

Y2P1 XMQs and MS

(Total: 30 marks)

1. P2_Specimen Q14. 8 marks - Y2P1 Algebraic methods
2. P2_2018 Q11. 7 marks - Y2P1 Algebraic methods
3. P1_2020 Q16. 4 marks - Y2P1 Algebraic methods
4. P1_2021 Q15. 6 marks - Y2P1 Algebraic methods
5. P1_2022 Q7 . 5 marks - Y2P1 Algebraic methods

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14. (i) Kayden claims that

$$3^x \geq 2^x$$

Determine whether Kayden's claim is always true, sometimes true or never true, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number.

(6)

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Question	Scheme	Marks	AOs
14 (i)	For an explanation or statement to show when the claim $3^x \dots 2^x$ fails This could be e.g. <ul style="list-style-type: none"> when $x = -1$, $\frac{1}{3} < \frac{1}{2}$ or $\frac{1}{3}$ is not greater than or equal to $\frac{1}{2}$ when $x < 0$, $3^x < 2^x$ or 3^x is not greater than or equal to 2^x 	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \dots 2^x$ is true. This could be e.g. <ul style="list-style-type: none"> $x = 2$, $9 \dots 4$ or 9 is greater than or equal to 4 when $x \dots 0$, $3^x \dots 2^x$ and a correct conclusion. E.g. <ul style="list-style-type: none"> so the claim $3^x \dots 2^x$ is sometimes true 	A1	2.4
		(2)	
(ii)	Assume that $\sqrt{3}$ is a rational number So $\sqrt{3} = \frac{p}{q}$, where p and q integers, $q \neq 0$, and the HCF of p and q is 1	M1	2.1
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$	M1	1.1b
	$\Rightarrow p^2$ is divisible by 3 and so p is divisible by 3	A1	2.2a
	So $p = 3c$, where c is an integer From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$	M1	2.1
	$\Rightarrow q^2 = 3c^2 \Rightarrow q^2$ is divisible by 3 and so q is divisible by 3	A1	1.1b
	As both p and q are both divisible by 3 then the HCF of p and q is not 1 This contradiction implies that $\sqrt{3}$ is an irrational number	A1	2.4
		(6)	
(8 marks)			

Question 14 Notes:	
(i)	
M1:	See scheme
A1:	See scheme
(ii)	
M1:	Uses a method of proof by contradiction by initially assuming that $\sqrt{3}$ is rational and expresses $\sqrt{3}$ in the form $\frac{p}{q}$, where p and q are correctly defined.
M1:	Writes $\sqrt{3} = \frac{p}{q}$ and rearranges to make p^2 the subject
A1:	Uses a logical argument to prove that p is divisible by 3
M1:	Uses the result that p is divisible by 3, (to construct the initial stage of proving that q is also divisible by 3), by substituting $p = 3c$ into their expression for p^2
A1:	Hence uses a correct argument, in the same way as before, to deduce that q is also divisible by 3
A1:	Completes the argument (as detailed on the scheme) that $\sqrt{3}$ is irrational.
	Note: All the previous 5 marks need to be scored in order to obtain the final A mark.

11.

$$\frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \equiv A + \frac{B}{(x - 3)} + \frac{C}{(1 - 2x)}$$

(a) Find the values of the constants A , B and C .

(4)

$$f(x) = \frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \quad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

(3)



Question	Scheme	Marks	AOs
11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using $-10x+10 \equiv B(1-2x) + C(x-3)$	A1	1.1b
	(4)		
(b)	$f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}$ { $= 3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$ }; $x > 3$		
	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2} \left\{ = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2} \right\}$	M1 A1ft	2.1 1.1b
	Correct $f'(x)$ and as $(x-3)^2 > 0$ and $(1-2x)^2 > 0$, then $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
(7 marks)			
Notes for Question 11			
(a)			
M1:	Way 1: Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3)$ in a complete method to find values for B and C . Note: Allow one slip in copying $1+11x-6x^2$ Way 2: Uses a correct identity $-10x+10 \equiv B(1-2x) + C(x-3)$ (which has been found from long division) in a complete method to find values for B and C		
B1:	$A = 3$		
M1:	Attempts to find the value of either B or C from their identity This can be achieved by either substituting values into their identity or by comparing coefficients and solving the resulting equations simultaneously		
A1:	See scheme		
Note:	Way 1: Comparing terms: $x^2: -6 = -2A$; $x: 11 = 7A - 2B + C$; constant: $1 = -3A + B - 3C$ Way 1: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4$; $x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$		
Note:	Way 2: Comparing terms: $x: -10 = -2B + C$; constant: $10 = B - 3C$ Way 2: Substituting: $x = 3: -20 = -5B \Rightarrow B = 4$; $x = \frac{1}{2}: 5 = -\frac{5}{2}C \Rightarrow C = -2$		

Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4, C=-2$ will get 1 st M0, 2 nd M1, 1 st A0
Note:	Way 1: You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	Way 2: You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\frac{-10x+10}{(x-3)(1-2x)} \equiv \frac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{f'(x) = \} \pm \lambda(x-3)^{-2} \pm \mu(1-2x)^{-2}; \lambda, \mu \neq 0$
A1ft:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $f'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2}; (\text{their } B), (\text{their } C) \neq 0$
A1:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$ and a correct explanation e.g. $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A = \dots$ or from $A = 0$ or no value of A found in part (a)

Notes for Question 11 Continued - Alternatives

(a)			
Note:	Be aware of the following alternative solutions, by initially dividing by " $(x-3)$ " or " $(1-2x)$ "		
	$\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{-6x-7}{(1-2x)} - \frac{20}{(x-3)(1-2x)} \equiv 3 - \frac{10}{(1-2x)} - \frac{20}{(x-3)(1-2x)}$ $\frac{20}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4, E = -8$ $\Rightarrow 3 - \frac{10}{(1-2x)} - \left(\frac{-4}{(x-3)} + \frac{-8}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$		
	$\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{3x-4}{(x-3)} + \frac{5}{(x-3)(1-2x)} \equiv 3 + \frac{5}{(x-3)} + \frac{5}{(x-3)(1-2x)}$ $\frac{5}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1, E = -2$ $\Rightarrow 3 + \frac{5}{(x-3)} + \left(\frac{-1}{(x-3)} + \frac{-2}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$		
(b)			
	Alternative Method 1:		
	$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)}, x > 3 \Rightarrow f(x) = \frac{1+11x-6x^2}{-2x^2+7x-3}; \begin{cases} u = 1+11x-6x^2 & v = -2x^2+7x-3 \\ u' = 11-12x & v' = -4x+7 \end{cases}$		
	$f'(x) = \frac{(-2x^2+7x-3)(11-12x) - (1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $f'(x)$	M1
		Correct differentiation	A1
	$f'(x) = \frac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $f'(x) = -\frac{(+ve)}{(+ve)} < 0$, so $f(x)$ is a decreasing {function}		A1
	Alternative Method 2:		
	Allow M1A1A1 for the following solution: Given $f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)} = 3 + \frac{4}{(x-3)} + \frac{2}{(2x-1)}$ as $\frac{4}{(x-3)}$ decreases when $x > 3$ and $\frac{2}{(2x-1)}$ decreases when $x > 3$ then $f(x)$ is a decreasing {function}		

16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p + q)(2p - q) = 25$	M1	2.1
	If true then $2p + q = 25$ or $2p + q = 5$ $2p - q = 1$ or $2p - q = 5$ Award for deducing either of the above statements	M1	2.2a
	Solutions are $p = 6.5, q = 12$ or $p = 2.5, q = 0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $2p + q = 25$ or $2p + q = 5$
 $2p - q = 1$ or $2p - q = 5$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $2p + q = 1$ or $2p - q = 25$ as this could not occur for positive p and q .

A1: For correctly solving one of the given statements,

For $2p + q = 25$ candidates only really need to proceed as far as $p = 6.5$ to show the contradiction.
For $2p - q = 1$

For $2p + q = 5$ candidates only really need to find either p or q to show the contradiction.
For $2p - q = 5$

Alt for $2p + q = 5$ candidates could state that $2p + q \neq 2p - q$ if p, q are positive integers.
For $2p - q = 5$

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question	Scheme	Marks	AOs
15(i)	$n = 1, 2^3 = 8, 3^1 = 3, (8 > 3)$ $n = 2, 3^3 = 27, 3^2 = 9, (27 > 9)$ $n = 3, 4^3 = 64, 3^3 = 27, (64 > 27)$ $n = 4, 5^3 = 125, 3^4 = 81, (125 > 81)$	M1	2.1
	So if $n \leq 4, n \in \mathbb{N}$ then $(n + 1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m = (2p \pm 1)$ and attempt $m^3 + 5 = (2p \pm 1)^3 + 5 = \dots$	M1	2.1
	$= 8p^3 + 12p^2 + 6p + 6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion <ul style="list-style-type: none"> reason for $8p^3 + 12p^2 + 6p + 6$ being even acceptable statement such as "this is a contradiction so if $m^3 + 5$ is odd then m must be even" 	A1	2.4
		(4)	
(6 marks)			
Notes			

(i)

M1: A full and rigorous argument that uses all of $n = 1, 2, 3$ and 4 in an attempt to prove the given result. Award for attempts at both $(n + 1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27 > 9$

Extra values, say $n = 0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for $n = 1, 2, 3$ and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept \checkmark or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m = 2p \pm 1$ and attempting to expand $(2p \pm 1)^3 + 5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p + 1)^3 + 5 = 8p^3 + 12p^2 + 6p + 6$ and **states** even.

Alternatively reaches $(2p - 1)^3 + 5 = 8p^3 - 12p^2 + 6p + 4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3 + 12p^2 + 6p + 6$ or $8p^3 - 12p^2 + 6p + 4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3 - 12p^2 + 6p + 4 = 2(4p^3 - 6p^2 + 3p + 2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if $m^3 + 5$ is odd then m is even"
- "this is contradiction, so proven."
- "So if $m^3 + 5$ is odd then m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a

counter example to the statement "if $m^3 + 5$ is odd then m must be even" such as when $m = \sqrt[3]{2}$ then they can score special case mark B1

7. (i) Given that p and q are integers such that

pq is even

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

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Question	Scheme	Marks	AOs
7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p = 2m + 1$ and $q = 2n + 1$ and then attempts $pq = (2m + 1)(2n + 1) = \dots$	M1	1.1b
	Obtains $pq = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so "if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x + y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
(5 marks)			
Notes:			

(i)

B1: For using the "correct"/ allowable language in setting up the contradiction.

Expect to see a minimum of

- "assume" or "let" or "there is " or other similar words
- " pq is even" and " p and q are (both) odd"

M1: Uses a correct algebraic form for p and q and attempting to multiply.

Allow any correct form so $p = 2n + 1$ and $q = 2m + 3$ would be fine to use

Different variables must be used for p and q , so $p = 2n + 1$ and $q = 2n - 1$ would be M0

A1*: Full argument .

This requires (1) a correct calculation for their pq

(2) a correct reason and conclusion that it is odd

$$\text{E.g. } (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1 = \text{odd}$$

$$\text{E.g. } (2m - 1)(2n + 1) = 4mn + 2m - 2n - 1 = \text{even} + \text{even} - \text{even} - 1 = \text{odd}$$

and (3) a minimal statement implying that they have proven what was required which could be QED, proven etc

Note that B0 M1 A1 is possible

(ii)

M1: For multiplying out and cancelling terms before proceeding to a correct intermediate line such as

$$2xy < 8x^2 \text{ o.e. such as } 2x(4x - y) > 0$$

A1*: Full and rigorous proof with reason shown as to why inequality reverses. The point at which it reverses must be correct and a correct reason given

See scheme

$$\text{Alt: } 2xy < 8x^2 \Rightarrow xy - 4x^2 < 0 \Rightarrow x(y - 4x) < 0$$

$$\text{as } x < 0, (y - 4x) > 0 \Rightarrow y > 4x \text{ scores M1 A1}$$

So, the following should be scored M1 A0 as line 3 is incorrect

$$2xy - 8x^2 < 0$$

$$\Rightarrow 2xy < 8x^2$$

$$\Rightarrow y < 4x$$

$$\Rightarrow y > 4x \text{ as } x < 0$$

There should be no incorrect lines in their proof