

Y2S3 XMQs and MS

(Total: 148 marks)

1. P3_2018 Q5 . 14 marks - Y2S3 The normal distribution
2. P3_Sample Q1 . 13 marks - Y2S3 The normal distribution
3. P3_Sample Q3 . 12 marks - Y2S3 The normal distribution
4. P3_Sample Q5 . 9 marks - Y2S3 The normal distribution
5. P3_Specimen Q1 . 14 marks - Y2S3 The normal distribution
6. P3_Specimen Q3 . 10 marks - Y2S3 The normal distribution
7. P3_Specimen Q5 . 8 marks - Y2S3 The normal distribution
8. P31_2019 Q2 . 11 marks - Y1S3 Representations of data
9. P31_2019 Q5 . 13 marks - Y2S3 The normal distribution
10. P31_2020 Q5 . 15 marks - Y2S3 The normal distribution
11. P31_2021 Q5 . 11 marks - Y2S3 The normal distribution
12. P31_2022 Q1 . 6 marks - Y1S6 Statistical distributions
13. P31_2022 Q2 . 12 marks - Y2S3 The normal distribution

Qu 5	Scheme	Marks	AO
(a)	$P(L > 16) = 0.69146\dots$	awrt 0.691	B1 (1) 1.1b
(b)	$P(L > 20 L > 16) = \frac{P(L > 20)}{P(L > 16)}$ $= \frac{0.308537\dots}{(a)} \text{ or } \frac{1-(a)}{(a)}, = 0.44621\dots$ <p>For calc to work require $(0.44621\dots)^4 = 0.03964\dots$</p>	awrt 0.0396	M1 3.1b A1ft, A1 1.1b dM1 2.1 A1 1.1b (5)
(c)	<p>Require: $[P(L > 4)]^2 \times [P(L > 20 L > 16)]^2$</p> $= (0.99976\dots)^2 \times (0.44621\dots)^2$ $= 0.19901\dots$	awrt 0.199 (*)	M1 1.1a A1ft 1.1b A1cso* 1.1b (3)
(d)	$H_0 : \mu = 18 \quad H_1 : \mu > 18$ $\bar{L} \sim N\left(18, \left(\frac{4}{\sqrt{20}}\right)^2\right)$ $P(\bar{L} > 19.2) = P(Z > 1.3416\dots) = 0.089856\dots$ <p>(0.0899 > 5%) <u>or</u> (19.2 < 19.5) <u>or</u> 1.34 < 1.6449 so not significant Insufficient evidence to support Alice's claim (or belief)</p>		B1 2.5 M1 3.3 A1 3.4 A1 1.1b A1 3.5a (5)
Notes			
(a)	B1 for evaluating probability using their calculator (awrt 0.691) Accept 0.6915		
(b)	<p>1st M1 for a first step of identifying a suitable conditional probability (either form)</p> <p>1st A1ft for a ratio of probabilities with numerator = awrt 0.309 or 1 – (a) and denom = their (a)</p> <p>2nd A1 for awrt 0.446 (o.e.) Accept 0.4465 (from $\frac{0.3085}{0.691} = 0.44645\dots$)</p> <p>NB $\frac{P(16 < L < 20)}{P(L > 16)} = 0.5538\dots$ scores M1A1A1 when they do $1 - 0.5538 = 0.4462\dots$</p> <p>2nd M1 (dep on 1st M1) for 2nd correct step i.e. (their 0.446...)⁴ <u>or</u> $X \sim B(4, "0.446")$ and $P(X = 4)$</p> <p>3rd A1 for awrt 0.0396</p>		
(c)	<p>1st M1 for a correct approach to solving the problem (May be implied by A1ft)</p> <p>1st A1ft for $P(L > 4) =$ awrt 0.9998 used <u>and</u> ft their 0.44621 in correct expression</p> <p>If use $P(L > 20) = 0.3085\dots$ as 0.446.. in (b) then M1 for $(0.3085\dots)^2 \times [P(L > 4)]^2$; A1ft as above</p> <p>* 2nd A1cso for 0.199 or better with clear evidence of M1 [NB $(0.4662\dots)^2 = 0.199\dots$ is M0A0A0] Must see M1 scored by correct expression in symbols or values (M1A1ft)</p>		
(d)	<p>B1 for both hypotheses in terms of μ.</p> <p>M1 for selecting a suitable model. Sight of <u>normal</u>, <u>mean</u> 18, <u>sd</u> $\frac{4}{\sqrt{20}}$ (o.e.) or <u>variance</u> = 0.8</p> <p>1st A1 for using the model correctly. Allow awrt 0.0899 <u>or</u> 0.09 from correct prob. statement</p>		
ALT	<p>CR $(\bar{L}) > 19.471\dots$ (accept awrt 19.5) <u>or</u> CV of 1.6449 (or better: calc 1.6448536..)</p> <p>2nd A1 for correct non-contextual conclusion. Wrong comparison or contradictions A0 Error giving 2nd A0 implies 3rd A0 but just a correct contextual conclusion can score A1A1</p> <p>3rd A1 dep on M1 and 1st A1 for a correct contextual conclusion mentioning <u>Alice's claim</u> /<u>belief</u> <u>or</u> there is insufficient evidence that the mean <u>lifetime</u> is more than 18 hours</p>		
(14 marks)			

SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. The number of hours of sunshine each day, y , for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leq y < 5$	$5 \leq y < 8$	$8 \leq y < 11$	$11 \leq y < 12$	$12 \leq y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leq y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

- (a) Find the width and the height of the $0 \leq y < 5$ group. (3)

- (b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures. (3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectively.

Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

- (c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief. (2)

- (d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean. (2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

- (e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean. (2)

- (f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model. (1)

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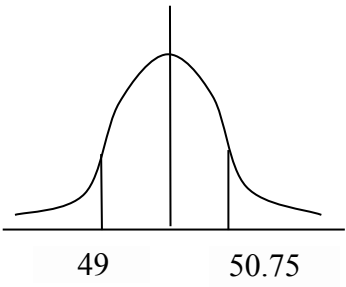
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Paper 3: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Area = $8 \times 1.5 = 12 \text{ cm}^2$ Frequency = 8 so $1 \text{ cm}^2 = \frac{2}{3} \text{ hour (o.e.)}$	M1	3.1a
	Frequency of 12 corresponds to area of 18 so height = $18 \div 2.5 = 7.2 \text{ (cm)}$	A1	1.1b
	Width = $5 \times 0.5 = 2.5 \text{ (cm)}$	B1cao	1.1b
		(3)	
(b)	$[\bar{y} =] \frac{205.5}{31} = \text{awrt } 6.63$	B1cao	1.1b
	$[\sigma_y =] \sqrt{\frac{1785.25}{31} - \bar{y}^2} = \sqrt{13.644641} = \text{awrt } 3.69$	M1	1.1a
	allow $[s =] \sqrt{\frac{1785.25 - 31\bar{y}^2}{30}} = \text{awrt } 3.75$	A1	1.1b
		(3)	
(c)	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		(2)	
(d)	$\bar{x} + \sigma \approx 10.3$ so number of days is e.g. $\frac{(11 - "10.3")}{3} \times 8 (+5)$	M1	1.1b
	= 6.86 so 7 days	A1	1.1b
		(2)	
(e)	$[H = \text{no. of hours}] \quad P(H > 10.3) \text{ or } P(Z > 1) = [0.15865\dots]$	M1	3.4
	Predict $31 \times 0.15865\dots = \underline{\underline{4.9 \text{ or } 5 \text{ days}}}$	A1	1.1b
		(2)	
(f)	(5 or) 4.9 days < (7 or) 6.9 days so model may not be suitable	B1	3.5a
		(1)	
(13 marks)			

Question 1 continued**Notes:****(a)****M1:** for clear attempt to relate the area to frequency. Can also award if their height \times their width = 18**A1:** for height = 7.2 (cm)**(b)****M1:** for a correct expression for σ or s , can ft their value for mean**A1:** awrt 3.69 (allow $s = 3.75$)**(c)****M1:** for a suitable comparison of standard deviations to comment on reliability.**A1:** for stating Hurn is south of Heathrow and a correct conclusion**(d)****M1:** for a correct expression – ft their $\bar{x} + \sigma \approx 10.3$ **A1:** for 7 days but accept 6 (rounding down) following a correct expression**(e)****M1:** for a correct probability attempted**A1:** for a correct prediction**(f)****B1:** for a suitable comparison and a compatible conclusion

Question	Scheme	Marks	AOs
Q3(a)			
	$P(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \frac{50.98 - \mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \mu = 50$	A1cao	1.1b
	$P(49 < L < 50.75)$	M1	3.4
	$= 0.9104\dots$ awrt 0.910	A1ft	1.1b
		(5)	
(b)	$S =$ number of strips that cannot be used so $S \sim B(10, 0.090)$	M1	3.3
	$= P(S \leq 3) = 0.991166\dots$ awrt 0.991	A1	1.1b
		(2)	
(c)	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ or $z = 1.936\dots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the mean length of strips is greater than 50.1	A1	2.2b
		(5)	
(12 marks)			

Question 3 continued**Notes:****(a)****1st M1:** for standardizing with μ and 0.5 and setting equal to a z value ($|z| > 1$)**2nd M1:** for attempting the correct probability for strips that can be used**2nd A1ft:** awrt 0.910 (allow ft of their μ)**(b)****M1:** for identifying a suitable binomial distribution**A1:** awrt 0.991 (from calculator)**(c)****B1:** hypotheses stated correctly**M1:** for selecting a correct model (stated or implied)**1st A1:** for use of the correct model to find $p =$ awrt 0.0264 (allow $z =$ awrt 1.94)**2nd A1:** for a correct calculation, comparison and correct statement**3rd A1:** for a correct conclusion in context mentioning “mean length” and 50.1

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	$[S = \text{no. of seeds out of 24 that germinate, } S \sim B(24, 0.55)]$		
	$T = \text{no. of trays with at least 15 germinating. } T \sim B(10, p)$	M1	3.3
	$p = P(S \geq 15) = 0.299126\dots$	A1	1.1b
	So $P(T \geq 5) = 0.1487\dots$ awrt 0.149	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geq 149.5) = P\left(Z \geq \frac{149.5 - 132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\dots$ awrt 0.0116	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			(9 marks)
Notes:			
(a) B1: cao			
(b) M1: for selection of an appropriate model for T 1 st A1: for a correct value of the parameter p (accept 0.3 or better) 2 nd A1: for awrt 0.149			
(c) B1: both correct conditions			
(d) B1: for correct normal distribution M1: for correct use of continuity correction A1: cso			
(e) B1: correct statement			

SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. *Kaff coffee* is sold in packets. A seller measures the masses of the contents of a random sample of 90 packets of *Kaff coffee* from her stock. The results are shown in the table below.

Mass w (g)	Midpoint y (g)	Frequency (f)
$240 \leq w < 245$	242.5	8
$245 \leq w < 248$	246.5	15
$248 \leq w < 252$	250	35
$252 \leq w < 255$	253.5	23
$255 \leq w < 260$	257.5	9

(You may use $\sum fy^2 = 5\,644\,171.75$)

A histogram is drawn and the class $245 \leq w < 248$ is represented by a rectangle of width 1.2 cm and height 10 cm.

- (a) Calculate the width and the height of the rectangle representing the class $255 \leq w < 260$ (3)
- (b) Use linear interpolation to estimate the median mass of the contents of a packet of *Kaff coffee* to 1 decimal place. (2)
- (c) Estimate the mean and the standard deviation of the mass of the contents of a packet of *Kaff coffee* to 1 decimal place. (3)

The seller claims that the mean mass of the contents of the packets is more than the stated mass.

Given that the stated mass of the contents of a packet of *Kaff coffee* is 250 g and the actual standard deviation of the contents of a packet of *Kaff coffee* is 4 g,

- (d) test, using a 5% level of significance, whether or not the seller's claim is justified. State your hypotheses clearly. (You may assume that the mass of the contents of a packet is normally distributed.) (5)
- (e) Using your answers to parts (b) and (c), comment on the assumption that the mass of the contents of a packet is normally distributed. (1)



9MA0/03 Mock Paper: Statistics and Mechanics mark scheme

Question	Scheme	Marks	AOs
1(a)	Width = $0.4 \times 5 = 2$ (cm)	B1	3.1a
	Area = 12 cm^2 Frequency = 15 so $1 \text{ cm}^2 = \frac{5}{4}$ packet o.e	M1	1.1b
	Frequency of 9 corresponds to area of 7.2 Height = $7.2 \div 2 = 3.6$ (cm)	A1	1.1b
		(3)	
(b)	$[Q_2 =] (248 +) \frac{22}{35} \times 4$ or (use of $(n+1)$) $(248 +) \frac{22.5}{35} \times 4$	M1	1.1a
	= awrt 250.5 (g) or 250.6	A1	1.1b
		(2)	
(c)	Mean = awrt 250.4 (g)	B1	1.1b
	$[\sigma_x =] \sqrt{\frac{5644171.75}{90} - \left(\frac{22535.5}{90}\right)^2} = \sqrt{15.64...}$	M1	1.1b
	= awrt 4.0 (g)	A1	1.1b
	Accept $\left(s_x = \sqrt{\frac{5644171.75 - 90\left(\frac{22535.5}{90}\right)^2}{89}} = 3.977... \right)$	(3)	
(d)	$H_0 : \mu = 250$ $H_1 : \mu > 250$	B1	2.5
	$\bar{X} \sim N\left(250, \frac{4^2}{90}\right)$ and $\bar{X} > 250.4$	M1	3.3
	$P(\bar{X} > 250.4) = 0.171...$	A1	3.4
	$0.171 > 0.05$ or $z = 0.9486... < 1.6449$	A1	1.1b
	There is insufficient evidence that the mean weight of coffee is greater than 250 g, or there is no evidence to support the sellers claim.	A1	2.2b
		(5)	
(e)	It is consistent as (the estimate of) the mean is close to (the estimate of) the median which is true for the normal distribution.	B1ft	3.5b
		(1)	
(14 marks)			

9MA0/03 Mock Paper: Statistics and Mechanics mark scheme

Notes:
<p>(a) B1: for correct width M1: for clear attempt to relate the area to frequency. May be implied by their height \times their width = 7.2 A1: for height = 3.6 cm</p>
<p>(b) M1: for $\frac{22}{35} \times 4$ or $\frac{22.5}{35} \times 4$ A1: awrt 250.5 or 250.6</p>
<p>(c) B1: awrt 250.4 M1: for a correct expression for σ or s, can ft their mean A1: awrt 4.0 (allow $s =$ awrt 4.0)</p>
<p>(d) B1: hypotheses stated correctly M1: for selecting a correct model, (stated or implied) A1: for use of the correct model to find $p =$ awrt 0.171 (allow $z =$ awrt 0.948) A1: for a correct calculation, comparison and correct statement A1: for a correct conclusion in context mentioning mean weight and 250</p>
<p>(e) B1: evaluating the validity of the model used in (d)</p>

9MA0/03 Mock Paper: Statistics and Mechanics mark scheme

Question	Scheme	Marks	AOs
3(a)	[$A =$ no. of bulbs that grow into plants with blue flowers,] $A \sim B(40, 0.36)$	M1	3.3
	$p = P(A \geq 21) = 0.0240$	A1	1.1b
	$C =$ no. of bags with more than 20 bulbs that grow into blue flowers, $C \sim B(5, p)$	M1	3.3
	So $P(C \leq 1) = 0.9945\dots$ awrt 0.995	A1	1.1b
		(4)	
(b)	[$T \sim$ number of bulbs that grow into blue flowers] $T \sim B(n, 0.36)$		
	T can be approximated by $N(0.36n, 0.2304n)$	B1	3.4
	$P\left(Z < \frac{244.5 - 0.36n}{\sqrt{0.2304n}}\right) = 0.9479$	M1	1.1b
	$\frac{244.5 - 0.36n}{\sqrt{0.2304n}} = 1.625$ or $\frac{244.5 - 0.36x^2}{0.48x} = 1.625$	M1 A1	3.4 1.1b
	$0.36n + 0.78\sqrt{n} - 244.5 = 0$	M1	1.1b
	$n = 625$	A1cso	1.1b
		(6)	
(10 marks)			
Notes:			
<p>(a) M1: for selecting an appropriate model for A A1: for a correct value of the parameter p for C M1: for selecting an appropriate model for C A1: for awrt 0.995</p>			
<p>(b) B1: for correct normal distribution M1: for correct use of continuity correction equal to a z value where $z > 1$ M1: for standardisation with their μ and σ A1: for a correct equation M1: using a correct method to solve their 3-term quadratic A1: 625 on its own cso</p>			

9MA0/03 Mock Paper: Statistics and Mechanics mark scheme

Question	Scheme	Marks	AOs
5(a)	$P(L_x > 160) = P\left(Z > \frac{160-150}{25}\right)$		
	$= P(Z > 0.4)$		
	$= 1 - 0.6554$		
	$= \text{awrt } 0.345 \quad 0.34457\dots$	B1	1.1b
	Expected number = $12 \times "0.345"$	M1	1.1b
	$= 4.13$ (allow 4.14)	A1	1.1b
		(3)	
(b)	$P(L_y < 180) = 0.841621\dots$	B1	3.4
	$\frac{180-160}{\sigma} = 0.8416$	M1	1.1b
	$\sigma = \text{awrt } 23.8$	A1	1.1b
		(3)	
(c)	The standard deviations for two companies are close but the mean for company Y is higher	M1	2.4
	therefore choose company Y	A1	2.2b
		(2)	
(8 marks)			
Notes:			
(a) B1: awrt 0.345 M1: for multiplying their probability by 12 A1: 4.13 (allow 4.14)			
(b) B1: for use of the correct model to find the correct value of z awrt 0.842 M1: for standardising = to a Z value $0.5 < Z < 1$ A1: awrt 23.8			
(c) M1: for a correct reason following their part(b) A1: for making an inference that follows their part(b)			

2.

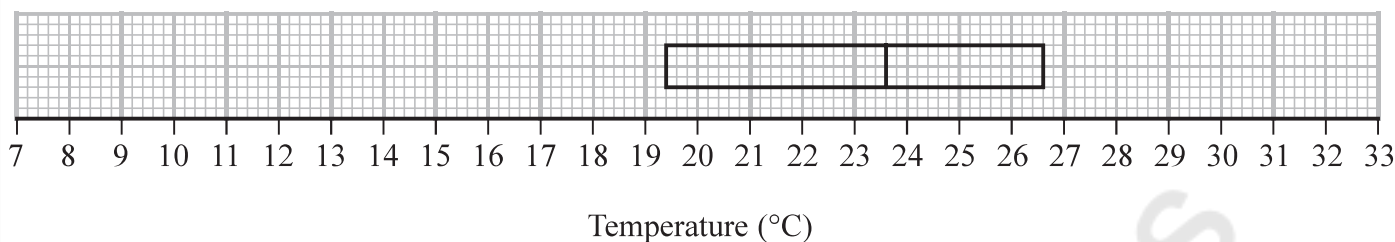


Figure 1

The partially completed box plot in Figure 1 shows the distribution of daily mean air temperatures using the data from the large data set for Beijing in 2015

An outlier is defined as a value
 more than $1.5 \times \text{IQR}$ below Q_1 or
 more than $1.5 \times \text{IQR}$ above Q_3

The three lowest air temperatures in the data set are 7.6°C , 8.1°C and 9.1°C

The highest air temperature in the data set is 32.5°C

(a) Complete the box plot in Figure 1 showing clearly any outliers. (4)

(b) Using your knowledge of the large data set, suggest from which month the two outliers are likely to have come. (1)

Using the data from the large data set, Simon produced the following summary statistics for the daily mean air temperature, $x^\circ\text{C}$, for Beijing in 2015

$$n = 184 \quad \sum x = 4153.6 \quad S_{xx} = 4952.906$$

(c) Show that, to 3 significant figures, the standard deviation is 5.19°C (1)

Simon decides to model the air temperatures with the random variable

$$T \sim N(22.6, 5.19^2)$$

(d) Using Simon's model, calculate the 10th to 90th interpercentile range. (3)

Simon wants to model another variable from the large data set for Beijing using a normal distribution.

(e) State two variables from the large data set for Beijing that are **not** suitable to be modelled by a normal distribution. Give a reason for each answer. (2)



Question	Scheme	Marks	AOs	
2(a)	IQR = 26.6 – 19.4 [= 7.2]	B1	2.1	
	19.4 – 1.5 × ‘7.2’ [= 8.6] or 26.6 + 1.5 × ‘7.2’ [= 37.4]	M1	1.1b	
	Plotting one upper whisker to 32.5 and one lower whisker to 8.6 or 9.1	A1	1.1b	
	Plotting 7.6 and 8.1 as the only two outliers	A1	1.1b	
		(4)		
(b)	October (since it is the month with the coldest temperatures between May and October in Beijing)	B1	2.4	
		(1)		
(c)	$[\sigma = \sqrt{\frac{4952.906}{184}} \quad \text{or e.g. } [\sigma = \sqrt{\frac{S_{xx}}{n}} = 5.188... \quad [=5.19*]$	B1cso*	1.1b	
		(1)		
(d)	$z = (\pm) 1.28(16)$	$[P_{90} =]29.251... \text{ or } [P_{10} =]15.948...$	B1	3.1b
	$2 \times 1.2816 \times 5.19$	‘29.251...’ – ‘15.948...’	M1	1.1b
		= awrt 13.3	A1	1.1b
			(3)	
(e)	Daily mean <u>wind speed</u> /Beaufort conversion since it is <u>qualitative</u>	B1	2.4	
	<u>Rainfall</u> since it is not symmetric/lots of days with 0 rainfall	B1	2.4	
		(2)		
(11 marks)				
Notes				
(a)	B1: for a correct calculation for the IQR (implied by 10.8 or 8.6 or 37.4 seen) M1: for a complete method for either lower outlier limit or upper outlier limit (allow ft on their IQR) (may be implied by the 1 st A1 or a lower whisker at 8.6) A1: both whiskers plotted correctly (allow ½ square tolerance) A1: only two outliers plotted, 7.6 and 8.1 (must be disconnected from whisker) NOTE: A fully correct box plot with no incorrect working scores 4/4			
	(c)	B1cso*: Correct expression with square root or correct formula and 5.188 or better Allow a complete correct method finding $\sum x^2 = \text{awrt } 98720$ and $\sigma = \sqrt{\frac{98715.9...}{184} - \left(\frac{4153.6}{184}\right)^2}$		
(d)	B1: Identifying z-value for 10th or 90th percentile (allow awrt (±) 1.28) or for identifying $[P_{90} =]29.251...$ (awrt 29.3) or $[P_{10} =]15.948...$ (awrt 15.9) (This may be implied by a correct answer awrt 13.3)			
	M1: for $2 \times z \times 5.19$ where $1 < z < 2$ or for their $P_{90} - P_{10}$ where $25 < P_{90} < 35$ and $10 < P_{10} < 20$ A1: awrt 13.3			
(e)	B1: for one variable identified and a correct supporting reason B1: for two variables identified and a correct supporting reason for each Allow any two of the following: <ul style="list-style-type: none"> • <u>Wind speed/Beaufort</u> since the data is <u>non-numeric</u> (o.e.). They need not mention Beaufort provided there is a description of the data as non-numeric (Do not allow wind direction/wind gust) • <u>Rainfall</u> as not symmetric/is skewed/is not bell shaped/lots of 0s /many days with no rain/mean≠mode or median • <u>Date</u> since each data value appears once/it is uniformly distributed • Daily mean <u>pressure</u> since it is not symmetric/is skewed/not bell shaped • Daily mean <u>wind speed</u> since it is not symmetric/is skewed/not bell shaped Do not allow ‘not continuous’ or ‘discrete’ as a supporting reason. Ignore extraneous non-contradicting statements			

5. A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

- (a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$ (5)

A random sample of 200 bottles is taken.

- (b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml (3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

- (c) Test Hannah's belief at the 5% level of significance. You should state your hypotheses clearly. (5)



Question	Scheme	Marks	AOs
5(a)	$\frac{24.63 - 25}{\sigma} = -1.0364$	M1	3.1b
	$[\sigma =]0.357$ (must come from compatible signs)	A1	1.1b
	$P(D > k) = 0.4$ or $P(D < k) = 0.6$	B1	1.1b
	$\frac{k - 25}{0.357} = 0.2533$	M1	3.4
	$k = \text{awrt } \underline{25.09}$	A1	1.1b
		(5)	
(b)	$[Y \sim B(200, 0.45) \rightarrow] W \sim N(90, 49.5)$	B1	3.3
	$P(Y < 100) \approx P(W < 99.5) \left[= P\left(Z < \frac{99.5 - 90}{\sqrt{49.5}}\right) \right]$	M1	3.4
	$= 0.9115\dots$ awrt <u>0.912</u>	A1	1.1b
		(3)	
(c)	$H_0 : \mu = 25$ $H_1 : \mu < 25$	B1	2.5
	$[\bar{D} \sim N\left(25, \frac{0.16^2}{20}\right)]$	M1	3.3
	$P(\bar{D} < 24.94) [= P(Z < -1.677\dots)] = 0.046766\dots$	A1	3.4
	$p = 0.047 < 0.05$ or $z = -1.677\dots < -1.6449$ or $24.94 < 24.94115\dots$	M1	1.1b
	or reject H_0 /in the critical region/significant		
	There is sufficient evidence to support <u>Hannah's belief</u> .	A1	2.2b
	(5)		
(13 marks)			
Notes			
(a)	M1: for standardising 24.63, 25 and ' σ ' (ignore label) and setting = to z where $1 < z < 2$ A1: [$\sigma =$] awrt 0.36. Do not award this mark if signs are not compatible. B1: for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram M1: for a correct expression with $z =$ awrt 0.253 (may be implied by correct answer) A1: awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)		
(b)	B1: setting up normal distribution approximation of binomial $N(90, 49.5)$ (may be implied by a correct answer) Look out for e.g. $\sigma = \frac{3\sqrt{22}}{2}$ or $\sigma =$ awrt 7.04 M1: attempting a probability using a continuity correction i.e. $P(W < 100.5)$, $P(W < 99.5)$ or $P(W < 98.5)$ condone \leq (The continuity correction may be seen in a standardisation). A1: awrt 0.912 [Note: 0.911299... from binomial scores 0 out of 3]		
(c)	B1: for both hypotheses in terms of μ M1: selecting suitable model must see $N(\text{ormal})$, mean 25, $sd = \frac{0.16}{\sqrt{20}}$ (o.e.) or $var = \frac{4}{3125}$ (o.e.) Condone $N(25, \frac{0.16}{\sqrt{20}})$ if $\frac{0.16}{\sqrt{20}}$ then used as s.d. A1: p value = awrt 0.047 or test statistic awrt -1.68 or CV awrt 24.941 (any of these values imply the M1 provided they do not come from Normal mean = 24.94) M1: a correct comparison (including compatible signs) or correct non-contextual conclusion (f.t. their p value, test statistic or critical value in the comparison) M1 may be implied by a correct contextual statement NB Any contradictory non contextual statements/comparisons score M0A0 e.g. ' $p < 0.05$, not significant' A1: correct conclusion in context mentioning <u>Hannah's belief</u> or the mean <u>amount/liquid</u> in each bottle is now <u>less than 25ml (dep on M1A1M1)</u>		

Qu 5	Scheme	Marks	AO
(a)	{Let $X =$ time spent, $P(X > 15) =$ } 0.105649... awrt 0.106	B1 (1)	1.1b
(b)	$H_0 : \mu = 10$ $H_1 : \mu > 10$ $\bar{X} \sim N\left(10, \left(\frac{4}{\sqrt{20}}\right)^2\right)$; $P(\bar{X} > 11.5) = 0.046766...$ [Condone 0.9532...] [This is significant (< 5%) so] there is evidence to support the complaint	B1 M1;A1 A1 (4)	2.5 3.3;3.4 2.2b
(c)(i)	[$P(T < 2) =$] 0.1956... awrt 0.196	B1 (1)	1.1b
(ii)	Require $\frac{P(0 < T < 2)}{P(T > 0)} = \frac{0.119119...}{0.923436...}$; = 0.1289955... awrt 0.129	M1 A1;A1 (3)	3.4 1.1bx2
(iii)	The current model suggests non-negligible probability of T values < 0 which is impossible	B1 (1)	3.5b
(d)	Require t such that $P(T > t T > 2) = 0.5$ or $P(T < t T > 2) = 0.5$ e.g. $\frac{P(T > t)}{P(T > 2)} = 0.5$; so $P(T > t) = 0.5 \times [1 - (c)(i)]$ or $P(T > t) = 0.5 \times 0.8043..$ [i.e. $P(T > t) = 0.40...$ implies] $\frac{t-5}{3.5} = 0.2533$ or $P(T < t) = "0.5978.."$ $t = 5.886...$ or from calculator 5.867... so awrt 5.9	M1 M1; A1ft M1 A1 (5)	3.1b 1.1b 3.4 1.1b 1.1b
Notes			
(a)	B1 for awrt 0.106 (from calculator) [Allow 10.6%]		
(b)	B1 for both hypotheses correct in terms of μ M1 for selection of a correct model (sight or use of correct normal- may not have label \bar{X}) 1 st A1 for use of this model to get probability allow 0.046~0.047 [Condone awrt 0.953] ALT OR test statistic $z = 1.677...$ (awrt 1.68) and cv of 1.64 (or better) or CR $\bar{X} > 11.47..$ 2 nd A1 (dep on 1 st A1 or at least $P(\bar{X} > 11.5) < 0.05$ (o.e.)) for a correct conclusion in context -must mention complaint/claim or time/mins is > 10 SC (M0 for $\bar{X} \sim N(11.5, ...)$ for correct probability and conclusion (score M0A0A1 on open)		
(c)(i)	B1 for awrt 0.196 (from calculator) [Allow 19.6%]		
(ii)	M1 for a correct probability ratio expression (may be implied by 1 st A1 scored) 1 st A1 for a correct ratio of probabilities (both correct or truncated to 2 dp) 2 nd A1 for awrt 0.129		
(iii)	B1 for a suitable explanation of why model is not suitable based on negative T values Must say that a significant proportion of values < 0 (o.e.) e.g. $P(T > 0)$ should be closer to 1 or Difference between $P(T < 2 T > 0)$ and $P(T < 2)$ is too big (o.e.)		
(d)	1 st M1 for a correct conditional probability statement to start the problem or $0.5 \times P(T > 2)$ 2 nd M1 for correct ratio of probability expressions [Must have $P(T > t)$ or $P(2 < T < t)$] 1 st A1ft for a correct equation for $P(T > t)$ (o.e.) ft their answer to part (c)[May be in a diagram] 3 rd M1 for attempt to find t (standardising and sight of 0.2533) or prepare to use calc (ft) Arriving at $P(T < \text{median}) = 1 - 0.5 \times$ "their 0.8043" will score 1 st 4 marks 2 nd A1 for awrt 5.9 Sight of awrt 5.9 and at least one M mark scores 5/5 [Answer only send to review]		
(15 marks)			

Qu 5	Scheme	Marks	AO
(a)	$\left[\text{Let } F \sim N(166.5, 6.1^2) \right] \quad P(F < k) = 0.01 \Rightarrow \frac{k - 166.5}{6.1} = -2.3263$ $k = 152.309... \quad \underline{\underline{152}} \text{ or awrt } \underline{\underline{152.3}}$	M1	3.4
		A1	1.1b
(b)	$[P(150 < F < 175) =] \quad 0.914840... \quad \text{awrt } \underline{\underline{0.915}}$	B1	1.1b
(c)	$P(F > 160 \mid 150 < F < 175)$ $= \frac{P(160 < F < 175)}{P(150 < F < 175)} \text{ or } \frac{P(160 < F < 175)}{"(b)"}$ $= \frac{0.7749487...}{"0.914840..."} = 0.84708... \text{ awrt } \underline{\underline{0.847}}$	(2)	
		M1	3.1b
		M1	1.1b
		A1ft	1.1b
(d)	$H_0 : \mu = 166.5 \quad H_1 : \mu < 166.5$ $[\text{Let } X = \text{height of female from 2}^{\text{nd}} \text{ country}] \quad \bar{X} \sim N\left(166.5, \left(\frac{7.4}{\sqrt{50}}\right)^2\right)$ $P(\bar{X} < 164.6) = 0.03472...$ $[0.0347... < 0.05 \text{ so significant or reject } H_0]$ <p style="text-align: center;">There is evidence to support Mia's belief</p>	A1	1.1b
		A1	1.1b
		(4)	
			(4)
			(11 marks)
Notes			
(a)	M1 for standardising (allow \pm) with k , 166.5 and 6.1 and set equal to a z value $2.3 < z < 2.4$ A1 for 152 or awrt 152.3 Ans only 2/2 [Condone poor use of notation e.g. $P\left(\frac{k-166.5}{6.1}\right) = -2.3263$] <p style="text-align: center;">Allow percentages instead of probabilities throughout.</p>		
(b)	B1 for awrt 0.915		
(c)	1 st M1 for interpreting demand as an appropriate conditional probability (\Rightarrow by 2 nd M1) 2 nd M1 for correct ratio of expressions (can fit their (b) on denominator) (\Rightarrow by 1 st A1ft) 1 st A1ft for a correct ratio of probs (can fit their "0.9148..." to 3sf from (b) if > 0.775) 2 nd A1 for awrt 0.847		
(d)	B1 for both correct hypotheses in terms of μ 1 st M1 for selecting the correct model (needn't use $\bar{X} \Rightarrow$ by standardisation or 1 st A1) 1 st A1 for correct use of the correct model i.e. awrt 0.035 (allow 0.04 if $P(" \bar{X} " < 164.6)$ seen) Condone $P(" \bar{X} " > 164.6) = 0.9652$ or awrt 0.97 <u>only if</u> comparison with 0.95 is made		
ALT	Use of z value: Need to see $Z = -1.8(15...)$ and cv of ± 1.6449 (allow 1.64 or better) for 1 st A1		
ALT	Use of CR or CV for \bar{X}: Need to see " \bar{X} " $< 164.7786...$ or CV = ... (awrt 164.8) for 1 st A1 Condone truncation i.e 164.7 or better		
	2 nd dA1 (dep on M1A1 only) for a correct inference in context. Must mention <u>Mia's belief</u> or <u>mean height of females/women</u> Do NOT award if contradictory statements about hypotheses made e.g. "not sig"		
SC	M0 for $\bar{X} \sim N(164.6, ...)$ If they achieve $p =$ awrt 0.035 (o.e. with z -value or CV of 166.3) and a correct conclusion in context is given score M0A0A1 [and SC for awrt 0.97 > 0.95 case]		

1. George throws a ball at a target 15 times.

Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable X represents the number of times George hits the target in 15 throws.

(a) Find

(i) $P(X = 3)$

(ii) $P(X \geq 5)$

(3)

George now throws the ball at the target 250 times.

- (b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.

(3)

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Question	Scheme		Marks	AOs
1(a)(i)	$X \sim B(15, 0.48)$		M1	3.3
	$P(X = 3) = 0.019668\dots$		awrt 0.0197	A1 3.4
(ii)	$[P(X \geq 5) = 1 - P(X \leq 4)] = 0.92013\dots$		awrt 0.920	A1 1.1b
			(3)	
(b)	Y is the number of hits	M is the number of misses		
	$Y \sim N(120, 62.4)$	$M \sim N(130, 62.4)$	B1	3.3
	$P(X > 110) \approx P(Y > 110.5)$	$P(X > 110) \approx P(M < 139.5)$	M1	3.4
	$\left[=P\left(Z > \frac{110.5 - "120"}{\sqrt{"62.4"}} \right) \right]$	$\left[=P\left(Z < \frac{139.5 - "130"}{\sqrt{"62.4"}} \right) \right]$		
	$= 0.88544\dots$		A1	1.1b
		(3)		
(6 marks)				
Notes:				
(a)	M1	Writing or using the binomial distribution in (i) or (ii) Allow for sight of $B(15, 0.48)$ or in words: <u>binomial</u> with $n = 15$ and $p = 0.48$ may be implied in (i) or (ii) by one correct answer to 3sf <u>or</u> sight of $P(X \leq 4) = 0.07986\dots$ i.e. awrt 0.0799. Allow for ${}^{15}C_3 \times 0.48^3 \times 0.52^{12}$ as this is "correct use" Condone $B(0.48, 15)$		
(i)	A1	awrt 0.0197		
(ii)	A1	awrt 0.920 (Allow 0.92)		
(b)	B1	Setting up a correct Normal model. Allow sight of $N(120, 62.4)$ or $N(130, 62.4)$ or $N\left(120, \frac{312}{5}\right)$ or $N\left(130, \frac{312}{5}\right)$ or may be awarded if used correctly in standardisation or in words: <u>Normal</u> with <u>mean</u> = 120/130 and <u>variance</u> = 62.4 or sd = $\sqrt{62.4}$ condone $N(120, \sqrt{62.4})$ or $N(130, \sqrt{62.4})$ or sd = 62.4 Look out for $\sigma = \frac{\sqrt{1560}}{5}$ or $\frac{2\sqrt{390}}{5}$ or awrt 7.90 (condone 7.9) This may be implied by sight of 0.897 or 0.8854...		
	M1	Sight of the continuity correction with a normal distribution		
		110.5 or 111.5 or 109.5	139.5 or 140.5 or 138.5	
		NB we will also allow 129.5 or 130.5 or 128.5	NB we will also allow 120.5 or 119.5 or 121.5	
		Continuity correction may be seen in standardisation NB No continuity correction(CC) gives awrt 0.897 which is M0 unless CC seen		
	A1	awrt 0.8854 or awrt 0.885 dependent on sight of >110.5 or <129.5 or <139.5 or >120.5 Allow \leq or \geq instead of $<$ or $>$ NB 0.885548... from $B(250, 0.48)$ scores M0A0		

2. A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places. (2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length. (1)

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leq L \leq 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.
Give your answer to the nearest pound. (5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim. (4)



Qu	Scheme		Marks	AOs
2(a)	$[P(L < 7.902) = 0.025 \Rightarrow] \frac{7.902 - 8}{x} = -1.96$ oe		M1	3.4
	$[x =] 0.05^*$		A1cso*	1.1b
	SC B1(mark as M0A1) for $\frac{7.902 - 8}{0.05} = -1.96 \Rightarrow 0.024998$			
			(2)	
(b)	$P(7.94 \leq L \leq 8.09) = 0.8490\dots$	awrt 0.849	B1	1.1b
			(1)	
(c)	$[P(L < 7.94) =] 0.115069\dots$ (awrt 0.115) or $[P(L > 8.09) =] 0.03593\dots$ (awrt 0.036)		B1	1.1b
	$[P(L < 7.94) =] 0.115069\dots$ (awrt 0.115) & $[P(L > 8.09) =] 0.03593\dots$ (awrt 0.036)		B1	1.1b
	Expected income per 500 rods = $\sum(\text{Income} \times \text{probability} \times 500)$ $(500 \times "0.849" \times 0.5) + (500 \times "0.1150\dots" \times 0.05) + (500 \times "0.03593\dots" \times 0.4)$ or		M1	3.4
	Expected profit per rod = $\sum(\text{Profit} \times \text{probability})$ $0.30 \times "0.849" + -0.15 \times "0.1150\dots" + 0.20 \times "0.03593\dots"$ [= 0.2446..]			
	Expected profit per 500 rods $500 \times \sum(\text{Profit} \times \text{probability})$ or $\sum(\text{Income} \times \text{probability} \times 500) - 500 \times 0.2$ $= 500 \times "0.2446\dots"$ or $= "222.3" - 500 \times 0.2$		M1d	3.1b
	$= [£]122.3\dots$ awrt [£]122		A1	1.1b
			(5)	
(d)	Let $X \sim B(200, 0.015)$		M1	3.3
	$P(X \leq 5) =$	$P(X \geq 6) =$	M1	1.1b
	0.9176...	0.0824	A1	1.1b
	Manufacturer is unlikely to achieve their aim since <u>0.9176 < 0.95</u>	Manufacturer is unlikely to achieve their aim since <u>0.0824 > 0.05</u>	A1ft	2.4
			(4)	
Notes:				(12 marks)
(a)	M1	Using the normal distribution to set up equation. Allow σ for x and awrt ± 1.96		
	A1*	cso For a correct expression for x followed by 0.05 or 0.05000... No incorrect working seen		
(b)	B1	awrt 0.849		
(c)	B1	awrt 0.115 (Implied by awrt 57.5 for number of rods) or awrt 0.036 (Implied by awrt 18 for number of rods)		
	B1	awrt 0.115 (Implied by awrt 57.5 for number of rods) and awrt 0.036 (Implied by awrt 18 for number of rods)		
	M1	Correct method to find the total income of 500 rods. Attempt at all 3 with at least two correct and no extras or Correct method to find sum of all three profits with at least two of 30, -15 or 20 correct. May work in pence but need to be consistent. Allow awrt 24.5 or 0.245		
	M1d	Dep on previous method for finding profit for 500 rods. May work in pence but need to be consistent. Allow " $0.2446\dots \times 500$ " or "their income" for 500 rods $- 500 \times 0.2$ (accept 499 or 501)		
	A1	All previous marks must be awarded for awrt 122 awrt 12200p NB if uses any integer values for numbers of rods then it is A0 other than for 18 for $L > 8.09$		
(d)	M1	Selecting the appropriate model. May be seen or used. Allow B(200, 0.985) or Po(3) Condone B(0.015, 200) or B(0.985, 200).		
	M1	Writing or using $P(X \leq 5)$ Do not accept $P(X < 6)$ unless found $P(X \leq 5)$	Writing or using $P(X \geq 6)$ Do not accept $P(X > 5)$ unless found $P(X \geq 6)$	
	A1	0.92 (Poisson 0.916...)	0.08 or better	
	A1ft	Need at least one of the method marks to be awarded. Correct conclusion with the comparison (may be in words). Ft "their $p = 0.9176\dots$ " as long as $p > 0.9$ If "their 0.9176..." < 0.95 must ... be unlikely... If "their 0.9176..." > 0.95 they must say ... be likely... To fit the alternative then $p < 0.1$		