Y2P5 XMQs and MS

(Total: 45 marks)

1. P1_Sample Q2 . 5 marks - Y2P5 Radians
2. P2_Specimen Q1 . 4 marks - Y2P5 Radians
3. P2_Specimen Q2 . 5 marks - Y2P5 Radians
4. P1_2018 Q1 . 3 marks - Y2P5 Radians
5. P1_2018 Q3 . 4 marks - Y2P5 Radians
6. P1_2019 Q2 . 5 marks - Y1P4 Graphs and transformations
7. P2_2019 Q3 . 3 marks - Y2P5 Radians
8. P1_2020 Q11. 8 marks - Y2P5 Radians
9. P2_2021 Q4 . 3 marks - Y2P5 Radians
10. P2_2021 Q6 . 5 marks - Y2P5 Radians

2.

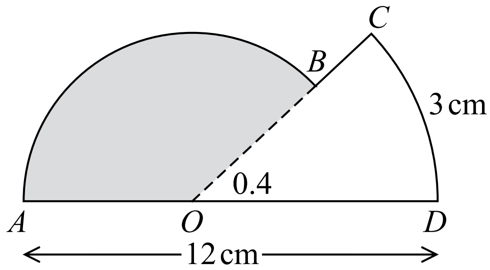

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, ∠*COD* = 0.4 radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

(Total for Question 2 is 5 marks)

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5 \text{ cm}$	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
		(5 marks)	

Notes:

(a)

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: *OD* = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of *AOB* = awrt 2.74 or uses radius is (12 – their '7.5')

M1: Follow through on their radius (12 – their *OD*) and their angle

A1ft: Allow awrt 27.8 cm^2. (Answer 27.75862562). Follow through on their (12 – their '7.5') Note: Do not follow through on a radius that is negative.

Answer ALL questions. Write your answers in the spaces provided.

1.

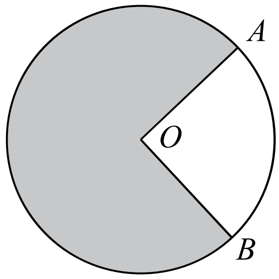

Figure 1

Figure 1 shows a circle with centre O. The points A and B lie on the circumference of the circle.

The area of the major sector, shown shaded in Figure 1, is $135\,\text{cm}^2$.

The reflex angle AOB is 4.8 radians.

Find the exact length, in cm, of the minor arc AB, giving your answer in the form $a\pi + b$, where a and b are integers to be found.

(4)

9MA0/02: Pure Mathematics Paper 2 Mark scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of minor arc $= 7.5(2\pi - 4.8)$	dM1	3.1a
	$= 15\pi - 36 \quad \{a = 15, b = -36\}$	A1	1.1b
		(4)	
1 Alt	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of major arc $= 7.5(4.8)\ \{= 36\}$		
	length of minor arc $= 2\pi(7.5) - 36$	dM1	3.1a
	$= 15\pi - 36 \quad \{a = 15, b = -36\}$	A1	1.1b
		(4)	
		(4 marks)	

Question 1 Notes:

M1:	Applies formula for the area of a sector with $\theta = 4.8$; i.e. $\frac{1}{2}r^2\theta$ with $\theta = 4.8$ **Note:** Allow M1 for considering ratios. E.g. $\frac{135}{\pi r^2} = \frac{4.8}{2\pi}$
A1:	Uses a correct equation $\left(\text{e.g. } \frac{1}{2}r^2(4.8) = 135\right)$ to obtain a radius of 7.5
dM1:	Depends on the previous M mark. A complete process for finding the length of the minor arc *AB*, by either • (their r)$\times(2\pi - 4.8)$ • 2π(their r) – (their r)(4.8)
A1:	Correct exact answer in its simplest form, e.g. $15\pi - 36$ or $-36 + 15\pi$

2. (a) Given that θ is small, use the small angle approximation for $\cos\theta$ to show that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$$

(3)

Adele uses $\theta = 5°$ to test the approximation in part (a).

Adele's working is shown below.

Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$

(b) (i) Identify the mistake made by Adele in her working.

(ii) Show that $8 - 5\theta^2$ can be used to give a good approximation to $1 + 4\cos\theta + 3\cos^2\theta$ for an angle of size 5°

(2)

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Question	Scheme	Marks	AOs
2(a)	Attempts to substitute $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into either $1 + 4\cos\theta$ or $3\cos^2\theta$	M1	1.1b
	$1 + 4\cos\theta + 3\cos^2\theta \approx 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \frac{1}{2}\theta^2\right)^2$		
	$= 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$	M1	1.1b
	$= 1 + 4 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$		
	$= 8 - 5\theta^2$ *	A1*	2.1
		(3)	
(b)(i)	E.g. • Adele is working in degrees and not radians • Adele should substitute $\theta = \frac{5\pi}{180}$ and not $\theta = 5$ into the approximation	B1	2.3
(b)(ii)	$8 - 5\left(\frac{5\pi}{180}\right)^2$ = awrt 7.962, so $\theta = 5°$ gives a good approximation.	B1	2.4
		(2)	
			(5 marks)

Question 2 Notes:

(a)(i)	
M1:	See scheme
M1:	Substitutes $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into $1 + 4\cos\theta + 3\cos^2\theta$ and attempts to apply $\left(1 - \frac{1}{2}\theta^2\right)^2$
	Note: It is not a requirement for this mark to write or refer to the term in θ^4
A1*:	Correct proof with no errors seen in working.
	Note: It is not a requirement for this mark to write or refer to the term in θ^4
(a)(ii)	
B1:	See scheme
(b)(i)	
B1:	See scheme
(b)(ii)	
B1:	Substitutes $\theta = \frac{5\pi}{180}$ or $\frac{\pi}{36}$ into $8 - 5\theta^2$ to give awrt 7.962 ***and*** an appropriate conclusion.

Answer ALL questions. Write your answers in the spaces provided.

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.
See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$$\to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} \text{ and attempts to simplify.}$$

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ
Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

....

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2\times(2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$
M1 For both of the above and attempts to simplify to a single term.
A1 $\frac{4}{3}$ oe

3.

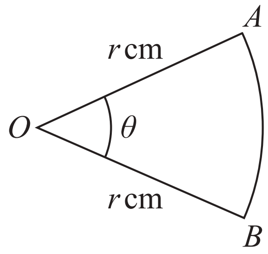


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O* and radius *r* cm.

The angle *AOB* is θ radians.
The area of the sector *AOB* is $11\,\text{cm}^2$

Given that the perimeter of the sector is 4 times the length of the arc *AB*, find the exact value of *r*.

(4)

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $...r = ...r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

....

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

2.

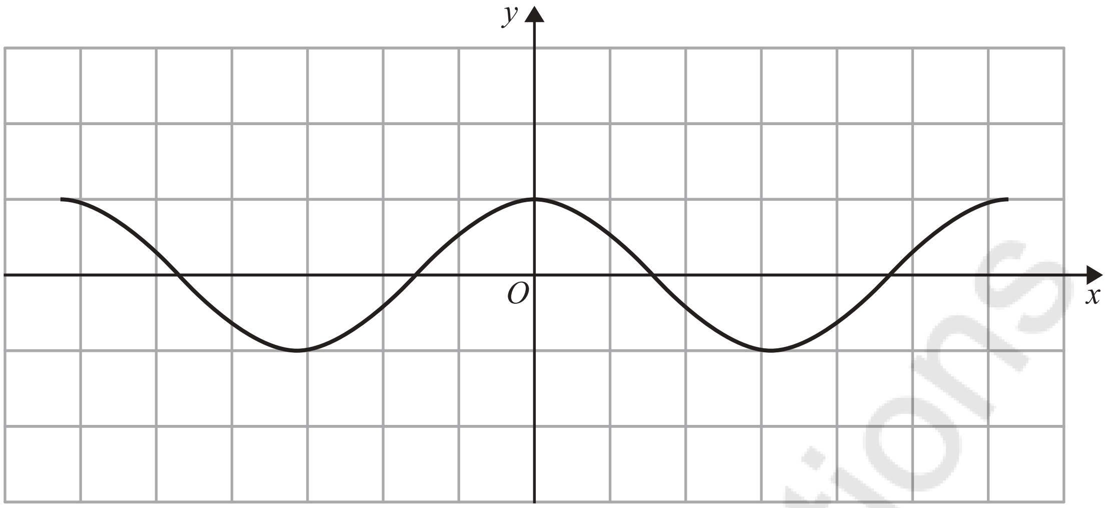


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

Question 2 continued

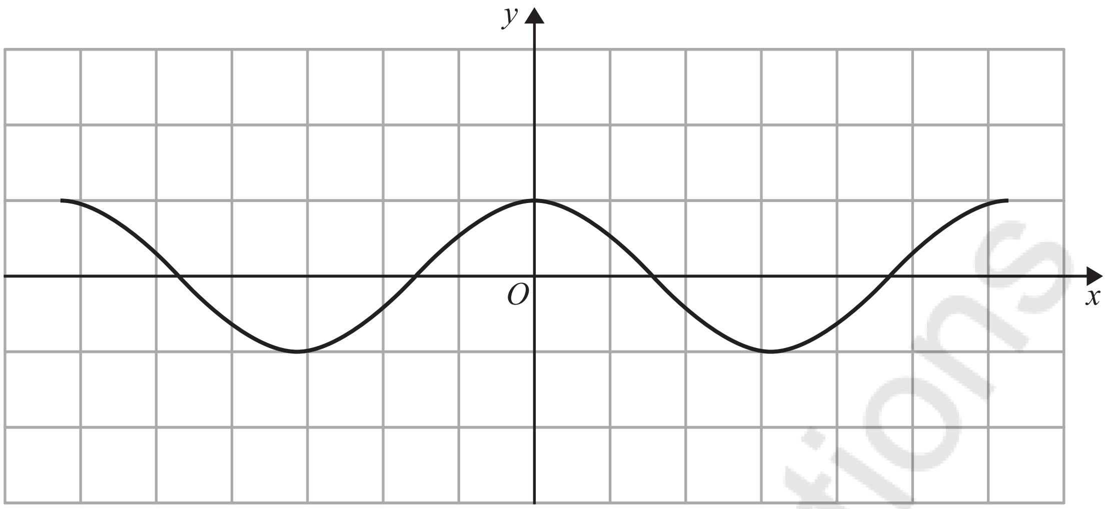
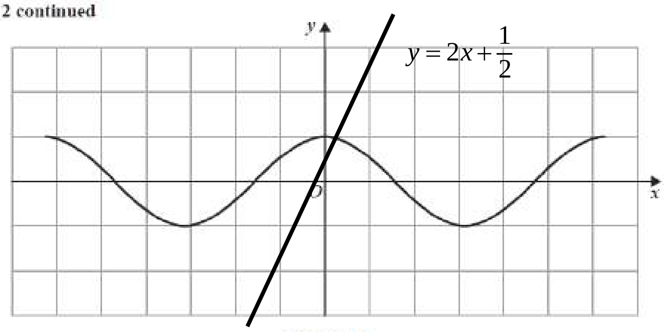


Diagram 1

(Total for Question 2 is 5 marks)

Question	Scheme	Marks	AOs
2(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2 + \sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2 + \sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

3.

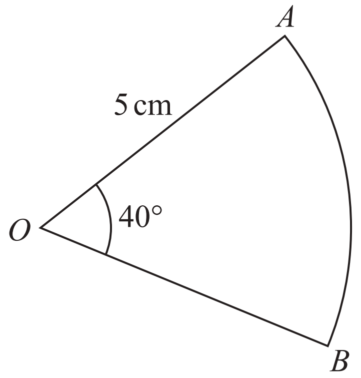


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O*, radius 5 cm and angle *AOB* = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) **Way 1**	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) **Way 2**	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
		(3 marks)	

Notes for Question 3

(a)	
B1:	Explains that the formula use is only valid when angle *AOB* is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

11.

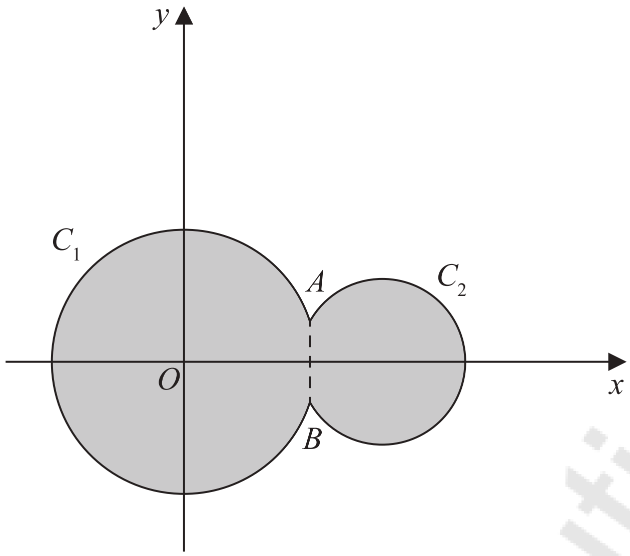


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

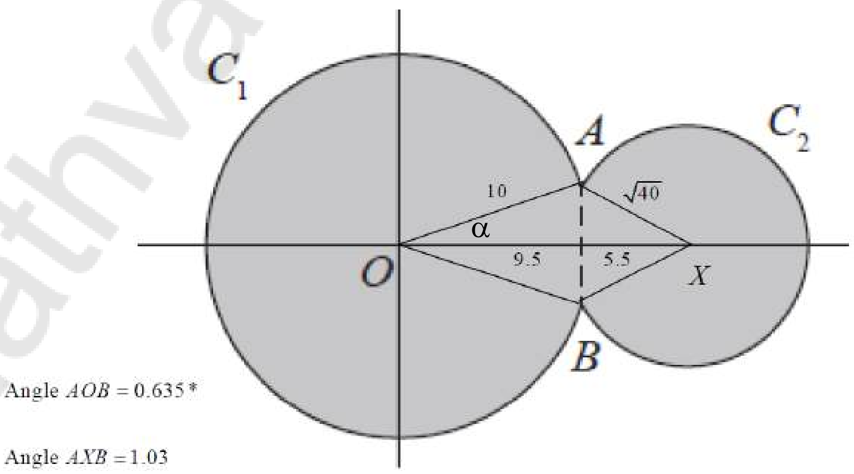
(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\frac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle *AOB* in circle C_1 Eg Attempts $\cos\alpha=\frac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle *AOB* = $2\times\arccos\left(\frac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle *AXB* or *AXO* in circle C_2 (see diagram) E.g. $\cos\beta=\frac{15-"9.5"}{\sqrt{40}}\Rightarrow\beta=...$ (Note *AXB* =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet. Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\frac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle *AOB* in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times10\times10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)
M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$
The formula may be embedded. You may see $2\pi 10 + 2\pi\sqrt{40} - 10\times 0.635...$ which is fine for this M1

M1: Attempts to use a correct method in order to find angle *AXB* or *AXO* in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.
A1: awrt 89.7

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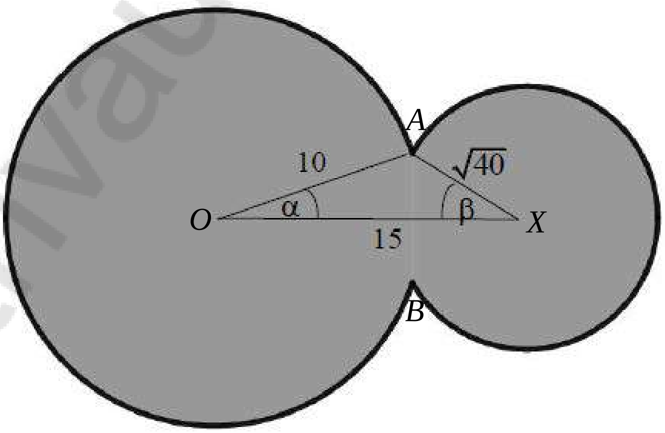

(a)
M1: For the key step in attempting to find all lengths in triangle *OAX*, condoning slips
A1: All three lengths correct
M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

....

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

Question	Scheme	Marks	AOs
4	**Examples:** $4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta = 3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$ $3\cos^2\theta = 3\frac{\left(\cos 2\theta+1\right)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\sin\frac{\theta}{2}+3\frac{\left(\cos 2\theta+1\right)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$= 2\theta+3\left(1-\theta^2+\ldots\right) = 3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

Notes

M1: Attempts to use at least one correct approximation **within the given expression**.

Either $\sin\frac{\theta}{2}\approx\frac{\theta}{2}$ or $\cos\theta\approx 1-\frac{\theta^2}{2}$ or e.g. $\sin\theta\approx\theta$ if they write $\cos^2\theta$ as $1-\sin^2\theta$ or e.g. $\cos 2\theta\approx 1-\frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2\theta$ as $\frac{1+\cos 2\theta}{2}$.

Allow sign slips only with any identities used but the appropriate approximations must be applied.

dM1: Attempts to use correct approximations **with the given expression** to obtain an expression in terms of θ only. **Depends on the first method mark.**

A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.

6.

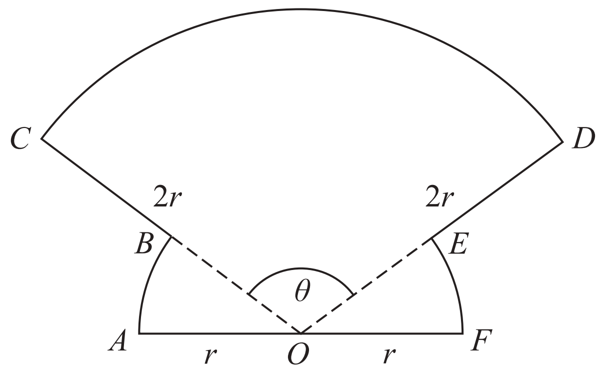

Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \frac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$	M1	2.1
	$=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)
B1: Deduces the correct expression for angle *AOB*
Note that $\frac{180-\theta}{2}$ scores B0

(b)
M1: Fully correct strategy for the area using their angle from (a) appropriately.
Need to see $2\times\frac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\frac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$
The first term must be seen expanded as e.g. $\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta$ or equivalent

(c)
M1: Fully correct strategy for the perimeter using their angle from (a) appropriately
Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ
A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\dfrac{180-\frac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times\pi r^2+\frac{\theta}{360}\times\frac{180}{\pi}\times\pi(2r)^2=\frac{1}{2}\pi r^2-\frac{1}{2}r^2\theta+2r^2\theta=\frac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times2\pi r+\frac{180\theta}{\pi}\times\frac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$