Answer ALL questions. Write your answers in the spaces provided. You must write down all the stages in your working.

1 The point P has coordinates (3, 4)
The point Q has coordinates (9, 16)

M is the midpoint of the line PQ

Find the coordinates of M

$M\left(\frac{3+9}{2}, \frac{4+16}{2}\right)$

(6 , 10)

(Total for Question 1 is 2 marks)

2 Solve the simultaneous equations

$$5x + y = 11$$
$$3x - y = 9$$

Show clear algebraic working.

$8x = 20$

$\div 8 \quad \div 8$

$x = \frac{5}{2}$

$3\left(\frac{5}{2}\right) - y = 9$

$\frac{15}{2} - y = 9$

$\frac{15}{2} - 9 = y$

$-\frac{3}{2} = y$

$x =$ $\frac{5}{2}$

$y =$ $-\frac{3}{2}$

(Total for Question 2 is 3 marks)

3 A box contains only

9 red bricks
43 blue bricks
and some yellow bricks

$\frac{7}{20}$ of the bricks are yellow bricks.

Each brick weighs 35 grams.

Work out the total weight of the yellow bricks.

let yellow bricks $= x$

Total $= 9 + 43 + x = 52 + x$

$P(\text{yellow}) = \frac{x}{52+x} = \frac{7}{20}$

$20 \times x = 7(52+x)$

$20x = 364 + 7x$

$13x = 364$

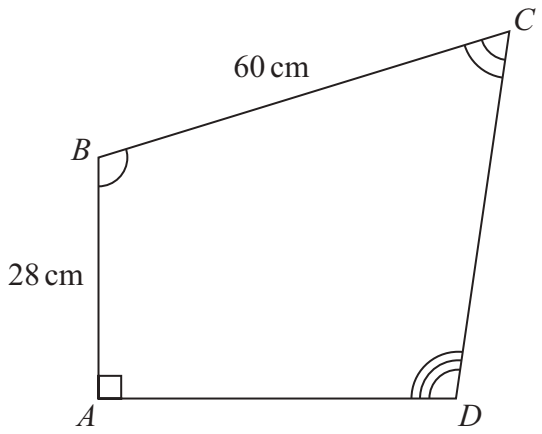
$x = 28$

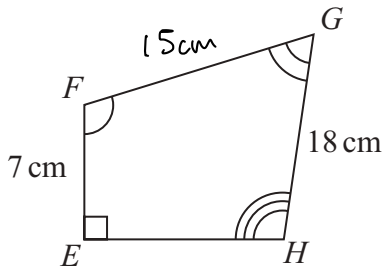
weight of yellow $= 35 \times 28$

980 grams

(Total for Question 3 is 4 marks)

4 Here are three similar quadrilaterals.





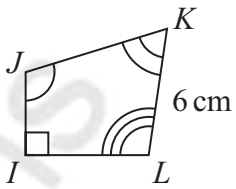


Diagram **NOT** accurately drawn

Work out the length of *JK*

Compare
BA and FE

$\frac{BA}{4} = FE$

$FG = \frac{1}{4} BC$

$FG = \frac{1}{4} \times 60 = 15cm$

$\times \frac{1}{4}$

$\times \frac{1}{3}$

Compare
GH and KL

$\frac{GH}{3} = KL$

$\frac{FG}{3} = JK$

$JK = \frac{15}{3} = 5cm$

5 cm

(Total for Question 4 is 3 marks)

5 Otis sells ice creams.

On Friday, Otis sells 75 ice creams.
On Saturday, Otis sells 87 ice creams.

(a) Work out the percentage increase in the number of ice creams Otis sells from Friday to Saturday.

$\frac{87 - 75}{75} \times 100$

16 %

(3)

Claudia buys an ice cream machine for 960 Swiss francs.

The value of the ice cream machine depreciates by 20% each year.

(b) Work out the value of the ice cream machine at the end of 3 years.

20% depreciation => 100% - 20% = 80% = 0.8

960×0.8^3

491.52 Swiss francs

(3)

(Total for Question 5 is 6 marks)

6 A large circle with centre O contains 3 identical small circles.
Each circle touches two other circles.

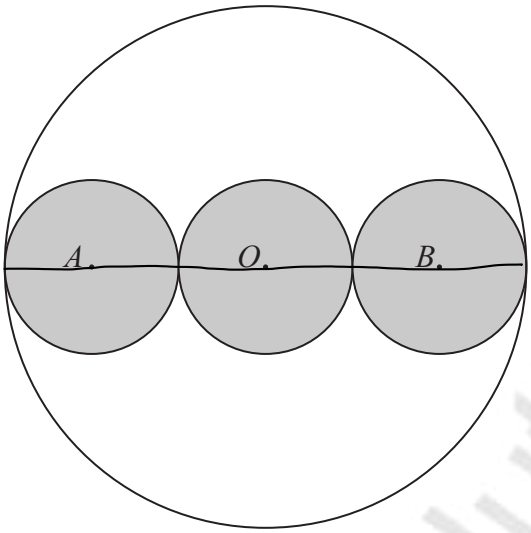


Diagram **NOT** accurately drawn

A, O and B are the centres of the small circles.
AOB is a straight line.

The circumference of the large circle is 160 cm

Work out the area of one small circle.
Give your answer correct to 3 significant figures.

$D_{big} = 3d_{small}$

$C = \pi \times D_{big}$

$160 = \pi D_{big}$

$D_{big} = \frac{160}{\pi}$ cm

$d_{small} = \frac{1}{3}\left(\frac{160}{\pi}\right)$ cm $= \frac{160}{3\pi}$ cm

$r_{small} = \frac{1}{2}\left(\frac{160}{3\pi}\right)$ cm $= \frac{80}{3\pi}$ cm

$A = \pi r^2$

$A_{small} = \pi \times \left(\frac{80}{3\pi}\right)^2$

$A_{small} = \pi \times \frac{6400}{9\pi^2}$

226 ..... cm^2

(Total for Question 6 is 5 marks)

7 (i) Factorise $x^2 + 2x - 48$

-48

$+48, -1$

$+12, -4$

$+8, -6$

$(x+8)(x-6)$

$(x+8)(x-6)$

(2)

(ii) Hence, solve $x^2 + 2x - 48 = 0$

$x + 8 = 0$ $\quad x = -8$

$x - 6 = 0$ $\quad x = 6$

$x = -8 \quad x = 6$

(1)

(Total for Question 7 is 3 marks)

8 (a) Write 5.76×10^4 as an ordinary number.

5.7600

57600

(1)

(b) Work out $\dfrac{3 \times 10^5 + 8 \times 10^3}{4 \times 10^{-2}}$

Give your answer in standard form.

$\dfrac{(3 \times 10^5) + (0.08 \times 10^5)}{4 \times 10^{-2}}$

$\dfrac{(3 + 0.08) \times 10^5}{4 \times 10^{-2}}$

$\dfrac{3.08 \times 10^5}{4 \times 10^{-2}}$

$0.77 \times 10^{5-(-2)}$

0.77×10^7

7.7×10^6

7.7×10^6

(2)

(Total for Question 8 is 3 marks)

9 Mark has three bags, bag A, bag B and bag C

Bag A contains 120 counters.
The probability of taking at random a red counter from bag A is 0.45

Bag B contains 80 counters.
The probability of taking at random a red counter from bag B is 0.3

Bag C is empty.

Mark puts all the counters from bag A and bag B into bag C

Mark takes at random a counter from bag C

Work out the probability that he takes a red counter.

Red in Bag A = $0.45 \times 120 = 54$

Red in Bag B = $0.3 \times 80 = 24$

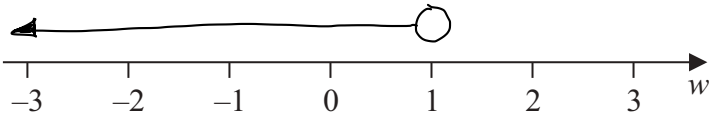
Bag C now has total of 78 red

Total in Bag C = $120 + 80 = 200$

$\frac{78}{200}$

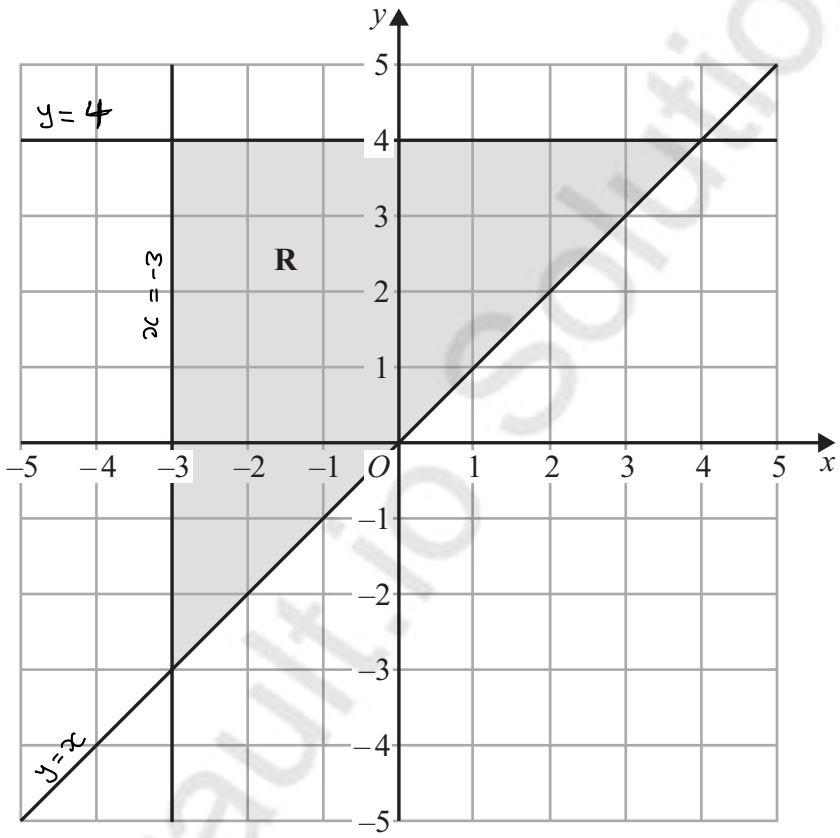
(Total for Question 9 is 3 marks)

10 (a) On the number line, represent the inequality $w < 1$



(1)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

$y \geqslant x$

$x \geqslant -3$

$y \leqslant 4$

(3)

(Total for Question 10 is 4 marks)

11 The diagram shows a right-angled triangle ABC

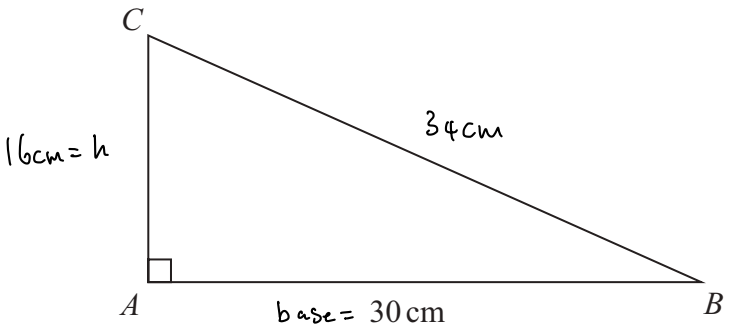


Diagram **NOT** accurately drawn

Area of triangle $ABC = 240\,\text{cm}^2$

Work out the perimeter of triangle ABC

$A = \frac{b \times h}{2}$

$240 = \frac{30 \times AC}{2}$

$\frac{240 \times 2}{30} = AC = 16\,\text{cm}$

$BC = \sqrt{30^2 + 16^2}$

$BC = 34\,\text{cm}$

Perimeter $= 34 + 30 + 16$

80 cm

(Total for Question 11 is 4 marks)

12 (a) Complete the table of values for $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$

At $x = 1$

$y = \frac{1}{2}\left(1 + \frac{4}{1}\right)$

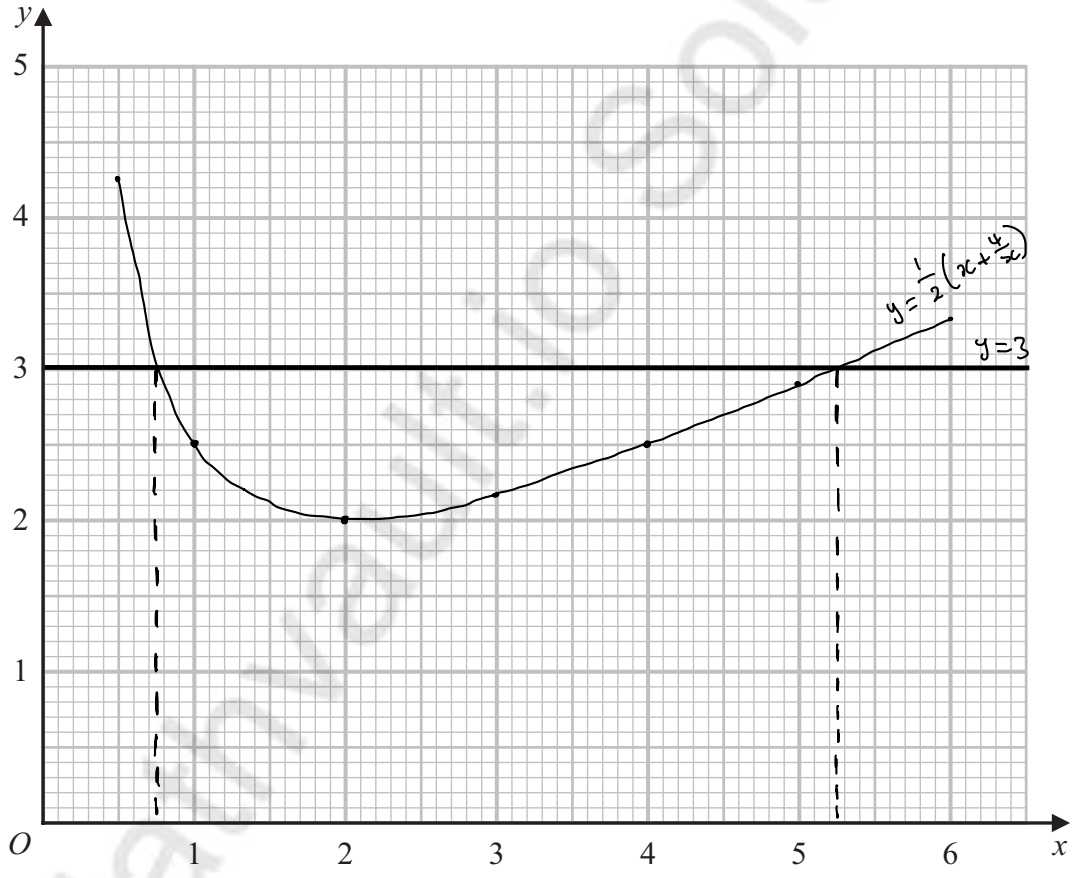
At $x = 2$

$y = \frac{1}{2}\left(2 + \frac{4}{2}\right)$

x	0.5	1	2	3	4	5	6
y	4.25	2·5	2	2.17	2.5	2.9	3.33

(1)

(b) Draw the graph of $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$ for $0.5 \leqslant x \leqslant 6$



(2)

(c) By drawing a suitable line on the grid, find estimates for the solutions of the equation $x + \frac{4}{x} = 6$

Give your answers correct to one decimal place.

$y = \frac{1}{2}\left(\underbrace{x + \frac{4}{x}}_{6}\right)$ $\quad x + \frac{4}{x} = 6$

$y = \frac{1}{2} \times 6$

$y = 3$ draw this on Graph

$x = 0.75 \quad x = 3.25$

$x = 0.8 \quad x = 3.3$

(2)

(Total for Question 12 is 5 marks)

13 (a) Simplify fully $\left(\frac{125x^6}{y^{15}}\right)^{\frac{2}{3}}$

$\frac{125^{2/3}(x^6)^{2/3}}{(y^{15})^{2/3}} = \frac{25x^{6\times 2/3}}{y^{15\times 2/3}} = \frac{25x^4}{y^{10}}$

$25x^4y^{-10}$

(2)

Given that $4^n = \frac{4^m}{64^p}$

(b) Express n in terms of m and p

$4^n = \frac{4^m}{(4^3)^p}$

$4^n = \frac{4^m}{4^{3p}}$

$4^n = 4^{m-3p}$

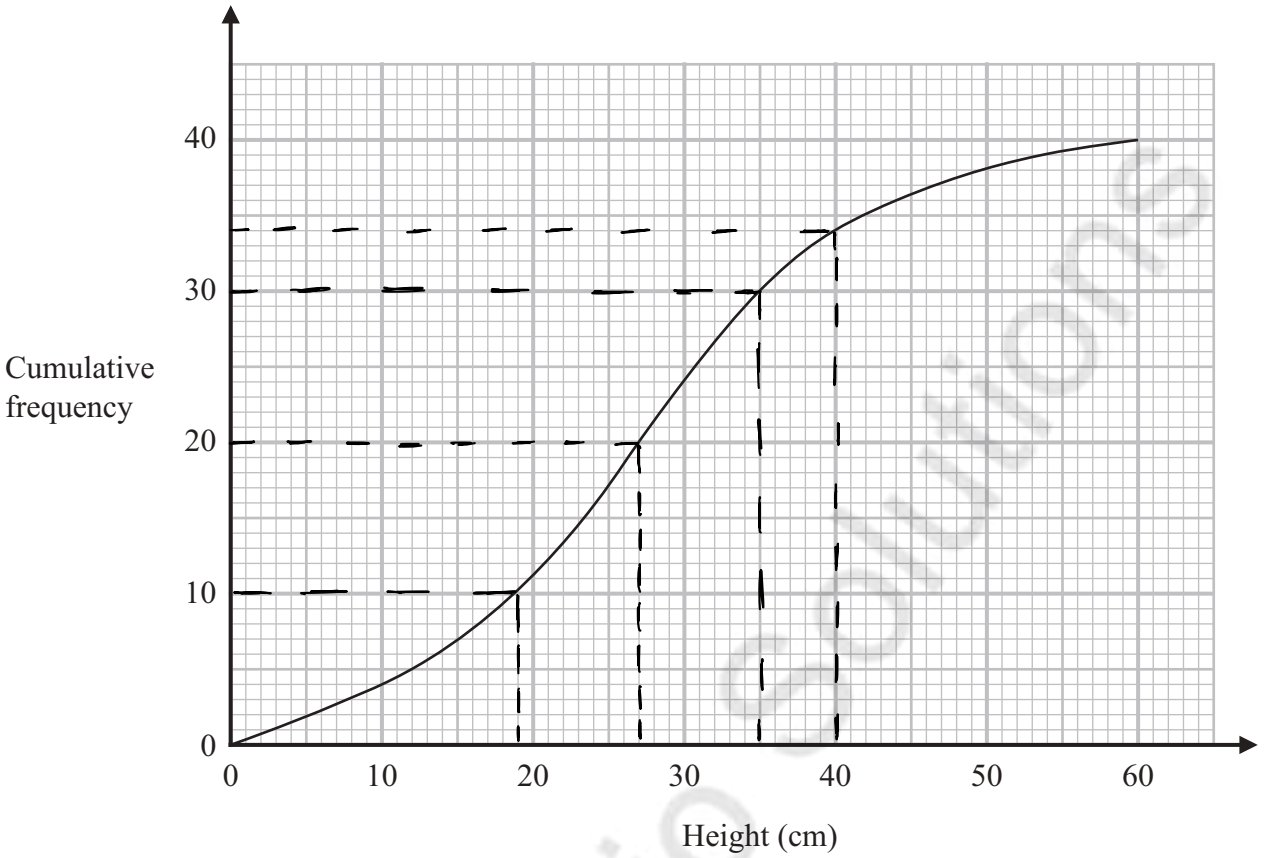
$n = m - 3p$

$n =$ $m - 3p$

(2)

(Total for Question 13 is 4 marks)

14 The cumulative frequency graph shows information about the heights of 40 plants that Greta has grown.



(a) Use the graph to find an estimate for the median height.

Median = $Q_2 = \frac{1}{2} \times 40 = 20$th

27 cm

(1)

(b) Use the graph to find an estimate for the interquartile range of the heights.

IQR = $Q_3 - Q_1$

$Q_3 = \frac{3}{4} \times 40 = 30$th

$Q_1 = \frac{1}{4} \times 40 = 10$th

$Q_3 = 35$cm

$Q_1 = 19$cm

IQR = 35 - 19

16 cm

(2)

Plants with a height greater than 40 cm are premium plants.
Greta sells all the premium plants for 30 euros each.

(c) Work out the total amount of money Greta receives for the premium plants.

34 plants less than 40cm

∴ 6 plants greater than 40cm

6 × 30

180 euros

(2)

(Total for Question 14 is 5 marks)

15 A, B, C and D are points on a circle with centre O

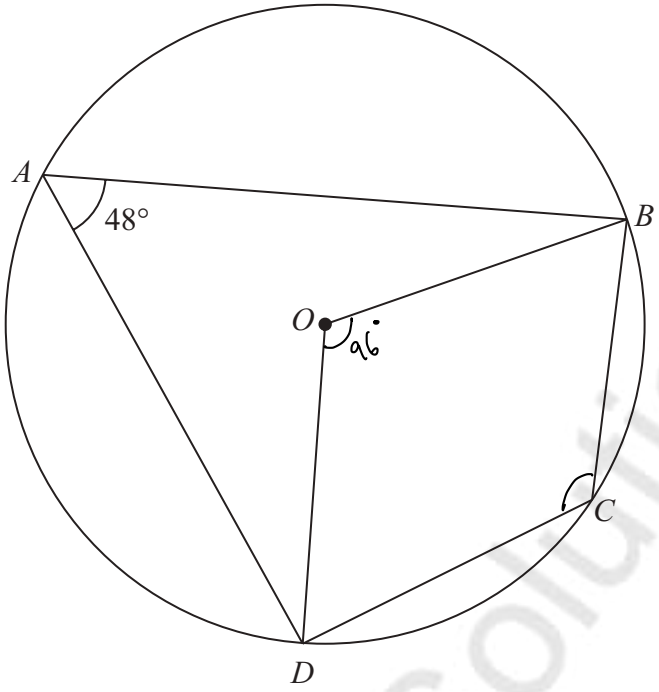


Diagram **NOT** accurately drawn

Angle $DAB = 48°$

(a) (i) Work out the size of the obtuse angle DOB

48 x 2

96 °

(1)

(ii) Give a reason for your answer.

Angle at Centre is twice angle at Circumference

(1)

(b) (i) Work out the size of angle BCD

180 − 48

132 °

(1)

(ii) Give a reason for your answer.

Opposite angles in a cyclic quadrilateral sum to 180°

(1)

(Total for Question 15 is 4 marks)

16 Use algebra to show that $0.3\dot{7}\dot{8} = \frac{25}{66}$

$x = 0.3\dot{7}\dot{8}$

$10x = 3.\dot{7}\dot{8}$ (×100)

$1000x = 378.\dot{7}\dot{8}$ (×100)

$1000x - 10x = 378.\dot{7}\dot{8} - 3.\dot{7}\dot{8}$

$990x = 375$

$x = \frac{375}{990}$ (÷15)

$x = \frac{25}{66}$

(Total for Question 16 is 2 marks)

17 T is inversely proportional to $\sqrt{m}$

$T = 15$ when $m = 36$

Find a formula for T in terms of m

$T \propto \frac{1}{\sqrt{m}}$

$T = \frac{k}{\sqrt{m}}$

$15 = \frac{k}{\sqrt{36}}$

$15 \times 6 = k$

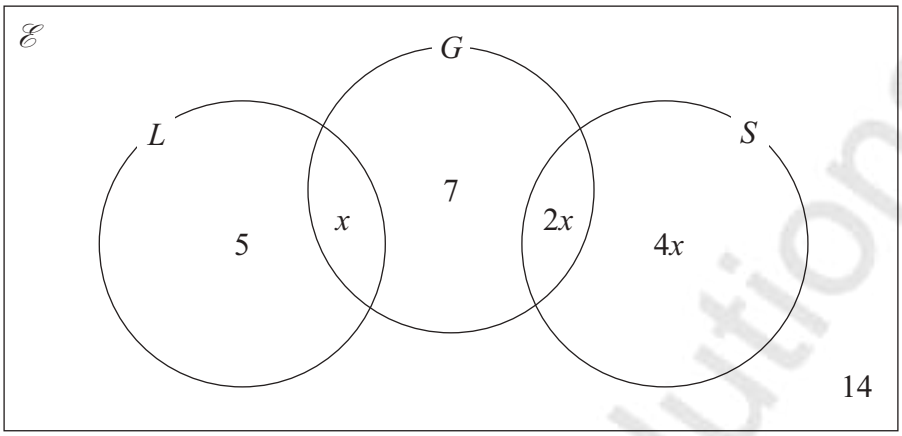
$90 = k$

$T = \frac{90}{\sqrt{m}}$

(Total for Question 17 is 3 marks)

18 Bianca asked 40 students which of the languages Latin (L), Greek (G) and Sanskrit (S) they study.

The Venn diagram gives some information about her results.
It shows the number of students in each subset.



One of these students is selected at random.
Work out the probability that this student studies Latin or Sanskrit.

$5 + x + 7 + 2x + 4x + 14 = 40$

$26 + 7x = 40$

$7x = 14$

$x = 2$

$\frac{5 + x + 2x + 4x}{40}$

$\frac{5 + 7x}{40}$

$\frac{5 + 7(2)}{40}$

$\frac{19}{40}$

(Total for Question 18 is 3 marks)

19 The functions f and g are such that

$$f(x) = 3x - 2 \qquad g(x) = \frac{x}{2x - 1}$$

(a) Find g(3)

$$g(3) = \frac{3}{2(3)-1}$$

$\frac{3}{5}$

(1)

(b) Find gf(x)
Give your answer in its simplest form.

$$\frac{3x - 2}{2(3x-2) - 1}$$

$$\frac{3x - 2}{6x-4 - 1}$$

$$\frac{3x-2}{6x - 5}$$

gf(x) = $\frac{3x-2}{6x-5}$

(2)

(Total for Question 19 is 3 marks)

20 Here is a quadratic equation.

$$ax^2 + 4x + c = 0$$

The solutions of this equation are given by $x = \dfrac{-4 \pm 2\sqrt{39}}{10}$

Find the value of a and the value of c
Show your working clearly.

$10x = -4 \pm 2\sqrt{39}$

$10x + 4 = \pm 2\sqrt{39}$

$(10x + 4)^2 = 156$

$100x^2 + 80x + 16 = 156$

$100x^2 + 80x - 140 = 0$

$\div 20 \quad \div 20 \quad \div 20 \quad \div 20$

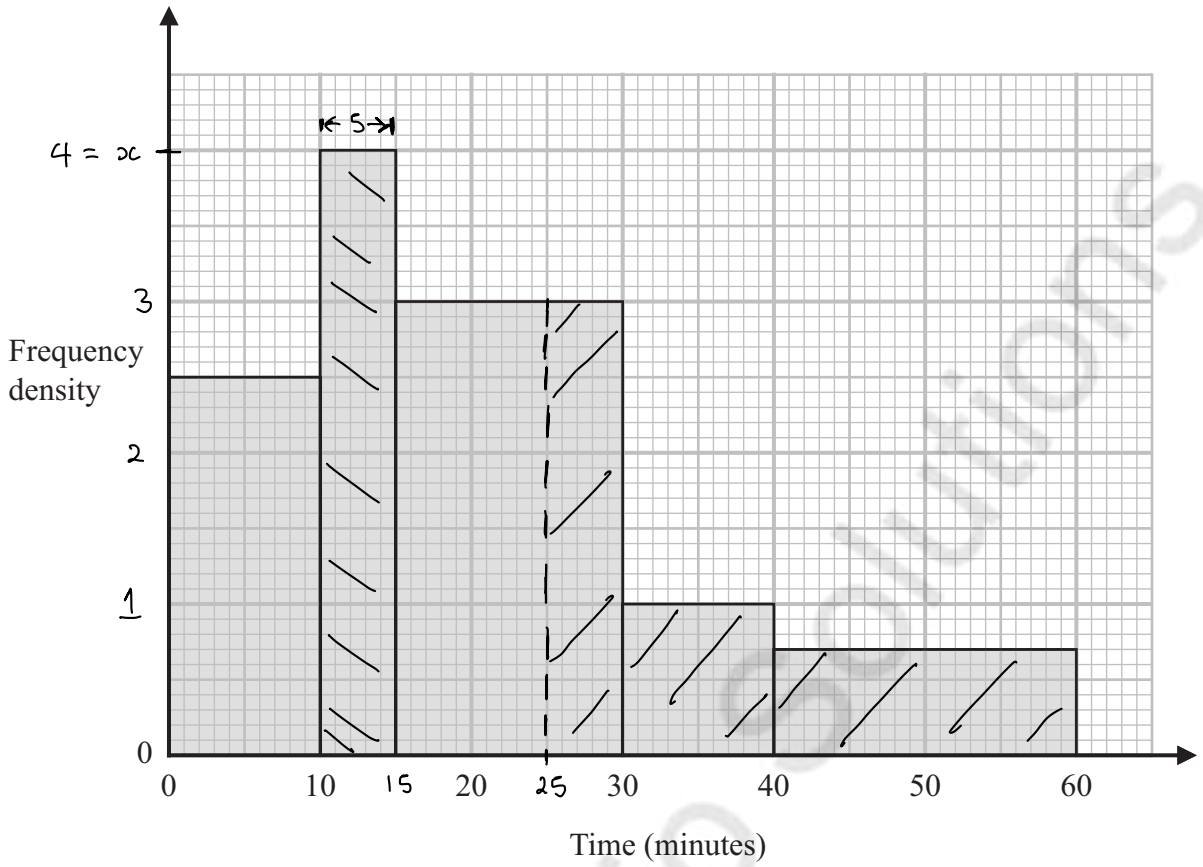
$5x^2 + 4x - 7 = 0$

$a =$ 5

$c =$ −7

(Total for Question 20 is 3 marks)

21 The histogram shows information about the times, in minutes, that some trains arrived late at a station one day.



20 of these trains arrived between 10 minutes late and 15 minutes late.

No trains arrived more than 60 minutes late.

Work out an estimate for the number of these trains that arrived at least 25 minutes late.

$20 = x \times 5$

$\frac{20}{5} = x = 4$

25 - 30 : $3 \times 5 = 15$

30 - 40 : $1 \times 10 = 10$

40 - 60 : $0.7 \times 20 = 14$

39

39

(Total for Question 21 is 3 marks)

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

$\frac{dy}{dx} = 3x^2 - 16$

$11 = 3x^2 - 16$

$27 = 3x^2$

$9 = x^2$

$x = 3 \quad x = -3$

$y = 3^3 - 16(3) + 7$ $\quad$ $y = (-3)^3 - 16(-3) + 7$

$y = -14$ $\quad$ $y = 28$

$-14 = 11(3) + b$ $\quad$ $28 = 11(-3) + b$

$-14 = 33 + b$ $\quad$ $28 = -33 + b$

$b = -47$ $\quad$ $b = 61$

$b = -47 \quad b = 61$

(Total for Question 22 is 6 marks)

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

P : Q

Area 200 : 450

÷10 ÷10

20 : 45

÷5 ÷5

Area 4 : 9

length $\sqrt{4}$: $\sqrt{9}$

length 2 : 3

Volume 2^3 : 3^3

8 : 27

672 : V_Q

$8 \times V_Q = 27 \times 672$

$V_Q = \frac{27 \times 672}{8}$

2268 cm^3

(Total for Question 23 is 3 marks)

24 $D = \dfrac{n}{p - q}$

n = 10.3 correct to 1 decimal place
p = 7.24 correct to 2 decimal places
q = 4.39 correct to 2 decimal places

By considering bounds, work out the value of D to a suitable degree of accuracy.
Show your working clearly.

10.2 ¦ 10.3 ¦ 10.4

$10.25 \leq n < 10.35$

7.23 ¦ 7.24 ¦ 7.25

$7.235 \leq p < 7.245$

4.38 ¦ 4.39 ¦ 4.40

$4.385 \leq q < 4.395$

$D_{min} = \dfrac{n_{min}}{p_{max} - q_{min}}$

$D_{min} = \dfrac{10.25}{7.245 - 4.385}$

$D_{min} = 3.583916084\ldots$

$D_{min} \approx 3.6$ (1 d.p.)

$D_{max} = \dfrac{n_{max}}{p_{min} - q_{max}}$

$D_{max} = \dfrac{10.35}{7.235 - 4.395}$

$D_{max} = 3.644366197\ldots$

$D_{max} \approx 3.6$ (1 d.p.)

3.6

(Total for Question 24 is 5 marks)

25 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$T_n = a+(n-1)d$

$a = 10$

$T_{20} = 86 = 10 + 19d$

$76 = 19d$

$d = 4$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$5194 = \frac{n}{2}\{2(10) + 4n - 4\}$

$5194 = \frac{n}{2}(16 + 4n)$

$10388 = 16n + 4n^2$

$0 = 4n^2 + 16n - 10388$

$0 = n^2 + 4n - 2597$

$0 = (n-49)(n+53)$

$n = 49$ $\quad n = -53$ reject

$N =$ 49

(Total for Question 25 is 5 marks)

Turn over for Question 26

26 A solid cone is joined to a solid hemisphere to make the solid shown below.

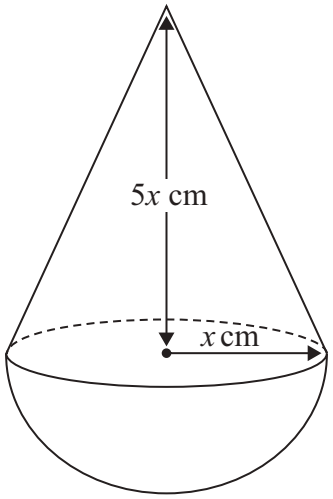


Diagram **NOT** accurately drawn

The cone is made from copper.
The density of copper is 9 g/cm^3

The hemisphere is made from a different metal.

The total mass of the solid is 4752π grams
The total volume of the solid is 504π cm^3

Work out the density of the hemisphere.
Show your working clearly.

$V_{cone} = \frac{1}{3}\pi \times x^2 \times 5x = \frac{5\pi x^3}{3}$

$V_{hemisphere} = \frac{2}{3}\pi x^3$

$V_{Total} = \left(\frac{2}{3}\pi + \frac{5\pi}{3}\right)x^3$

$V_{Total} = \frac{7}{3}\pi x^3 = 504\pi$

$x^3 = \frac{504 \times 3}{7}$

$x = 216^{1/3} = 6$

$V_{hemisphere} = \frac{2}{3}\pi \times 6^3 = 144\pi \text{ cm}^3$

$V_{cone} = \frac{5\pi \times 6^3}{3} = 360\pi \text{ cm}^3$

$D = \frac{M}{V}$, $M_{cone} = D_{cone} \times V_{cone}$

$M_{cone} = 9 \times 360\pi$

$M_{cone} = 3240\pi \text{ g}$

$M_{hemisphere} = 4752\pi - 3240\pi = 1512\pi \text{ g}$

$D_{hemisphere} = \frac{1512\pi \text{ g}}{144\pi \text{ cm}^3} = 10.5 \text{ g/cm}^3$

10·5 g/cm^3

(Total for Question 26 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS