

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Tuesday 20 May 2025

Afternoon (Time: 2 hours)

Paper
reference

4PM1/01

Further Pure Mathematics PAPER 1



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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P 7 2 8 6 5 A 0 3 3 6

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 A particle P moves in a straight line. At time t seconds, the velocity, v m/s, of P is given by

$$v = 3t^2 - 9t + 7$$

- (a) Show that P never comes to rest. (2)

- (b) Find the acceleration of P , in m/s^2 , when $t = 4$ (2)

Using algebra

- (c) find the distance, in m, that P travels in the interval $0 \leq t \leq 5$ (3)

(a) $0 = 3t^2 - 9t + 7$

If $b^2 - 4ac < 0$ Then P never comes to rest

$$(-9)^2 - 4(3)(7)$$

$$-3$$

$-3 < 0 \therefore P$ never comes to rest

(b) $a = \frac{dv}{dt} = 6t - 9$

At $t=4$, $a = 6(4) - 9$

$$a = \underline{15 \text{ m/s}^2}$$

(c) $d = \int_0^5 3t^2 - 9t + 7 dt$

$$d = \left[\frac{3t^3}{3} - \frac{9t^2}{2} + 7t \right]_0^5$$

$$d = \left[t^3 - \frac{9}{2}t^2 + 7t \right]_0^5$$

$$d = \left[(5)^3 - \frac{9}{2}(5)^2 + 7(5) \right] - \left[(0)^3 - \frac{9}{2}(0)^2 + 7(0) \right]$$

$$d = \underline{\underline{\frac{95}{2} \text{ m}}}$$



Question 1 continued

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(Total for Question 1 is 7 marks)



- 2 The point A has coordinates $(3, 2)$, the point B has coordinates $(8, 3)$ and the point C has coordinates $(4, 7)$

(a) Show that ABC is an isosceles triangle.

(2)

The midpoint of BC is M

(b) Find an equation of the line that passes through A and M

Give your answer in the form $y = mx + c$

(3)

The points A , C and D are collinear such that $AD = kAC$ ($k > 1$)

Given that $\angle ABD = 90^\circ$

(c) find the coordinates of D

Show your working clearly.

(8)

$$\textcircled{a} \quad AB = \sqrt{(8-3)^2 + (3-2)^2} = \sqrt{26} \quad D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(8-4)^2 + (3-7)^2} = 4\sqrt{2}$$

$$AC = \sqrt{(4-3)^2 + (7-2)^2} = \sqrt{26}$$

$$AB = AC = \sqrt{26}, \quad \Delta ABC \text{ is an isosceles triangle.}$$

$$\textcircled{b} \quad M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad B(8, 3) \quad C(4, 7)$$

$$M \left(\frac{8+4}{2}, \frac{3+7}{2} \right), \quad M(6, 5) \quad A(3, 2)$$

$$\text{gradient, } m_{AM} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AM} = \frac{5-2}{6-3} = \frac{3}{3} = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x - 3)$$

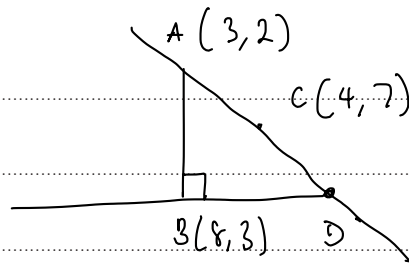
$$y = x - 3 + 2$$

$$y = x - 1 \quad \text{Equation of line AM}$$



Question 2 continued

(c)



$$m_{AB} = \frac{3-2}{8-3} = \frac{1}{5}$$

$$m_{BD} = -5$$

Equation of line BD : $y - 3 = -5(x - 8)$

$$y = -5x + 40 + 3$$

$$y = -5x + 43$$

$$m_{AC} = m_{AD} = \frac{7-2}{4-3} = 5$$

Equation of line AD = $y - 7 = 5(x - 4)$

$$y = 5x - 20 + 7$$

$$y = 5x - 13$$

$$\begin{array}{r} -5x + 43 = 5x - 13 \\ +5x \qquad \qquad +5x \end{array}$$

$$\begin{array}{r} 43 = 10x - 13 \\ +13 \qquad \qquad +13 \end{array}$$

$$\begin{array}{r} 56 = 10x \\ \div 10 \qquad \qquad \div 10 \end{array}$$

$$5.6 = x$$

$$y = 5(5.6) - 13$$

$$y = 15$$

$$\underline{\underline{D(5.6, 15)}}$$

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Question 2 continued

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Question 2 continued

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(Total for Question 2 is 13 marks)



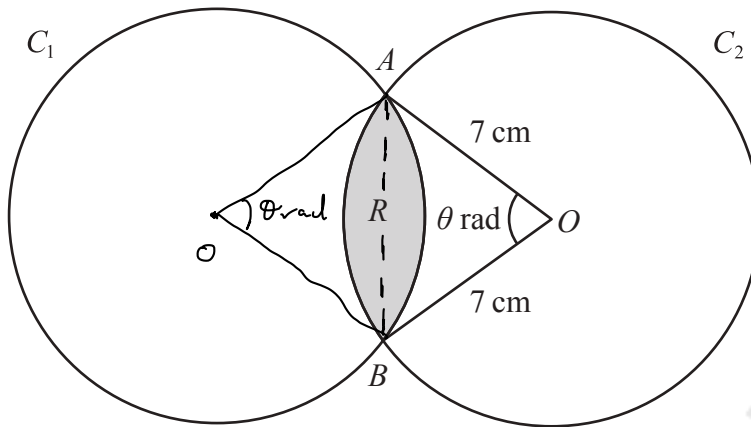


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Figure 1

Figure 1 shows circle C_1 with radius 7 cm and circle C_2 with centre O and radius 7 cm

The circles C_1 and C_2 intersect at the point A and at the point B

The size of angle AOB is θ radians

The perimeter of the region R shown shaded in Figure 1 is $\frac{28\pi}{9}$ cm

(a) Find the exact value of θ

(2)

(b) Find the area, in cm^2 to 3 significant figures, of the region R

(4)

(a) $L = \theta r$

$$\left. \begin{array}{l} \text{left Arc } AB = 7\theta \\ \text{right Arc } AB = 7\theta \end{array} \right\} \text{perimeter} = 7\theta + 7\theta = 14\theta$$

$$\div 14 \quad 14\theta = \frac{28}{9}\pi \quad \div 14$$

$$\theta = \frac{28}{9}\pi \times \frac{1}{14}$$

$$\theta = \frac{2}{9}\pi \text{ rad}$$

(b) Area of Sector = $\frac{r^2\theta}{2} = 7^2 \times \frac{2}{9}\pi \times \frac{1}{2} = \frac{49}{9}\pi \text{ cm}^2$

Area of triangle $AOB = \frac{1}{2}ab\sin C = \frac{1}{2} \times 7 \times 7 \times \sin \frac{2}{9}\pi = 15.748 \text{ cm}^2$

Half of area of region $R = \frac{49}{9}\pi - 15.748 = 1.356 \text{ cm}^2$

Area of region $R = 1.356 \times 2 \approx \underline{\underline{2.71 \text{ cm}^2}} \text{ (3 s.f.)}$



Question 3 continued

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(Total for Question 3 is 6 marks)



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4 (a) On the grid opposite, draw the graph of the line with equation

(i) $3x + 2y = 18$

(ii) $x + 3y - 6 = 0$

(iii) $y = 3x$

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$3x + 2y \leq 18$

$x + 3y - 6 \geq 0$

$y \leq 3x$

(1)

For all points in R , with coordinates (x, y)

$$P = 2x - y$$

(c) find the least value of P

(4)

ai) $3x + 2y = 18$

ii) $x + 3y - 6 = 0$

iii) $y = 3x$

$2y = -3x + 18$

$3y = -x + 6$

At $x=0, y=0$

$y = -\frac{3}{2}x + 9$

$y = -\frac{1}{3}x + 2$

At $x=3, y=9$

At $y=0, \frac{3}{2}x = 9$

At $y=0, \frac{1}{3}x = 2$

$(0, 0) (3, 9)$

$3x = 18$

$x = 6$

$x = 6$

$(6, 0)$

$(6, 0)$

At $x=0, y=2$

At $x=0, y=9$

$(0, 2)$

$(0, 9)$

b) $y \leq -\frac{3}{2}x + 9$

$y \geq -\frac{1}{3}x + 2$

$y \leq 3x$

c) $P_{\min} = 2x_{\min} - y_{\max}$

$2(0.6) - 1.8$

$2(6) - 0$

$2(2) - 6$

$1.2 - 1.8$

$12 - 0$

$4 - 6$

-0.6

12

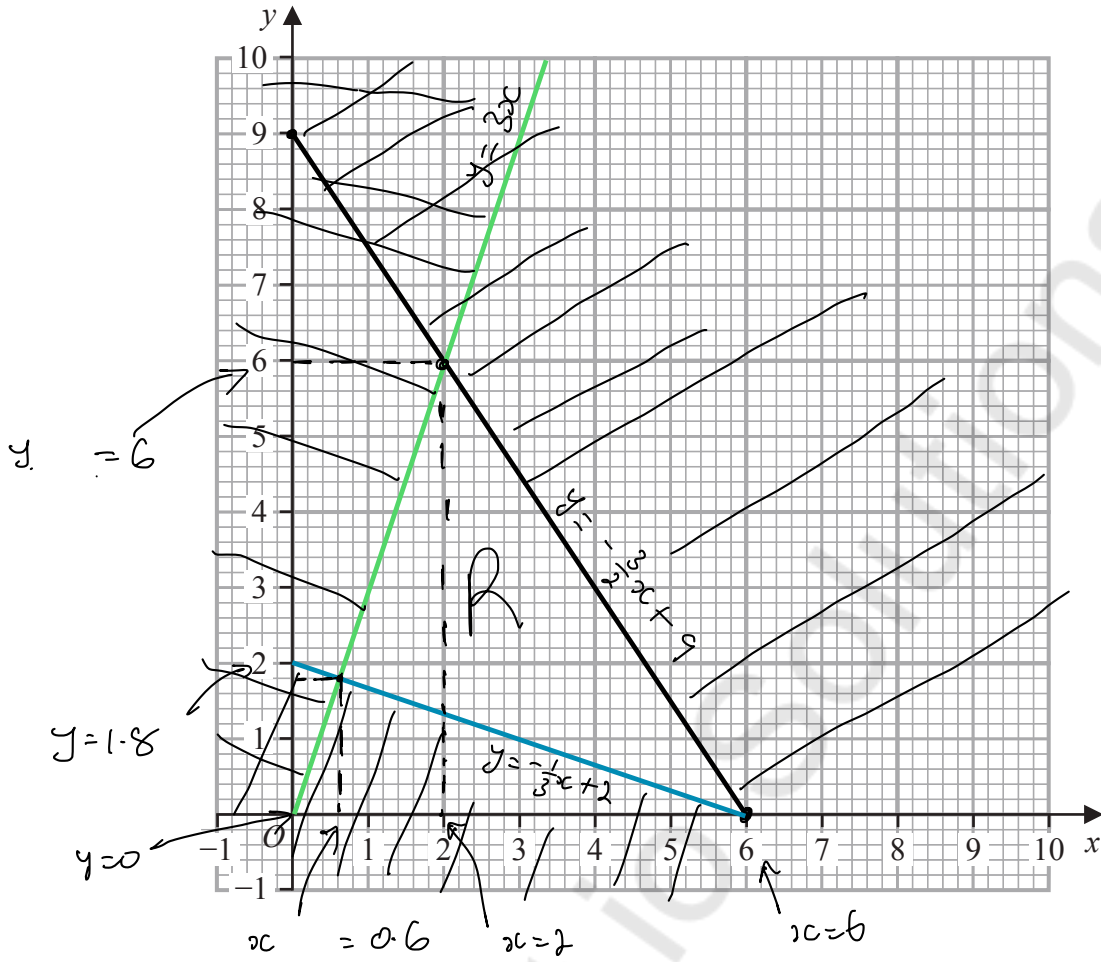
-2

lowest value

$P_{\min} = -2$



Question 4 continued



$(0.6, 1.8)$ $(6, 0)$ $(2, 6)$

Turn over for a spare grid if you need to redraw your graph.

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Question 4 continued

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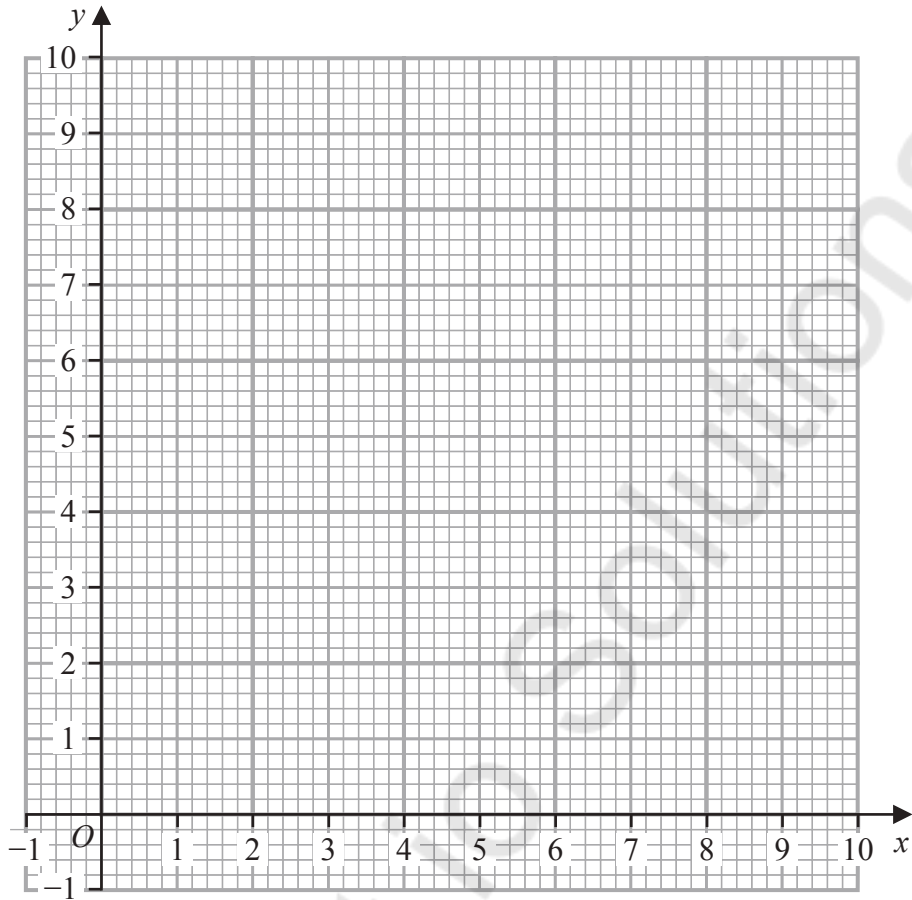
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Question 4 continued

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(Total for Question 4 is 8 marks)



5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$ (2)

The quadratic equation $2x^2 - 6x - 7 = 0$ has roots α and β

Without solving the equation

(b) form a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ (6)

(a) $(\alpha + \beta)(\alpha + \beta)^2 - 3\alpha\beta(\alpha + \beta)$

$$(\alpha + \beta)(\alpha^2 + 2\alpha\beta + \beta^2) - 3\alpha^2\beta - 3\alpha\beta^2$$

$$\alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3 - 3\alpha^2\beta - 3\alpha\beta^2$$

$$\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 - 3\alpha^2\beta - 3\alpha\beta^2 + \beta^3$$

$$\underline{\underline{\alpha^3 + \beta^3}} \quad \text{Shown.}$$

(b) $(x - \frac{\alpha^2}{\beta})(x - \frac{\beta^2}{\alpha})$

$(x - \alpha)(x - \beta)$

$$x^2 - \frac{\beta^2}{\alpha}x - \frac{\alpha^2}{\beta}x + \frac{\alpha^2\beta^2}{\alpha\beta}$$

$$x^2 - \alpha x - \beta x + \alpha\beta$$

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

$$x^2 - \left(\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}\right)x + \frac{\alpha^2\beta^2}{\alpha\beta}$$

$$2x^2 - 6x - 7$$

$$x^2 - \left[\frac{\beta^3}{\alpha\beta} + \frac{\alpha^3}{\alpha\beta}\right]x + \alpha\beta$$

$$2\left[x^2 - 3x - \frac{7}{2}\right]$$

$$x^2 - \left[\frac{\alpha^3 + \beta^3}{\alpha\beta}\right]x + \alpha\beta$$

$$\alpha + \beta = 3 \quad \alpha\beta = -\frac{7}{2}$$

$$x^2 - \left[\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}\right]x + \alpha\beta$$

$$x^2 - \left[\frac{(3)^3 - 3(-\frac{7}{2})(3)}{3}\right]x + (-\frac{7}{2})$$

$$x^2 - \left[-\frac{117}{7}\right]x - \frac{7}{2}$$

$$x^2 + \frac{117}{7}x - \frac{7}{2}$$

$$\underline{14x^2 + 234x - 49 = 0}$$



Question 5 continued

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(Total for Question 5 is 8 marks)



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6 (a) Show that $\sqrt{4-x} = A\left(1-\frac{x}{B}\right)^{\frac{1}{2}}$ where A and B are integers to be found. (2)

(b) Hence expand $\sqrt{4-x}$ in ascending powers of x up to and including the term in x^3
Give each coefficient as an exact fraction in its lowest terms. (3)

(c) Use your expansion with a suitable value of x to obtain an estimate of $\frac{\sqrt{305}}{10}$
Give your answer correct to 5 decimal places. (3)

(a)
$$\sqrt{4\left(1-\frac{x}{4}\right)}$$

$$\sqrt{4} \sqrt{1-\frac{x}{4}}$$

$$2 \sqrt{1-\frac{x}{4}}$$

$$2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$$

$A=2, B=4$

(b)
$$2\left[1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^3\left(\frac{1}{6}\right) + \dots\right]$$

$$2\left[1 - \frac{x}{8} - \frac{x^2}{128} - \frac{x^3}{1024}\right]$$

$$\sqrt{4-x} \approx 2 - \frac{x}{4} - \frac{x^2}{64} - \frac{x^3}{512} + \dots$$

(c)
$$\sqrt{4-x} = \frac{\sqrt{305}}{10} \quad \left|-\frac{x}{4}\right| < 1$$

$$4-x = \frac{305}{100} \quad |x| < 4$$

$4 - 3.05 = x \quad 0.95 < 4$

$x = 0.95 = \frac{95}{100}$ Thus valid value of x

$$\frac{\sqrt{305}}{10} \approx 2 - \frac{0.95}{4} - \frac{(0.95)^2}{64} - \frac{(0.95)^3}{512} \approx \underline{\underline{1.74672}} \quad (5d.p.)$$



Question 6 continued

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(Total for Question 6 is 8 marks)



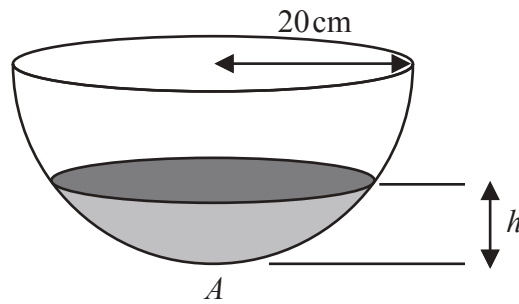


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Figure 2

Figure 2 shows a hollow hemisphere with radius 20 cm

The hemisphere contains liquid, which is dripping out of a small hole at the lowest point A at a constant rate of $k \text{ cm}^3/\text{s}$

At time t seconds after the liquid starts to drip from the hemisphere, the height of the liquid is h cm above A

The volume $V \text{ cm}^3$ of liquid in the hemisphere is given by

$$V = \frac{\pi}{3} h^2 (60 - h)$$

When $h = 12$, the height of the liquid is decreasing at a rate of $\frac{1}{60} \text{ cm/s}$

Find the value of k

Give your answer in terms of π

(6)

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$V = \frac{60\pi}{3} h^2 - \frac{\pi}{3} h^3$$

$$\frac{dV}{dh} = \frac{120}{3} \pi h - \pi h^2$$

$$\text{At } h=12, \quad \frac{dV}{dh} = \frac{120}{3} \pi (12) - \pi (12)^2$$

$$\frac{dV}{dh} = 336\pi$$

$$\text{At } h=12, \quad \frac{dh}{dt} = \frac{1}{60}$$

$$\frac{dV}{dt} = 336\pi \times \frac{1}{60}$$

$$\frac{dV}{dt} = \frac{28}{5} \pi \text{ cm}^3/\text{s}$$

$$k = \frac{28}{5} \pi$$



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 6 marks)



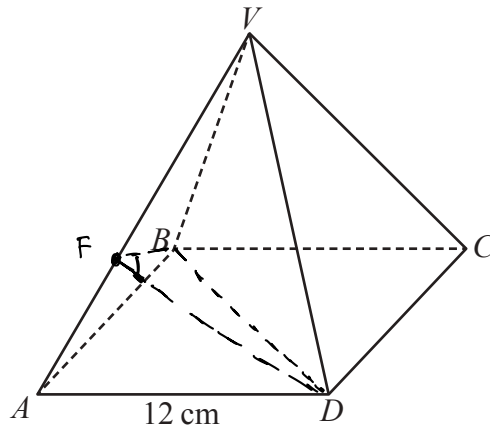


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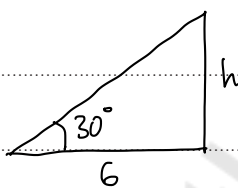
Figure 3

Figure 3 shows a right pyramid with vertex V and square base, $ABCD$, of side 12 cm

The size of the angle between the plane VAB and the base $ABCD$ is 30°

- (a) Show that the height of the pyramid is $2\sqrt{3}$ cm (2)
- (b) Find, in cm, the exact length of VA (3)
- (c) Find, in cm^2 , the exact area of triangle VAD (3)
- (d) Find, in cm, the exact length of the perpendicular from D to VA (2)
- (e) Find, in degrees to one decimal place, the size of the obtuse angle between the plane VAB and the plane VAD (3)

(a)

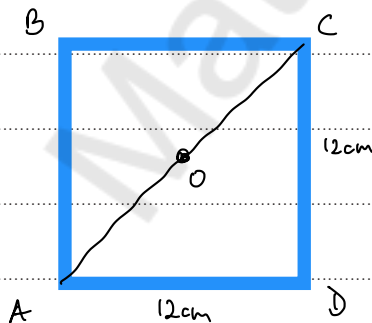


$$\tan 30 = \frac{h}{6}$$

$$\frac{\sqrt{3}}{3} = \frac{h}{6}$$

$$h = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ cm}$$

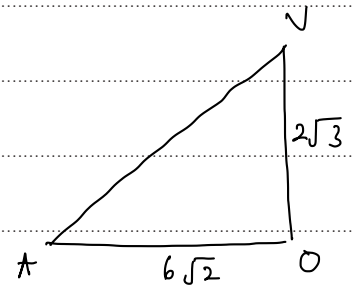
(b)



$$AC^2 = 12^2 + 12^2$$

$$AC = \sqrt{288}$$

$$AO = \frac{\sqrt{288}}{2} = 6\sqrt{2} \text{ cm}$$



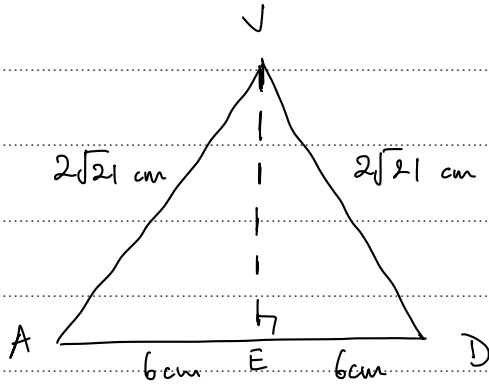
$$VA^2 = (6\sqrt{2})^2 + (2\sqrt{3})^2$$

$$VA = \sqrt{(6\sqrt{2})^2 + (2\sqrt{3})^2} = \underline{\underline{2\sqrt{21} \text{ cm}}}$$



Question 8 continued

(c)



$$VA = VD = 2\sqrt{21} \text{ cm}$$

Square based pyramid.

$$VE^2 + 6^2 = (2\sqrt{21})^2$$

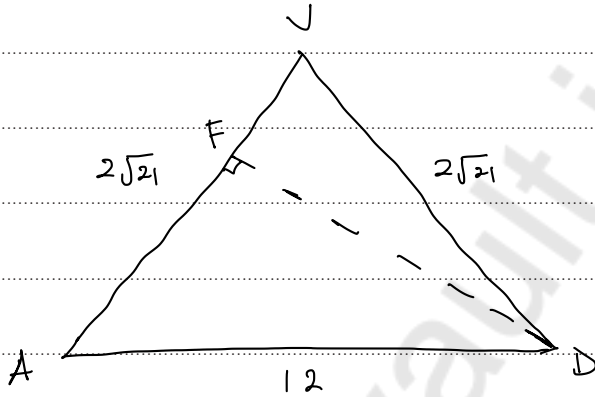
$$VE^2 = (2\sqrt{21})^2 - 6^2$$

$$VE = \sqrt{(2\sqrt{21})^2 - 6^2}$$

$$VE = 4\sqrt{3} \text{ cm}$$

$$\text{Area of } \triangle VAD = \frac{12 \times 4\sqrt{3}}{2} = \underline{\underline{24\sqrt{3} \text{ cm}^2}}$$

(d)



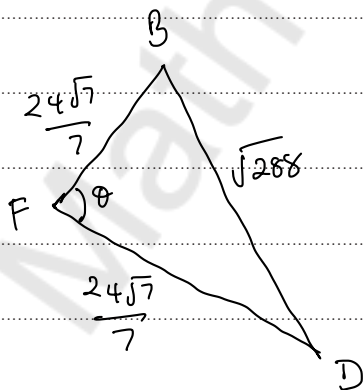
$$24\sqrt{3} = \frac{2\sqrt{21} \times DF}{2}$$

$$48\sqrt{3} = 2\sqrt{21} \times DF$$

$$\frac{48\sqrt{3}}{2\sqrt{21}} = DF$$

$$\underline{\underline{\frac{24\sqrt{7}}{7} \text{ cm} = DF}}$$

(e)



$$BD = AC = \sqrt{288}$$

$$(\sqrt{288})^2 = \left(\frac{24\sqrt{7}}{7}\right)^2 + \left(\frac{24\sqrt{7}}{7}\right)^2 - 2\left(\frac{24\sqrt{7}}{7}\right)\left(\frac{24\sqrt{7}}{7}\right)\cos \hat{BFD}$$

$$\hat{BFD} = \cos^{-1} \left[\frac{\left(\frac{24\sqrt{7}}{7}\right)^2 + \left(\frac{24\sqrt{7}}{7}\right)^2 - (\sqrt{288})^2}{2 \left(\frac{24\sqrt{7}}{7}\right)\left(\frac{24\sqrt{7}}{7}\right)} \right]$$

$$\hat{BFD} \approx \underline{\underline{138.6^\circ}} \text{ (1 d.p.)}$$



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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 13 marks)



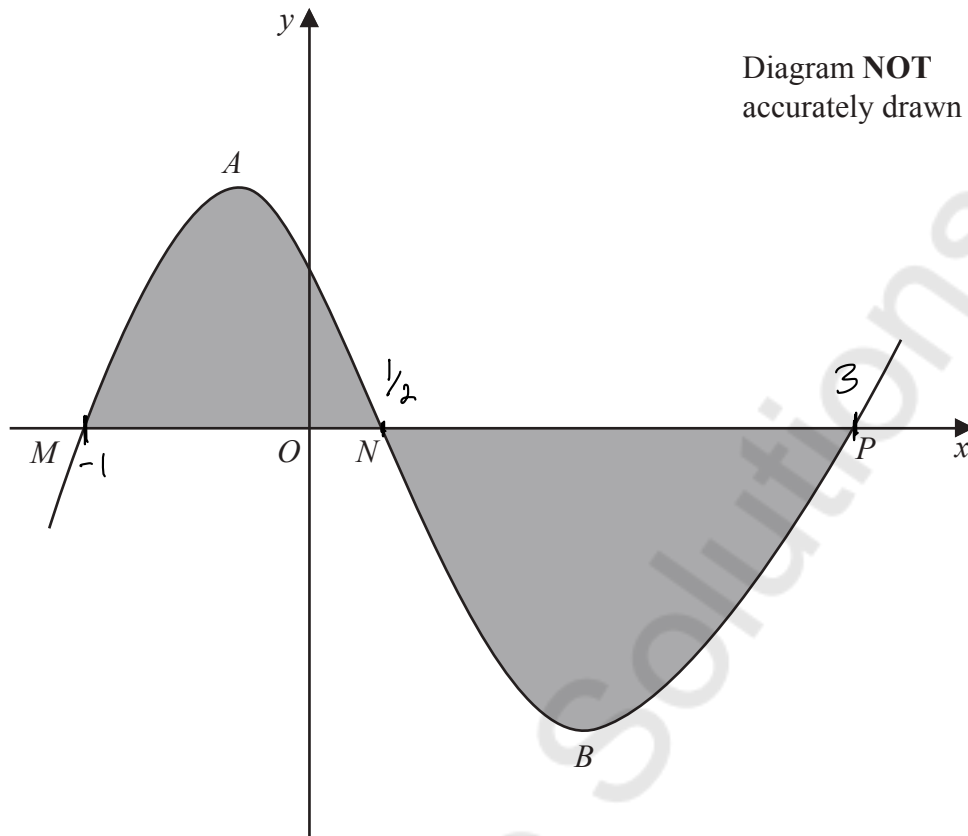


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = 2x^3 + ax^2 + bx + c$$

The curve C has a maximum at the point A with coordinates $\left(-\frac{1}{3}, \frac{100}{27}\right)$ and a minimum at the point B with coordinates $(2, -9)$

Given that a , b and c are integers

(a) show that $a = -5$, $b = -4$ and $c = 3$ (5)

(b) (i) Show that $(x+1)$ is a factor of $f(x)$ (1)

(ii) Hence, or otherwise, use algebra to factorise $f(x)$ completely. (3)

The curve C crosses the x -axis at the points M , N and P
The finite regions shown shaded in Figure 4 are bounded by the curve C and parts of the x -axis from M to N and from N to P

(c) Use algebraic integration to determine the total area of the shaded regions.
Give your answer as an exact fraction. (4)



Question 9 continued

$$(a) f(x) = 2x^3 + ax^2 + bx + c$$

$$f'(x) = 6x^2 + 2ax + b$$

$$f'(2) = 0, \quad 6(2)^2 + 2a(2) + b = 0$$

$$24 + 4a + b = 0, \quad b = -24 - 4a$$

$$f'(-\frac{1}{3}) = 0, \quad 6(-\frac{1}{3})^2 + 2a(-\frac{1}{3}) + b = 0$$

$$\frac{6}{9} - \frac{2a}{3} + b = 0, \quad b = \frac{2a}{3} - \frac{2}{3}$$

$$-24 - 4a = \frac{2a}{3} - \frac{2}{3}$$

$$-24 + \frac{2}{3} = \frac{2}{3}a + 4a$$

$$-72 + 2 = 2a + 12a$$

$$-70 = 14a$$

$$-5 = a$$

$$b = -24 - 4(-5) = -24 + 20$$

$$b = -4$$

$$f(2) = -9, \quad 2(2)^3 + (-5)(2)^2 + (-4)(2) + c = -9$$

$$16 - 20 - 8 + c = -9$$

$$c = -9 - 16 + 20 + 8$$

$$c = 3$$

$$f(x) = 2x^3 - 5x^2 - 4x + 3$$

$$(b) i) f(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$$

$$f(-1) = -2 - 5 + 4 + 3$$

$$f(-1) = 0 \quad \therefore x + 1 \text{ is a factor of } f(x)$$



Question 9 continued

$$\begin{array}{r}
 \text{ii)} \quad \begin{array}{r} 2x^2 - 7x + 3 \\ x+1 \overline{) 2x^3 - 5x^2 - 4x + 3} \\ \underline{-(2x^3 + 2x^2)} \\ -7x^2 - 4x \\ \underline{-(-7x^2 - 7x)} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array}
 \end{array}$$

$$f(x) = (x+1)(2x^2 - 7x + 3)$$

$$f(x) = (x+1)(x-3)(2x-1)$$

$$\text{c)} \quad x = -1 \quad x = 3 \quad x = \frac{1}{2} \\
 M(-1, 0) \quad P(3, 0) \quad N\left(\frac{1}{2}, 0\right)$$

$$\int_{-1}^{\frac{1}{2}} (2x^3 - 5x^2 - 4x + 3) dx$$

$$\left[\frac{2x^4}{4} - \frac{5x^3}{3} - \frac{4x^2}{2} + 3x \right]_{-1}^{\frac{1}{2}}$$

$$\left[\frac{x^4}{2} - \frac{5}{3}x^3 - 2x^2 + 3x \right]_{-1}^{\frac{1}{2}}$$

$$\left[\left(\frac{1}{2}\right)^4 - \frac{5}{3}\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) \right] - \left[\frac{(-1)^4}{2} - \frac{5}{3}(-1)^3 - 2(-1)^2 + 3(-1) \right]$$

$$\frac{117}{32}$$

$$\int_{\frac{1}{2}}^3 (2x^3 - 5x^2 - 4x + 3) dx$$

$$\left[\frac{x^4}{2} - \frac{5}{3}x^3 - 2x^2 + 3x \right]_{\frac{1}{2}}^3$$

$$\left[\frac{(3)^4}{2} - \frac{5}{3}(3)^3 - 2(3)^2 + 3(3) \right] - \left[\frac{(\frac{1}{2})^4}{2} - \frac{5}{3}\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) \right]$$

$$- \frac{1375}{96}$$

$$A = \frac{117}{32} + \left| -\frac{1375}{96} \right| = \frac{863}{48}$$



Question 9 continued

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(Total for Question 9 is 13 marks)



10 Solve the equation

$$(i) \log_4(6y-5) = 3 \quad (2)$$

$$(ii) \log_4(4-3x)^2 - \log_2(x^2-5) - 3 = 0$$

Show clear algebraic working.
Give your answer to 3 significant figures. (9)

$$i) \quad 4^3 = 6y - 5$$

$$64 = 6y - 5$$

$$69 = 6y$$

$$\frac{69}{6} = y$$

$$\underline{\underline{\frac{23}{2} = y}}}$$

$$ii) \quad 2 \log_4(4-3x) - \log_2(x^2-5) - 3 \log_2 2 = 0$$

$$2 \log_2(4-3x) - \log_2(x^2-5) - 3 \log_2 2 = 0$$

$$\frac{1}{2} \times 2 \log_2(4-3x) - \log_2(x^2-5) - \log_2 2^3 = 0$$

$$\log_2(4-3x) - \log_2(x^2-5) - \log_2 8 = 0$$

$$\log_2 \left[\frac{4-3x}{(x^2-5)(8)} \right] = 0$$

$$2^0 = \frac{4-3x}{8x^2-40}$$

$$1(8x^2-40) = 4-3x$$

$$8x^2 + 3x - 40 - 4 = 0$$

$$8x^2 + 3x - 44 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(8)(-44)}}{2(8)}$$

$$x = \frac{-3 + \sqrt{1417}}{16} \approx 2.1651 \quad \text{reject} \quad x = \frac{-3 - \sqrt{1417}}{16} \approx \underline{\underline{-2.540}}$$



Question 10 continued

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(Total for Question 10 is 11 marks)



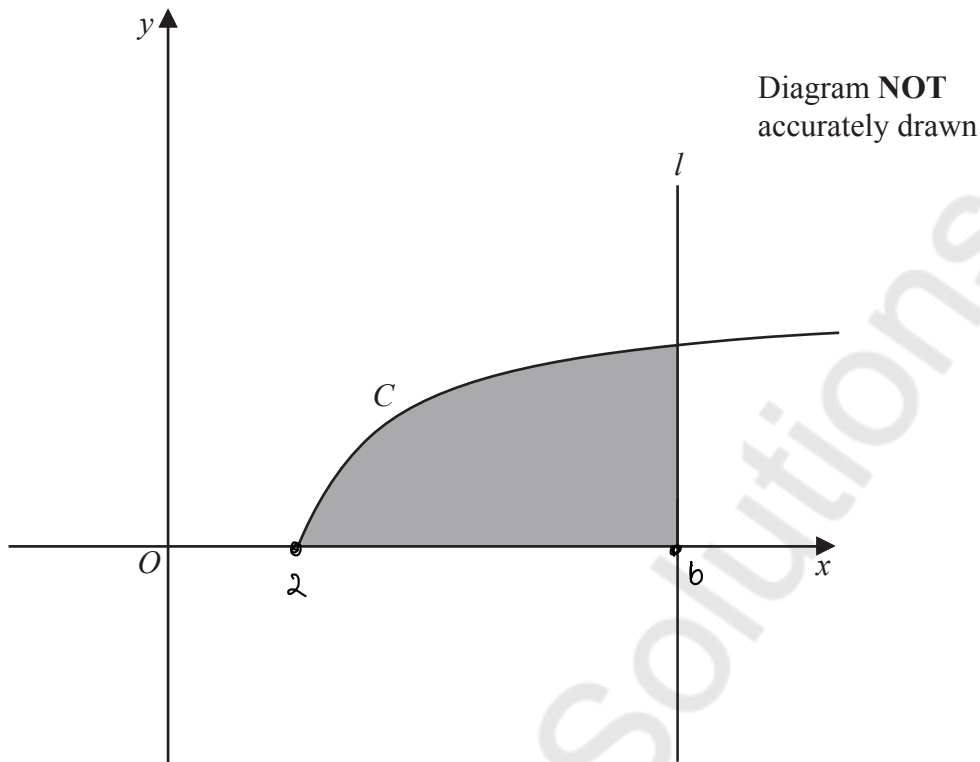


Figure 5

Figure 5 shows part of the curve C with equation $y = \sqrt{4x-8}$ and the line l with equation $x = b$ where $b > 0$

The finite region bounded by the curve C , the x -axis and the line l , shown shaded in Figure 5, is rotated through 360° about the x -axis.

Given that the volume of the solid formed is 50π units³

find the value of b

(7)

$$\begin{aligned} \text{At } y=0 \quad 0 &= \sqrt{4x-8} \\ 0 &= 4x-8 \\ 4x &= 8, \quad x=2 \end{aligned}$$

$$\pi \int_2^b [(4x-8)^{1/2}]^2 dx = 50\pi$$

$$\pi \int_2^b (4x-8) dx = 50\pi$$

$$\pi [2x^2 - 8x]_2^b = 50\pi$$

$$(2b^2 - 8b) - (2(2)^2 - 8(2)) = 50$$

$$\begin{aligned} 2b^2 - 8b - 8 + 16 - 50 &= 0 \\ 2b^2 - 8b - 42 &= 0 \\ b^2 - 4b - 21 &= 0 \\ (b+3)(b-7) &= 0 \\ b &= -3 \quad \text{reject} \quad -3 < 2 \\ b &= 7 \quad \checkmark \quad 7 > 2 \end{aligned}$$



Question 11 continued

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