1. For any appropriate equivalent fully correct comment or statement. E.g. maximum value of

x is $\frac{5a}{3}$

4.

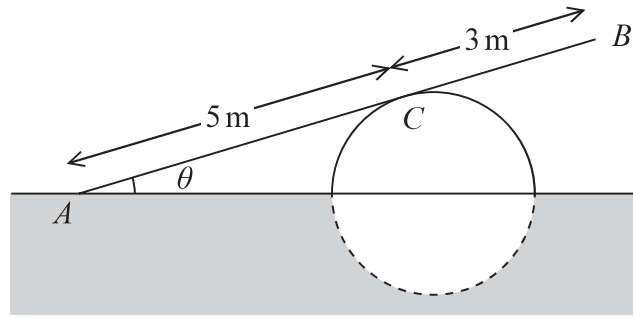


Figure 2

A ramp, AB , of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A .

The point of contact between the ramp and the drum is C , where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan \theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp. (1)

(b) Find the magnitude of the resultant force acting on the ramp at A . (9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B ,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C . (1)



Question	Scheme	Marks	AO
4(a)	Drum smooth , or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	N.B. In (b), for a moments equation, if there is an extra $\sin \theta$ or $\cos \theta$ on a length, give M0 for the equation e.g. $M(A): 20g \times 4 \cos \theta = 5N \sin \theta$ would be given M0A0		
	Possible equns	M1	3.3
	(↗): $F \cos \theta + R \sin \theta = 20g \sin \theta$	A1	1.1b
	(↖): $N + R \cos \theta = 20g \cos \theta + F \sin \theta$	M1	3.4
	(↑) $R + N \cos \theta = 20g$	A1	1.1b
	(→) $F = N \sin \theta$	M1	3.4
	$M(A): 20g \times 4 \cos \theta = 5N$		
	$M(B): 3N + R \times 8 \cos \theta = F \times 8 \sin \theta + 20g \times 4 \cos \theta$		
	$M(C): R \times 5 \cos \theta = F \times 5 \sin \theta + 20g \times \cos \theta$	A1	1.1b
	$M(G): R \times 4 \cos \theta = F \times 4 \sin \theta + N$		
	(The values of the 3 unknowns are: $N = 150.528; F = 42.14784; R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y)	M1	3.3
	Possible equations:		
	(↗): $X = 20g \sin \theta$	A1	1.1b
	(↖): $Y + N = 20g \cos \theta$	M1	3.4
	(↑): $X \sin \theta + Y \cos \theta + N \cos \theta = 20g$		
	(→): $X \cos \theta = Y \sin \theta + N \sin \theta$	A1	1.1b
	$M(A): 20g \times 4 \cos \theta = 5N$		
	$M(B): 20g \times 4 \cos \theta = 8Y + 3N$	M1	3.4
	$M(C): 20g \times \cos \theta = 5Y$		
	$M(G): 4Y = N \times 1$	A1	1.1b
	(The values of the 3 unknowns are: $N = 150.528; X = 54.88; Y = 37.632$)		

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		<p>Generally 3 independent equations required so at least one moments equation.: M1A1M1A1M1A1.</p> <p>More than 3 equations, give marks for the best 3. For each:</p> <p>M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion.</p> <p>A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. $\cos(24/25)$ if they recover</p> <p><u>Enter marks on ePEN in order in which equations appear.</u></p> <p>N.B. If reaction at C is not perpendicular to the ramp, can only score marks for M(C)</p> <p>Allow use of (μR) for F</p>
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find F and R using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find X and Y using only TWO equations, the 1 st and 7 th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find H and S using only TWO equations, the 1 st and 7 th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark <u>but must use 3 equations</u> (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an <u>independent</u> M mark but must have found a value for F (or X) and a value for R (or Y)) OR a complete method to find magnitude e.g. cosine rule but must have found a value for H and a value for S
	A1	Correct answer only
	B1	Ignore reasons

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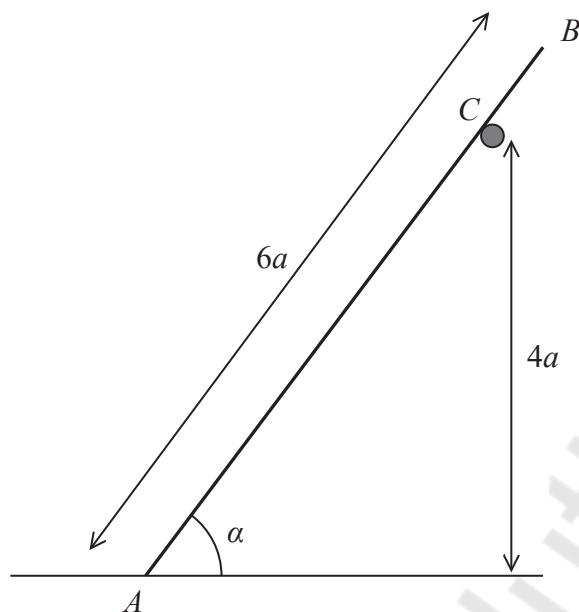


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C .

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$ (3)

(b) Hence, or otherwise, find the value of μ . (7)



Question	Scheme	Marks	AOs
4(a)	Take moments about A	M1	3.3
	$N \times \frac{4a}{\sin \alpha} = Mg \times 3a \cos \alpha$	A1	1.1b
	$\frac{9Mg}{25} *$	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow) F = \frac{9Mg}{25} \sin \alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow) R + \frac{9Mg}{25} \cos \alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R \cos \alpha + \frac{9Mg}{25} = Mg \cos \alpha + F \sin \alpha$ $(\nearrow), Mg \sin \alpha = F \cos \alpha + R \sin \alpha$ M(C), $Mg \cdot 2a \cos \alpha + F \cdot 5a \sin \alpha = R \cdot 5a \cos \alpha$ M(G), $\frac{9Mg}{25} \cdot 2a + F \cdot 3a \sin \alpha = R \cdot 3a \cos \alpha$ M(B), $Mg \cdot 3a \cos \alpha + F \cdot 6a \sin \alpha = R \cdot 6a \cos \alpha + \frac{9Mg}{25} a$ $(F = \frac{36Mg}{125}, R = \frac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow), Y + \frac{9Mg}{25} = Mg \cos \alpha + X \sin \alpha$ $(\nearrow), Mg \sin \alpha = X \cos \alpha$ $(\uparrow), \frac{9Mg}{25} \cos \alpha + Y \cos \alpha = Mg$ $(\rightarrow), Y \sin \alpha + \frac{9Mg}{25} \sin \alpha = X$		

		$M(C), Mg.2a \cos \alpha + X.5a \sin \alpha = Y.5a$ $M(G), \frac{9Mg}{25}.2a + X.3a \sin \alpha = Y.3a$ $M(B), Mg.3a \cos \alpha + X.6a \sin \alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y \sin \alpha = \mu Y \cos \alpha$ Eliminate X and Y and solve for μ	M1A1 M1A1 M1 M1		
		$\mu = \frac{18}{49}$ (0.3673.....accept 0.37 or better)		A1	2.2a
				(7)	
(10 marks)					
Notes:					
4a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of: $\frac{4a}{\sin \alpha}, \frac{4a}{\cos \alpha}, 5a$ but use of any of: $6a, 5a \sin \alpha, 4a \cos \alpha, \dots$ or anything involving $\tan \alpha$ is M0 Also M0 if no a 's in their first equation.			
	A1	Correct equation, trig does not need to be substituted			
	A1*	Given answer correctly obtained.			
4b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors			
	A1	Correct equation, trig does not need to be substituted but N does.			
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors			
	A1	Correct equation, trig does not need to be substituted but N does.			
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.			
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)			
	M1	Must have 3 equations (and all 3 previous M marks)			
	A1	Accept 0.37 or better			

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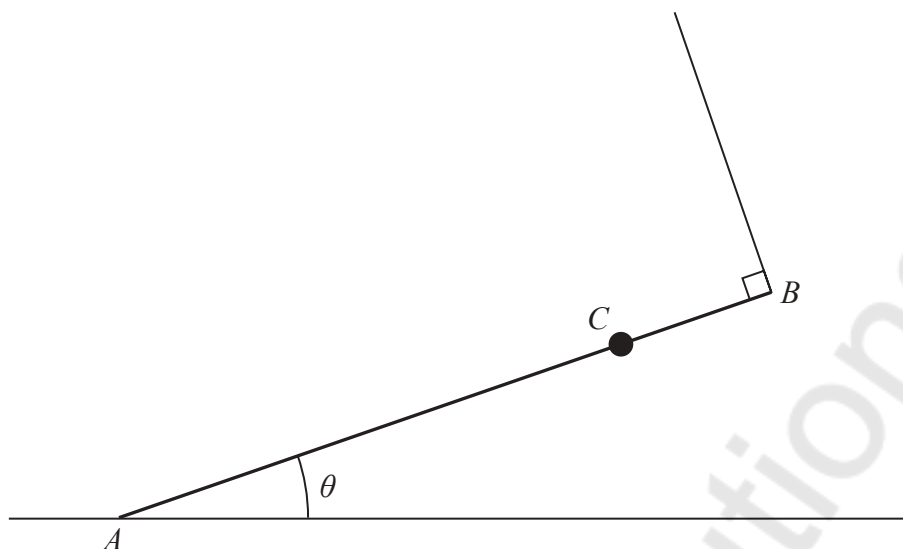


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C , where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

- (a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

- (b) Show that $T = 2Mg \cos \theta$

(3)

Given that $\cos \theta = \frac{3}{5}$

- (c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

- (d) show that $\mu = \frac{8}{19}$

(4)



Question	Scheme	Marks	AOs
4(a)	The horizontal component of T acts to the left and since the only other horizontal force is friction, it must act to the right oe	B1	2.4
		(1)	
4(b)	Take moments about A or any other complete method to obtain an equation in T, M and θ only. (see possible equations below that they may use)	M1	3.1b
	$T.2a = Mga \cos \theta + 2Mg \times 1.5a \cos \theta$ (A0 if a 's missing)	A1	1.1b
	Other possible equations but F and R would need to be eliminated. $(\nwarrow), R \cos \theta + T = F \sin \theta + Mg \cos \theta + 2Mg \cos \theta$ $(\nearrow), R \sin \theta + F \cos \theta = Mg \sin \theta + 2Mg \sin \theta$ $(\rightarrow), F = T \sin \theta$ $M(B), R.2a \cos \theta = Mga \cos \theta + 2Mg \times 0.5a \cos \theta + F.2a \sin \theta$ $M(G), Fa \sin \theta + Ta = Ra \cos \theta + 2Mg \times 0.5a \cos \theta$ $M(C), R \times 1.5a \cos \theta = T \times 0.5a + Mg \times 0.5a \cos \theta + F \times 1.5a \sin \theta$		
	$T = 2Mg \cos \theta^*$	A1*	1.1b
		(3)	
4(c)	e.g. Resolve vertically	M1	3.4
	$(\uparrow), R + T \cos \theta = Mg + 2Mg$	A1	1.1b
	$R = \frac{57Mg}{25}^*$	A1*	1.1b
		(3)	
	Other possible equations but F would need to be eliminated. $(\nwarrow), R \cos \theta + T = F \sin \theta + Mg \cos \theta + 2Mg \cos \theta$ $(\nearrow), R \sin \theta + F \cos \theta = Mg \sin \theta + 2Mg \sin \theta$ $(\rightarrow), F = T \sin \theta$ $M(B), R.2a \cos \theta = Mga \cos \theta + 2Mg \times 0.5a \cos \theta + F.2a \sin \theta$ $M(G), Fa \sin \theta + Ta = Ra \cos \theta + 2Mg \times 0.5a \cos \theta$ $M(C), R \times 1.5a \cos \theta = T \times 0.5a + Mg \times 0.5a \cos \theta + F \times 1.5a \sin \theta$		
4(d)	Find an equation containing F e.g. Resolve horizontally	M1	3.4
	$(\rightarrow), F = T \sin \theta$	A1	1.1b
	Other possible equations		

		$(\nwarrow), R \cos \theta + T = F \sin \theta + Mg \cos \theta + 2Mg \cos \theta$ $(\nearrow), R \sin \theta + F \cos \theta = Mg \sin \theta + 2Mg \sin \theta$ $(\rightarrow), F = T \sin \theta$ $M(B), R.2a \cos \theta = Mga \cos \theta + 2Mg \times 0.5a \cos \theta + F.2a \sin \theta$ $M(G), Fa \sin \theta + Ta = Ra \cos \theta + 2Mg \times 0.5a \cos \theta$ $M(C), R \times 1.5a \cos \theta = T \times 0.5a + Mg \times 0.5a \cos \theta + F \times 1.5a \sin \theta$		
		$F = \mu R$ used i.e. both F and R are substituted.	M1	3.1b
		$\mu = \frac{8}{19} *$	A1*	2.2a
			(4)	
(11 marks)				
Notes:				
4a	B1	Any equivalent explanation		
4b	M1	Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors		
	A1	Correct equation, trig does not need to be substituted (Allow: $T.2a = Mga \cos \theta + 3Mga \cos \theta$)		
	A1*	Given answer correctly obtained with <u>no wrong working seen</u> . Allow $2Mg \cos \theta = T$ But not $T = 2 \cos \theta Mg$		
4c	M1	For an equation in R, M, T and θ only Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved		
	A1	Correct equation, T and trig do not need to be substituted		
	A1*	Given answer correctly obtained with <u>no wrong working seen</u>		
4d	M1	For any equation with F in it Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved		
	A1	Correct equation, trig does not need to be substituted		
	M1	Must be used i.e M0 if merely quoting it.		
	A1*	Given answer correctly obtained with <u>no wrong working seen</u>		