Mark Scheme (Results)

Summer 2025

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown, then correct answers may score full marks

  If no working is shown, then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: e.g. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

   $(x^2+bx+c)=(x+p)(x+q)$ where $|pq|=|c|$ leading to $x=\ldots$

   $(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots$

2. Formula:

   Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=\ldots$

3. Completing the square:

   $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration:

   Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in
the substitution of values
or, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this e.g. in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	**Scheme**	**Marks**
1 (a)	$b^2-4ac=\left(-9\right)^2-4\times3\times7=-3$ $-3<0$ so no (real) solutions for v	M1 A1 [2]
(b)	$\left\{\dfrac{\mathrm{d}v}{\mathrm{d}t}\right\}=6t-9\Rightarrow a=6\times4-9=15\left(\text{m/s}^2\right)$	M1A1 [2]
(c)	$s=\int_0^5(3t^2-9t+7)\,\mathrm{d}t=\left[t^3-\dfrac{9}{2}t^2+7t\right]_0^5$ $=5^3-\dfrac{9}{2}\times5^2+7\times5=\dfrac{95}{2}\text{(m) oe}$	M1 M1A1 [3]
		Total 7 marks

Part	**Mark**	**Additional Guidance**
(a)	M1	Applies the discriminant, or any other valid method. e.g. Attempts to solve the 3TQ by quadratic formula, Completing square, achieves $3\left(t-\dfrac{3}{2}\right)^2+\dots$ Attempts to solve v = 0, finds complex roots
	A1	Verifies that the equation for v has no (real) solutions By completing square, it must be correct, $3\left(t-\dfrac{3}{2}\right)^2+\dfrac{1}{4}$, states the graph is above x axis, or > 0 Finds correct complex roots $\dfrac{9\pm\sqrt{3}i}{6}$, states imaginary numbers/roots **Note**: Just stating "error" is not enough
(b)	M1	Attempts to differentiate the expression for v. Power of t to decrease in at least one term and increase in none.
	A1	Substitutes t = 4 and obtains correct acceleration, with or without unit.
(c)	M1	Attempt to integrate the expression for v. Power of t to increase in at least one term and decrease in none. Ignore limits if shown.
	M1	Clear substitution of the **given** limits into their **integrated** expression for distance, no need to see substitution of zero, can be implied by the correct distance found from a **correct integrated** expression
	A1	$\dfrac{95}{2}$ (m) oe e.g. 47.5, ignore unit

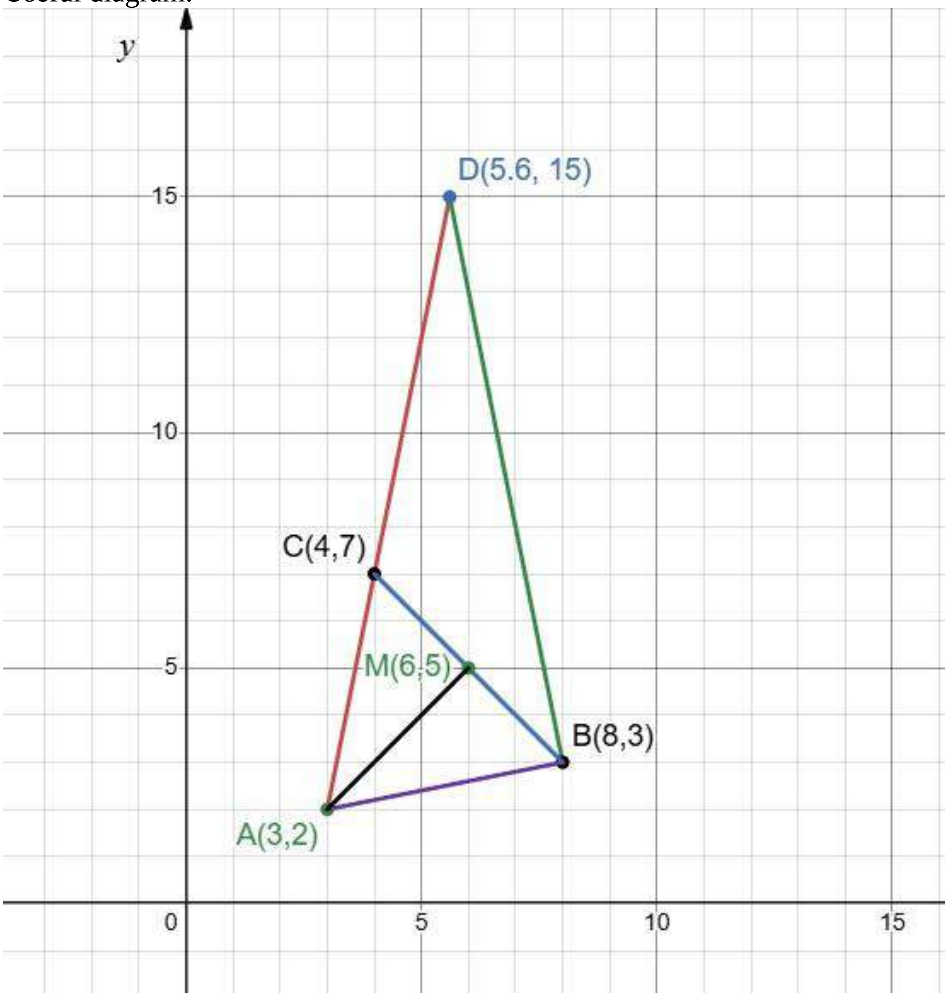
Question number	Scheme	Marks
2(a)	$\sqrt{(8-3)^2+(3-2)^2}$ or $\sqrt{(4-3)^2+(7-2)^2}$ or $\sqrt{(8-4)^2+(3-7)^2}$	M1
	$AB=AC=\sqrt{26}$ hence isosceles	A1*cso [2]
(b)	Midpoint $M=(6,5)$ or gradient of $AM=1$	B1
	e.g. $y-2=\left[\frac{5-2}{6-3}\right](x-3)$ or $y-2=\text{"1"}\times(x-3)\Rightarrow y=x-1$	M1A1 [3]
(c)	Equation of AC $y-2=\left(\frac{7-2}{4-3}\right)(x-3)\Rightarrow[y=5x-13]$	M1A1
	Gradient of AB $m=\frac{3-2}{8-3}\left[=\frac{1}{5}\right]$	M1
	Gradient of DB $m_p=-\frac{1}{'1/5'}[=-5]$	M1
	Equation of BD $y-3=-5(x-8)\Rightarrow[y=-5x+43]$	M1A1
	$5x-13='-5x+43'$ $\Rightarrow x=....$ or $y=...$	dM1
	$\left(\frac{56}{10},15\right)$ oe	A1 [8]
ALT I (c) Last 6 marks	$D(x,'5x-13')$	M1
	$BD^2=(x-8)^2+(5x-13-3)^2$ or $AD^2=(x-3)^2+(5x-13-2)^2$	M1
	$26+(x-8)^2+(5x-13-3)^2=(x-3)^2+(5x-13-2)^2$	M1A1
	$26-16x+64-160x+256=-6x+9-150x+225\Rightarrow x=...$	dM1
	$\left(\frac{56}{10},15\right)$ oe	A1

ALT II (c)	$y-2=\left(\frac{7-2}{4-3}\right)(x-3)\Rightarrow\left[y=5x-13\right]$ or $\overrightarrow{AC}=\begin{pmatrix}1\\5\end{pmatrix}$ oe	M1A1
	$\left('4\sqrt{2}'\right)^2='\sqrt{26}'^2+'\sqrt{26}'^2-2\times'\sqrt{26}'\times'\sqrt{26}'\cos\angle BAC\Rightarrow\cos\angle BAC=...$ or $\sin\frac{\angle BAC}{2}=\frac{'2\sqrt{2}'}{'\sqrt{26}'}$	M1
	$\cos\angle BAC=\frac{5}{13}$ or $\angle BAC=67.4$(awrt)	M1
	$\frac{\sqrt{26}}{k\sqrt{26}}=\frac{5}{13}$ or $\cos 67.38...=\frac{\sqrt{26}}{k\sqrt{26}}$	M1
	$k=2.6$ exact	A1
	$3+1\times'2.6'=...$ and $2+5\times'2.6'=....$ or $\sqrt{(x-3)^2+('5x-13'-2)^2}='2.6\sqrt{26}'\Rightarrow x=...,y=...$	dM1
	$(5.6,15)$	A1
		Total 13 marks

Part	Mark	Additional Guidance
(a)	M1	For correct use of Pythagoras' theorem to find the $\text{length}/\text{length}^2$ of one of the sides. Must have a subtraction of *x* coordinates and a subtraction of *y* coordinates with appropriate substitution into Pythagoras' theorem. Or at least one correct vector for AC, AB, BC, or the other way around
	A1	For showing **both** *AB* and *AC* have $\text{length} = \sqrt{26}$ or $\text{length}^2 = 26$ with conclusion that the triangle is isosceles. e.g. stating *AC*=*AB* Correct vectors for *AB* and *AC*, followed by $\lvert AC\rvert = \lvert AB\rvert$ oe Allow statements such as 'hence shown' or symbols to indicate conclusion of the demonstration such as QED, // or □.
(b)	B1	For finding the midpoint *M* of line *BC* Or finds the correct gradient of $AM = 1$
	M1	For finding the equation of the line *AM* in any form using any correct valid method: e.g. $\begin{cases} 3m + c = 2 \\ '6'm + c = '5' \end{cases} \Rightarrow m = \ldots, c = \ldots \Rightarrow y = \ldots$
	A1	For $y = x - 1$
(c)	M1	For correct method of finding the equation of line *AC*
	A1	For the correct equation of *AC*.
	M1	For attempting to find the gradient of *AB*
	M1	For negative reciprocal of **their** gradient *AB*
	M1	For attempting to find the equation of the line perpendicular to *AB* and passing through *B*, their gradient of *BD* might not be $-\dfrac{1}{m_{AB}}$, but must be a changed gradient
	A1	For the correct equation of *BD*.
	dM1	For attempting to solve their line *BD* and *AC* simultaneously. Minimum required for a correct method is to obtain an equation in either *x* or *y* (by elimination method or substitution method) and finds a value for *x* or *y* Dependent on the previous M mark only
	A1	For (5.6, 15) oe, accept given as $x = 5.6 \quad y = 15$
ALT I (c) Last 6 marks	M1	Use the fact that *D* is on the line *AC*, sets up the coordinate of $D(x, '5x-13')$ or $D(x, y)$ where *y* is replaced by their $'5x-13'$ later
	M1	For correct attempt to find the length/length^2 expression of *BD* or *AD* using $D(x, '5x-13')$ or $D(x, y)$
	M1	For correct use of Pythagoras' theorem to set up equation $\lvert AB\rvert^2 + \lvert BD\rvert^2 = \lvert AD\rvert^2$ in terms of *x* only
	A1	Correct equation, simplified/ unsimplified
	dM1	For expanding and solving their equation to find *x* =… Dependent on the previous M mark only
	A1	For (5.6, 15) oe accept given as $x = 5.6 \quad y = 15$

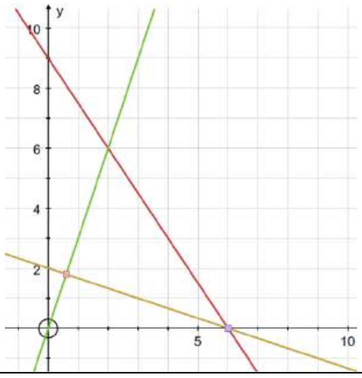
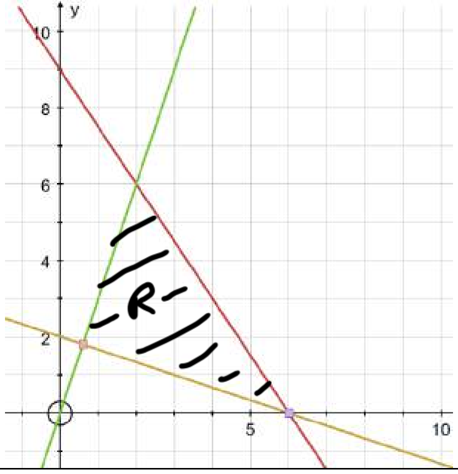
ALT II (c)	M1	Correct method of finding the equation of line *AC* or Vector *AC*
	A1	Correct equation of *AC* or correct vector *AC*
	M1	Correct use of cosine rule or trigonometry to set up an equation involving angle *BAC*
	M1	Obtains a value of $\cos\angle BAC$ or $\angle BAC$ from a correct cosine rule or trigonometry equation
	M1	Correct equation in *k*
	A1	Correct *k* value, must be exact/rounded to 2.6
	dM1	Uses their *k* value to find *x* =… and *y* =… Dependent on the previous M mark only
	A1	Correct coordinates from an exact value of *k* found, not a rounded value

Useful diagram:



Question number	Scheme	Marks
3 (a)	$2\times 7\times\theta=\frac{28\pi}{9}$ oe or $2\times\pi\times 7\times\frac{angle}{360}=\frac{14\pi}{9}$ oe **and** attempts to change back to radians	M1
	$\theta=\frac{2\pi}{9}$	A1 [2]
(b)	Area of sector $A=\frac{1}{2}\times 7^2\times{}'\frac{2\pi}{9}{}'$	M1
	Area of triangle $A=\frac{1}{2}\times 7^2\times\sin{}'\frac{2\pi}{9}{}'$	M1
	Area of segment $[A=][2\times]\left(\frac{1}{2}\times 7^2\times{}'\frac{2\pi}{9}{}'-\frac{1}{2}\times 7^2\times\sin{}'\frac{2\pi}{9}{}'\right)=2.71$ (awrt)	ddM1A1 [4]
		Total marks 6

Part	Mark	Additional Guidance
(a)	M1	For forming a correct equation using arc length and *r* =7 If attempted by degrees, must see attempts to change degrees to radians
	A1	For $\theta=\frac{2\pi}{9}$ oee (or exact equivalent)
(b)	M1	For a correct method finding area of the sector using **their** θ $A=\frac{1}{2}\times 7^2\times{}'\frac{2\pi}{9}{}'$ or $A=\pi\times 7^2\times\frac{'40'}{360}$ $\left(\text{Note}:\frac{2\pi}{9}=40°\right)$
	M1	For a fully correct method finding the area of a triangle using their θ, can be implied by the correct area of triangle 15.7 awrt
	ddM1	For finding the difference between the area of the sector and the area of the triangle. The subtraction must be shown, or it could be implied by correct final answer. Dependent on **both** previous M marks
	A1	For 2.71 (awrt) ignore unit

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4 (a)</td><td>$3x+2y=18$ Intersections with axes at (0,9) (6,0)
$x+3y-6=0$ Intersections with axes at (0,2) (6,0)
$y=3x$ Coordinates at (0,0) (2, 6)
 </td><td>B1
B1
B1
[3]</td></tr>
<tr><td>(b)</td><td>For the correct region shaded in or out
 </td><td>B1ft
[1]</td></tr>
<tr><td>(c)</td><td>Points of intersection are:<table><tr><td>(0.6,1.8)</td><td>(2,6)</td><td>(6,0)</td></tr></table><table><tr><td>Vertex</td><td>(0.6,1.8)</td><td>(2,6)</td><td>(6,0)</td></tr><tr><td>$P=2x-y$</td><td>-0.6</td><td>-2</td><td>12</td></tr><tr><td></td><td></td><td>Least</td><td></td></tr></table>For $P=-2$</td><td>M1
A1
dM1
A1
[4]</td></tr>
<tr><td></td><td colspan="2">ALT – objective line approach</td></tr>
<tr><td></td><td>Slope of objective line is 2
(2,6)
$P=2\times2-6=-2$
For $P=-2$</td><td>[M1
A1
dM1
A1]</td></tr>
<tr><td colspan="3" align="right">Total 8 marks</td></tr>
</table>

Part	Mark	Additional Guidance
(a)	B1B1B1	B1 for one line correct, B1B1 for two lines correct, B1B1B1 for all three lines correct: $3x+2y=18$ Intersections with axes at (0,9) (6,0) $x+3y-6=0$ Intersections with axes at (0,2) (6,0) $y=3x$ Intersections with axes at (0,0) (2, 6)
(b)	B1ft	For the correct region marked – allow shaded in or out. Ft for shading the **closed** region from their three lines.
(c)	M1	For attempting to find at least **one** coordinate pair of their intersections: Either by reading values from their graph (might be written on the graph), **or** by solving simultaneous equations leading x=… y=....
	A1	Finds all **three** correct points of intersection: Allow point $\left(0.6,1.8\right)$ to be $\left(0.6\pm0.1,1.8\pm0.1\right)$ (0.6,1.8) | (2,6) | (6,0)
	dM1	For substituting any **one** point of intersection (could be from reading from the graph or solved by simultaneous equations) into the given P ft their coordinates. Any correct P value: $P=-2$, $P=-0.6$ or $P=12$ implies this mark. Dependent on the previous M mark
	A1	For $P=-2$ from testing at least **two** points, **and** all **three** points of intersection must be correct $\left(0.6,1.8\right)$, $\left(2,6\right)$ and $\left(6,0\right)$
ALT – objective line approach		
	M1	For an attempt to use objective line approach. Identified that the slope of the objective line is 2. Identified that the intersection of $3x+2y=18$ and $y=3x$ is where P is the least.
	A1	For finding the correct coordinates (2,6)
	dM1	For substituting their (2,6) into P.
	A1	For identifying $P=-2$

Question number	**Scheme**	**Marks**
5(a)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$ $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=$ $\beta^3\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3-3\alpha^2\beta-3\alpha\beta^2=\alpha^3+\beta^3$	M1 A1cso [2]
ALT (a)	$(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ $=(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)$ $=\alpha^3+\alpha\beta^2-\alpha^2\beta+\beta\alpha^2+\beta^3-\beta^2\alpha$ $=\alpha^3+\beta^3$	M1 A1 cso [2]
(b)	$\alpha\beta=-\frac{7}{2}$ and $\alpha+\beta=3$ Product: $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=-\frac{7}{2}$ Sum: $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\left\{\frac{\alpha^3+\beta^3}{\alpha\beta}\right\}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{3^3-3\times-\frac{7}{2}\times3}{-\frac{7}{2}}=-\frac{117}{7}$ Equation: $x^2+\frac{117}{7}x-\frac{7}{2}=0\Rightarrow14x^2+234x-49=0$	B1 B1ft M1A1 M1A1 [6]
		Total 8 marks

Part	**Mark**	**Additional Guidance**
(a)	M1	For correctly expanded $(\alpha+\beta)^3$ $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$
	A1cso	For obtaining the given result without any errors
ALT(a)	M1	Factorises $(\alpha+\beta)$ first then attempts to expand all brackets
	A1cso	For obtaining the given result without any errors
(b)	B1	For the correct sum **and** product of the given equation Allow calling α,β as $x_{1,}x_2$ or other letters
	B1ft	For the product $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta='-\frac{7}{2}'$, follows through their value of $\alpha\beta$
	M1	For the sum $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{(\alpha+\beta)^3-3\alpha\beta\left(\alpha+\beta\right)}{\alpha\beta}=\frac{('3')^3-(3)\times\left('-\frac{7}{2}'\right)\times('3')}{\left('-\frac{7}{2}'\right)}=\left(-\frac{117}{7}\right)$ Correct algebra and **correct** substitution of values of their $\alpha\beta$ and $\alpha+\beta$
	A1	For the correct sum $-\frac{117}{7}$
	M1	For correctly forming an equation with their sum and product $x^2-\text{sum }x+\text{product}=0$ $x^2\,'+\frac{117}{7}'x'-\frac{7}{2}'=0$ **Condone the absence of = 0** for this mark.
	A1	For the correct equation $14x^2+234x-49=0$ oe Must be integers coefficients

Question number	**Scheme**	**Marks**
6 (a)	$\sqrt{4-x}=2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ or A = 2, B = 4	B1B1 [2]
(b)	$2\left\{1+\left(\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2}{2!}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^3}{3!}+...\right\}$ $=2-\frac{x}{4}-\frac{x^2}{64}-\frac{x^3}{512}+...$	M1A1ft A1 [3]
(c)	$\frac{\sqrt{305}}{10}=\sqrt{4-x}\Rightarrow x=0.95$ $2-\frac{0.95}{4}-\frac{0.95^2}{64}-\frac{0.95^3}{512}=1.7467238......\approx 1.74672$	B1 M1A1 [3]
		Total 8 marks

Part	**Mark**	**Additional Guidance**
(a)	B1B1	B1B0 for one correct value, B1B1 For the correct values of both *A* and *B* *A* and *B* must be integers from correct working **Do not accept** *A*= $4^{\frac{1}{2}}$ or $\sqrt{4}$
(b)	M1	Attempt to expand their $'2'\left(1-\frac{x}{'4'}\right)^{\frac{1}{2}}$ up to x^3 term (with or without ‘2’) Must have: • First term is 1 for their $\left(1-\frac{x}{'4'}\right)^{\frac{1}{2}}$ expansion • Correct powers of $\pm\frac{x}{'4'}$ for each term No simplification needed. Ignore terms beyond x^3
	A1ft	Two correct algebraic terms in their expansion, with or without’2’ Not necessarily in lowest terms. Ignore terms beyond x^3.
	A1	All terms correct and in lowest terms. Ignore terms beyond x^3.
(c)	B1	Identifies $x=0.95$ oe. e.g. $\frac{19}{20}$
	M1	Substitutes their *x* which must come from some working (provided $\lvert x\rvert<4$) into each of the algebraic terms of their expansion.
	A1	1.74672 (must be 5dp) **Note:** Calculator value for $\frac{\sqrt{305}}{10}$ is 1.74642492

Question number	Scheme	Marks
7	$\frac{\mathrm{d}V}{\mathrm{d}t}=-k$	B1
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h}\right\}=40\pi h-\pi h^2$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{k}{40\pi h-\pi h^2}$	M1A1
	$-\frac{1}{60}=-\frac{k}{40\times\pi\times12-\pi\times12^2}\left\{=-\frac{k}{336\pi}\right\}\Rightarrow k=\frac{28}{5}\pi\left(\mathrm{cm}^3\,/\,\mathrm{s}\right)$	dM1A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
	B1	For identifying that $\frac{\mathrm{d}V}{\mathrm{d}t}=-k$ seen/embedded in a correct chain rule, accept $\pm k$
	M1	Attempt to find $\frac{\mathrm{d}V}{\mathrm{d}h}$, condone if $\frac{\mathrm{d}V}{\mathrm{d}h}$ is not present or incorrect If attempted by expanding the bracket first, they need to achieve $Ph\pm Qh^2$ If attempted by product rule, score for $Ah(60-h)\pm Bh^2$
	M1	A correct chain rule formed by using $\frac{\mathrm{d}V}{\mathrm{d}h},\frac{\mathrm{d}h}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}$ (in any order) stated or implied
	A1	Obtains the correct chain rule having at least two terms with values/expressions substituted in, no errors seen. e.g. $-k=\left(40\pi h-\pi h^2\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=\left(40\pi h-\pi h^2\right)\times-\frac{1}{60}$
	dM1	Substitutes $h=12,\ \frac{\mathrm{d}V}{\mathrm{d}t}=\pm k,\ \frac{\mathrm{d}h}{\mathrm{d}t}=\pm\frac{1}{60}$ and their $\frac{\mathrm{d}V}{\mathrm{d}h}='40\pi h-\pi h^2'$ to a correct chain rule to find $k=...$ Dependent on the 2nd M mark only
	A1	Correct answer only for using correct signs throughout $\frac{28}{5}\pi$ or 5.6π, ignore unit

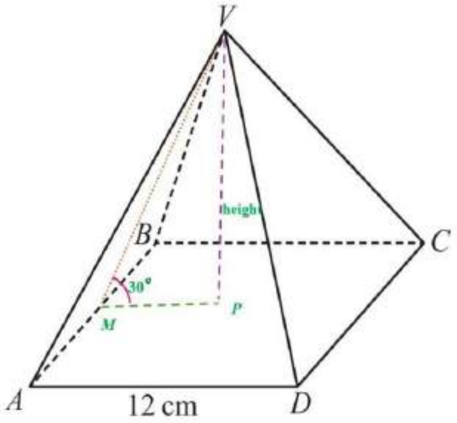
Question number	**Scheme**	**Marks**
8(a)	$\tan 30° = \frac{VP}{6}$ where *P* is the centre of the base $VP = 6\tan 30° = 2\sqrt{3}$ *	M1 A1*cso [2]
(b)	$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}$ or $AC = \frac{12}{\sin 45}$ or $AC = \frac{12}{\cos 45}$ $VA^2 = \left(2\sqrt{3}\right)^2 + \left(6\sqrt{2}\right)^2 (= 84)$ $VA = 2\sqrt{21}$ (cm) oee	M1 oe M1 oe A1 [3]
(c)	$VX = \sqrt{\left('2\sqrt{21}'\right)^2 - 6^2} \left(= 4\sqrt{3}\right)$ or $VX = \frac{6}{\cos 30}$ or $VX = \sqrt{6^2 + \left(2\sqrt{3}\right)^2}$ where *X* is the midpoint of the side *AD* $\text{Area} = \frac{1}{2} \times 4\sqrt{3} \times 12 = 24\sqrt{3}$ $\left(\text{cm}^2\right)$ oee	M1 M1A1 [3]
(d)	$'24\sqrt{3}' = \frac{1}{2} \times '2\sqrt{21}' \times DY$ where *Y* is the foot of the perpendicular from *D* to *VA* $DY = \frac{24\sqrt{7}}{7}$ oee	M1 A1 [2]
(e)	$\cos\theta = \dfrac{\left('\frac{24\sqrt{7}}{7}'\right)^2 + \left('\frac{24\sqrt{7}}{7}'\right)^2 - \left('12\sqrt{2}'\right)^2}{2 \times '\frac{24\sqrt{7}}{7}' \times '\frac{24\sqrt{7}}{7}'} = -\frac{3}{4}$ $\theta = 138.6°$	M1A1 A1 [3]
		Total is 13 marks

Part	Mark	Additional Guidance
(a)	M1	For forming a correct equation using trigonometry. e.g. $\tan 60^\circ = \frac{6}{VP}$ or $\frac{h}{\sin 30^\circ} = \frac{6}{\sin 60^\circ}$ (If attempted by ratio, send to review)
	A1*cso	For **rearranging** to show the required result. Minimum need to see one intermediate step from their equation to the answer ,if the height is not the subject of their equation: e.g. $\tan 30^\circ = \frac{VP}{6} \Rightarrow VP = 6\tan 30^\circ = 2\sqrt{3}$ is M1A1 $VP = 6\tan 30^\circ = 2\sqrt{3}$ is M1A1 $\tan 30^\circ = \frac{VP}{6} \Rightarrow \frac{\sqrt{3}}{3} = \frac{VP}{6} \Rightarrow VP = 2\sqrt{3}$ is M1A1 $\tan 30^\circ = \frac{VP}{6} \Rightarrow VP = 2\sqrt{3}$ is M1A0
(b)	M1oe	Correct use of Pythagoras' theorem or trigonometry to obtain the length/half length of the diagonal of the base AC or $\frac{1}{2}AC$ Or other valid method to find *VM*, (Let *M* to be the midpoint of *AB*) e.g.: $VM = \frac{6}{\cos 30}\left(= 4\sqrt{3}\right)$ or $VM = \sqrt{6^2 + \left(2\sqrt{3}\right)^2}$ $\left(= 4\sqrt{3}\right)$
	M1oe	Use Pythagoras' theorem to obtain a numerical value for VA / VA^2 Any correct method. e.g. $VA^2 = \left(4\sqrt{3}\right)^2 + \left(6\right)^2$
	A1	Correct exact length $2\sqrt{21}$ or $\sqrt{84}$ oee (or exact equivalent)
(c)	M1	Correct use of Pythagoras' theorem or trigonometry to obtain the length of *VX*, which is the same as *VM*, could be found in part b Or correct use of cosine rule (ft their *VA*) to find the value of cosine value of any angle in triangle *VAD* e.g. $AD^2 = VA^2 + VA^2 - 2VA \times VA \times \cos\angle AVD$ $12^2 = \left('2\sqrt{21}'\right)^2 + \left('2\sqrt{21}'\right)^2 - 2\times '2\sqrt{21}'\times '2\sqrt{21}'\times \cos\angle AVD$ $\left\{\Rightarrow \cos\angle AVD = \frac{1}{7}\right\}$
	M1	Correct method to find the area of triangle *VAD* (Do not have to use exact values) Accept any correct valid method. e.g. $\left\{\cos\angle AVD = \frac{1}{7} \Rightarrow \sin\angle AVD = \frac{4\sqrt{3}}{7}\right\} \Rightarrow A = \frac{1}{2}\times '2\sqrt{21}'\times '2\sqrt{21}'\times '\frac{4\sqrt{3}}{7}' = 24\sqrt{3}$
	A1	Correct **exact** value for area
(d)	M1	Use of area of triangle with *VA* as the base to form an equation for *DY* Or other valid method: e.g. using trigonometry ratio: $\sin\angle VAD = \frac{'4\sqrt{3}'}{'2\sqrt{21}} = \frac{DY}{12}$

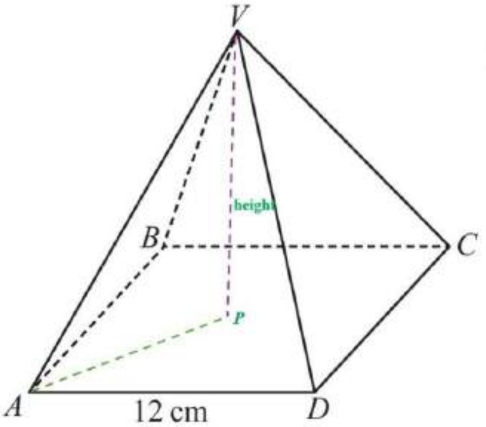
	A1	Correct exact length $DY = \frac{24\sqrt{7}}{7}$ oee, e.g. $DY = \frac{8\sqrt{63}}{7}$ or $24\sqrt{\frac{1}{7}}$
(e)	M1	Correct use of cosine rule: $\cos\theta = \frac{"DY"^2 + "BY"^2 - "BD"^2}{2\times "DY" \times "BY"}$ with their values Or other valid method, e.g. $\sin\frac{\theta}{2} = \frac{"\frac{BD}{2}"}{"DY"} = \frac{"6\sqrt{2}"}{"\frac{24\sqrt{7}}{7}"}$
	A1	Obtains $\cos\theta = -\frac{3}{4}$ or $\sin\frac{\theta}{2} = \frac{\sqrt{14}}{4}$
	A1	Correct angle found

Useful diagrams:

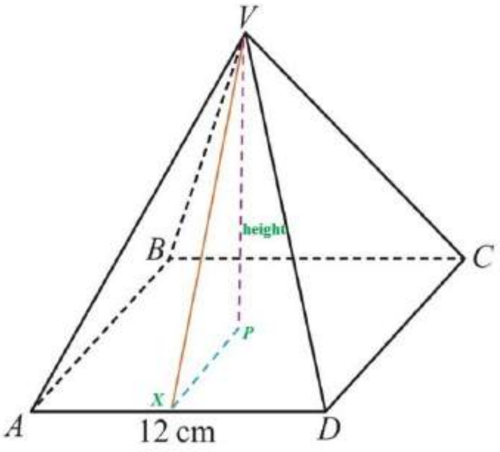
1



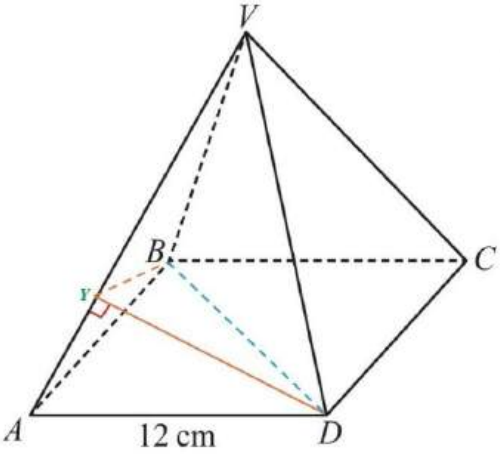
2



3



4



Question number	Scheme	Marks
9(a)	$\mathrm{f}'(x)=\lambda(x-2)(3x+1)$	M1
	$\mathrm{f}(x)=\int\lambda(x-2)(3x+1)\,\mathrm{d}x=\left[\int\lambda\left(3x^2-5x-2\right)\right]\mathrm{d}x=\lambda\left(\frac{3x^3}{3}-\frac{5x^2}{2}-2x+k\right)$	M1A1
	At $(2,-9)$	
	$-9=2(2^3)-5(2^2)-4(2)+c\Rightarrow c=3$	M1
	$a=-5,b=-4,c=3\Rightarrow\left\{\mathrm{f}(x)=2x^3-5x^2-4x+3\right\}$ *	A1*cso [5]
	ALT	
	$\mathrm{f}(x)=2x^3+ax^2+bx+c$	
	$\mathrm{f}'(x)=6x^2+2ax+b$	[M1
	Both $\begin{cases}0=6(2^2)+2(2)a+b\\ 0=6\times\left(-\frac{1}{3}\right)^2+2\times\left(-\frac{1}{3}\right)\times a+b\end{cases}$	M1A1
	$\begin{cases}72+12a+3b=0 \text{ and } 12-12a+18b=0\Rightarrow a=\ldots,b=\ldots\\ \text{At }(2,9)\quad -9=2(2^3)-5(2^2)-4(2)+c\Rightarrow c=\ldots\end{cases}$	M1
	$a=-5,b=-4,c=3\Rightarrow\left\{\mathrm{f}(x)=2x^3-5x^2-4x+3\right\}$ *	A1*cso]
(b)(i)	$\mathrm{f}(-1)=2(-1)^3-5(-1)^2-4(-1)+3=0$ oe	B1 [1]
(b)(ii)	$\begin{array}{r}2x^2-7x+3\\ x+1\overline{)2x^3-5x^2-4x+3}\end{array}$	M1
	$2x^2-7x+3=(2x-1)(x-3)\Rightarrow(x+1)(x-3)(2x-1)$	M1A1 [3]
(c)	$A=\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,\mathrm{d}x+\left\lvert\int_{\frac{1}{2}}^{3}\right\rvert(2x^3-5x^2-4x+3)\,\mathrm{d}x\Big\rvert\quad\Big\rvert$	M1
	$A=\left[\frac{1}{2}x^4-\frac{5}{3}x^3-2x^2+3x\right]_{-1}^{\frac{1}{2}}+\left\lvert\left[\frac{1}{2}x^4-\frac{5}{3}x^3-2x^2+3x\right]_{\frac{1}{2}}^{3}\right\rvert$	M1
	$A=\left\{\left[\frac{1}{2}.\left(\frac{1}{2}\right)^4-\frac{5}{3}.\left(\frac{1}{2}\right)^3-2.\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)\right]-\left[\frac{1}{2}.(-1)^4-\frac{5}{3}.(-1)^3-2.(-1)^2+3.(-1)\right]\right\}$ $+\left\lvert\left\{\left[\frac{1}{2}.(3)^4-\frac{5}{3}.(3)^3-2.(3)^2+3(3)\right]-\left[\frac{1}{2}.\left(\frac{1}{2}\right)^4-\frac{5}{3}.\left(\frac{1}{2}\right)^3-2.\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)\right]\right\}\right\rvert$	M1
	$A=\left\{\frac{117}{32}+\left\lvert-\frac{1375}{96}\right\rvert\right\}=\frac{863}{48}$	A1
		[4]
		Total 13 marks

Part	Mark	Additional Guidance
(a)	M1	For forming an expression for $f'(x)$ using the two turning points given. Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	M1	Attempts to integrate, see General Guidance for the definition of an attempt Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	A1	Correct integration including +c, Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	M1	$\lambda = 2$ is stated or implied, $\lambda k = c$ Substitution of $(2, -9)$ to their $f(x)$ (Does not have to be simplified) or substitution of $\left(-\frac{1}{3}, \frac{100}{27}\right)$ (Does not have to be simplified)
	A1*cso	Obtains the **given** answer with no errors in the working
	ALT - starts with $f(x) = 2x^3 + ax^2 + bx + c$	
	M1	Attempts to differentiate, see General Guidance for the definition of an attempt
	M1	Equates differentiated expression to 0 and substitutes $x = 2$ **and** equates differentiated expression to 0 and substitutes $x = -\frac{1}{3}$
	A1	Two correct equations, simplified or unsimplified
	M1	Attempts to solve two **correct** equations simultaneously (including using calculators) to find *a*, *b* **and** substitution of their *a*,*b*, $(2, -9)$ to $f(x)$ to find *c* or substitution of their *a*,*b*, $\left(-\frac{1}{3}, \frac{100}{27}\right)$ to $f(x)$ to find *c*
	A1*cso	Obtains the **given** answer with no errors in the working
(b)(i)	B1	Demonstrating that $x+1$ is a factor $f(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3 = 0$ (factor theorem) If shown via polynomial division, the division must be fully correct and obtains 0 as the remainder.
(b)(ii)	M1	Dividing by $(x+1)$ to obtain a three-term quadratic quotient: $2x^2 - 7x + ...$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line : e.g. $(x+1)(x-3)(2x-1)$
	ALT I – equates coefficients	
	M1	For stating $2x^3 - 5x^2 - 4x + 3 = (x+1)(Ax^2 + Bx + C) \Rightarrow$ $2x^3 - 5x^2 - 4x + 3 = Ax^3 + (A+B)x^2 + (B+C)x + C \Rightarrow$ $A = 2, B = -7, C = ...$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line: e.g. $(x+1)(x-3)(2x-1)$
	ALT II - inspection	
	M1	For finding the quadratic factor. Minimum required is $(x+1)(2x^2 - 7x + C)$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line: e.g. $(x+1)(x-3)(2x-1)$

(c)	M1	For identifying the correct limits, uses a correct strategy to find the area above the *x*-axis and below the *x*-axis: $\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,\mathrm{d}x+\left\|\int_{\frac{1}{2}}^{3}(2x^3-5x^2-4x+3)\,\mathrm{d}x\right\|$ or $\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,\mathrm{d}x-\int_{\frac{1}{2}}^{3}(2x^3-5x^2-4x+3)\,\mathrm{d}x$ or implied by their working
	M1	For an attempt to integrate $2x^3-5x^2-4x+3$ See General Guidance for the definition of an attempt.
	M1	For clear substitution of **correct** limits into any **one** of their integrated expressions (must be a changed expression). Can be implied by **one** of the **correct exact** areas found from a **correctly** integrated expression
	A1	For correctly obtaining $\frac{863}{48}$ or $17\frac{47}{48}$

Question number	Scheme	Marks
10(i)	$\log_4\left(6y-5\right)=3 \Rightarrow 6y-5=64 \Rightarrow y=\frac{23}{2}$ oe	M1A1 [2]
(ii)	$\log_4(4-3x)^2-\frac{\log_4(x^2-5)}{\log_4 2}=3$ or $\frac{\log_2(4-3x)^2}{\log_2 4}-\log_2(x^2-5)=3$ $\frac{1}{2}\log_4(4-3x)^2-\log_4(x^2-5)=\frac{3}{2}$ or $\frac{1}{2}\log_2(4-3x)^2-\log_2(x^2-5)=3$ $\log_4(4-3x)-\log_4(x^2-5)=\frac{3}{2}$ or $\log_2(4-3x)-\log_2(x^2-5)=3$ $\log_4\left(\frac{4-3x}{x^2-5}\right)=\frac{3}{2}$ or $\log_2\left(\frac{4-3x}{x^2-5}\right)=3$ $\frac{4-3x}{x^2-5}=4^{\frac{3}{2}}=8$ or $\frac{4-3x}{x^2-5}=2^3=8$ $8x^2+3x-44=0$ $x=\frac{-3\pm\sqrt{3^2-4\times8\times-44}}{2\times8}=\frac{-3\pm\sqrt{1417}}{16}\Rightarrow\left\{x=2.1651...\text{ or }-2.540...\right\}$ $\therefore x=-2.54\ \left(\text{awrt}\right)$	M1 B1 M1 M1 M1 A1 M1A1 A1 [9]
		Total 11 marks

Part	Mark	Additional Guidance
(i)	M1	For a complete correct method to find the value of y
	A1	For the correct value
For part (ii) award the marks when you see them		
(ii)	M1	For change of base correctly to either 4 or 2
	B1	For replacing $\log_2 4$ with 2 or replacing $\log_4 2$ with $\frac{1}{2}$
	M1	For applying the power law correctly e.g.: $\frac{1}{2}\log_4(4-3x)^2 \Rightarrow \log_4(4-3x)$ or $\frac{1}{2}\log_2(4-3x)^2 \Rightarrow \log_2(4-3x)$
	M1	For applying the subtraction/addition law correctly
	M1	For correctly removed logs
	A1	For the correct 3TQ, “=0 “could be implied by later work
	M1	For a correct clear method to solve their 3TQ, allow one/two solutions from a calculator correct to 1dp (awrt) from a **correct** 3TQ
	A1	For two correct solutions, **exact** (allow embedded in a quadratic formula) or $x = 2.17$(awrt), $x = -2.54$(awrt) or JUST $x = -2.54$(awrt) seen only, from a correct 3TQ, scores M1A1A1(rejected 2.17 is implied)
	A1	For the correct solution. Accept awrt -2.54 The positive root must be rejected as it is not feasible.

Question number	Scheme	Marks
11	Intersection of the curve with the x-axis to find lower limit $\left\{y=\sqrt{4x-8}=0 \Rightarrow 4x-8=0\right\} \Rightarrow x=2$	B1
	Volume of rotation	
	$V=\pi\int_2^b\left(\sqrt{4x-8}\right)^2\,\mathrm{d}x=\left\{\pi\int_2^b(4x-8)\,\mathrm{d}x\right\}$	M1
	$V=\pi\left[\frac{4x^2}{2}-8x\right]_2^b$	M1
	$50\pi=\pi\left[\left(\frac{4\times b^2}{2}-8\times b\right)-\left(\frac{4\times 2^2}{2}-8\times 2\right)\right]$	M1
	$50=2b^2-8b+8 \Rightarrow 0=b^2-4b-21$ $0=(b+3)(b-7)$ $\Rightarrow b=7$	dddM1 ddddM1 A1 [7]
		Total 7 marks

Mark	**Additional Guidance**
B1	Finds the lower limit of the integral: For x = 2 or (2,0) seen/embedded in their integral/on the graph
M1	Sets up a correct expression for the volume with their lower limit (a numerical value) found earlier and the upper limit of b, must have π
M1	Correctly integrates $4x-8$, with or without π
M1	Substitutes their lower limit (a numerical value) and upper limit b correctly into **their integrated** expression (with or without π), and equates to $50\pi / 50$
dddM1	Forms a 3TQ in b/x Dependent on previous three M marks
ddddM1	Solves the 3TQ with any correct method Can be implied by their correct b value from a correct 3TQ Dependent on all previous M marks
A1	For b = 7 only, from correct working, must be b=7, not x =7
Note: If attempted the volume of revolution about the y-axis, only B1 could be possibly awarded, no other marks could be scored.	

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