

## Y2P6 XMQs and MS

(Total: 8 marks)

1. P1\_2020      Q12. 8 marks - Y2P6 Trigonometric functions

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12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for  $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ) \quad (5)$$

DO NOT WRITE IN THIS AREA

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Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1	1.2
	$\operatorname{cosec} \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$	M1	2.1
	$= \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \times \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50^\circ)$		
	$\cot x = \cot(3x - 50^\circ) \Rightarrow x = 3x - 50^\circ$	M1	3.1a
	$x = 25^\circ$	A1	1.1b
	Also $\cot x = \cot(3x - 50^\circ) \Rightarrow x + 180^\circ = 3x - 50^\circ$	M1	2.1
	$x = 115^\circ$	A1	1.1b
	Deduces $x = 90^\circ$	B1	2.2a
		(5)	
			(8 marks)
<b>Notes:</b>			

(a) **Condone a full proof in  $x$  (or other variable) instead of  $\theta$ 's here**

**B1:** States or uses  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$  Do not accept  $\operatorname{cosec} \theta = \frac{1}{\sin}$  with the  $\theta$  missing

**M1:** For the key step in forming a single fraction/common denominator

E.g.  $\operatorname{cosec} \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$ . Allow if written separately  $\frac{1}{\sin \theta} - \sin \theta = \frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta}$

Condone missing variables for this M mark

**A1\*:** Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) **Condone  $\theta$ 's instead of  $x$ 's here**

**M1:** Uses part (a), cancels or factorises out the  $\cos x$  term, to establish that one solution is found when  $x = 3x - 50^\circ$ .

You may see solutions where  $\cot A - \cot B = 0 \Rightarrow \cot(A - B) = 0$  or  $\tan A - \tan B = 0 \Rightarrow \tan(A - B) = 0$ .

As long as they don't state  $\cot A - \cot B = \cot(A - B)$  or  $\tan A - \tan B = \tan(A - B)$  this is acceptable

**A1:**  $x = 25^\circ$

**M1:** For the key step in realising that  $\cot x$  has a period of  $180^\circ$  and a second solution can be found by solving  $x + 180^\circ = 3x - 50^\circ$ . The sight of  $x = 115^\circ$  can imply this mark provided the step  $x = 3x - 50^\circ$  has been seen. Using reciprocal functions it is for realising that  $\tan x$  has a period of  $180^\circ$

**A1:**  $x = 115^\circ$  Withhold this mark if there are additional values in the range  $(0, 180)$  but ignore values outside.

**B1:** Deduces that a solution can be found from  $\cos x = 0 \Rightarrow x = 90^\circ$ . Ignore additional values here.

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Solutions with limited working. The question demands that candidates show all stages of working.

SC:  $\cos x \cot x = \cos x \cot(3x - 50^\circ) \Rightarrow \cot x = \cot(3x - 50^\circ) \Rightarrow x = 25^\circ, 115^\circ$

They have shown some working so can score B1, B1 marked on open as 11000

**Alt 1- Right hand side to left hand side**

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot \theta = \frac{\cos \theta}{\sin \theta}$	B1	1.2
	$\cos \theta \cot \theta = \frac{\cos^2 \theta}{\sin \theta} = \frac{1 - \sin^2 \theta}{\sin \theta}$	M1	2.1
	$= \frac{1}{\sin \theta} - \sin \theta = \operatorname{cosec} \theta - \sin \theta$ *	A1*	2.1
		<b>(3)</b>	

**Alt 2- Works on both sides**

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot \theta = \frac{\cos \theta}{\sin \theta}$ or $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1	1.2
	$LHS = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$ $RHS = \cos \theta \cot \theta = \frac{\cos^2 \theta}{\sin \theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta$ o.e. (condone = for $\equiv$ )	A1*	2.1
		<b>(3)</b>	

**Alt (b)**

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x - 50^\circ) \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos(3x - 50^\circ)}{\sin(3x - 50^\circ)}$ $\sin(3x - 50^\circ)\cos x - \cos(3x - 50^\circ)\sin x = 0$ $\sin((3x - 50^\circ) - x) = 0$ $2x - 50^\circ = 0$	M1	3.1a
	$x = 25^\circ$	A1	1.1b
	Also $2x - 50^\circ = 180^\circ$	M1	2.1
	$x = 115^\circ$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90^\circ$	B1	2.2a
		<b>(5)</b>	