Answer ALL questions. Write your answers in the spaces provided. You must write down all the stages in your working.

1 The table gives information about the number of kilometres that Ted cycled on each of the 30 days in April.

Number of kilometres (K)	Frequency (f)	midpoint (x)	fx
$0 \leqslant K < 5$	8	2·5	20
$5 \leqslant K < 10$	7	7·5	52·5
$10 \leqslant K < 15$	3	12·5	37·5
$15 \leqslant K < 20$	10	17·5	175
$20 \leqslant K < 25$	2	22·5	45
	30		330

Calculate an estimate for the mean number of kilometres that Ted cycled on each day in April.

mean Estimate $= \frac{330}{30} = 11$ km

11 kilometres

(Total for Question 1 is 4 marks)

2 By rounding each number to one significant figure, work out an estimate for the value of

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}}$$

Show your working clearly.

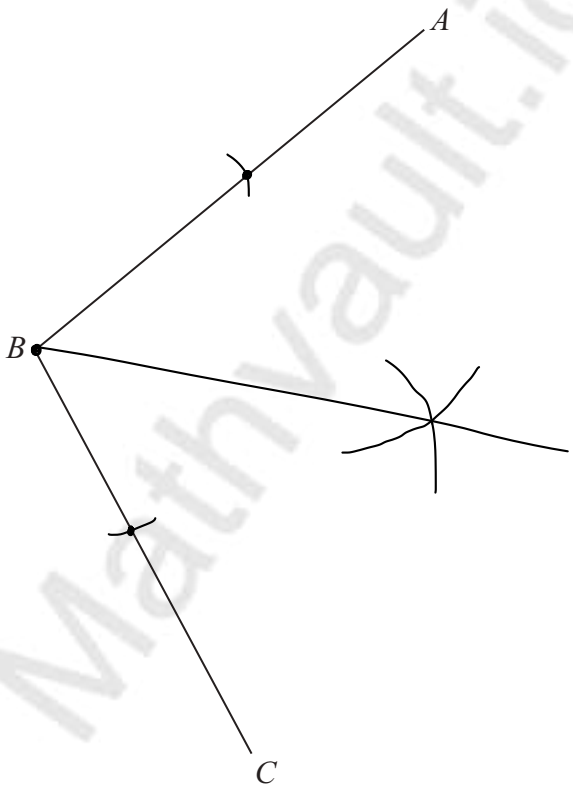
$$\frac{2^2 \times 60}{\sqrt{9}}$$

$$\frac{4 \times 60}{3}$$

80

(Total for Question 2 is 2 marks)

3 Using ruler and compasses only, construct the bisector of angle *ABC*
Show all your construction lines.



(Total for Question 3 is 2 marks)

4 e, f and g represent integers such that

$$e \quad f \quad 7 \quad g \quad 12 \quad 17$$

is a list of integers written in order of size.

The integers have

a range of 15
a median of 8.5
a mean of 9

Work out the value of e, the value of f and the value of g

$15 = 17 - e$

$e = 17 - 15$

$e = 2$

$\frac{7+g}{2} = 8.5$

$7 + g = 17$

$g = 10$

$\frac{2 + f + 7 + 10 + 12 + 17}{6} = 9$

$48 + f = 54$

$f = 6$

$e =$ 2

$f =$ 6

$g =$ 10

(Total for Question 4 is 3 marks)

5 Show that $3\frac{5}{7} + 1\frac{2}{3} = 5\frac{8}{21}$

$\frac{26}{7} + \frac{5}{3}$

$\frac{26^{\times 3}}{7 \times 3} + \frac{5^{\times 7}}{3 \times 7}$

$\frac{78}{21} + \frac{35}{21}$

$\frac{113}{21}$

$5\frac{8}{21}$ Shown.

(Total for Question 5 is 3 marks)

6 Eli and Peta share \$275 in the ratio 2 : 3

5 parts total

Eli gives $\frac{3}{11}$ of his share to charity.

Peta gives 0.32 of her share to charity.

Work out the total amount that Eli and Peta give to charity.

Eli = $\frac{2}{5} \times 275 = \$110$

Peta = $\frac{3}{5} \times 275 = \$165$

Eli gives charity = $\frac{3}{11} \times 110 = \$30$

Peta gives charity = $0.32 \times 165 = \$52.80$

$\$30 + \$52.80$

$\$82.80$

\$ 82.80

(Total for Question 6 is 4 marks)

7 The weight of a bag of apples is 475 g correct to the nearest g

(a) Write down the lower bound of the weight.

474 | 475 | 476

$474.5 \leq w < 475.5$

474.5 g

(1)

The height of a box is 120 cm correct to the nearest 10 cm

(b) Write down the upper bound for the height.

110 cm | 120 cm | 130 cm

$115\text{cm} \leq h < 125\text{cm}$

125 cm

(1)

(Total for Question 7 is 2 marks)

8 $\dfrac{8^{-2} \times 8^{9}}{8^{10}} = 8^{n}$

Work out the value of n

$\dfrac{8^{-2+9}}{8^{10}} = 8^{n}$

$\dfrac{8^{7}}{8^{10}} = 8^{n}$

$8^{7-10} = 8^{n}$

$8^{-3} = 8^{n}$

$-3 = n$

$n =$ -3

(Total for Question 8 is 2 marks)

9 Hari scored 140 points playing a game on Tuesday.
This was 12% more points than he scored playing the game on Monday.

Work out the number of points that Hari scored playing the game on Monday.

Monday ⟶ Tuesday

Percentage 100% $\xrightarrow{+12\%}$ 112%

Points M $\xrightarrow{+P}$ 140

100 : 112

M : 140

$M \times 112 = 100 \times 140$

$112M = 14000$

$M = \dfrac{14000}{112} = 125$

125

(Total for Question 9 is 3 marks)

10 (a) Simplify $(6m)^0$

1

(1)

(b) Solve $5y + 20 < 7y + 1$

$-5y \quad -5y$

$20 < 2y + 1$

$-1 \quad -1$

$19 < 2y$

$9.5 < y$

$9.5 < y$

(2)

(c) Factorise fully $15w^2x^5 + 25w^3x^2$

$5w^2x^2(3x^3 + 5w)$

$5w^2x^2(3x^3+5w)$

(2)

(d) Simplify fully $\dfrac{36a^5c^7}{12a^2c^3}$

$3a^{5-2}c^{7-3}$

$3a^3c^4$

(2)

(Total for Question 10 is 7 marks)

11 ABC is a right-angled triangle.

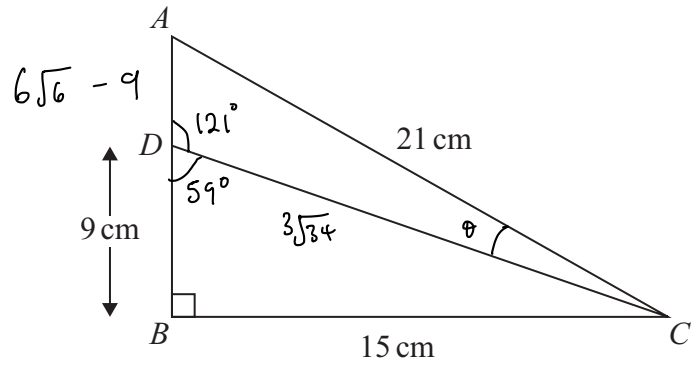
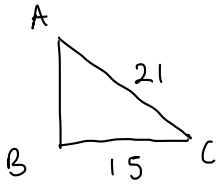


Diagram **NOT** accurately drawn

$AC = 21$ cm $BC = 15$ cm angle $ABC = 90°$

The point D lies on AB such that $DB = 9$ cm

Work out the size of angle ACD
Give your answer correct to one decimal place.



$AB = \sqrt{21^2 - 15^2}$

$AB = 6\sqrt{6}$ cm

$AD = 6\sqrt{6} - 9$ cm

$CD = \sqrt{15^2 + 9^2}$

$CD = 3\sqrt{34}$

$B\hat{D}C = \tan^{-1}\left(\frac{15}{9}\right) \approx 59°$

$A\hat{D}C = 180 - 59 = 121°$

Using Sine Rule:

$$\frac{\sin A\hat{C}D}{6\sqrt{6} - 9} = \frac{\sin 121}{21}$$

$$A\hat{C}D = \sin^{-1}\left[\frac{(6\sqrt{6} - 9) \times \sin 121}{21}\right]$$

$A\hat{C}D \approx 13.4°$ (1 d.p.)

13.4 °

(Total for Question 11 is 4 marks)

12

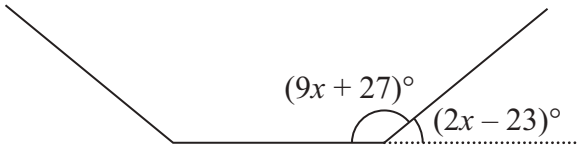


Diagram **NOT** accurately drawn

The diagram shows part of a regular n-sided polygon with

an interior angle of $(9x + 27)°$
an exterior angle of $(2x – 23)°$

Work out the value of n

$2x - 23 = \frac{360}{n}$

$2(16) - 23 = \frac{360}{n}$

$32 - 23 = \frac{360}{n}$

$9 = \frac{360}{n}$

$n = \frac{360}{9}$

$n = 40$

$9x + 27 + 2x - 23 = 180$

$11x + 4 = 180$

$11x = 176$

$x = 16$

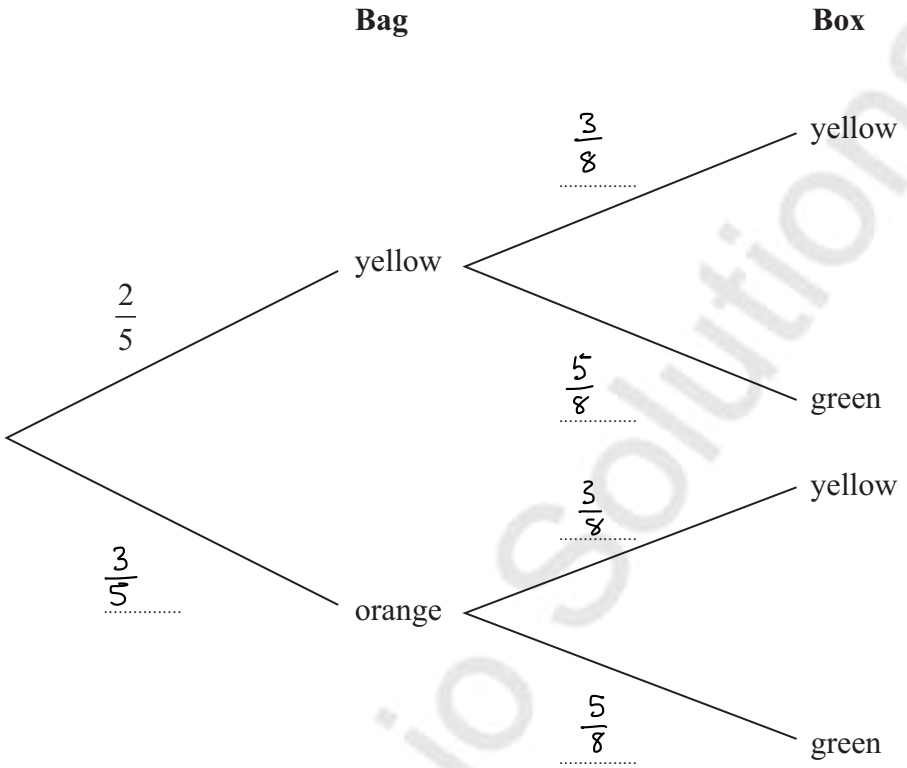
$n =$ 40

(Total for Question 12 is 4 marks)

13 In a bag, there are only 2 yellow beads and 3 orange beads.
In a box, there are only 3 yellow beads and 5 green beads.

Chi takes at random a bead from the bag and a bead from the box.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Chi takes two beads of different colours.

$P(YG) + P(OY) + P(OG)$

$\left(\frac{2}{5} \times \frac{5}{8}\right) + \left(\frac{3}{5} \times \frac{3}{8}\right) + \left(\frac{3}{5} \times \frac{5}{8}\right)$

$\frac{17}{20}$

(3)

(Total for Question 13 is 5 marks)

14 (a) Expand and simplify $7x(3x + 2)(2x - 5)$

	$3x$	$+2$
$2x$	$6x^2$	$+4x$
-5	$-15x$	-10

$7x(6x^2 + 4x - 15x - 10)$

$7x(6x^2 - 11x - 10)$

$42x^3 - 77x^2 - 70x$

$42x^3 - 77x^2 - 70x$

(3)

(b) Solve $\frac{9}{2y} + \frac{5}{7} = 5$

Show clear algebraic working.

$\frac{9^{\times 7}}{2y_{\times 7}} + \frac{5^{\times 2y}}{7 \times 2y} = 5$

$\frac{63}{14y} + \frac{10y}{14y} = 5$

$\frac{63 + 10y}{14y} = 5$

$63 + 10y = 70$

$10y = 7$

$y = \frac{7}{10}$

$y =$ 0.7

(3)

(Total for Question 14 is 6 marks)

15 AOB is a sector of a circle with centre O and radius 12 cm

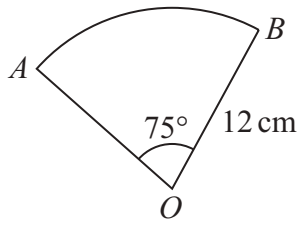


Diagram **NOT** accurately drawn

Work out the area of the sector.
Give your answer correct to 3 significant figures.

$\frac{75}{360} \times \pi \times 12^2 = 30\pi \text{ cm}^2$

94.2 cm^2

(Total for Question 15 is 2 marks)

16 Ivan asked 15 people how many books they read last year.

Here are his results.

1 1 3 | 4 | 6 7 8 | 9 | 10 12 15 | 25 | 30 37 50

Q_1 Q_2 Q_3

Work out the interquartile range of the number of books read.

$IQR = Q_3 - Q_1$

$IQR = 25 - 4 = 21$

21

(Total for Question 16 is 2 marks)

17 Show that $\dfrac{2\sqrt{7}+2}{\sqrt{7}-3}$ can be written in the form $a - \sqrt{b}$ where a and b are integers.

Show your working clearly.

$$\frac{(2\sqrt{7}+2)(\sqrt{7}+3)}{(\sqrt{7}-3)(\sqrt{7}+3)}$$

$$\frac{14+6\sqrt{7}+2\sqrt{7}+6}{7-9}$$

$$\frac{20+8\sqrt{7}}{-2}$$

$$-10-4\sqrt{7}$$

$a = -10$

$b = 4$

(Total for Question 17 is 3 marks)

18 Make y the subject of $p = \sqrt{\dfrac{y+w}{3y-t}}$

$$p^2 = \frac{y+w}{3y-t}$$

$$p^2(3y-t) = y+w$$

$$3p^2y - p^2t = y+w$$

$$3p^2y - y - p^2t = w$$

$$3p^2y - y = w + p^2t$$

$$y[3p^2-1] = w+p^2t$$

$$y = \frac{w+p^2t}{3p^2-1}$$

$y = \dfrac{w+p^2t}{3p^2-1}$

(Total for Question 18 is 4 marks)

19 The diagram shows triangle ABC

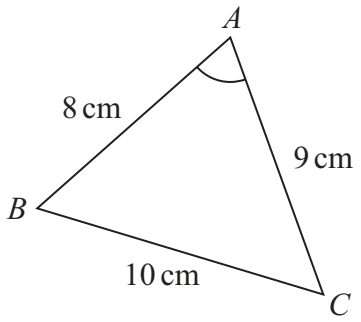


Diagram **NOT** accurately drawn

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$ $CA = 9\,\text{cm}$

Work out the size of angle BAC
Give your answer correct to one decimal place.

$10^2 = 8^2 + 9^2 - 2(8)(9)\cos B\hat{A}C$

$100 = 145 - 144\cos B\hat{A}C$

$144\cos B\hat{A}C = 45$

$\cos B\hat{A}C = \frac{45}{144}$

$B\hat{A}C = \cos^{-1}\left(\frac{45}{144}\right) \approx 71.8°$

71.8 °

(Total for Question 19 is 3 marks)

20 The straight line **L** has equation $y = 4x + 7$

The straight line **M** is perpendicular to **L** and passes through the point with coordinates (8, 1)

Find an equation for **M**
Give your answer in the form $y = mx + c$

L : $y = 4x + 7$

$m_L = 4$

$m_M = -\frac{1}{4}$ $\quad (8, 1)$ x, y

$y = mx + c$

$1 = -\frac{1}{4}(8) + c$

$1 = -2 + c$

$3 = c$

M : $y = -\frac{1}{4}x + 3$

$y = -\frac{1}{4}x + 3$

(Total for Question 20 is 3 marks)

21 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

$5(x^2 - 4x) + 23$

$5[(x-2)^2 - 2^2] + 23$

$5[(x-2)^2 - 4] + 23$

$5(x-2)^2 - 20 + 23$

$5(x-2)^2 + 3$

$5(x-2)^2 + 3$

(Total for Question 21 is 3 marks)

22 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

$P(\text{No pink left}) = P(RPP) + P(BPP) + P(PPR) + P(PPB) + P(PRP) + P(PBP)$

$P(\text{No pink left}) = \left(\frac{3}{15} \times \frac{2}{14} \times \frac{1}{13}\right) \times 3 + \left(\frac{10}{15} \times \frac{2}{14} \times \frac{1}{13}\right) \times 3 =$

$\frac{3}{455} + \frac{2}{91}$

$P(\text{No pink left}) = \frac{1}{35}$

$P(\text{At least 1 pink left}) = 1 - \frac{1}{35} = \frac{34}{35}$

$\frac{34}{35}$

(Total for Question 22 is 3 marks)

23 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$$a = \frac{2\left[\frac{5-2y}{3y}\right]+5}{1-\frac{5-2y}{3y}}$$

$$a = \frac{\frac{10-4y}{3y}+5}{\frac{3y-(5-2y)}{3y}}$$

$$a = \frac{\frac{10-4y+15y}{3y}}{\frac{3y-5+2y}{3y}}$$

$$a = \frac{10+11y}{3y} \times \frac{3y}{5y-5}$$

$$a = \frac{10+11y}{5(y-1)}$$

$m = 10 \quad n = 11 \quad p = 5$

$a = \dfrac{10+11y}{5(y-1)}$

(Total for Question 23 is 3 marks)

24 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (4, 3)

Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x + 5)$

(−1 , 3)

(1)

(ii) $y = f(x) + 7$

(4 , 10)

(1)

(iii) $y = f(2x)$

(2 , 3)

(1)

(Total for Question 24 is 3 marks)

$2x - 28 = -56$

$+8, -7$

25 Solve the inequality $2x^2 + x - 28 > 0$
Show clear algebraic working.

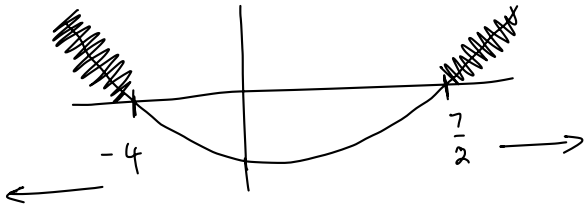
$2x^2 + 8x - 7x - 28 > 0$

$2x(x+4) - 7(x+4) > 0$

$(2x-7)(x+4) > 0$

critical values

$x = \frac{7}{2} \quad x = -4$



$x < -4$

$x > \frac{7}{2}$

$x < -4 \quad x > \frac{7}{2}$

(Total for Question 25 is 3 marks)

Turn over for Question 26

26 Here is a prism *ABCDEFGH* with a horizontal, rectangular base *ABGF*

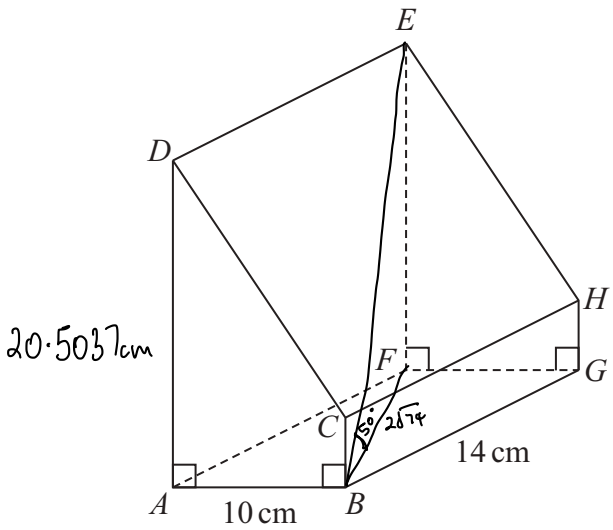
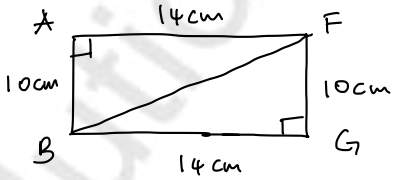


Diagram **NOT** accurately drawn



$AB = 10\,\text{cm}$ $BG = 14\,\text{cm}$

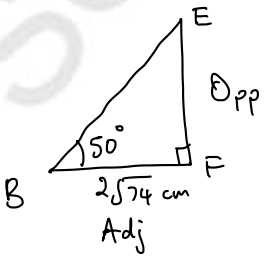
angle DAB = angle ABC = angle EFG = angle $FGH = 90°$

$BC = \frac{1}{5}AD$

The angle of elevation of E from B is 50°

Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
Show your working clearly.

$BF = \sqrt{14^2 + 10^2} = 2\sqrt{74}$ cm



$\tan 50 = \frac{EF}{2\sqrt{74}}$

$2\sqrt{74} \times \tan 50 = EF$

$EF \approx 20.5037\ldots$ cm

$EF = AD = 20.5037\ldots$ cm

$BC = \frac{1}{5} \times 20.5037\ldots$ cm

$BC \approx 4.10074\ldots$ cm

Area of Cross Section $= \dfrac{(4.10074 + 20.5037) \times 10}{2}$

Area of Cross Section $= 123.0222\ldots\ \text{cm}^2$

Volume of prism $= 123.0222\ldots \times 14 \approx 1722.3108\ \text{cm}^3$

1720 cm^3

(Total for Question 26 is 5 marks)

Turn over for Question 27

27

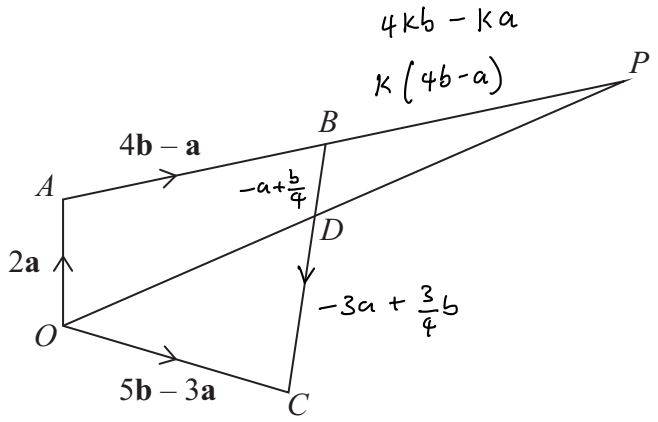


Diagram **NOT** accurately drawn

OABC is a quadrilateral.
ABP and *ODP* are straight lines.

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{AB} = 4\mathbf{b} - \mathbf{a}$ $\overrightarrow{OC} = 5\mathbf{b} - 3\mathbf{a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC}$
$\overrightarrow{BC} = -\overrightarrow{AB} - \overrightarrow{OA} + \overrightarrow{OC}$
$\overrightarrow{BC} = -(4b-a) - 2a + 5b - 3a$
$\overrightarrow{BC} = -4b + a - 2a + 5b - 3a$
$\overrightarrow{BC} = -4a + b$

$-4a + b$

(2)

The point *D* lies on *BC* such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of *n*

$\overrightarrow{BD} = \frac{1}{4}\overrightarrow{BC} = \frac{1}{4}(-4a+b) = -a + \frac{1}{4}b$
$\overrightarrow{DC} = \frac{3}{4}\overrightarrow{BC} = \frac{3}{4}(-4a+b) = -3a + \frac{3}{4}b$
$\overrightarrow{OP} = n\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AP}$
$n\overrightarrow{OD} = 2a + 4b - a + 4kb - ka$
$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$
$\overrightarrow{OD} = 5b - 3a - (-3a + \frac{3}{4}b)$
$\overrightarrow{OD} = 5b - 3a + 3a - \frac{3}{4}b$
$\overrightarrow{OD} = \frac{17}{4}b$

$\frac{17}{4}nb = 2a + 4b - a + 4kb - ka$
$\frac{17}{4}nb = a - ka + 4b + 4kb$
$\frac{17}{4}nb = (1-k)a + (4+4k)b$
$1 - k = 0$ $4 + 4k = \frac{17}{4}n$
$k = 1$ $4 + 4 = \frac{17}{4}n$
$8 = \frac{17}{4}n$
$\frac{8 \times 4}{17} = n = \frac{32}{17}$ //

$n =$ $\frac{32}{17}$

(4)

(Total for Question 27 is 6 marks)

Turn over for Question 28

28 The curve **C** has equation $x^2 + y^2 = d - 11x$ where d is an integer.
The line **L** has equation $y = 3x + e$ where e is an integer.

C and **L** intersect at the point A and at the point B

The coordinates of A are (0.2, 2.6)

Work out the coordinates of B
Show your working clearly.

To find d

$(0.2)^2 + (2.6)^2 = d - 11(0.2)$

$6.8 = d - 2.2$

$9 = d$

C : $x^2 + y^2 = 9 - 11x$

To find e

$2.6 = 3(0.2) + e$

$2.6 - 0.6 = e$

$2 = e$

L : $y = 3x + 2$

$x^2 + (3x+2)^2 = 9 - 11x$

$x^2 + 9x^2 + 12x + 4 = 9 - 11x$

$10x^2 + 12x + 11x + 4 - 9 = 0$

$10x^2 + 23x - 5 = 0$

$10x^2 + 25x - 2x - 5 = 0$

$5x(2x+5) - 1(2x-5) = 0$

$(5x-1)(2x-5) = 0$

$5x - 1 = 0 \quad 2x - 5 = 0$

$x = \frac{1}{5} \quad x = \frac{5}{2}$

$y = 3\left(\frac{5}{2}\right) + 2$

$y = \frac{19}{2}$

$B\left(\frac{5}{2}, \frac{19}{2}\right)$

$10x - 5 = -50$

$50, -1$

$25, -2$

($\frac{5}{2}$, $\frac{9}{2}$)

(Total for Question 28 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS