Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

Pearson Edexcel International GCSE

Tuesday 10 June 2025

Afternoon (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics

PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Solve the equation

$$\sqrt{3}\tan A = 2\sin A \qquad \text{for} \qquad 0^\circ \leqslant A \leqslant 360^\circ$$

(5)

$\sqrt{3}\dfrac{\sin A}{\cos A} = 2\sin A$

$\sqrt{3}\sin A = 2\sin A\cos A$

$\sqrt{3}\sin A - 2\sin A\cos A = 0$

$\sin A[\sqrt{3} - 2\cos A] = 0$

$\sin A = 0$ $\qquad$ $\sqrt{3} - 2\cos A =$

$A = \sin^{-1}(0)$ $\qquad$ $\sqrt{3} = 2\cos A$

$A = \underline{\underline{0^\circ}}, \underline{\underline{180^\circ}}, \underline{\underline{360^\circ}}$ $\qquad$ $\dfrac{\sqrt{3}}{2} = \cos A$

$\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = A$

$\underline{\underline{30^\circ}}, \underline{\underline{330^\circ}} = A$

(Total for Question 1 is 5 marks)

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2 Find the set of values for x for which

(a) $9-3x>11x+2$ **(1)**

(b) $10x^2+7x<12$ **(3)**

(c) **both** $9-3x>11x+2$ **and** $10x^2+7x<12$ **(1)**

(a) $9-3x>11x+2$

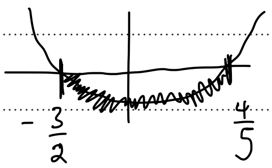
$7 \quad 7 \; 14x$

$\frac{1}{2} > x$

(b) $10x^2+7x-12<0$

$(5x-4)(2x+3)<0$

Critical Values: $x=-\frac{3}{2}$, $x=\frac{4}{5}$



$-\frac{3}{2}<x<\frac{4}{5}$

(c) $-\frac{3}{2}<x<\frac{1}{2}$

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Question 2 continued

(Total for Question 2 is 5 marks)

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3 Given that $\dfrac{a+b\sqrt{5}}{6-2\sqrt{5}} = \dfrac{9+4\sqrt{5}}{c}$ where a, b and c are prime numbers,

find the value of a, the value of b and the value of c

(5)

$$\frac{(a+b\sqrt{5})(6+2\sqrt{5})}{(6-2\sqrt{5})(6+2\sqrt{5})} = \frac{9+4\sqrt{5}}{c}$$

$$\frac{6a+2a\sqrt{5}+6b\sqrt{5}+10b}{36-20} = \frac{9+4\sqrt{5}}{c}$$

$$\frac{6a+10b+(2a+6b)\sqrt{5}}{16} = \frac{9+4\sqrt{5}}{c}$$

$$\frac{2[(3a+5b)+(a+3b)\sqrt{5}]}{16} = \frac{9+4\sqrt{5}}{c}$$

$$\frac{(3a+5b)+(a+3b)\sqrt{5}}{8} = \frac{9+4\sqrt{5}}{c}$$

$3a+5b=9$

$a+3b=4$

$a=\frac{7}{4} \quad b=\frac{3}{4}$

$$\frac{4\left(\frac{7}{4}+\frac{3}{4}\sqrt{5}\right)}{6-2\sqrt{5}} = \frac{4(9+4\sqrt{5})}{c}$$

$$\frac{7+3\sqrt{5}}{6-2\sqrt{5}} = \frac{4(9+4\sqrt{5})}{c}$$

$$\frac{9+4\sqrt{5}}{2} = \frac{4(9+4\sqrt{5})}{c}$$

$$\frac{1}{2} = \frac{4}{c}$$

$8 = c$

$$\frac{7+3\sqrt{5}}{6-2\sqrt{5}} = \frac{(9+4\sqrt{5})}{c/4}$$

$8/4 = 2$ $\therefore$ Prime Values are

$a=7 \quad b=3 \quad c=2$

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Question 3 continued

(Total for Question 3 is 5 marks)

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4 A geometric series G has first term a and common ratio r

The third term of G is 10 and the seventh term of G is 40

Given that the second term of G is negative,

(a) find

(i) the exact value of r

(ii) the value of a

(5)

(b) Explain why G does not have a sum to infinity.

(1)

a i) $T_n = ar^{n-1}$

$T_3 = 10 = ar^2$ — 1

$T_7 = 40 = ar^6$ — 2

2 ÷ 1 $\quad 4 = r^4$

$(4)^{1/2} = (r^4)^{1/2}$

$2 = r^2$

$\pm\sqrt{2} = r$

$-\sqrt{2} = r \qquad \sqrt{2} = r$ reject

ii) $10 = a(-\sqrt{2})^2$

$\frac{10}{2} = a = 5$

b $\quad S_\infty = \frac{a}{1-r} \qquad |r| < 1$

$|-\sqrt{2}| > 1$

Thus the series is not convergent.

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Question 4 continued

(Total for Question 4 is 6 marks)

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5 In triangle ABC, $AC = 12$ cm, $BC = 14$ cm, $AB = x$ cm and angle $ABC = 30°$

(a) Show that $x = P\sqrt{3} \pm \sqrt{95}$ where P is a prime number.

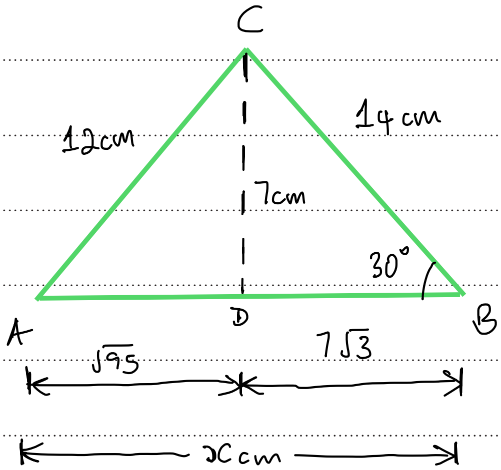
(5)

(b) Hence or otherwise, find in cm², the difference between the two possible areas of triangle ABC

Give your answer in the form $m\sqrt{n}$ where m is a prime number and n is an integer.

(3)

(a)



$\cos 30 = \frac{BD}{14}$

$\frac{\sqrt{3}}{2} = \frac{BD}{14}$

$BD = 7\sqrt{3}$

$14^2 = (7\sqrt{3})^2 + CD^2$

$196 = 147 + CD^2$

$49 = CD^2$

$7\text{cm} = CD$

$12^2 = AD^2 + 7^2$

$144 = AD^2 + 49$

$95 = AD^2$

$\sqrt{95} = AD$

$AB = x = 7\sqrt{3} \pm \sqrt{95}$ $\therefore P = 7$

(b) $A_1 = \frac{(7\sqrt{3} + \sqrt{95}) \times 7}{2}$ $A_2 = \frac{(7\sqrt{3} - \sqrt{95}) \times 7}{2}$

$A_1 = \frac{49\sqrt{3}}{2} + \frac{7\sqrt{95}}{2}$ $A_2 = \frac{49\sqrt{3}}{2} - \frac{7\sqrt{95}}{2}$

$A_1 - A_2 = \frac{49\sqrt{3}}{2} + \frac{7\sqrt{95}}{2} - \frac{49\sqrt{3}}{2} + \frac{7\sqrt{95}}{2}$

Difference in the area of the two possible triangles ABC is $7\sqrt{95}$ $m = 7$ $n = 95$

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Question 5 continued

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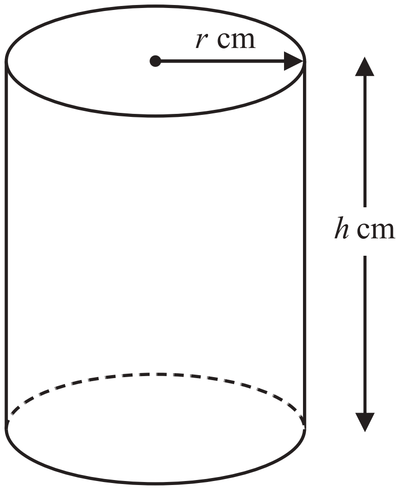

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a solid right circular cylinder with radius r cm and height h cm

The total surface area of the cylinder is 700π cm^2

The volume of the cylinder is V cm^3

(a) Show that $V = \pi r\left(350 - r^2\right)$ **(4)**

Given that r can vary and using calculus,

(b) find, in cm to 3 significant figures, the value of r for which V is a maximum. Justify that this value of r gives a maximum value of V **(5)**

(c) Find, to 3 significant figures, the height h cm for which V is a maximum. **(1)**

(a) $V = \pi r^2 h$ $TSA = 2\pi r^2 + 2\pi r h$

$V = \pi r^2\left[\frac{350}{r} - r\right]$ $700\pi = 2\pi r^2 + 2\pi r h$

$V = \pi r[350 - r^2]$ $700 - 2r^2 = 2rh$

Shown. $\frac{700}{2r} - \frac{2r^2}{2r} = h$

$\frac{350}{r} - r = h$

(b) $\frac{dV}{dr} = 0$ occurs at V_{max}

$V = 350\pi r - \pi r^3$

$\frac{dV}{dr} = 350\pi - 3\pi r^2$

$0 = 350\pi - 3\pi r^2$

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Question 6 continued

$$3\pi r^2 = 350\pi$$

$$3r^2 = 350$$

$$r^2 = \frac{350}{3}$$

$$r = \sqrt{\frac{350}{3}}$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

At $r = \sqrt{\frac{350}{3}}$, $\frac{d^2V}{dr^2} = -6\pi\left(\sqrt{\frac{350}{3}}\right)$ which is negative

Thus local maximum.

(c) $h = \frac{350}{r} - r$

$$h = \frac{350}{\sqrt{\frac{350}{3}}} - \sqrt{\frac{350}{3}}$$

$h \approx \underline{21.6 \text{ cm}}$ (3 s.f.)

(Total for Question 6 is 10 marks)

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7 (a) Complete the table of values for $y = \log_3(4 - x) + 3$ giving your answers to 2 decimal places.

x	0	0.5	1	1.5	2	2.5	3	3.5
y	4.26	4.14	4	3.83	3.63	3.37	3	2.37

(2)

(b) On the grid opposite, draw the graph of $y = \log_3(4 - x) + 3$ in the interval $0 \leqslant x \leqslant 3.5$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate,

to one decimal place, of the root of the equation $3^{2x-5} - (4 - x)^3 = 0$ in the interval $0 \leqslant x \leqslant 3.5$

(6)

(a) At $x = 0.5$
$y = \log_3(4-0.5)+3$
$y = 4.14$

At $x = 1.5$
$y = \log_3(4-1.5)+3$
$y = 3.83$

At $x = 2.5$
$y = \log_3(4-2.5)+3$
$y = 3.37$

(c) $3^{2x-5} - (4-x)^3 = 0$

$3^{2x-5} = (4-x)^3$

$\log_3 3^{2x-5} = \log_3 (4-x)^3$

$2x - 5 = 3\log_3(4-x)$

$\frac{2x}{3} - \frac{5}{3} = \log_3(4-x)$

$\frac{2x}{3} - \frac{5}{3} + 3 = \log_3(4-x) + 3$

$\frac{2x}{3} + \frac{4}{3} = \log_3(4-x) + 3$

Plot $y = \frac{2x}{3} + \frac{4}{3}$

$(1, 2)$ $(2.5, 3)$

At $y=2$, $2 = \frac{2x}{3} + \frac{4}{3}$
$6 = 2x + 4$
$2 = 2x$
$1 = x$

At $y=3$, $3 = \frac{2x}{3} + \frac{4}{3}$
$9 = 2x + 4$
$5 = 2x$
$2.5 = x$

$x = 2.8$ as this is the x-coordinate of the point of intersection between $y = \log_3(4-x)+3$ and $y = \frac{2x}{3} + \frac{4}{3}$

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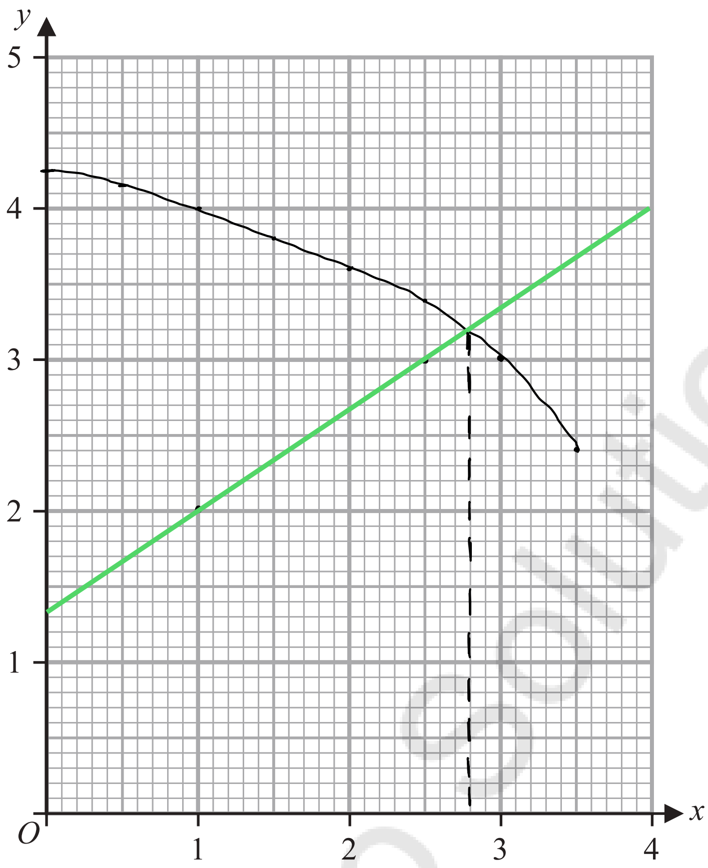
Question 7 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 7 continued

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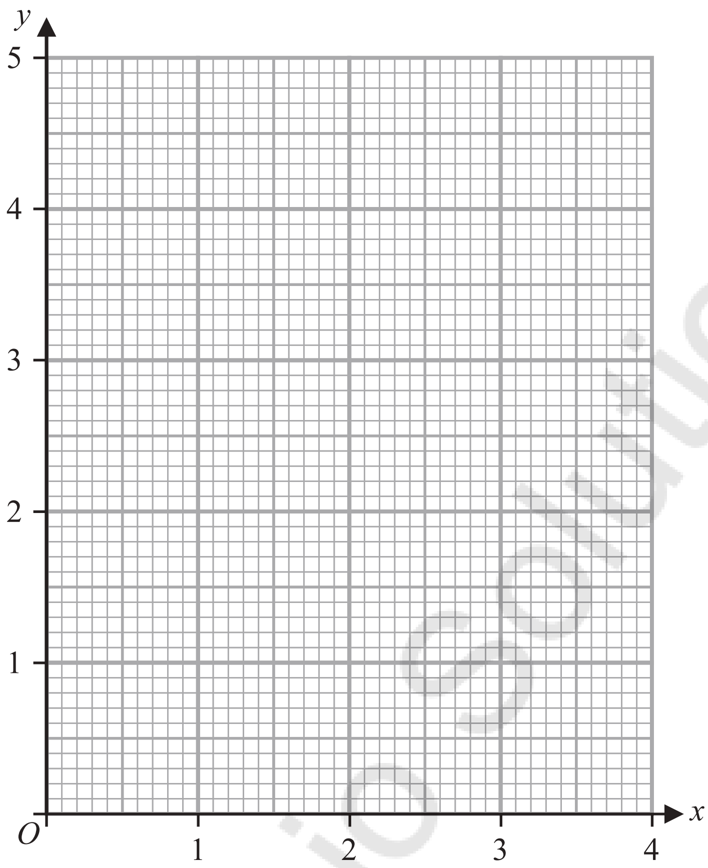
Question 7 continued

Only use this grid if you need to redraw your graph.



(Total for Question 7 is 10 marks)

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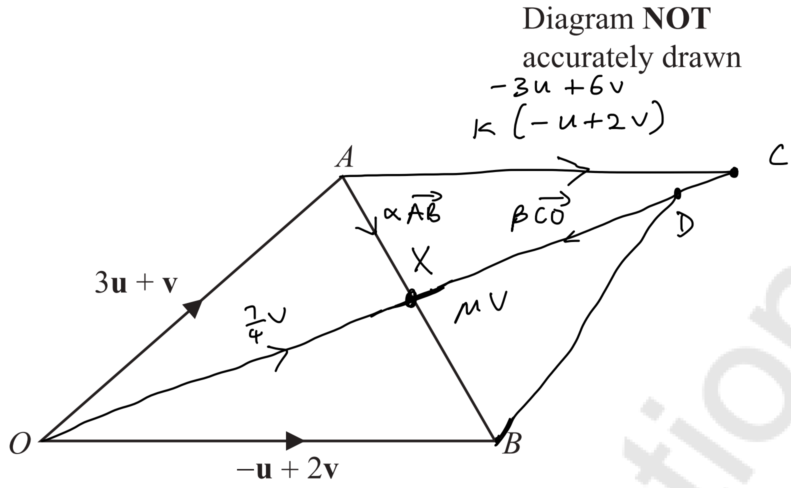

Figure 2

Figure 2 shows triangle OAB with

$$\overrightarrow{OA} = 3\mathbf{u}+\mathbf{v} \qquad \overrightarrow{OB} = -\mathbf{u}+2\mathbf{v}$$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{u}$ and $\mathbf{v}$

(2)

Point C is such that AC is parallel to OB

Given that $\overrightarrow{OC} = \mu\mathbf{v}$

(b) find the value of μ

(4)

The lines OC and AB intersect at point X

Point D lies on $\overrightarrow{OC}$ such that

area of triangle BOX : area of triangle $BXD = 2:3$

Using a vector method,

(c) find $\overrightarrow{BD}$ as a simplified expression in terms of $\mathbf{u}$ and $\mathbf{v}$

(5)

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$\overrightarrow{AB} = -(3u+v) + (-u+2v)$

$= -3u - v - u + 2v$

$\overrightarrow{AB} = -4u + v$

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Question 8 continued

(b) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$\overrightarrow{OC} \quad 3u+v+k(-u+2v)$

$\overrightarrow{OC} = 3u-ku+v+2kv$

$\overrightarrow{OC} = (3-k)u + (1+2k)v$

$\overrightarrow{OC} = Mv$

$\therefore 3-k=0 , \quad k=3$

$M = 1+2k$

$M = 1+2(3)$

$M = 1+6$

$M = 7$

(c) To find $\overrightarrow{BD}$, need to find $\overrightarrow{OX}$ first

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$\overrightarrow{OX} = 3u+v+\alpha\overrightarrow{AB}$

$\overrightarrow{OX} = 3u+v+\alpha(-4u+v)$

$\overrightarrow{OX} = (3-4\alpha)u + (1+\alpha)v$

$\therefore 3-4\alpha = 0$

$\alpha = \frac{3}{4}$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CX}$

$\overrightarrow{OX} = 3u+v-3u+6v+\beta\overrightarrow{CO}$

$\overrightarrow{OX} = 7v + \beta(-7v)$

$\overrightarrow{OX} = (7-7\beta)v$

$1+\alpha = 7-7\beta$

$1+\frac{3}{4} = 7-7\beta$

$\frac{7}{4} = 7-7\beta$

$7\beta = 7-\frac{7}{4}$

$7\beta = \frac{21}{4}$

$\beta = \frac{3}{4}$

$\therefore \overrightarrow{OX} = \left[7-7\left(\frac{3}{4}\right)\right]v$

$\overrightarrow{OX} = \frac{7}{4}v$

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Question 8 continued

$\overrightarrow{OX} : \overrightarrow{XD} = 2:3$ (Both triangles have the same base BX)

$\frac{7}{4}v : \overrightarrow{XD} = 2:3$

$\frac{\frac{7}{4}v}{XD} = \frac{2}{3}$

$\frac{7}{4}v \times \frac{3}{2} = \overrightarrow{XD}$

$\frac{21}{8}v = \overrightarrow{XD}$

$\overrightarrow{AX} = \frac{3}{4}\overrightarrow{AB}$, $\overrightarrow{XB} = \frac{1}{4}\overrightarrow{AB}$

$\overrightarrow{BD} = \overrightarrow{BX} + \overrightarrow{XD}$

$\overrightarrow{BD} = -\overrightarrow{XB} + \overrightarrow{XD}$

$\overrightarrow{BD} = -\frac{1}{4}(-4u+v) + \frac{21}{8}v$

$\overrightarrow{BD} = u - \frac{1}{4}v + \frac{21}{8}v$

$\overrightarrow{BD} = \underline{u + \frac{19}{8}v}$

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Question 8 continued

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9

$$y = e^{-4t} \cos 2t$$

(a) Show that $2e^{-4t}\sin 2t = -\dfrac{dy}{dt} - 4y$

(3)

Given that $\dfrac{d^2y}{dt^2} + M\dfrac{dy}{dt} + Ny = 0$ where M and N are integers

(b) find the value of M and the value of N

(5)

(a) $y = e^{-4t}\cos 2t$

$u = e^{-4t}$ $\qquad v = \cos 2t$

$\frac{du}{dt}$: $u = e^{m}$ $\quad m = -4t$ $\qquad \frac{dv}{dt}$: $v = \cos n$ $\quad n = 2t$

$\frac{du}{dm} = e^{m}$ $\quad \frac{dm}{dt} = -4$ $\qquad \frac{dv}{dn} = -\sin n$ $\quad \frac{dn}{dt} = 2$

$\frac{du}{dt} = \frac{du}{dm} \times \frac{dm}{dt}$ $\qquad \frac{dv}{dt} = \frac{dn}{dt} \times \frac{dv}{dn}$

$\frac{du}{dt} = e^{M} \times -4$ $\qquad \frac{dv}{dt} = 2 \times -\sin n$

$\frac{du}{dt} = -4e^{m}$ $\qquad \frac{dv}{dt} = -2\sin n$

$\frac{du}{dt} = -4e^{-4t}$ $\qquad \frac{dv}{dt} = -2\sin 2t$

$y = uv$ $\qquad \frac{dy}{dt} = u\frac{dv}{dt} + v\frac{du}{dt}$

$\frac{dy}{dt} = (e^{-4t})(-2\sin 2t) + (\cos 2t)(-4e^{-4t})$

$\frac{dy}{dt} = -2e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$

$\frac{dy}{dt} + 2e^{-4t}\sin 2t = -4e^{-4t}\cos 2t$

$2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4\underbrace{e^{-4t}\cos 2t}_{y}$

$2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ $\qquad$ Shown.

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Question 9 continued

(b) $\frac{dy}{dt} = -2e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$

$2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ —— (*)

$\frac{d}{dt}\left[2e^{-4t}\sin 2t\right] = -\frac{d^2y}{dt^2} + \frac{d}{dt}(-4y)$

$u = 2e^{-4t}$ $v = \sin 2t$

$\frac{du}{dt} = -8e^{-4t}$ $\frac{dv}{dt} = 2\cos 2t$

$\frac{d}{dt}(2e^{-4t}\sin 2t) = 4e^{-4t}\cos 2t - 8e^{-4t}\sin 2t$

$\underbrace{4e^{-4t}\cos 2t}_{y} - 8e^{-4t}\sin 2t = \frac{-d^2y}{dt^2} - 4\frac{dy}{dt}$

$4y - 8e^{-4t}\sin 2t = \frac{-d^2y}{dt^2} - 4\frac{dy}{dt}$

$2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$

$\times -4$ $\times -4$

$-8e^{-4t}\sin 2t = 4\frac{dy}{dt} + 16y$

$4y + 4\frac{dy}{dt} + 16y = \frac{-d^2y}{dt^2} - 4\frac{dy}{dt}$

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4\frac{dy}{dt} + 4y + 16y = 0$

$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 20y = 0$

$M = 8$ $N = 20$

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 8 marks)

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10 A curve C has equation

$$y = \frac{7-x}{2x+3} \qquad x \neq -\frac{3}{2}$$

(a) Write down an equation of the asymptote to C that is

(i) parallel to the y-axis

(ii) parallel to the x-axis

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using the axes on the opposite page, sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

C passes through the point A with coordinates $\left(\frac{1}{2}, \frac{13}{8}\right)$

(d) Show that the gradient of C at A is $-\frac{17}{16}$

(3)

C also passes through the point P

Given that the tangent to C at P is parallel to the tangent to C at A

(e) find an equation of the tangent to C at P
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(7)

(a) i) When is denominator zero?

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

ii) When x becomes really large

$y = \frac{7-10^6}{2(10^6)+3} \approx \frac{-10^6}{2(10^6)}$

$y \approx \frac{-10^6}{2(10^6)} \approx -\frac{1}{2}$

$x \to \infty \quad y = -\frac{1}{2}$

(b) At $x=0$, $y = \frac{7-0}{2(0)+3}$ $\quad (0, \frac{7}{3})$

$y = \frac{7}{3}$

At $y=0$, $0 = \frac{7-x}{2x+3}$

$0 = 7 - x$

$x = 7$ $\quad (7, 0)$

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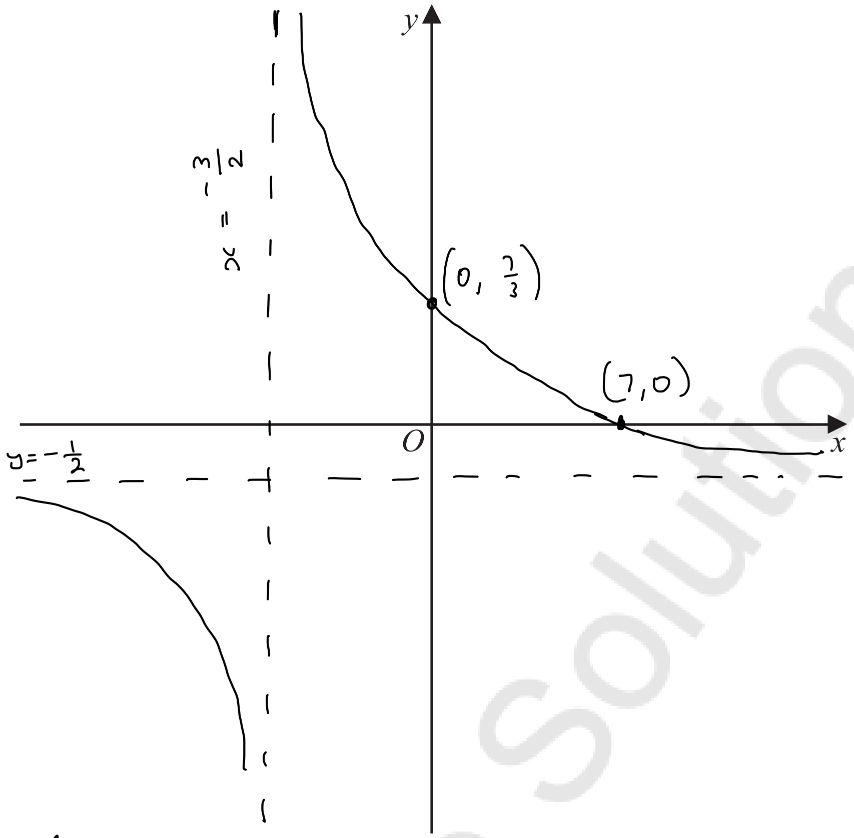
Question 10 continued

(c)



(d) $y = \dfrac{7-x}{2x+3}$

$u = 7 - x$ $\qquad$ $v = 2x+3$

$\dfrac{du}{dx} = -1$ $\qquad$ $\dfrac{dv}{dx} = 2$

$\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{(2x+3)(-1) - [7-x](2)}{(2x+3)^2}$

$\dfrac{dy}{dx} = \dfrac{-2x-3-14+2x}{(2x+3)^2}$

$\dfrac{dy}{dx} = \dfrac{-17}{(2x+3)^2}$

$m_A = \dfrac{-17}{[2(\frac{1}{2})+3]^2}$ $\quad$ At $A(\frac{1}{2}, \frac{13}{8})$

$m_A = -\dfrac{17}{16}$

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Question 10 continued

(e) $\frac{dy}{dx} = \frac{-17}{(2x+3)^2}$

At $m = -\frac{17}{16}$ $\quad -\frac{17}{16} = \frac{-17}{(2x+3)^2}$

$16 = (2x+3)^2$

$4 = 2x+3 \qquad -4 = 2x+3$

$1 = 2x \qquad -7 = 2x$

$\frac{1}{2} = x \qquad -\frac{7}{2} = x$

$P(-\frac{7}{2}, y) \qquad y = \frac{7-x}{2x+3}$

At $x = -\frac{7}{2}$, $y = \frac{7-(-\frac{7}{2})}{2(-\frac{7}{2})+3}$

$y = -\frac{21}{8}$

$P(-\frac{7}{2}, -\frac{21}{8})$

$m_A = m_P = -\frac{17}{16}$

$y - y_1 = m(x - x_1)$

$y - (-\frac{21}{8}) = -\frac{17}{16}(x - (-\frac{7}{2}))$

$y + \frac{21}{8} = -\frac{17}{16}x - \frac{119}{32}$

$32y + 84 = -34x - 119$

$34x + 32y + 203 = 0$

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Question 10 continued

(Total for Question 10 is 17 marks)

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11 (a) Show that $\cos^4\theta - \sin^4\theta = \cos 2\theta$

(4)

(b) Hence, or otherwise, solve the equation

$$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\cos^4\left(\theta + \frac{\pi}{8}\right) - 2\sin^4\left(\theta + \frac{\pi}{8}\right) \quad \text{for} \quad 0 \leqslant \theta < \pi$$

Give your solutions to 2 decimal places.

(7)

(c) Using calculus, find the exact value of $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \left(\cos^4 2x - \sin^4 2x - 8\sin 4x\right) dx$

Give your answer in the form $a - b\sqrt{2}$ where a and b are rational numbers.

(4)

(a) $\cos^4\theta - \sin^4\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$\cos^2\theta + \sin^2\theta \equiv 1$

$\cos^2\theta - \sin^2\theta$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

If $A = B = \theta$

$\cos(\theta + \theta) \equiv \cos 2\theta \equiv \cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$

$\cos 2\theta$ shown.

(b) $8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\cos^4\left(\theta + \frac{\pi}{8}\right) - 2\sin^4\left(\theta + \frac{\pi}{8}\right)$

$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\left[\cos^4\left(\theta + \frac{\pi}{8}\right) - \sin^4\left(\theta + \frac{\pi}{8}\right)\right]$

$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\cos 2\left[\theta + \frac{\pi}{8}\right]$

$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\cos\left(2\theta + \frac{\pi}{4}\right)$

$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 2\cos\left(2\theta + \frac{\pi}{4}\right) - 3 = 0$

let $Y = \cos\left(2\theta + \frac{\pi}{4}\right)$

$8Y^2 - 2Y - 3 = 0$

$(4Y - 3)(2Y + 1) = 0$

$Y = \frac{3}{4} \quad Y = -\frac{1}{2}$

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Question 11 continued

$\cos\left(2\theta + \frac{\pi}{4}\right) = \frac{3}{4}$ $\qquad$ $\cos\left(2\theta + \frac{\pi}{4}\right) = -\frac{1}{2}$

$0 \leq \theta < \pi$

$0 \leq 2\theta < 2\pi$

$\frac{\pi}{4} \leq 2\theta + \frac{\pi}{4} < \frac{9\pi}{4}$

$2\theta + \frac{\pi}{4} = 0.7227$, $2\pi - 0.7227$, $2\pi + 0.7227$

reject

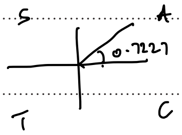
$\theta = \frac{1}{2}\left(2\pi - 0.7227 - \frac{\pi}{4}\right), \frac{1}{2}\left(2\pi + 0.7227 - \frac{\pi}{4}\right)$

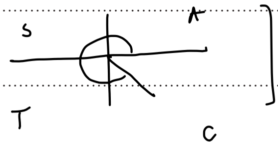
$\underline{\underline{\theta = 2.39}}, \underline{\underline{3.11}}$

$2\theta + \frac{\pi}{4} = \frac{2\pi}{3}, \frac{4\pi}{3}$

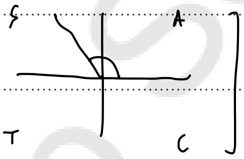
$2\theta = \frac{5}{12}\pi, \frac{13}{12}\pi$

$\theta = \underline{\underline{\frac{5}{24}\pi}}, \underline{\underline{\frac{13}{24}\pi}}$

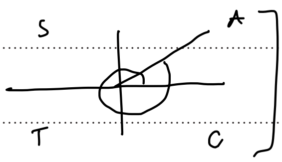




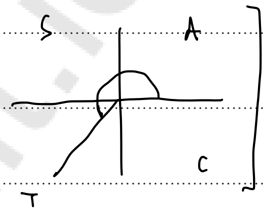
$2\pi - 0.7227$



$\frac{2\pi}{3}$



$2\pi + 0.7227$



$\frac{4\pi}{3}$

(c) $\displaystyle\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \cos^4 2x - \sin^4 2x - 8\sin 4x \, dx$

$\cos^4 2x - \sin^4 2x = \underbrace{(\cos^2 2x + \sin^2 2x)}_{1}\underbrace{(\cos^2 2x - \sin^2 2x)}_{\cos 4x}$

$\cos^2 x - \sin^2 x = \cos 2x$

$\cos^2 2x - \sin^2 2x = \cos 4x$

$\displaystyle\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} (\cos 4x - 8\sin 4x)\, dx$

$\left[\frac{\sin 4x}{4} + \frac{8\cos 4x}{4}\right]_{\frac{\pi}{16}}^{\frac{\pi}{8}}$

$\left[\frac{\sin 4(\pi/8)}{4} + \frac{8\cos 4(\pi/8)}{4}\right] - \left[\frac{\sin 4(\pi/16)}{4} + \frac{8\cos 4(\pi/16)}{4}\right]$

$\left(\frac{1}{4} + 0\right) - \left(\frac{\sqrt{2}}{8} + \sqrt{2}\right) = \underline{\frac{1}{4} - \frac{9\sqrt{2}}{8}}$

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Question 11 continued

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TOTAL FOR PAPER IS 100 MARKS

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