Differentiation Exam Questions Mark Scheme

Topic Test and Revision

Mathvault.io

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes $(2, 8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)

Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $x > 2$ and $x < 6$ $\{x : x > 2\} \cap \{x : x < 6\}$ $x \in (2, 6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2, 6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2, 6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $f(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{\ \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2, 8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the ($y =$..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

Alternative I part (c):

Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.

There are various versions of this but can be marked similarly

M1: Realises that the equation of C is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in a, b and c. Condone with a =1 for this mark.

Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until d is set equal to 0.

There are four equations that could be formed, only two are necessary for this mark.

Condone slips

Using (6, 0) $\Rightarrow 216a+36b+6c=0$

Using (2, 8) $\Rightarrow 8a+4b+2c=8$

Using $\frac{dy}{dx}=0$ at $x = 2$ $\Rightarrow 12a+4b+c=0$

Using $\frac{dy}{dx}=0$ at $x = 6 \Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using (2, 8) to find values for a, b and c. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for C $y=\frac{1}{4}x^3-3x^2+9x$ o.e.

ISW after a correct answer. Condone $f(x)=\frac{1}{4}x^3-3x^2+9x$

Alternative II part (c)

Using the gradient and integrating

M1: Realises that the gradient of C is zero at 2 and 6 so sets $f'(x)=k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with $k = 1$

dM1: Substitutes $x=2, y=8$ into $f(x)=k(...x^3+...x+...)$ and finds a value for k

A1: Uses all of the information to form a correct equation for C $y=\frac{3}{4}\left(\frac{1}{3}x^3-4x^2+12x\right)$ o.e.

ISW after a correct answer. Condone $f(x)=\frac{3}{4}\left(\frac{1}{3}x^3-4x^2+12x\right)$

(**Q06 9MA0/01, June 2022**)

Q2.

Question	Scheme	Marks	AOs
(a)	$8(4)-4^{\frac{5}{2}}=32-32=0$	B1	1.1b
		(1)	
(b)	$8-\frac{5}{2}x^{\frac{3}{2}}$	B1	1.1b
	$x=4\Rightarrow\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}8-\frac{5}{2}\times 8=-12$ $\Rightarrow y\{-0\}="-12"(x-4)$	M1	1.1b
	$12x+y=48$ *	A1*	1.1b
		(3)	
(c)	Attempts to find one of the coordinates of the point of intersection $y=8x,\ 12x+y=48\Rightarrow y=19.2$ (or $x=2.4$)	M1	1.1b
	Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$ or $\int_0^{"2.4"} 8x\ \mathrm{d}x+\int_{"2.4"}^{4} "(48-12x)"\ \mathrm{d}x$	dM1	3.1a
	$\int\left(8x-x^{\frac{5}{2}}\right)\mathrm{d}x=4x^2-\frac{2}{7}x^{\frac{7}{2}}$	B1	1.1b
	$A=38.4-\left["4x^2-\frac{2}{7}x^{\frac{7}{2}}"\right]_0^4=38.4-64+\frac{256}{7}$	ddM1	3.1a
	$=\frac{384}{35}$	A1	1.1b
		(5)	
			(9 marks)

Notes

(a)

B1: Substitutes $x=4$ into the equation of the curve and verifies that $y=0$. Accept "$8(4)-4^{\frac{5}{2}}=0$"

Alternatively, sets $8x-x^{\frac{5}{2}}=0$ and solves with correct processing to achieve $x=4$.

As a minimum accept e.g. $8x-x^{\frac{5}{2}}=0\Rightarrow x^{\frac{3}{2}}=8\Rightarrow\{x=\}\ 4$ which may follow factorisation.

(b)

B1: Correct differentiation. The $\frac{\mathrm{d}y}{\mathrm{d}x}=$ need not be present.

M1: Correct method for finding the equation of the tangent at $A(4, 0)$.

Requires substitution of $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and an attempt at the equation of the line using this gradient. If using $y-y_1=m(x-x_1)$ then condone the omission of the -0.

If $y=mx+c$ is used they must proceed as far as $c=\ldots$

Accept $\frac{\mathrm{d}y}{\mathrm{d}x}=-12$ or $m=-12$ without explicit substitution of $x=4$ provided $8-\frac{5}{2}x^{\frac{3}{2}}$ is seen.

A1*: Correct work leading to the given equation having scored B1M1.
Condone $y+12x=48$ and apply isw once seen.
Do not condone $12x+y-48=0$ (unless a correct equation = 48 is seen).

(c) **Note:** Condone poor notation such as missing dx or spurious $\int$ symbols throughout.

M1: Attempts to find either the x or y coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for the area of the triangle. e.g. Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$

If integration is attempted then condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x \, dx + \int_{"2.4"}^{4} "(48-12x)" \, dx \quad \left\{=\frac{576}{25}+\frac{384}{25}=23.04+15.36\right\}$

B1: Correct integration of **curve** ignoring limits, i.e. $4x^2-\frac{2}{7}x^{\frac{7}{2}}$ but condone e.g. $\frac{8x^{1+1}}{2}-\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
Implied by $38.4-\frac{192}{7}$ or a correct final answer $\frac{384}{35}$
Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence of substitution (which need not be evaluated).

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

Alternative using lines – curve:

M1: Attempts to find either the x or y coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for at least one part (0 to "2.4" **or** "2.4" to 4) of the area of R including limits.
Condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x-\left(8x-x^{\frac{5}{2}}\right) dx$ or $\int_{"2.4"}^{4} "(48-12x)"-\left(8x-x^{\frac{5}{2}}\right) dx$ (or a sum of both)

B1: Correct integration of **both** regions ignoring limits. May be completed as a sum or separately.

Condone e.g. $\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$ in place of $\frac{2}{7}x^{\frac{7}{2}}$ Note that each integral may have been simplified.

$\int_{...}^{...} x^{\frac{5}{2}} \, dx$ and {+} $\int_{...}^{...} 48-20x+x^{\frac{5}{2}} \, dx \rightarrow \left[\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$ and {+} $\left[48x-10x^2+\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
This approach requires:

- substitution of 0, their 2.4 and 4 in the correct places
- the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ to be cancelled (may be implied by a correct final answer $\frac{384}{35}$)

Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence that the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ has been cancelled e.g. ~~6.118...~~ − ~~6.118...~~

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

Q3.

Question	Scheme	Marks	AOs
	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1 A1	1.1b 1.1b
	(As $h\to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)\to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{dy}{dx}=\cos x$ *	A1*	2.5
			(5 marks)

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently
There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).
Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**
Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.
e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}\to\dfrac{-\dfrac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ **at some point in their solution and linking** $\frac{dy}{dx}$ **with** $\cos x$ e.g.

- $\frac{dy}{dx} = ... = \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{dy}{dx} = ... = \lim_{h\to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{dy}{dx} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $f'(x)$ or y' in place of $\frac{dy}{dx}$

Give final A0 for no evidence of limiting arguments:

e.g. when $h = 0$ $\frac{dy}{dx} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

(Q12 9MA0/01, June 2023)

Q4.

Question	Scheme	Marks	AOs
	$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$	M1	2.1
	$= \frac{2xh + h^2}{h}$	A1	1.1b
	$\frac{dy}{dx} = \lim_{h\to 0}\frac{2xh + h^2}{h} = \lim_{h\to 0}(2x + h) = 2x$*	A1*	2.5
		(3)	
		(3 marks)	

	Notes
Note:	Throughout the question allow use of δx for h or any other letter e.g. a if used consistently. If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's
M1:	Begins the process by writing down the gradient of the chord and attempts to expand the correct squared bracket – you can condone "poor" squaring e.g. $(x+h)^2 = x^2 + h^2$ but the $-x^2$ must be present.
A1:	Reaches a correct fraction o.e. with the x^2 terms cancelled out and with no algebraic errors, e.g. $\dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$, $2x + h$ is correct.
A1*:	Completes the process by applying a limiting argument and deduces that $\dfrac{dy}{dx} = 2x$ with no errors seen. They must have $= 2x$ and not just $\lim_{h\to 0} 2x$ to complete the proof. $\dfrac{dy}{dx} =$ or an equivalent e.g. $f'(x) =$ or "Gradient =" must be evident somewhere in their working or final line. If $f'(x)$ is used then there is no requirement to see f (x) defined first. Condone e.g. $\dfrac{dy}{dx} \to 2x$ or $f'(x) \to 2x$. Condone missing brackets to allow e.g. $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{2xh + h^2}{h} = \lim_{h\to 0} 2x + h = 2x$ Do not allow $h = 0$ if there is never a reference to $h \to 0$. e.g. $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{2xh + h^2}{h} = \lim_{h\to 0} 2x + 0 = 2x$ is acceptable but e.g. $\dfrac{dy}{dx} = \dfrac{2xh + h^2}{h} = 2x + 0 = 2x$ is not unless $h \to 0$ is seen. The $h \to 0$ does not need to be present throughout the proof e.g. appear on every line but must appear at least once. They must reach $2x + h$ at the end and not $\dfrac{2xh + h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

(**Q04 9MA0/01, June 2024**)

Q5.

Question	Scheme	Marks	AOs
	$\dfrac{\sin(\theta+h)-\sin\theta}{h}$	B1	2.1
	$\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	M1	1.1b
		A1	1.1b
	(As $h\to 0$), $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)\to 0\times\sin\theta+1\times\cos\theta$	M1	2.1
	so $\dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

Throughout the question allow the use of $h=\delta\theta$ if used consistently

B1: Gives the correct fraction such as $\dfrac{\sin(\theta+h)-\sin\theta}{\theta+h-\theta}$ or $\dfrac{\sin\theta-\sin(\theta+h)}{-h}$ or $\dfrac{\sin(\theta+h)-\sin(\theta-h)}{2h}$ or $\dfrac{\sin(\theta-h)-\sin\theta}{\theta-h-\theta}$. Condone invisible brackets.

May be implied by $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$

M1: Uses the compound angle formula for $\sin(\theta\pm h)$ to give $\sin\theta\cos h\pm\cos\theta\sin h$

A1: Achieves $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent (may be implied by further work).

Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**

Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)=\sin\theta\times 0+\cos\theta\times 1$

Accept as a minimum $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)=\cos\theta$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from e.g. $\dfrac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}$ to $0\times\sin\theta+\cos\theta$ (or e.g. $0+\cos\theta$) then this can be implied and score dM1

$\dfrac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}=\cos\theta$ is dM0

Condone if limit notation remains within their expression after the limits have been applied. e.g. $\lim\limits_{h\to 0}(\sin\theta\times 0+\cos\theta\times 1)$

Alternatively, condone use of the small angle approximations such that

$$\frac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}\rightarrow\frac{-\frac{h^2}{2}\sin\theta+h\cos\theta}{h}=-\frac{h}{2}\sin\theta+\cos\theta \text{ and replaces } \frac{h}{2} \text{ with } 0$$

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\cos\theta$ with no errors seen. (cso)

We need to see $h\rightarrow 0$ at some point in their solution e.g.

- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\lim_{h\to 0}\left(\sin\theta\left(\frac{\cos h-1}{h}\right)+\cos\theta\left(\frac{\sin h}{h}\right)\right)=\cos\theta$
- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\lim_{h\to 0}\left(-\frac{h}{2}\sin\theta+\cos\theta\right)=0\times\sin\theta+\cos\theta=\cos\theta$ (using small angle approximations)
- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\frac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}=\sin\theta\times 0+1\times\cos\theta=\cos\theta$ as $h\rightarrow 0$

Condone $f'(\theta)$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ in place of $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}$

Give final A0 for no evidence of limiting arguments:

e.g. when $h=0$ $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\sin\theta\left(\frac{\cos h-1}{h}\right)+\cos\theta\left(\frac{\sin h}{h}\right)=\sin\theta\times 0+\cos\theta\times 1=\cos\theta$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h}=1$ and $\frac{\cos h-1}{h}=0$ and attempting to apply these (without seeing e.g. $h\rightarrow 0$ at some point in their solution)

If they work in another variable (e.g. x) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin\theta(\cos h-1)$ e.g. $\frac{\sin\theta\cos h-1+\cos\theta\sin h}{h}$ but accept terms written as e.g. $\sin\theta\frac{\cos h-1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

(Q11 9MA0/02/M, June 2025)

Q6.

Question	Scheme	Marks	AOs
(a)	$\{f'(x)=\}...x^2+...x+...\Rightarrow\{f''(x)=\}...x+...$	M1	1.1b
	$\{f'(x)=\}3x^2+4x-8\Rightarrow\{f''(x)=\}6x+4$	A1cso	1.1b
		(2)	
(b)(i)	$"6x+4"=0\Rightarrow x="-\frac{2}{3}"$	B1ft	1.1b
(ii)	$x \leqslant "-\frac{2}{3}"$ or $x<"-\frac{2}{3}"$	B1ft	2.2a
		(2)	
			(4 marks)

Notes

(a)

M1: For attempting to differentiate twice.

It can be scored for any of: $x^3\rightarrow...x^2\rightarrow....x$ or $2x^2\rightarrow...x\rightarrow k$ or $-8x\rightarrow k\rightarrow 0$ where … are constants.

You can ignore the lhs so do not be concerned what they call the first and/or second derivative, just look for their expressions.

The indices do not need to be processed for this mark so allow for e.g. $x^3\rightarrow...x^{3-1}\rightarrow....x^{3-1-1}$

A1cso: $(f''(x)=)\ 6x+4$ Correct second derivative from fully correct work. The "$f''(x)=$" is not required.

Allow $6x^1$ for $6x$ but not $4x^0$ for 4 unless the $4x^0$ becomes 4 later, e.g. in part (b).

Do **not** apply isw so mark their final answer. E.g. if $6x+4$ becomes $3x+2$ score A0

(b)

(i)

B1ft: $ax+b=0\Rightarrow(x=)-\frac{b}{a}$. This mark is for obtaining $x=-\frac{2}{3}$ **or** $x=-\frac{b}{a}$ which has come from solving an equation of the form $ax+b,\ a,b\neq 0$ where $ax+b$ is their attempt to differentiate twice in part (a)

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$ or an exact decimal and isw.

(ii)

B1ft: Deduces $x\leqslant-\frac{2}{3}$ or follow through their single value of x from part (i) obtained from their attempt to solve an equation of the form $ax+b=0,\ a,b\neq 0$ where $ax+b$ was their attempt to differentiate twice in part (a) . Do not isw and mark their final answer.

If 2 inequalities are given e.g. $x<"-\frac{2}{3}",\ x>"-\frac{2}{3}"$ without indicating which is their answer score B0

Condone $<$ for $\leqslant$ and allow equivalent inequalities e.g. $-\frac{2}{3}>x$

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$

Allow equivalent notation so these are all acceptable:

$x\leqslant"-\frac{2}{3}",\ x<"-\frac{2}{3}",\ \left(-\infty,"-\frac{2}{3}"\right],\ \left(-\infty,"-\frac{2}{3}"\right),\ \left\{x:x\leqslant"-\frac{2}{3}"\right\},\ \left\{x:x<"-\frac{2}{3}"\right\}$

Ignore any reference to values of y.

Allow ft decimal answers from (i) which may be inexact.

Correct answers in part (b) with no working in (a) can score 0011.

(Q01 9MA0/02, June 2023)

Q7.

Question	Scheme	Marks	AOs
(a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{dy}{dx}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\frac{dy}{dx}=\frac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\frac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{((x+1)^2)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$ but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{dy}{dx}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow\frac{dy}{dx}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
			(5 marks)

(**Q03 9MA0/01, June 2019**)

Q8.

Question	Scheme	Marks	AOs
(a)(i)	$y = 4x^3 - 7x^2 + 5x - 10 \Rightarrow \left(\frac{dy}{dx} =\right) 12x^2 - 14x + 5$	M1 A1	1.1b 1.1b
(ii)	$\left(\frac{d^2y}{dx^2} =\right) 24x - 14$	A1ft	1.1b
		(3)	
(b)	$24x - 14 = 0 \Rightarrow x = ...$	M1	1.1b
	$x = \frac{7}{12}$ oe e.g. $x = \frac{14}{24}$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)(i) If "$+ c$" is included with either derivative penalise it only once on the first occurrence.

M1: Award for $x^3 \to x^2$ or $x^2 \to x$ or $5x \to 5$ or $-10 \to 0$

Indices may be unprocessed e.g. $x^3 \to x^{3-1}$ or $x^2 \to x^{2-1}$ or $5x \to 5x^0$

A1: Correct simplified expression with indices processed $12x^2 - 14x + 5$.

Do **not** allow x^1 for x or $5x^0$ for 5.

Apply isw if necessary once a correct answer is seen.

The "$\frac{dy}{dx} =$" is **not** required.

(ii)

A1ft: Correct simplified second derivative $24x - 14$ or follow through their first derivative.

Must be simplified so do **not** allow e.g. x^1 for x or x^0 for 1 as above.

Apply isw if necessary once a correct answer is seen.

The "$\frac{d^2y}{dx^2} =$" is **not** required.

(b)

M1: Sets their **second derivative** of the form $ax + b,\ a, b \neq 0$ equal to 0 and proceeds to a value for x. Condone slips in rearranging as long as a value for x is obtained.

This may be implied by their value of x or may be implied by their working e.g.

$\left(\frac{d^2y}{dx^2} =\right) 24x - 14 \to 24x = 14 \Rightarrow x = ...$

Condone one slip in copying their second derivative.

Also condone if they "cancel" e.g. $\left(\frac{d^2y}{dx^2} =\right) 24x - 14 \to 12x - 7 = 0 \Rightarrow x = ...$

A1: Correct value from correct work and a correct second derivative but allow recovery if they "cancel" their second derivative to obtain e.g. $12x - 7$.

Allow exact equivalents e.g. $\frac{14}{24}$ but not rounded decimals e.g. 0.583

Allow recurring decimal if clearly indicated e.g. $0.58\dot{3}$

Correct answer only from a correct second derivative (or correctly cancelled second derivative) scores both marks.

Isw after a correct answer is seen.

(Q01 9MA0/02, June 2024)

Q9.

Question	Scheme	Marks	AOs
(a)	$x^4 \to \ldots x^3$ or $\ldots x^3 \to \ldots x^2$ or $\ldots x^2 \to \ldots x$ or $ax \to a$	M1	1.1b
	$(\mathrm{f}'(x)=)\ 4x^3+x^2-16x+a$	A1	1.1b
	$\left(\mathrm{f}'\left(-\frac{1}{4}\right)=\right)\ 4\left(-\frac{1}{4}\right)^3+\left(\pm\frac{1}{4}\right)^2-16\left(-\frac{1}{4}\right)+a=0$ $\Rightarrow a=\ldots$	dM1	3.1a
	$a=-4$ *	A1*	2.1
		(4)	
(b)	$\frac{4735}{768}$	B1	1.1b
		(1)	

Notes

(a) **The first two marks are the same for all approaches.**

M1: Reduces the power by one for any term in $\mathrm{f}(x)$. Look for $x^n \to x^{n-1}$ and allow for $ax \to a$ including $ax^1 \to ax^0$ or $-4x \to -4$

A1: Correct differentiation $(\mathrm{f}'(x)=)\ 4x^3+x^2-16x+a$ or $4x^3+x^2-16x-4$

Ignore the absence of the LHS. May be in terms of a or have -4 substituted at this point as above.

Main Scheme:

dM1: Substitutes $x=-\frac{1}{4}$ into their $\mathrm{f}'(x)$, sets $=0$ ($=0$ may be implied) and solves for a.

Condone $4+a=0$ as evidence of substitution for the dM1 only.

This mark is not available if their $\mathrm{f}'(x)$ does not have a term with a in it using this approach.

A1*: cso Requires:

- a correct derivative
- correct substitution of $x=-\frac{1}{4}$ into $\mathrm{f}'(x)$ and set $=0$ (now the $=0$ must be seen somewhere for this mark). Each term does not need to be evaluated.
- achieves $a=-4$

For the A1*, evidence of correct substitution might be $-\frac{1}{16}+\frac{1}{16}+4+a=0$ but not $4+a=0$

Condone invisible brackets around $-\frac{1}{4}$ provided this is recovered before the given answer e.g. $-\frac{1}{4}^2$ recovered to $+\frac{1}{16}$

Alt 1: Verification

dM1: Substitutes $x=-\frac{1}{4}$ and $a=-4$ into their $\mathrm{f}'(x)$, and finds a value.

The $a=-4$ may already have been substituted into f (x) as above.

A1*: cso Requires:

- a correct derivative

- correct substitution of $x=-\frac{1}{4}$ into $f'(x)$ and use of $a=-4$ (at some stage) to achieve $f'\left(-\frac{1}{4}\right)=...=0$. The substitution must be embedded in $f'(x)$ **or** evaluated in an intermediate step e.g. $-\frac{1}{16}+\frac{1}{16}+4-4$ before the $=0$
- a minimal conclusion e.g. "hence proven" or "$\therefore a=-4$" or e.g. a tick.
  but **not** e.g. "max $=-\frac{1}{4}$" or just "$0=0$"

Alt 2: Algebraic division

dM1: Attempts to divide their $f'(x)$ by $(4x+1)$ $\left(\text{or}\left(x+\frac{1}{4}\right)\right)$, achieves a linear remainder in a only, sets $=0$ and achieves a value for a. Condone slips in their calculations.
As a minimum, expect to see $x^2+\lambda x+\mu$ $\left(\text{or } 4x^2+\lambda x+\mu\right)$ as their quotient (with $\mu\neq 0$) leading to a linear remainder in a only set $=0$ leading to a value for a.

A1*: cso Requires:
- a correct derivative
- correct quotient x^2-4 $\left(\text{or}\left(4x^2-16\right)\right)$ and correct remainder $a+4$ set $=0$
- achieves $a=-4$

(b)

B1: $\frac{4735}{768}$ cao Allow exact equivalents e.g. $6\frac{127}{768}$ or the recurring decimal $6.16536458\dot{3}$
There is no need to see a calculation. Decimal approximations e.g. 6.1653645833... score B0.

Question	Scheme	Marks	AOs
(c)	$\left(f'(x)=\right)\ 4x^3+x^2-16x-4=(4x+1)\left(x^2+...\right)$	M1	3.1a
	$=(4x+1)\left(x^2-4\right)$ or e.g. $\dfrac{4x^3+x^2-16x-4}{4x+1}=x^2-4$	A1	1.1b
	$f("-2")=...(-5)$ $\Rightarrow "-5" < k < "\dfrac{4735}{768}"$	M1	2.1
	$\left\{k\in\mathbb{R}: -5 < k < "\dfrac{4735}{768}"\right\}$	A1ft	2.5
		(4)	
			(9 marks)

Notes

(c) **Note: Candidates who do not score the first M mark may score maximum M0A0M1A1ft via the special case at the end of the mark scheme for part (c).**

M1: For the key step in using the factor theorem to take $(4x+1)$ or $\left(x+\dfrac{1}{4}\right)$ out as a factor of their $f'(x)$, **not** $f(x)$. Look for $(4x+1)\left(x^2+\lambda x-4\right)$ or $(4x+1)\left(x^2\pm\mu\right)$ or $\left(x+\dfrac{1}{4}\right)\left(4x^2\pm\mu\right)$ or $\left(x+\dfrac{1}{4}\right)\left(4x^2\pm\lambda x-16\right)$ (where the λ or μ is a constant) either by inspection or division.

If they attempt division in (a) then they must use their quotient in (c) to score the marks.

They may "spot" that one of $f'(\pm 2)=0$ and factor out $(x\pm 2)\left(4x^2+\lambda x\pm 2\right)$ without evidence that $f'(\pm 2)=0$

There must be some factorisation or algebraic division present – not just roots stated from a calculator.

A1: Correct factorisation of the cubic to a linear and quadratic product, so

$(4x+1)(x^2-4)$ or $\left(x+\frac{1}{4}\right)(4x^2-16)$ or $(x-2)(4x^2+9x+2)$ or $(x+2)(4x^2-7x-2)$

or for the correct quadratic seen e.g. x^2-4 or $4x^2-16$ (which may be the quotient in their algebraic division and may come from their work in (a) provided it is used in (c)).
There is no need to complete the long division if the appropriate quotient is found.
Note that proceeding directly to $(4x+1)(x-2)(x+2)$ does not score either mark without sight of the e.g. $(4x+1)(x^2-4)$ but they can score the final 2 marks via the special case.

Similarly, proceeding directly to $\left(x+\frac{1}{4}\right)(2x-4)(2x+4)$ or $4\left(x+\frac{1}{4}\right)(x-2)(x+2)$ does not score either mark without sight of $\left(x+\frac{1}{4}\right)(4x^2-16)$ but they can score marks via the SC.

M1: Solves their quadratic factor = 0, substitutes one of these solutions (not $x=-\frac{1}{4}$) into $f(x)$ to find a value **and** selects the inside region between this and their answer to (b).
If their quadratic is of the form Ax^2-B then they can write down their solution for x but if it is a 3TQ then they must find their value of x using the usual non-calculator rules.
Condone "incorrect" factorisation following the correct quadratic leading to $x=\pm 2$

e.g. $\left(x+\frac{1}{4}\right)(4x^2-16)\rightarrow\left(x+\frac{1}{4}\right)(x-2)(x+2)=0\rightarrow x=\pm 2$

y (or k) $=-5$ (or $-\frac{47}{3}$) with nothing else can imply substitution of $x=-2$ (or 2)

Note that if they "spot" that $f'(\pm 2)=0$ earlier then they can go straight to substitution.

Condone the use of y but not x in their region. Condone e.g. $k>-5, k<"\frac{4735}{768}"$ for this mark.

Condone the use of $\le$ in place of $<$ for this mark. Note that $\left(2,-\frac{47}{3}\right)$ is the other minimum.

A1ft:	$\left\{k : -5 < k < "\frac{4735}{768}"\right\}$ Requires correct use of set notation but condone absence of $\in \mathbb{R}$ Follow through on their upper limit from (b) which may be a decimal but must be positive. Must be strict inequalities i.e. not include the end points and must now be in terms of k. Other acceptable notation includes: $k \in \left(-5, "\frac{4735}{768}"\right)$, $\{k, k > -5\} \cap \left\{k, k < "\frac{4735}{768}"\right\}$ Condone $\left\{k : k > -5 \cap k < "\frac{4735}{768}"\right\}$ or $\left\{-5 < k < "\frac{4735}{768}"\right\}$ Do not allow e.g. $\{k : k > -5\} \cup \left\{k : k < "\frac{4735}{768}"\right\}$ or just $-5 < k < "\frac{4735}{768}"$
SC:	Candidates that have not scored the first M1 may go on to score the final two marks by finding an acceptable range between -5 and $"\frac{4735}{768}"$. Use of their "-5" or "$-\frac{47}{3}$", coming from $x = \pm 2$, (or -5 or $-\frac{47}{3}$ directly from a calculator) with $"\frac{4735}{768}"$ will score M1 if the inside region is selected. A correct $\left\{k : -5 < k < "\frac{4735}{768}"\right\}$ will score A1ft There is no need to see any working to achieve the -5 (or $-\frac{47}{3}$). All the relevant guidance regarding the region in the M1 and A1ft notes above continues to apply. e.g. for the M1 we will condone the use of y but not x in their region while for the A1ft we require use of k. For example: • $\left\{k : -5 < k < "\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A1ft • $"-5" < k < "\frac{4735}{768}"$ without scoring the first M scores M0A0M1A0ft • $\left\{k : "-\frac{47}{3}" < k < "\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A0ft

(Q08 9MA0/02, June 2025)

Q10.

Question	Scheme	Marks	AOs
(a)	$\frac{1}{2}r^2\theta + \frac{1}{10}r^2 = 240 \Rightarrow r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r}$ or $\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r}$ into $(P =)\ r\theta + 2r + \frac{1}{5}r$	dM1	3.4
	$P = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} + 2r + \frac{1}{5}r = \frac{480}{r} - \frac{1}{5}r + 2r + \frac{1}{5}r = 2r + \frac{480}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dP}{dr} =\right) 2 - \frac{480}{r^2}$	M1	1.1b
	Sets $\frac{dP}{dr} = 0 \Rightarrow r^2 = 240$ $r =$ awrt 15.5	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{d^2P}{dr^2} =\right) \frac{960}{r^3}$	M1	1.1b
	$\left(\frac{d^2P}{dr^2} =\right)$ awrt $0.26 > 0$ proving a minimum value of P	A1	1.1b
		(2)	
			(9 marks)

Notes:

(a) **Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks.**

M1: Uses area formulae to form an equation of the form $\alpha r^2\theta + \beta r^2 = 240$ o.e. ($\alpha, \beta \neq 0$) and rearranges to make $r\theta$, θ or $r\theta + \frac{1}{5}r$ the subject. Look for:

$r\theta = \frac{M \pm Nr^2}{r}\left(= \frac{M}{r} \pm Nr\right)$ o.e. or $\theta = \frac{M \pm Nr^2}{r^2}\left(= \frac{M}{r^2} \pm N\right)$ o.e. where $M, N \neq 0$

or $r\theta + \frac{1}{5}r = \frac{L}{r}$ $L \neq 0$ o.e. May work in degrees.

A1: A correct rearrangement for θ or $r\theta$ or $r\theta + \frac{1}{5}r$ which may be unsimplified (may be in degrees)

$r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r}$ o.e. e.g. $r\theta = \frac{2400 - r^2}{5r}$ or $r\theta = \frac{480 - 0.2r^2}{r}$

or $r\theta + \frac{1}{5}r = \frac{480}{r}$ o.e.

or $\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2}$ o.e. e.g. $\theta = \frac{2400 - r^2}{5r^2}$ or $\theta = \frac{480}{r^2} - \frac{1}{5}$ or $\theta = 480r^{-2} - 0.2$

dM1: Substitutes their $r\theta = \frac{M \pm Nr^2}{r}$ o.e. or $\theta = \frac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \frac{1}{5}r = \frac{L}{r}$ into an expression of the form $(P =)\ r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \frac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ, $r\theta$ or $r\theta + \frac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \frac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
$P =$, Perimeter = must be seen at least once in their solution in the correct place.

(b)	**Mark (b) and (c) together. There is no requirement to see the notation $\frac{dP}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$. Allow use of e.g. P' or e.g. y'**
M1:	$\left(\frac{dP}{dr}=\right)p\pm\frac{q}{r^2}$ where p and q are non-zero constants
dM1:	Sets or implies that their $\frac{dP}{dr}=0$ and proceeds to $mr^{\pm 2}=n,\quad m\times n>0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p-\frac{q}{r^2}=0$. You may need to check this on your calculator.
A1:	$r=$ awrt 15.5 or $\sqrt{240}\ \left(=4\sqrt{15}\right)$ Do not accept $\pm$ (ignore any units if given)
(c)	**Condone other letters used instead of *P* and *r* for $\frac{d^2P}{dr^2}$ e.g. $\frac{d^2y}{dx^2}$ for M1 only. Just using $\frac{dP}{dr}$ and considering a sign change is M0A0**
M1:	Differentiates and finds $\left(\frac{d^2P}{dr^2}=\right)\pm\frac{f}{r^3}$ (do not be concerned about the sign)
A1:	**Note if they score A0 in (b) then this mark cannot be scored.** Requires • a correct a correct expression for $\frac{d^2P}{dr^2}$ • a correct value for $\left(\frac{d^2P}{dr^2}=\right)\frac{960}{r^3}=$ awrt 0.26 using awrt 15.5 (but allow 0.23(43..) if using 16) • a correct comparison with 0 **and** a conclusion e.g. minimum **The expression for the second derivative does not need to be labelled but if it is then it must be $\frac{d^2P}{dr^2}$ o.e. or accept e.g. P'' BUT $\frac{d^2y}{dx^2}$ used in their conclusion is A0**

(Q15 9MA0/01, June 2025)

Q11.

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
		(7 marks)	

(a)

M1: Attempts to differentiate $x = 4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy} = k\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$ or $1 = k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = ...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8\sin y\cos y \Rightarrow \frac{dx}{dy} = \pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy} = 8\cos 2x \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x = 8y$ oe such as $x = 4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x = 4\sin 2y \rightarrow y = \ldots\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx} = \frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

Q12.

Question	Scheme	Marks	AOs
(a)(i)	$(f'(x)=)8xe^{4x^2-1}$ or e.g. $\frac{8xe^{4x^2}}{e}$ oe	B1	1.1b
(ii)	$(g'(x)=)\frac{8}{x}$ or e.g. $8x^{-1}$ oe	B1	1.2
		(2)	
(b)	$8xe^{4x^2-1}=\frac{8}{x} \Rightarrow e^{4x^2-1}=\frac{1}{x^2} \Rightarrow 4x^2-1=\ln\frac{1}{x^2}$	M1	1.1b
	$4x^2-1=\ln\frac{1}{x^2} \Rightarrow 4x^2-1=-2\ln x$ $\Rightarrow 4x^2+2\ln x-1=0$*	A1*	2.1
		(2)	
(c)(i)	$x_1=0.6 \Rightarrow x_2=\sqrt{\frac{1-2\ln 0.6}{4}}$	M1	1.1b
	$(x_2=)0.7109$	A1	1.1b
(ii)	$(\alpha=)0.6706$	B1 (A1 In ePEN)	1.1b
		(3)	
			(7 marks)

(a)(i)

B1: Correct derivative in any form. "$f'(x)=$" is not required. Apply isw if necessary.

(ii)

B1: Correct derivative in any form. "$g'(x)=$" is not required. Apply isw if necessary.

(b)

M1: Eliminates e by setting their $f'(x)=$ their $g'(x)$ where $f'(x)=Axe^{4x^2-1}$ oe and $g'(x)=\frac{B}{x}$ oe with $A\times B>0$ and proceeds via $e^{4x^2-1}=\frac{...}{x^2}$ or equivalent work (see below) to obtain $4x^2-1=\ln\frac{...}{x^2}$ oe e.g. $\ln x+4x^2-1=\ln\frac{1}{x}$

Allow if they use α for x.

Note that there are various alternatives for this mark but the derivatives must be of the form defined above and the processing must be correct with coefficient/sign slips only.

<u>**Examples of equivalent work:**</u>

$$8xe^{4x^2-1}=\frac{8}{x} \Rightarrow x^2e^{4x^2-1}=1 \Rightarrow \ln x^2+\ln e^{4x^2-1}=0 \Rightarrow \ln e^{4x^2-1}=-\ln x^2 \Rightarrow 4x^2-1=-2\ln x$$

$$\frac{8xe^{4x^2}}{e}=\frac{8}{x} \Rightarrow \frac{1}{e}e^{4x^2}=\frac{1}{x^2} \Rightarrow e^{4x^2}=\frac{e}{x^2} \Rightarrow \ln e^{4x^2}=\ln\frac{e}{x^2} \Rightarrow 4x^2=\ln\frac{e}{x^2}=1-2\ln x$$

A1*: Obtains the printed answer with sufficient working and no errors.
Sufficient work would require the "e" eliminated before the given answer.
Must follow correct derivatives in part (a).
Condone $4x^2+2\ln|x|-1=0$ and condone $4\alpha^2+2\ln\alpha-1=0$ or $4\alpha^2+2\ln|\alpha|-1=0$

Note that if both derivatives in (a) **are correct** we will allow fully correct work using the equation in (b) to work backwards to verify that $p\mathrm{f}'(x) = q\mathrm{g}'(x)$ for M1 then obtains $\mathrm{f}'(x) = \mathrm{g}'(x)$ with a minimal conclusion for A1

If either derivative in (a) is incorrect or missing, candidates who work backwards score no marks in (b).

(c)(i)/(ii)

M1: Attempts to use the iterative formula with $x_1 = 0.6$

Award this mark for e.g. $(x_2 =)\sqrt{\dfrac{1-2\ln 0.6}{4}}$ or may be implied by awrt 0.71 provided no incorrect working is seen.

Candidates sometimes find x_3 (or possibly subsequent terms) rather than x_2 in which case the M1 can be implied. (See table below for first few iterations)

A1: $(x_2 =)$ awrt 0.7109

Sight of $(x_2 =)$ awrt 0.7109 scores M1A1

B1(A1 on ePEN): $(\alpha =)$ 0.6706 (4dp)

Must be this value and **not** awrt 0.6706

For reference:

x_1	0.6
x_2	0.7109239143
x_3	0.6485329086
x_4	0.6830236199
x_5	0.6637868021
x_6	0.6744606223
. . .	. . .
α	0.6706416243

(**Q06 9MA0/02, June 2024**)

Q13.

Question	Scheme	Marks	AOs
(a)	$0 = 15T - T\,\mathrm{e}^{0.2T} \Rightarrow \mathrm{e}^{0.2T} = 15$	M1	3.4
	$(T =)$ awrt 13.5	A1	1.1b
		(2)	
(b)	Attempts to differentiate using the product rule $\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(t\,\mathrm{e}^{0.2t}\right)=\right)\ 0.2t\,\mathrm{e}^{0.2t} + \mathrm{e}^{0.2t}$	M1	1.1b
	$v = 15t - t\,\mathrm{e}^{0.2t} \Rightarrow \left(\frac{\mathrm{d}v}{\mathrm{d}t} =\right) 15 - \left(0.2t\,\mathrm{e}^{0.2t} + \mathrm{e}^{0.2t}\right)$	A1	1.1b
	Sets their $\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow \mathrm{e}^{0.2t}(0.2t+1) = 15 \Rightarrow \mathrm{e}^{0.2t} = \frac{15}{0.2t+1}$	dM1	3.1b
	$\Rightarrow t = 5\ln\left(\frac{75}{t+5}\right)$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = 5\ln\left(\frac{75}{8+5}\right)$	M1	1.1b
	awrt 8.478	A1	1.1b
	(ii) awrt 8.55 seconds **(including units)**	A1	3.2a
		(3)	
			(9 marks)

Notes:

(a) May use t or another variable which is acceptable.

M1: Uses the model with $v = 0$ and proceeds to $\mathrm{e}^{0.2T} = 15$ Do not be concerned by the use of an inequality sign instead of an equals.
May be implied by awrt 13.5

A1: $(T =)$ awrt 13.5 units are not required but if given they must be seconds (or e.g. secs or s)

(b) If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b)

M1: Attempts to use the product rule to differentiate $t\,e^{0.2t}$ achieving the form $Ae^{0.2t}+Bte^{0.2t}$ (A and B both non zero but may be 1) which may be unsimplified. It is likely to be part of an expression.

A1: $\left(\frac{dv}{dt}=\right)15-\left(0.2t\,e^{0.2t}+e^{0.2t}\right)$ o.e. which may be unsimplified. (Condone a missing trailing bracket)
Do not allow recovery of signs to score this mark if they initially write
e.g. $15-0.2t\,e^{0.2t}+e^{0.2t}=0$ and on a later line correct this. e.g. $15-0.2t\,e^{0.2t}-e^{0.2t}=0$

dM1: Sets $15\pm Ae^{0.2t}\pm Bte^{0.2t}=0$ (the =0 may be implied), attempts to make $e^{\pm 0.2t}$ (or $Ce^{\pm 0.2t}$) the subject **and** proceeds to the form
$Ce^{0.2t}=\frac{D}{E+Ft}$ or $Ce^{-0.2t}=\frac{E+Ft}{D}$ (where C can be 1 and $A, B, C, D, E, F\neq 0$)
It is dependent on the previous method mark.
May see $15-"0.2t\,e^{0.2t}-e^{0.2t}"=0\Rightarrow e^{0.2t}=\frac{15}{0.2t+1}$ which scores dM1
They must take out a factor of $e^{\pm 0.2t}$ (or $Ce^{\pm 0.2t}$) and divide by their bracket. Condone sign slips in their rearrangement, however, if they take logs of both sides first, the rearrangement must be correct (with no sign slips).
Allow invisible brackets to be implied by further work which is not the given answer.

A1*: Achieves the given answer with **no errors seen including use of invisible brackets** (but condone a missing trailing bracket) All previous marks in (b) must have been scored. The = 0 must have been seen somewhere in their solution. Do not allow this mark to be scored for proceeding directly from
$e^{0.2t}(0.2t+1)=15\Rightarrow t=5\ln\left(\frac{75}{t+5}\right)$ which is A0*
We must see either $e^{0.2t}=\frac{15}{0.2t+1}$ o.e. or an unsimplified expression for t
e.g. $t=5\ln\left(\frac{15}{0.2t+1}\right)$ before achieving the given answer. Condone $t=5\ln\frac{75}{t+5}$

(c) (i) Check by the question. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

M1: Attempts to use the iteration formula at least once. $t_2=5\ln\left(\frac{75}{8+5}\right)$. May be implied by awrt 8.76 or awrt 8.48 or awrt 8.58. It is not implied by awrt 8.55

A1: awrt 8.478 (on its own can score M1A1)

(c)(ii) This mark can only be scored provided in (c)(i) M1 has been scored so M0A0A1 is not a possible mark profile.

A1: awrt 8.55 **seconds** (e.g. s or secs) **Requires units.**
If the candidate lists their iterations but does not select an answer then take the final value, which still requires units to be stated (which in many cases is likely to be omitted)
Note that awrt 8.55 does not imply M1.

Q14.

Question	Scheme	Marks	AOs
(a)	Uses $t=0, V=20\,000 \Rightarrow 20\,000 = 1500 + A\mathrm{e}^{0}$	M1	3.4
	$A = 18500$	A1	1.1b
	$t=2.5, V=12\,000 \Rightarrow 12\,000 = 1500 + 18\,500\mathrm{e}^{-k\times 2.5}$ $\Rightarrow 10500 = 18500\mathrm{e}^{-k\times 2.5} \Rightarrow k = ...\left(= -\frac{2}{5}\ln\frac{21}{37} = \text{awrt } 0.227\right)$	dM1	3.1b
	$V = 1500 + 18500\,\mathrm{e}^{-0.227t}$	A1	3.3
		(4)	
(b)	Achieves $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -kA\mathrm{e}^{-kt}$ or $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	B1	1.1b
	Substitutes $A\mathrm{e}^{-kt} = V - 1500$ into $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -kA\mathrm{e}^{-kt}$ or $"18500"\mathrm{e}^{"-0.227"t} = V - 1500$ into $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	M1	3.4
	Rate of change in value of car is $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -k(V-1500)$ or $"-0.227"(V-1500)$ *	A1*	2.1
		(3)	
(c)	Suggests a suitable limitation of the model (see notes)	B1	3.5b
		(1)	
			(8 marks)

Notes:

Mark (a) and (b) together

(a)

M1: Uses the equation of the model with $t = 0, V = 20\,000 \Rightarrow 20000 = 1500 + Ae^{0}$ o.e.
May be implied by 18500

A1: $A = 18500$ (18500 with no working seen scores M1A1) Ignore £ if present.

dM1: Attempts to use the equation of the model $t = 2.5, V = 12\,000$
$\Rightarrow 12000 = 1500 + "18500"e^{-2.5k}$ and proceeds to $Ce^{\pm k\times 2.5} = D$ (where $C \times D > 0$ and allow $C = 1$) **before proceeding** to find a value for k. Allow them to have a non-numerical C for this mark.
Note it cannot be implied by their awrt ± 0.227 so
$12000 = 1500 + "18500"e^{-2.5k} \Rightarrow k = 0.227$ scores dM0A0 as we need to see the intermediate stage $Ce^{\pm k\times 2.5} = D$. (typically look for $18\,500e^{-k\times 2.5} = 10\,500 \Rightarrow k = ...$ or condone to be implied by a correct expression involving logarithms for their A)
It is dependent on the previous method mark.

A1: $V = 1500 + 18500\,e^{\text{awrt}-0.227t}$ o.e. e.g. $t = \dfrac{\ln\left(\dfrac{V-1500}{18500}\right)}{\text{awrt}-0.227}$ Allow k to be exact.

(b)

B1: Differentiates to a form $\left(\dfrac{dV}{dt} =\right) -kAe^{-kt}$ where k and A may be their values from (a)
e.g. $"-0.227"\times"18500"e^{"-0.227"t}$ May just see e.g. (using an exact k) $"-4191.3..."e^{"-0.227"t}$ or e.g. (using k to 3sf) $"-4199.5"e^{"-0.227"t}$ but do not be too concerned by over rounding provided the intention is clear that it is their $-kA$
Do not be too concerned by the left hand side / poor labelling of the derivative.

M1: Substitutes $Ae^{-kt} = V - 1500$ into their $\left(\dfrac{dV}{dt} =\right) \pm kAe^{-kt}$ to form an expression for $\dfrac{dV}{dt}$ in terms of V. May see e.g. $"-0.227"\times"18500"e^{"-0.227"t} \Rightarrow "-0.227"(V-1500)$.
Condone recovery of a sign slip on the index on $e^{-kt} \to e^{kt}$ provided it is before they substitute $Ae^{-kt} = V - 1500$ for this mark.
Using the given answer: substitutes $V - 1500 = Ae^{-kt}$ and shows that $-k(V-1500) = -kAe^{-kt}$ (may be in terms of their A and k)

A1*: Full and complete proof with sight of $\dfrac{dV}{dt}$ oe seen somewhere in their solution and no errors seen. Must see $-kAe^{-kt}$ (or using their values) before proceeding to the given answer which may be written using their numerical value for k
Using the given answer they must conclude that $\dfrac{dV}{dt} = -k(V-1500)$ which may be written using their numerical value for k

---

Alt (b) Separating the variables – may be in terms of their numerical value for k

B1ft: Separates the variables correctly and integrates to $\ln(V-1500) = -kt$ with or without $+c$

$$\int \frac{1}{(V-1500)} \mathrm{d}V = \int -k\,\mathrm{d}t \Rightarrow \ln(V-1500) = -kt \ \ (+c) \ \ \text{oe}$$

M1: Proceeds from $...\ln(V-1500) = ...kt + c$ o.e. and rearranges to make V the subject. Condone slips.

e.g. $\ln(V-1500) = -kt + c \Rightarrow V - 1500 = Ae^{-kt} \Rightarrow V = 1500 + Ae^{-kt}$ or $V = 1500 + e^{-kt+c}$
(just look for proceeding to V=… for this mark though.) May be in terms of their numerical values for k and A

A1*: Achieves $V = 1500 + Ae^{-kt}$ with no errors seen and concludes that $\frac{\mathrm{d}V}{\mathrm{d}t} = -k(V-1500)$

May be in terms of their numerical values for k and A

Alt (b) Rearranging to make t the subject

B1ft: For a correct rearrangement to $t = -\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)$ ft on their k

M1: Differentiates $...\ln(V-1500)$ to $\frac{...}{V-1500}$ and then finding $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{\left(\frac{\mathrm{d}t}{\mathrm{d}V}\right)}$

Typically look for $t = -\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right) \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}V} = -\frac{1}{k}\left(\frac{1}{V-1500}\right)$ o.e. so they may have an unsimplified version of this e.g.

$$t = -\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right) \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}V} = -\frac{1}{k}\left(\frac{\frac{1}{18500}}{\frac{V-1500}{18500}}\right) \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = -k\left(\frac{\frac{V-1500}{18500}}{\frac{1}{18500}}\right)$$

May be in terms of their numerical k. Condone slips.

A1*: Achieves the given answer with no errors and $\frac{\mathrm{d}V}{\mathrm{d}t}$ seen somewhere in their solution

(c)	**Note the question asks for a limitation of the model. If there is ambiguity over whether the response is referring to the model then try putting "the model suggests" or "the model" in front of their comment to see if this is a valid limitation.** **Ignore comments which do not contradict a valid limitation.** **If values are given then they must be correct and if it is the value of the car then it must have units (£ or pounds)**
B1:	**Suitable limitations in context referring to the limitation of the model which score B1** e.g. (the model suggests) the value/price of the car will never go below £1500 e.g. (the model suggests) the rate of decrease of the value of the car is proportionally the same each year e.g. (the model suggests) after a certain period of time the car will no longer lose value (condone this property of the model that it will tend to a limit) e.g. (the model) only takes account of age e.g. (the model) does not take into account damage / alteration to the car/ mileage e.g. (the model) predicts that the car's value will always go down e.g. after many years the car may become worthless whereas the model does not allow for this (valid limitation comparing the car to the model) e.g. the value of the car may go up where as it only decreases according to the model **Do not accept vague/incorrect/irrelevant or non-contextual comments which score B0** e.g. after many years the car may become worthless e.g. the value of the car may increase e.g. damage or alterations to the car may impact the value e.g. the car will still have value when it is very old (should refer to £1500) e.g. (the model suggests) the minimum value is 1500 (no units for money) e.g. (the model suggests) the (value of the) car cannot be negative e.g. car value will fluctuate which the model will not show (a model is not for this purpose) e.g. the rate of decrease is proportionally the same each year (no context)

(**Q11 9MA0/01, June 2025**)

Q15.

Question	Scheme		Marks	AOs
(a)	$\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1	3.3
	$\int r^2\,dr = \int \pm k\,dt \Rightarrow ...$ (for k or a numerical k)		M1	2.1
	$\frac{1}{3}r^3 = \pm kt \; \{+c\}$		A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
			(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$		M1	3.4
	time = 5 minutes 6 seconds		A1	1.1b
			(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1	3.5b
			(1)	
				(8 marks)

Notes for Question	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{dr}{dt} = f(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2\, dr = \int \pm k\, dt$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,dt \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t = 0, r = 5$ and $t = 4, r = 3$, or • $t = 0, r = 5$ and $t = 240, r = 3$, *on their integrated equation* to find their constants k and c *and* obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250-49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250-2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000-49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000-120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1

(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = ...$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

(**Q10 9MA0/02, June 2018**)

Q16.

Question	Scheme	Marks	AOs
	$N=\dfrac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0;\quad \dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}\right)-3\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS}=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e., $\text{RHS}=\dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2}=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7e^{-0.25T}}\Rightarrow e^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question	
(b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{dN}{dt} = \ldots$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = \ldots$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = \ldots$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
	$N=\frac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0;\ \ \frac{dN}{dt}=\frac{N(300-N)}{1200}$		
(b) Way 3	$\int\frac{1}{N(300-N)}\,dN=\int\frac{1}{1200}\,dt$	M1	2.1
	$\int\frac{1}{300}\left(\frac{1}{N}+\frac{1}{300-N}\right)dN=\int\frac{1}{1200}\,dt$ $\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t\ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90\Rightarrow\}\ c=\frac{1}{300}\ln(90)-\frac{1}{300}\ln(210)\Rightarrow c=\frac{1}{300}\ln\left(\frac{3}{7}\right)$ $\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t+\frac{1}{300}\ln\left(\frac{3}{7}\right)$ $\ln N-\ln(300-N)=\frac{1}{4}t+\ln\left(\frac{3}{7}\right)$ $\ln\left(\frac{N}{300-N}\right)=\frac{1}{4}t+\ln\left(\frac{3}{7}\right)\Rightarrow\frac{N}{300-N}=\frac{3}{7}e^{\frac{1}{4}t}$	dM1	2.1
	$7N=3e^{\frac{1}{4}t}(300-N)\Rightarrow 7N+3Ne^{\frac{1}{4}t}=900e^{\frac{1}{4}t}$ $N(7+3e^{\frac{1}{4}t})=900e^{\frac{1}{4}t}\Rightarrow N=\frac{900e^{\frac{1}{4}t}}{7+3e^{\frac{1}{4}t}}\Rightarrow N=\frac{900}{3+7e^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7e^{-0.25t})=900\Rightarrow e^{-0.25t}=\frac{1}{7}\left(\frac{900}{N}-3\right)\Rightarrow e^{-0.25t}=\frac{900-3N}{7N}$	M1	2.1
	$\Rightarrow t=-4(\ln(900-3N)-\ln(7N))$ $\Rightarrow\frac{dt}{dN}=-4\left(\frac{-3}{900-3N}-\frac{7}{7N}\right)$	A1	1.1b
	$\frac{dt}{dN}=4\left(\frac{1}{300-N}+\frac{1}{N}\right)\Rightarrow\frac{dt}{dN}=4\left(\frac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\frac{dt}{dN}=\left(\frac{1200}{N(300-N)}\right)\Rightarrow\frac{dN}{dt}=\frac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give ln terms $=kt\ \{+c\},\ k\neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t\ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0,\ N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t}=f(N);\ \lambda\neq 0$ or $\lambda e^{-\frac{1}{4}t}=f(N);\ \lambda\neq 0$
A1*:	Correct manipulation leading to $N=\frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN}=-4\left(\frac{-3}{900-3N}-\frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt}=\frac{N(300-N)}{1200}$ *

Q17.

Question	Scheme	Marks	AOs
	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right)$ $\{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e}, m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}$ $\frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question	
M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\text{e}, \text{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \text{e} = m_N(x - \text{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\text{e}$, allowing un-simplified values for x such as $x = 2\text{e} + \text{e}\ln \text{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\text{e}} x\ln x\,dx = [\ ...\ ]_1^{\text{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • or Area under line $= \frac{1}{2}((\text{their } x_A) - \text{e})\text{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2 \ln x - \int B\left(\frac{x^2}{x}\right)\{dx\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\text{e}^2 + \frac{1}{4}$
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) dx = [\ ...\ ]_{\text{e}}^{\text{their } x_A} = ...$
Note:	**Calculator approach with no algebra, differentiation or integration seen:** • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

(**Q13 9MA0/02, June 2018**)

Q18.

Question	Scheme	Marks	AOs
	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\frac{B}{(x-3)}+\frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\text{f}(x)=3+\frac{4}{(x-3)}-\frac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\text{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\frac{4}{(x-3)^2}-\frac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $\text{f}'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\text{f}'(x)=-(+\text{ ve})-(+\text{ ve})<0$, so $\text{f}(x)$ is a decreasing function	A1	2.4
		(3)	
			(7 marks)

Notes for Question	
(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for B and C. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for B and C
B1:	$A=3$
M1:	Attempts to find the value of either B or C from their identity This can be achieved by *either* substituting values into their identity *or* by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: -6=-2A;\quad x: 11=7A-2B+C;\quad \text{constant}: 1=-3A+B-3C$ **Way 1:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: -10=-2B+C;\quad \text{constant}: 10=B-3C$ **Way 2:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4,\ C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	**Way 1:** You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \dfrac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	**Way 2:** You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\dfrac{-10x+10}{(x-3)(1-2x)} \equiv \dfrac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{f'(x)=\}\ \pm\lambda(x-3)^{-2} \pm \mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $f'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2}$; $(\text{their } B), (\text{their } C) \neq 0$
A1:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\dfrac{4}{(x-3)^2} - \dfrac{4}{(1-2x)^2}$ **and** a correct explanation e.g. $f'(x) = -(+\text{ve}) - (+\text{ve}) < 0$, so $f(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A = ...$ or from $A = 0$ or no value of A found in part (a)

Notes for Question Continued - Alternatives			
(a)			
Note:	Be aware of the following alternative solutions, by initially dividing by "$(x-3)$" or "$(1-2x)$" • $\dfrac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \dfrac{-6x-7}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)} \equiv 3 - \dfrac{10}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)}$ $\dfrac{20}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x)+E(x-3) \Rightarrow D=-4,\ E=-8$ $\Rightarrow 3 - \dfrac{10}{(1-2x)} - \left(\dfrac{-4}{(x-3)} + \dfrac{-8}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$ • $\dfrac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \dfrac{3x-4}{(x-3)} + \dfrac{5}{(x-3)(1-2x)} \equiv 3 + \dfrac{5}{(x-3)} + \dfrac{5}{(x-3)(1-2x)}$ $\dfrac{5}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x)+E(x-3) \Rightarrow D=-1,\ E=-2$ $\Rightarrow 3 + \dfrac{5}{(x-3)} + \left(\dfrac{-1}{(x-3)} + \dfrac{-2}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$		
(b)			
	Alternative Method 1:		
	$f(x) = \dfrac{1+11x-6x^2}{(x-3)(1-2x)},\ x>3 \Rightarrow f(x) = \dfrac{1+11x-6x^2}{-2x^2+7x-3};\ \begin{cases} u = 1+11x-6x^2 & v = -2x^2+7x-3 \\ u' = 11-12x & v' = -4x+7 \end{cases}$		
	$f'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	Uses quotient rule to find $f'(x)$	M1
		Correct differentiation	A1
	$f'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $f'(x) = -\dfrac{(+\text{ve})}{(+\text{ve})} < 0$, so $f(x)$ is a decreasing {function}		A1
	Alternative Method 2:		
	Allow M1A1A1 for the following solution: Given $f(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$ as $\dfrac{4}{(x-3)}$ decreases when $x>3$ and $\dfrac{2}{(2x-1)}$ decreases when $x>3$ then $f(x)$ is a decreasing {function}		

(**Q11 9MA0/02, June 2018**)

Q19.

Question	Scheme		Marks	AOs
(a)	$\frac{dV}{dt}=0.45$ or $\frac{dV}{dt}=\pm 0.3V$		M1	3.1b
	$\frac{dV}{dt}=0.45-\frac{3}{10}V$ $20\frac{dV}{dt}=9-6V$ *		A1*	2.1
			(2)	
(b)	e.g. $\frac{1}{9-6V}\frac{dV}{dt}=\frac{1}{20}\rightarrow\int\frac{1}{9-6V}dV=\int\frac{1}{20}dt$		B1	1.1b
	$\frac{1}{9-6V}\rightarrow\ldots\ln\lvert 9-6V\rvert$		M1	1.1b
	$-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}\quad(+c)$		A1	1.1b
	$-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}+c$ $9-6V=Ae^{-\frac{3t}{10}}$ $t=0,\ V=0.25\Rightarrow A=(7.5)$	$-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}+c$ $t=0,\ V=0.25\Rightarrow c=\left(-\frac{1}{6}\ln 7.5\right)$	dM1	3.1a
	$9-6V=7.5e^{-\frac{3t}{10}}$ $V=\frac{3}{2}-\frac{5}{4}e^{-\frac{3t}{10}}$	$\frac{1}{6}\ln 7.5-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}$ $\ln\frac{7.5}{9-6V}=0.3t$ $9-6V=7.5e^{-0.3t}$ $V=\frac{3}{2}-\frac{5}{4}e^{-0.3t}$	A1	2.1
			(5)	

Notes	
(a)	**Marks for part (a) may not be scored in part (b)**
M1:	Either $\frac{dV}{dt}=0.45$ or $\frac{dV}{dt}=\pm 0.3V$ o.e. e.g. $\frac{dV}{dt}=\frac{9}{20}$ or $\frac{dV}{dt}=\pm\frac{3}{10}V$ seen or implied by e.g. $\frac{dV}{dt}=0.45-\frac{3}{10}V$ (but not implied by just stating the given answer). Condone use of $\dot{V}$ It may be seen as part of their $\frac{dV}{dt}$ e.g. $\frac{dV}{dt}=0.45+V+0.3V$ scores M1A0* Condone e.g. change in volume = (inflow – outflow =) $0.45-0.3V$ for this mark.
A1*:	Achieves $20\frac{dV}{dt}=9-6V$ with no errors, following $\frac{dV}{dt}=0.45-\frac{3}{10}V$ o.e. (including the $\frac{dV}{dt}$ or $\dot{V}$ but note that it must be $\frac{dV}{dt}$ in the final line and not $\dot{V}$). change in volume $=0.45-0.3V \rightarrow 20\frac{dV}{dt}=9-6V$ scores M1A0*. Ignore any units used in their working for both marks.
(b)	
B1:	Separates the variables correctly, e.g., $\int \frac{1}{9-6V}dV=\int \frac{1}{20}dt$ or $\int \frac{20}{9-6V}dV=\int \{1\}\,dt$ o.e. The integral symbol and/or dV and/or dt may be implied if they go on to integrate **both** sides to the correct form $...\ln\left\|\alpha(9-6V)\right\|=...t \quad (+c)$ with or without the modulus brackets.
M1:	Attempts to integrate the reciprocal term $\frac{\beta}{9-6V}\rightarrow ...\ln\left\|9-6V\right\|$ or $\rightarrow ...\ln\left\|\alpha(6V-9)\right\|$ for some constant β (and α if used). Condone e.g. $\frac{20}{9-6V}\rightarrow ...\ln 9-6V$ or $\rightarrow ...\ln 6V-9$
A1:	Correct integration for both sides. They do not need the $+c$ for this mark. Note scoring this mark implies the earlier B1 (unless it is a verification attempt – see SC). Note that e.g. $-\frac{1}{6}\ln\left\|3-2V\right\|=\frac{t}{20} \quad (+c)$ or $-\frac{10}{3}\ln\left\|2V-3\right\|=t \quad (+c)$ are also correct. $-\frac{1}{6}\ln(9-6V)=\frac{t}{20} \quad (+c)$ is also correct. Condone log being used in place of ln.
dM1:	Requires constant of integration now. Substitutes (or states) $t=0$ and $V=0.25$ and finds a value for c, which may be "A" = e^c if they rearrange first to eliminate ln terms. Dependent on the previous method mark. Do not be concerned about their processing to find c or "A" = e^c and does not need to be exact.

A1:	Achieves the required form e.g. $V=\frac{3}{2}-\frac{5}{4}e^{-\frac{3t}{10}}$ with no errors and clear working. Allow equivalent fractions or decimals e.g. $V=1.5-1.25e^{-\frac{6}{20}t}$
SC:	**Attempts by verification may score maximum B0M1A1dM1A0 – see below.**
Alt:	**Use of an integrating factor – see below.**
(b)	**Special Case: Attempts by verification may score maximum B0M1A1dM1A0**
B0:	This mark may not be scored via this approach.
M1:	Differentiates $V=P-Qe^{-kt}$ to the form $\frac{dV}{dt}=\alpha e^{-kt}$ where α is a constant (note it should be $\frac{dV}{dt}=Qke^{-kt}$) **and** substitutes both this and $V=P-Qe^{-kt}$ into $20\frac{dV}{dt}=9-6V$ **and** deduces a value for P **or** k by comparing coefficients.
A1:	Correct values for both P **and** k.
dM1:	Substitutes (or states) $t=0$ and $V=0.25$ and finds a value for Q. Requires a value for P to have been found using the above approach.
A0:	This mark may not be scored via this approach.
	Alternative: Using Integrating Factor (Further Maths)
B1:	Deduces the correct integrating factor for the equation, $e^{0.3t}$ This should come from $\frac{dV}{dt}+0.3V=0.45\Rightarrow \text{I.F.}=e^{\int 0.3dt}=e^{0.3t}$ May be implied by sight of $\frac{d\left(Ve^{0.3t}\right)}{dt}=...$
M1:	Fully multiplies through by their integrating factor and integrates both sides. Score for $Ve^{kt}=\int ...e^{kt}dt=...e^{kt}$ Condone missing dt
A1:	Correct integration $Ve^{0.3t}=\int 0.45e^{0.3t}dt=\frac{3}{2}e^{0.3t}\left(+c\right)$
dM1:	As main scheme.
A1:	As main scheme.

Question	Scheme	Marks	AOs
(c)	**Examples:** (1) $\frac{dV}{dt}=0 \Rightarrow V=(1.5)$ (or e.g. max V is 1.5) (2) As $t \to \infty$, $e^{-"0.3"t} \to 0$ (or $V \to$ "1.5") (3) Flow in = flow out at max V so $0.3V=0.45 \Rightarrow V=1.5$ (4) As $e^{-"0.3"t}>0$, $V<"1.5"$ (5) When $V>1.5$, $\frac{dV}{dt}<0$ (6) $V=2 \Rightarrow \frac{dV}{dt}=-0.15$ **or** compares $\frac{dV_{out}}{dt}$ $(=0.6)$ against $\frac{dV_{in}}{dt}$ $(=0.45)$ at $V=2$ (7) $V=2 \Rightarrow "1.5"-"1.25"e^{-"0.3"t}=2 \Rightarrow e^{-"0.3"t}<0$ (8) $"1.5"-"1.25"e^{-"0.3"t}=2 \Rightarrow \ln(-0.4)$ is undefined (condone e.g. gives a maths error)	M1	3.2a
	• The (upper) limit for V is 1.5 (m^3) so no (the container will not become full) (first 4 bullets) • If $V=2$ (or $V>1.5$), it would be emptying so no (it can never be full) (bullets 5, 6) • No, as the equation cannot be solved (or is not true/doesn't work) when $V=2$ (bullets 7, 8)	A1ft	2.4
		(2)	
			(9 marks)

Notes	
(c)	
M1:	See main scheme. If using the answer to part (b) it must be of the form $V=P-Qe^{-kt}$ but there is no limitation on the values of their P, Q or k. Substitution of a large value for t may score this mark but it is unlikely to be recovered to score the A1 unless they reference e.g. V_{max} being "1.5". Reference to an (upper) limit of "1.5" or their P can imply the method mark. If setting $V=2$ in their equation they must reach either ln(–ve) **or** solve the equation to reach a value for t to score this mark.
A1ft:	Must conclude "no" or equivalent e.g. "the container will not become full". Makes a correct interpretation for their method (see bullets 1-8) with a clear conclusion e.g. "no". To score this mark through ft, their V must be of the form $V=P-Qe^{-kt}$ with $k>0$, $Q>0$ **and** $0<P<2$ if used (but note that they can still use the answer to part (a) to score both marks via bullets 1, 3, 5 or 6). Allow "it" in place of the "container"/"tank". Just stating "the equation cannot be solved when $V=2$" without any evidence is M0A0. There must be no incorrect working if solving their equation or contradictory statements such as "t cannot be negative" but condone notational errors provided the intention is clear.

(**Q10 9MA0/02, June 2025**)

Q20.

Question	Scheme	Marks	AOs
(a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \pi r h = \frac{6}{r} - \frac{2}{3}\pi r^2,\ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi r h + 2\pi r^2 \{\Rightarrow A = 3\pi r^2 + 2\pi r h\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2 \Rightarrow A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\} \frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{dA}{dr} = 0 \Rightarrow\right\} -\frac{12}{r^2} + \frac{10}{3}\pi r = 0 \Rightarrow -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \ldots \left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736\ldots \Rightarrow r = 1.05$ (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	(4)	
(c)	$A_{\min} = \frac{12}{(1.046\ldots)} + \frac{5}{3}\pi(1.046\ldots)^2$	M1	3.4
	$\{A_{\min} = 17.20\ldots \Rightarrow\}\ A = 17\ (\text{m}^2)$ or $A = $ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question	
(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = \ldots$ or $\pi h = \ldots$ or $\pi r h = \ldots$ or $rh = \ldots$, where '...' $= \text{f}(r)$ into an expression for the surface area which is of the form $A = \lambda \pi r^2 + \mu \pi r h;\ \lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\ 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = \ldots$ or $S = \ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{dA}{dr} =\right\} -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{dA}{dr} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \rightarrow r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r = 1.05$ (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$
(c)	
M1:	Substitutes their $r = 1.046...$, found from solving $\frac{dA}{dr} = 0$ in part (b), into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their r which has been found from solving $\frac{d^2A}{dr^2} = 0$ or from using $\frac{d^2A}{dr^2}$ into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
A1ft:	$\{A =\}$ 17 or $\{A =\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of r for $0.6 \le$ their $r \le 1.3$ (and where their r has been found from solving $\frac{dA}{dr} = 0$ in part (b))

r	A	A (nearest integer)
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Note:	Give M1 A1 for $A = 17\ (\text{m}^2)$ or $A =$ awrt $17\ (\text{m}^2)$ from no working

(Q13 9MA0/02, June 2019)

Q21.

Question	Scheme	Marks	AOs
(a)	$5000-5000\mathrm{e}^{-0.075\times3}=\ldots$	M1	3.4
	1007	A1	1.1b
		(2)	
(b)	$5000-5000\mathrm{e}^{-0.075T}=3000\rightarrow5000\mathrm{e}^{-0.075T}=2000$	M1	1.1b
	$\Rightarrow T=\dfrac{1}{-0.075}\ln\dfrac{2000}{5000}$	dM1	1.1b
	$\Rightarrow T=12.22$	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}=\right)-5000\times-0.075\mathrm{e}^{-0.075\times3}=\ldots$	M1	3.4
	299 (car sales per month)	A1	1.1b
		(2)	
(d)	Change the constant 5000 to 6500.	B1	3.5c
		(1)	
			(8 marks)

Notes

(a)

M1: Substitutes $t=3$ into the given model and proceeds to a value.
Implied by awrt 1007, 1008 or 1010
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000 – in such cases you may need to check their calculation if the substitution isn't shown.

A1: 1007 but allow 1008 or 1010 (3sf). Must be a whole number.
Correct answer only scores both marks. A0 for e.g. 1007.4
Do not ISW if they go on to sum their values of N from $t=1$ to 3

(b)

M1: Sets $5000-5000\mathrm{e}^{-0.075T}$ equal to 3000 and proceeds to either $A\mathrm{e}^{-0.075T}=B$ or $\mathrm{e}^{-0.075T}=k$ where A, B, k are constants with no restrictions for this mark.
May use t instead of T. Condone slips.
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

dM1: Uses the correct order of operations and correct log work from an equation of the form $A\mathrm{e}^{-0.075T}=B$ with $AB>0$ or $\mathrm{e}^{-0.075T}=k$ with $k>0$ to proceed to a value for T or t which may be a numerical expression.

e.g. $A\mathrm{e}^{-0.075T}=B\Rightarrow\ln A-0.075T=\ln B\Rightarrow T=\dfrac{\ln A-\ln B}{0.075}$ or $=\dfrac{\ln\left(A/B\right)}{0.075}$

Condone log being used in place of ln unless there is clear evidence that they have used an inconsistent base.

May not be scored if "recovered" from e.g. $\mathrm{e}^{-0.075T}=-\dfrac{2}{5}$

Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

A1: 12.22 cao but must see a correct equation following use of logs, e.g., $-0.075T=\ln\dfrac{2}{5}$ or

$T=\dfrac{1}{-0.075}\times-0.916$ which may have intermediate rounding. Ignore any units given.
This mark is available following the occasional slip in writing –0.075 as –0.75 or –0.0075 provided it is not consistently used.

(c)	
M1:	Differentiates once to the form $\lambda e^{-0.075t}$, λ a constant (even ±5000), and substitutes in $t = 3$ Award for an expression of the form $ke^{-0.075\times 3}$ with constant k if no incorrect working is seen. The substitution of $t = 3$ may be implied by a correct value for their derivative of the required form, provided the derivative is seen. Do not be concerned with what they call their derivative.
A1:	awrt 299 (car sales per month). Answer only scores no marks. Condone 300 following 299.4 Requires a correct derivative to be seen, e.g., $375e^{-0.075t}$ o.e., with or without the substitution of $t = 3$ present. Units may be omitted, but score A0 if an incorrect time frame is given e.g. cars per week
(d)	
B1:	Acceptable refinement. Some examples: • Change the 5000 to 6500 (condone ambiguity about which 5000) • Change the first 5000 to 6500 • Change both 5000s to 6500s • Multiply the model by 1.3 (or e.g. $\frac{65}{50}$ or $\frac{13}{10}$) • Add 1500 to the model • Change the constant to 6500 The statement of an acceptable refined model e.g. $(N =)\ 6500 - 6500e^{-0.075t}$ or e.g. $(N =)\ 6500 - 5000e^{-0.075t}$ scores B1. Ignore any extra unnecessary refinements such as increase/decrease the 0.075. The following score B0: • Change 5000 in the model (not specific enough – they haven't said to 6500) • Change the second 5000 to 6500 (incorrect)

(**Q09 9MA0/02, June 2025**)

Q22.

Question	Scheme	Marks	AOs
(a)	$3(x+y)^2\left(1+\frac{dy}{dx}\right)=6x-3\frac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\left(3(x+y)^2+3\right)\frac{dy}{dx}=6x-3(x+y)^2 \Rightarrow \frac{dy}{dx}=...$	M1	2.1
	$\frac{dy}{dx}=\frac{6x-3(x+y)^2}{3(x+y)^2+3}$ $\left(\text{oe e.g. } \frac{2x-(x+y)^2}{(x+y)^2+1}\right)$	A1	1.1b
		(5)	
	Alternative – expands $(x+y)^3$ before differentiating $(x+y)^3=x^3+3x^2y+3xy^2+y^3$		
	$\Rightarrow 3x^2+3x^2\frac{dy}{dx}+6xy+6xy\frac{dy}{dx}+3y^2+3y^2\frac{dy}{dx}=6x-3\frac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\left(3x^2+6xy+3y^2+3\right)\frac{dy}{dx}=6x-3x^2-6xy-3y^2 \Rightarrow \frac{dy}{dx}=...$	M1	2.1
	$\left(3x^2+6xy+3y^2+3\right)\frac{dy}{dx}=6x-3x^2-6xy-3y^2$ $\Rightarrow \frac{dy}{dx}=\frac{6x-3x^2-6xy-3y^2}{3x^2+6xy+3y^2+3}$ $\left(\text{oe e.g. } \frac{2x-x^2-2xy-y^2}{x^2+2xy+y^2+1}\right)$	A1	1.1b

(a) Notes

(a) Some candidates have a spurious "$\frac{dy}{dx}=$" appearing as their intention to differentiate e.g.

$$\left(\frac{dy}{dx}=\right)3(x+y)^2\left(1+\frac{dy}{dx}\right)=6x-3\frac{dy}{dx}$$

This can be condoned for the first 3 marks in both versions.

Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Award this mark for one of:

- $(x+y)^3 \to k(x+y)^2\left(\lambda+\frac{dy}{dx}\right)$ where λ is 1, x or 0 but condone missing brackets e.g. $3(x+y)^2 1+\frac{dy}{dx}$
- $3x^2-3y-2 \to ...x+...\frac{dy}{dx}$ but condone $3x^2-3y-2 \to ...x+...\frac{dy}{dx}-2$

A1: **Either** $3(x+y)^2\left(1+\frac{dy}{dx}\right)$ **or** $6x-3\frac{dy}{dx}$ oe

May be implied if e.g. they collect terms to one side initially.
Do not condone missing brackets unless they are implied by subsequent work.

A1: $3(x+y)^2\left(1+\frac{dy}{dx}\right)$ **and** $6x-3\frac{dy}{dx}$ (seen separately or equated)

If they collect terms to one side initially then the signs must be correct.

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$, one coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of "$-3y$"

Note that here, 2 **different** terms means terms such as $3\frac{dy}{dx}$ and $3(x+y)^2\frac{dy}{dx}$ and not e.g. $3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$

Look for $(...\pm...)\frac{dy}{dx}=...\Rightarrow\frac{dy}{dx}=...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx}=...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that from $3(x+y)^2\left(1+\frac{dy}{dx}\right)=6x-3\frac{dy}{dx}$, candidates may expand the brackets before rearranging, in which case they would need 4 **different** $\frac{dy}{dx}$ terms coming from the appropriate places.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions are satisfied.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Apply isw as soon as a correct expression is seen.

(a) alternative by expanding:

M1: Award this mark for one of:

- $3x^2-3y-2\rightarrow...x+...\frac{dy}{dx}$ but condone $3x^2-3y-2\rightarrow...x+...\frac{dy}{dx}-2$
- Expanding $(x+y)^3$ to obtain either an x^2y term or an xy^2 term and then uses the product rule to obtain $...x^2y\rightarrow...x^2\frac{dy}{dx}+...xy$ or $...xy^2\rightarrow...xy\frac{dy}{dx}+...y^2$

A1: **Either** $3x^2+3x^2\frac{dy}{dx}+6xy+6xy\frac{dy}{dx}+3y^2+3y^2\frac{dy}{dx}$ **or** $6x-3\frac{dy}{dx}$.

May be implied if e.g. they collect terms to one side initially.

A1: $3x^2+3x^2\frac{dy}{dx}+6xy+6xy\frac{dy}{dx}+3y^2+3y^2\frac{dy}{dx}$ **and** $6x-3\frac{dy}{dx}$ oe. (seen separately or equated) If they collect terms to one side initially then the signs must be correct.

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 4 **different** terms in $\frac{dy}{dx}$, 3 coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of "$-3y$"

Note that here, 4 **different** terms means terms such as $x^2\frac{dy}{dx}$ and $6xy\frac{dy}{dx}$ and not e.g.

$3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$

Look for $(...\pm...\pm...\pm...)\frac{dy}{dx}=...\Rightarrow\frac{dy}{dx}=...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx}=...$ at the start, they may incorporate this in their rearrangement in which case they will have 5 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions are satisfied. E.g. if they have an incorrect term such as $6x\frac{dy}{dx}$, this mark is still available.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Condone e.g. $3x2y$ for $6xy$.

Apply isw as soon as a correct expression is seen.

Alternative making y the subject in (a):

$$(x+y)^3=3x^2-3y-2$$

$$x+y=\left(3x^2-3y-2\right)^{\frac{1}{3}}\Rightarrow y=\left(3x^2-3y-2\right)^{\frac{1}{3}}-x$$

$$\frac{dy}{dx}=\frac{1}{3}\left(3x^2-3y-2\right)^{-\frac{2}{3}}\left(6x-3\frac{dy}{dx}\right)-1$$

$$\frac{dy}{dx}\left(1+\left(3x^2-3y-2\right)^{-\frac{2}{3}}\right)=2x\left(3x^2-3y-2\right)^{-\frac{2}{3}}-1$$

$$\frac{dy}{dx}=\frac{2x\left(3x^2-3y-2\right)^{-\frac{2}{3}}-1}{1+\left(3x^2-3y-2\right)^{-\frac{2}{3}}}$$

Score as follows:

M1: Cube roots both sides and makes $x+y$ or y the subject then award for

- $\left(3x^2-3y-2\right)^{\frac{1}{3}}\rightarrow...\left(3x^2-3y-2\right)^{-\frac{2}{3}}$ or
- $3x^2-3y-2\rightarrow...x+...\frac{dy}{dx}$ but condone $3x^2-3y-2\rightarrow...x+...\frac{dy}{dx}-2$

A1: For the $\frac{1}{3}\left(3x^2-3y-2\right)^{-\frac{2}{3}}$ or $6x-3\frac{dy}{dx}$

A1: Fully correct

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$

A1: Correct expression

Using partial derivatives in (a):

$$(x+y)^3 = 3x^2 - 3y - 2 \rightarrow \mathrm{f}(x,y) = (x+y)^3 - 3x^2 + 3y + 2$$

$$\frac{\partial \mathrm{f}}{\partial x} = 3(x+y)^2 - 6x \qquad \frac{\partial \mathrm{f}}{\partial y} = 3(x+y)^2 + 3$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\partial \mathrm{f}}{\partial x} \div \frac{\partial \mathrm{f}}{\partial y} = \frac{6x - 3(x+y)^2}{3(x+y)^2 + 3}$$

or

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = 3x^2 - 3y - 2$$

$$\mathrm{f}(x,y) = x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2 + 3y + 2$$

$$\frac{\partial \mathrm{f}}{\partial x} = 3x^2 + 6xy + 3y^2 - 6x \qquad \frac{\partial \mathrm{f}}{\partial y} = 3x^2 + 6xy + 3y^2 + 3$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\partial \mathrm{f}}{\partial x} \div \frac{\partial \mathrm{f}}{\partial y} = \frac{-3x^2 - 6xy - 3y^2 + 6x}{3x^2 + 6xy + 3y^2 + 3}$$

Score as follows:

M1: Correct structure for either partial derivative:

Doesn't expand: $\frac{\partial \mathrm{f}}{\partial x} = ...(x+y)^2 + ...x$ or $\frac{\partial \mathrm{f}}{\partial y} = ...(x+y)^2 + ...$

or

Expands: $\frac{\partial \mathrm{f}}{\partial x} = ...x^2 + ...xy + ...y^2 + ...x$ or $\frac{\partial \mathrm{f}}{\partial y} = ...x^2 + ...xy + ...y^2 + ...$

Where "..." are non-zero constants

A1: Correct $\frac{\partial \mathrm{f}}{\partial x}$ or correct $\frac{\partial \mathrm{f}}{\partial y}$

A1: Correct $\frac{\partial \mathrm{f}}{\partial x}$ and correct $\frac{\partial \mathrm{f}}{\partial y}$

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\partial \mathrm{f}}{\partial x} \div \frac{\partial \mathrm{f}}{\partial y}$

A1: Correct expression

(b)	$\frac{dy}{dx} = \frac{6(1)-3(0+1)^2}{3(0+1)^2+3} = \frac{1}{2}$ or e.g. $\frac{dy}{dx} = \frac{6(1)-3(1)^2-6(1)(0)-3(0)^2}{3(1)^2+6(1)(0)+3(0)^2+3} = \frac{1}{2}$ $\Rightarrow y-0=-2(x-1)$ **or** $\Rightarrow y=-2x+c \Rightarrow 0=-2+c \Rightarrow c=...$	M1	2.1
	$y=-2x+2$*	A1*	1.1b
		(2)	

(b) Notes

(b) **Note that the gradient of $\frac{1}{2}$ could have been deduced from the given equation so you will need to check their solution carefully.**

M1: Substitutes $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ to obtain the tangent gradient **and** then uses the negative reciprocal and $x = 1$ and $y = 0$ in a correct straight line method to obtain the normal equation with $x = 1$ and $y = 0$ correctly placed.
Note that when finding the normal gradient, they may find the negative reciprocal of their expression from part (a) and then substitute $x = 1$ and $y = 0$ which is fine.
If using $y = mx + c$ they must proceed as far as finding a value for c.
If no substitution of $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ is seen you will need to check their value. If they **just** state a value for $\frac{dy}{dx}$ then it must follow their $\frac{dy}{dx}$ with $x = 1$ and $y = 0$

A1*: Correct equation with no errors **following a correct** $\frac{dy}{dx}$ from part (a) (unless they start again which is unlikely)
Be aware that some incorrect expressions for $\frac{dy}{dx}$ from part (a) may fortuitously give $\frac{dy}{dx} = \frac{1}{2}$ and would generally score A0
In general A1* must follow the final A1 in (a) or correct differentiation in (a)

(c)	$y=-2x+2 \Rightarrow (x-2x+2)^3 = 3x^2-3(-2x+2)-2$ **or** $x=\frac{2-y}{2} \Rightarrow \left(\frac{2-y}{2}+y\right)^3 = 3\left(\frac{2-y}{2}\right)^2 - 3y - 2$	M1	1.1b
	$x^3-3x^2+18x-16=0$ **or** $y^3+60y=0$	A1	1.1b
	$\Rightarrow (x-1)(x^2-2x+16)=0$ ($x = 1$ is known) or $\Rightarrow y(y^2+60)=0$ ($y = 0$ is known)	dM1	2.1
	For $x^2-2x+16=0$, $b^2-4ac=4-4\times1\times16$ or For $y^2+60=0$, $y^2 \neq -60$	ddM1	2.1
	As $b^2-4ac<0$ or as $y^2 \neq -60$ there are no other real roots and so the normal does not meet C again.	A1	2.4
		(5)	

(c) Notes

(c)

M1: Uses the equation from part (a) and substitutes $y = \pm 2x \pm 2$ or $x = \frac{\pm 2 \pm y}{2}$ to obtain an equation in one variable (usually x) (not necessarily a cubic equation). Allow slips in rearranging to obtain x in terms of y (or y in terms of x) as long as the intention is clear.

A1: Correct cubic equation with terms collected and "= 0" seen or implied.
Note that both $-x^3 + 3x^2 - 18x + 16 = 0$ and $-y^3 - 60y = 0$ are correct equations.

To access any of the following marks, candidates must attempt to use either the factor of $(x - 1)$ with their cubic in x or the factor of y in their cubic in y to obtain a quadratic expression in x or y.
Attempts that just use a calculator to solve the cubic equation score no more marks in this part.

dM1: Uses the fact that $(x - 1)$ or y is a factor in an attempt to establish the quadratic factor.
For the cubic in x, it must be of the form $ax^3 + bx^2 + cx + d = 0 \quad a, b, c, d \neq 0$
For the cubic in y, it must be of the form $ay^3 + by = 0 \quad a, b \neq 0$
For the cubic in x, the attempt at the quadratic factor using $(x - 1)$ may be via inspection or e.g. long division to obtain a 3 term quadratic expression. There may or may not be a remainder but they must obtain 3 terms.
For the cubic in y, they would need to take out a factor of y (or divide through by y) to obtain a factor of the form $k(y^2 + \alpha)$

ddM1: This mark requires:

- a correct cubic equation in x or y
- the correct quadratic factor or a multiple of it e.g. $k(x^2 - 2x + 16)$ or $k(y^2 + 60)$
- an attempt to show that the quadratic factor has no real roots

<u>For the quadratic in x this could be:</u>

<u>Attempts discriminant</u>: e.g. $b^2 - 4ac = 4 - 4 \times 1 \times 16$ (may be embedded in the quadratic formula)

<u>Attempts to complete the square</u>: e.g. $x^2 - 2x + 16 = (x - 1)^2 - 1 + 16$

<u>Uses calculus to find the turning point</u>: e.g. $\frac{\mathrm{d}(x^2 - 2x + 16)}{\mathrm{d}x} = 2x - 2 = 0 \Rightarrow x = 1 \Rightarrow y = \ldots$

<u>Attempts to solve:</u> e.g. $x^2 - 2x + 16 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4 \times 16}}{2}$ or from a calculator $x = 1 \pm \sqrt{15}\mathrm{i}$

<u>For the quadratic in y this is likely to be:</u>

<u>Attempts to solve:</u> e.g. $y^2 + 60 = 0 \Rightarrow y^2 = -60 \Rightarrow \ldots$

A1: Fully correct argument that requires:

- fully correct work
- a justification depending on their strategy
- a conclusion depending on their strategy

Via discriminant: $4-4\times1\times16<0$ so no real roots so they do not meet again

Via completing the square: $\rightarrow(x-1)^2+15$ which has a minimum value of 15 so no real roots so they do not meet again.

Via calculus: $x=1\Rightarrow y=15$ is the minimum so no real roots so they do not meet again.

Via solving: $x=1\pm\sqrt{15}\text{i}$ or e.g. $x=1\pm3.87\text{i}$ or e.g. $x=1\pm\sqrt{-15}$ so math error so they do not meet again.

For y it is likely to be more straightforward e.g. $y^2\neq-60$ which cannot be solved so they do not meet again.

Allow equivalent statements for "they do not meet again" e.g. so they only meet once.

(But do **not** condone incorrect statements such as "therefore P does not meet C again")

A minimum justification could be:

$$x^2-2x+16=0\rightarrow b^2-4ac=(-2)^2-4\times1\times16 \quad \textbf{ddM1}$$

$$4-4\times1\times16<0 \text{ so no more roots so no more intersections} \quad \textbf{A1}$$

Do not allow e.g.

"$x^2-2x+16=0$ gives a math error so they do not meet again"

as there has been no attempt to show why the "math error" occurs – this scores M0A0

Alternative to (c) by showing the cubic is strictly increasing (or decreasing):

M1A1: As in the main scheme then

$$\text{f}(x)=x^3-3x^2+18x-16\Rightarrow\text{f}'(x)=3x^2-6x+18$$

$$3x^2-6x+18=3(x^2-2x+6)=3(x-1)^2+15$$

$$3(x-1)^2+15>0 \text{ so f}(x)\text{ is an increasing function}$$

Hence there can only be one intersection (at $x=1$) so the normal and curve do not intersect again.

dM1: Differentiates their cubic of the form $ax^3+bx^2+cx+d=0$ $a,b,c,d\neq0$ to obtain a 3 term quadratic expression with only coefficient errors on the non-constant terms.

ddM1: This mark requires:

- a correct cubic equation in x
- the correct derivative or a multiple of it
- an attempt to show that the quadratic expression is always positive (or negative)

A1: Fully correct concluding argument e.g. that as the derivative is always positive (or always negative) the function is strictly increasing (or decreasing) and therefore there can only be one intersection (at $x=1$) so the normal and curve do not meet again.

(12 marks)

(**Q15 9MA0/02, June 2024**)

Q23.

Question	Scheme	Marks	AOs
(a)	$x^3 \to ...x^2$ and $3y^2 \to ...y\frac{dy}{dx}$	M1	1.1b
	$2xy \to 2y + 2x\frac{dy}{dx}$	B1	1.1b
	$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$	M1	2.1
	$\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$	A1	1.1b
		(4)	
(b)	$\frac{dy}{dx} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{dy}{dx} + 2\times 5 + 6\times 5\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = ...\left(-\frac{11}{13}\right)$	M1	1.1b
	$y - 5 = "\frac{13}{11}"(x+2)$	dM1	1.1b
	$13x - 11y + 81 = 0$	A1	2.2a
		(3)	
			(7 marks)
	Notes		

(a) Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Attempts to differentiate $x^3 \to ...x^2$ **and** $3y^2 \to ...y\frac{dy}{dx}$ where … are constants

B1: Correct application of the product rule on $2xy$: $2xy \to 2x\frac{dy}{dx} + 2y$

Note that some candidates have a spurious $\frac{dy}{dx} = ...$ at the start (as their intention to differentiate) and this can be ignored for the first 2 marks

M1: For a valid attempt to make $\frac{dy}{dx}$ the subject, with exactly 2 different terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$. Look for $(... \pm ...)\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$ oe e.g. $\frac{dy}{dx} = \frac{-2y-3x^2}{2x+6y}, \frac{2y+3x^2}{-2x-6y}$ Isw once a correct expression is seen.

Note that it is sometimes unclear if the minus sign(s) is/are correctly placed and you may have to use your judgement. Evidence may be available in part (b) to help you decide if they have the correct expression.

(b)

M1: Substitutes $x = -2$ and $y = 5$ into $\frac{dy}{dx} = "-\frac{2y+3x^2}{2x+6y}"$

They must have x's and y's in their $\frac{dy}{dx}$ but condone slips in substitution provided the intention is clear.

As a minimum look for at least one x and at least one y substituted correctly.

Note that this mark may be implied by their value for $\frac{dy}{dx}$ and may be implied if, for example, they find the negative reciprocal or the reciprocal of $"-\frac{2y+3x^2}{2x+6y}"$ and then substitute $x = -2$ and $y = 5$

Alternatively, substitutes $x = -2$ and $y = 5$ into their attempt to differentiate and then rearranges to find a value or numerical expression for $\frac{dy}{dx}$

dM1: Attempts to find the equation of the normal using their gradient of the tangent and $x = -2$ and $y = 5$ correctly placed. Score for an expression of the form $(y-5) = "\frac{13}{11}"(x+2)$ or if they use $y = mx + c$ they must proceed as far as $c = \ldots$ Must be using the **negative reciprocal** of the tangent gradient.

Note that $y - 5 = \frac{2x+6y}{2y+3x^2}(x+2)$ is not a correct method unless the gradient is evaluated first *before* expanding.

A1: $13x - 11y + 81 = 0$ or any integer multiple of this equation including the "= 0", not just a, b, c given.

e.g., $26x - 22y + 162 = 0$ is likely if they don't cancel down their gradient.

(Q07 9MA0/02, June 2023)

Q24.

Question	Scheme	Marks	AOs
(a)	$\{f(3.6)=\}\ 3.6+\tan\left(\frac{1}{2}(3.6)\right) = -0.686... < 0$ **and** $\{f(3.7)=\}\ 3.7+\tan\left(\frac{1}{2}(3.7)\right) = 0.211... > 0$	M1	1.1b
	Change of sign and function is continuous in the interval $\Rightarrow$ conclusion e.g. "there is a root in [3.6, 3.7]" *	A1*	2.4
		(2)	
(b)	Use of $\tan\left(\frac{1}{2}x\right) \to ...\sec^2\left(\frac{1}{2}x\right)$	M1	1.1b
	$\{f'(x)=\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	A1	1.1b
		(2)	
(c)	Attempts $3.7-\dfrac{3.7+\tan\left(\frac{1}{2}(3.7)\right)}{"1+\frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)"} = ...$ (N.B. $f(3.7)=0.211...$ and $f'(3.7)=7.58...$)	M1	1.1b
	α = awrt 3.672	A1	1.1b
		(2)	
			(6 marks)

Notes
(a)
M1: Attempts both f(3.6) and f(3.7) or a narrower interval that contains the root 3.672… (which may be implied by sight of f(3.6) = … and f(3.7) = … with at least one correct) **and** obtains at least one correct to 1 significant figure (rounded or truncated) for their interval **and** considers their signs. Use of degrees is M0. Some examples for consideration of sign (which are also sufficient for the change of sign part of reasoning for the A1): • $f(3.6) = -0.6 < 0$ and $f(3.7) = 0.2 > 0$ • $f(3.6)\times f(3.7) < 0$ • $f(3.6) = -0.7$, $f(3.7) = 0.2$ "change in sign" For reference $f(3.6) = -0.68626...$ and $f(3.7) = 0.21194...$
A1*: This mark requires: • both f(3.6) and f(3.7) correct to 1 significant figure (rounded or truncated) (or their values correct to 1 significant figure if using a narrower interval) • a reference to sign change • a reference to continuity {of $f(x)$} • a (minimal) conclusion, e.g. "hence root", "proved", ✓, #, QED, $3.6 < \alpha < 3.7$ Accept as a minimum, "change of sign, continuous, root". Do not condone "change in sign therefore continuous" or other incorrect statements such as "x is continuous", "the interval is continuous" – these score A0. Condone "the graph is continuous". Condone reference to x in place of α in their conclusion, e.g. "hence x lies in the interval". Condone statements such as "there is at least one root" in place of their conclusion.

(b) **Note:** Their answer to (b) may be seen in part (c) provided that they have not clearly attempted part (b) incorrectly, e.g., an attempt at $\mathrm{f}^{-1}(x)$ in (b).

M1: For $\tan\left(\frac{1}{2}x\right) \to ...\sec^2\left(\frac{1}{2}x\right)$ o.e. The brackets are not required. You may see attempts at the quotient rule but the method should be correct and they should reach something equivalent to $...\sec^2\left(\frac{1}{2}x\right)$.

e.g. $\tan\left(\frac{1}{2}x\right) = \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} \to \frac{k\cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right) - -k\sin\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ where k is a **positive** constant scores M1. If the formula is seen it must be correct.

A1: $\{\mathrm{f}'(x) =\}\ 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ o.e. which may be unsimplified and apply isw.

The brackets are not required. There is no need for $\mathrm{f}'(x) =$ just look for the expression.

Note that $\{\mathrm{f}'(x) =\}\ \frac{3}{2} + \frac{1}{2}\tan^2\left(\frac{1}{2}x\right)$ is correct and appears occasionally.

$\{\mathrm{f}'(x) =\}\ 1 + \frac{1}{2}\sec\frac{1}{2}x^2$ is condoned for M1A0 only but $1 + \frac{1}{2}\left(\sec\frac{1}{2}x\right)^2$ scores M1A1.

(c)

M1: Attempts $3.7 - \frac{\mathrm{f}(3.7)}{\mathrm{f}'(3.7)}$ and obtains a value following through on their f ′(x) as long as it is a "changed" function in terms of x.

Just stating $3.7 - \frac{\mathrm{f}(3.7)}{\mathrm{f}'(3.7)} = ...$ without evidence of use of 3.7 in f(x) (note that this evidence might come from part (a)) **and** in their f ′(x) is M0 unless implied by a correct value for both f(x) **and** f ′(x) **or** by their final answer.

Must be a correct N-R formula used – you may need to check their values – accuracy of at least 3s.f. rounded or truncated required.

Allow if attempted in degrees. For reference in degrees f (3.7) = 3.73… and f ′(3.7) = 1.50… and gives α = 1.21…

Note that the full N-R accuracy is 3.672051617.

For reference, the value of α is approximately 3.673194406… and scores M0A0 without other valid work.

A1: For awrt 3.672 Ignore any subsequent iterations.

(**Q03 9MA0/01, June 2024**)

Q25.

Question	Scheme	Marks	AOs
(a) Way 1	$\{y = x^x \Rightarrow\}\ \ln y = x\ln x$	B1	1.1a
	$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	(5)	
(a) Way 2	$\{y = x^x \Rightarrow\}\ y = e^{x\ln x}$	B1	1.1a
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	(5)	
(b) Way 1	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) — Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
			(11 marks)

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$ $\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{dy}{dx};\ \lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\ \ k \neq 0$

Question	Scheme	Marks	AOs
(b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question

(a)	Way 1
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	Way 2
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1 + \ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question Continued	
(b)	Way 1
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	Way 2
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	Way 3
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	Way 4
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

(**Q11 9MA0/02, June 2019**)

Q26.

Question	Scheme	Marks	AOs
(a)	$-3=\frac{t-1}{2}\Rightarrow t=-5\Rightarrow y=5("-5"+2)^4$	M1	1.1b
	$(y=)\,405$	A1	1.1b
		(2)	
(b)	$x=\frac{t-1}{2}\Rightarrow t=2x+1\Rightarrow y=5("2x+1"+2)^4$	M1	1.1b
	$y=5(2x+3)^4$	A1	1.1b
		(2)	
(c)	$("2x+3")^n\rightarrow ...("2x+3")^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}=\right)40(2(-3)+3)^3$	dM1	1.1b
	$\left(\frac{dy}{dx}=\right)-1080$	A1	1.1b
		(3)	
			(7 marks)

Notes	
(a)	**Mark parts (a) and (b) together.**
M1:	Substitutes $x=-3$ into $x=\frac{t-1}{2}$, attempts to find *t*, and substitutes their *t* into $y=5(t+2)^4$ Condone slips. May be implied by substitution of $t=-5$ into *y* or by 405 or by $y=5(-3)^4$ Alternatively, they may substitute $x=-3$ into their $y=f(x)$
A1:	cao Answer only scores full marks and ISW after seeing 405 e.g. (–5, 405) Accept (–3, 405) or e.g. $P=405$
(b)	**Mark parts (a) and (b) together.**
M1:	Attempts to make *t* the subject of $x=\frac{t-1}{2}$ using the correct **order** of operations and substitutes into $y=5(t+2)^4$ Condone slips e.g. dividing by 2 first: $t=\frac{x}{2}+1$ or missing the +2 in $y=5(t+2)^4$ **Alt 1:** Writes $t=\left(\frac{y}{5}\right)^{\frac{1}{4}}-2$, substitutes into $x=\frac{t-1}{2}$ and attempts to make *y* the subject using the correct **order** of operations, i.e., $2x=\left(\frac{y}{5}\right)^{\frac{1}{4}}-3\Rightarrow(2x+3)^4=\frac{y}{5}\Rightarrow y=5(2x+3)^4$ **Alt 2:** Makes *t* the subject and then substitutes into an expanded $5(t+2)^4$ Do not be too concerned about their expansion, but it must contain t^4 and a constant term.
A1:	cao and ISW after a correct answer seen. May be scored for $y=5(2x+1+2)^4$ Allow e.g. $y=80x^4+480x^3+1080x^2+1080x+405$ or e.g. $y=5(2x+1)^4+40(2x+1)^3+120(2x+1)^2+160(2x+1)+80$ Their RHS may be unsimplified but do not allow if e.g. binomial coefficients are still present. Do not accept f(*x*) = … It must be *y* = …

(c)	**Note: Differentiation seen in (a) or (b) can score marks if used in (c)**
M1:	Reduces the power of their $("2x+3")^n$ by one to $Q("2x+3")^{n-1}$ where Q is a constant and could be 1. There should be no other terms using this method. Alternatively, attempts to expand their $("2x+3")^n$ (may have been expanded in (b)) and reduces the power of x by one in at least one term. They may use parametric differentiation, i.e., $\frac{dy}{dt} = a(t+2)^3$ and $\frac{dx}{dt} = b$ where a and b are constants, leading to $\frac{dy}{dx} = \frac{a(t+2)^3}{b}$ i.e. they must divide the correct way round. Condone attempts at the chain rule that reach e.g. $y' = \ldots u^3$ but make a slip when substituting back in for x or t, e.g., $\frac{dy}{dx} = (5x+3)^3$ or $\frac{dy}{dt} = (t+2)^2$, provided the intention is clear.
dM1:	Substitutes $x = -3$ into their $\frac{dy}{dx}$ in terms of x which may be implied by their answer. Using parametric differentiation, substitutes their t (found from an attempt at substituting $x = -3$ into $x = \frac{t-1}{2}$ which may have been seen in (a)) into their $\frac{dy}{dx}$ in terms of t. Note for reference, if correct, the parametric differentiation is $\frac{dy}{dt} = 20(t+2)^3$ and $\frac{dx}{dt} = \frac{1}{2}$ leading to $\frac{dy}{dx} = 40(t+2)^3$ They may substitute their value of t into $\frac{dy}{dt}$ first before using the chain rule to reach $\frac{dy}{dx}$ which is acceptable and implies the first M mark.
A1:	$\left(\frac{dy}{dx} =\right) -1080$ Correct answer only scores full marks. May be seen labelled as m or something else.

(**Q05 9MA0/02, June 2025**)

Q27.

Question	Scheme	Marks	AOs
(a)	$4=2+4\cos t \Rightarrow \cos t=\frac{1}{2} \Rightarrow t=-\frac{\pi}{3} \Rightarrow y=...$	M1	2.1
	$y=3\times\left(-\frac{\pi}{3}\right)+2\sin\left(-\frac{\pi}{3}\right)=-\pi-\sqrt{3}$	A1	1.1b
		(2)	
(b)	Attempts $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3+2\cos t}{-4\sin t}=-\frac{3}{2}$ $\Rightarrow 6\sin t-2\cos t=3$	M1 A1	2.1 1.1b
	E.g. $6\sin t-2\cos t=R\sin(t-\alpha)$ $\Rightarrow R=\sqrt{6^2+2^2}=\sqrt{40},\ \tan\alpha=\frac{1}{3}\Rightarrow \alpha=0.322...\text{ or }18.4°$	M1 A1	3.1a 1.1b
	$\sqrt{40}\sin(t-0.322)=3\Rightarrow t=0.322+\sin^{-1}\left(\frac{3}{\sqrt{40}}\right)$	dM1	2.1
	$t=$ awrt 0.816	A1	1.1b
		(6)	
			(8 marks)

Notes:

(a)

M1: For the key step in finding y when $t=-\frac{\pi}{3}$

A1: $-\pi-\sqrt{3}$

(b)

M1: For setting up an equation in t using all of the information given.
The key steps required to score this mark are

- Finding $\frac{dy}{dx}$ by dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$
- Setting $\frac{dy}{dx}=-\frac{3}{2}$ and proceeding to an equation of the form $a\sin t+b\cos t=c$

A1: $6\sin t-2\cos t=3$ or exact equivalent

M1: Chooses a suitable method to solve the equation.
Look for a correct overall method condoning slips.

A1: Correct values of "R" and "α"

dM1: Full and complete method to find t using correct order of operations.
It is dependent upon <u>all</u> previous M marks.

A1: For awrt 0.816 and no other values.

Alternative for the final 4 marks:

$$6\sin t-2\cos t=3\Rightarrow 6\sin t=3+2\cos t\Rightarrow 36\sin^2 t=9+12\cos t+4\cos^2 t$$

$$\Rightarrow 36-36\cos^2 t=9+12\cos t+4\cos^2 t\Rightarrow 40\cos^2 t+12\cos t-27=0$$

$$\Rightarrow \cos t=\frac{-3\pm3\sqrt{31}}{20}(0.685...,\ -0.985...)\Rightarrow t=0.816$$

Q28.

Question	Scheme	Marks	AOs
(a) Way 1	$x=(t+3)^2-25$	M1	1.1b
	$\Rightarrow x+25=(t+3)^2 \Rightarrow (x+25)^{\frac{1}{2}}=(t+3) \Rightarrow y=\ldots$	M1	2.1
	$y=6\ln(x+25)^{\frac{1}{2}} \Rightarrow y=3\ln(x+25)$	A1cso	1.1b
		(3)	
	(a) Way 2		
	$y=6\ln(t+3)=3\ln(t+3)^2$	M1	1.1b
	$y=3\ln(t+3)^2=3\ln(t^2+6t+9)=3\ln(x+16+9)$	M1	2.1
	$y=3\ln(x+25)$	A1cso	1.1b
	(a) Way 3		
	$y=6\ln(t+3) \Rightarrow \frac{y}{6}=\ln(t+3) \Rightarrow t+3=e^{\frac{y}{6}} \Rightarrow t=e^{\frac{y}{6}}-3$	M1	1.1b
	$x=\left(e^{\frac{y}{6}}-3\right)^2+6\left(e^{\frac{y}{6}}-3\right)-16 \Rightarrow y=\ldots$ or $x=\left(e^{\frac{y}{6}}-3+8\right)\left(e^{\frac{y}{6}}-3-2\right) \Rightarrow y=\ldots$	M1	2.1
	$y=3\ln(x+25)$	A1cso	1.1b
	(a) Way 4		
	$x=(t+3)^2-25$	M1	1.1b
	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{3}{(t+3)^2}=\frac{3}{x+25} \Rightarrow y=3\ln(x+25)(+c)$	M1	2.1
	e.g. $t=0 \Rightarrow x=-16, y=6\ln 3 \Rightarrow 6\ln 3=3\ln(9) \Rightarrow c=0$ $y=3\ln(x+25)$	A1cso	1.1b

(b)	$x=0,\ y=3\ln 25$ oe e.g. $6\ln 5$	B1ft	2.2a
	$\frac{dy}{dx}=\frac{3}{x+"25"} \Rightarrow \frac{dy}{dx}=\frac{3}{0+"25"} \quad \left(=\frac{3}{25}\right)$ or $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{2+3}}{2\times 2+6} \quad \left(=\frac{6}{50}=\frac{3}{25}\right)$	M1	2.1
	$y-"3\ln 25"="\frac{3}{25}"(x\{-0\})$	dM1	3.1a
	$25y-3x=150\ln 5$	A1	2.2a
		(4)	
			(7 marks)
Notes			
Choose the mark scheme that best matches their chosen method.			

(a) **Way 1**

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm ...$ where $... \neq 0$

M1: Rearranges their $x=(t+3)^2-25$ to either $(t+3)=...$ or $(t+3)^2=...$ and then substitutes correctly their expression into the parametric equation for y. So e.g., $t=\sqrt{x+25}-3 \to y=6\ln\left(\sqrt{x+25}-3\right)$ is M0.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 2

M1: Attempts to use the power rule for logarithms $y=6\ln(t+3)=...\ln(t+3)^2$ where $... \neq 6$

M1: Writes $y=6\ln(t+3)$ as $3\ln(t+3)^2$ and then multiplies out and substitutes correctly in for t to obtain a Cartesian equation for C

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 3

M1: Attempts to make t the subject for $y=6\ln(t+3)$ to obtain $t=e^{\frac{y}{6}} \pm ...$ where $... \neq 0$

M1: Substitutes $t=e^{f(y)} \pm ...$ correctly into $x=t^2+6t-16$ and rearranges to make y the subject.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 4

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm ...$ where $... \neq 0$

M1: Attempts to find $\frac{dy}{dx}$ where $\frac{dy}{dx}=\frac{\left(\frac{...}{t+3}\right)}{at+b}$, $a,b \neq 0$ and uses the completed square form to find $\frac{dy}{dx}$ in terms of x and then integrates to obtain a Cartesian equation for C

A1cso: A complete method using any correct point on the curve to show that $c=0$ and obtain $y=3\ln(x+25)$ with all stages of working shown. The "$y=$" must appear at some point.

Note that a common incorrect approach in (a) is:

$$x=t^2+6t-16=(t-2)(t+8) \Rightarrow x=t-2 \Rightarrow t=x+2 \Rightarrow y=6\ln(x+5)$$

which scores no marks.

(b)

B1ft: Deduces $y = 3\ln 25$ oe e.g $y = 6\ln 5$ but allow follow through on their Cartesian equation with $x = 0$ and apply isw after a correct value or ft value for y

M1: Attempts to find $\frac{dy}{dx}$ when $x = 0$ so score for obtaining $\frac{dy}{dx} = \frac{...}{x + "25"}$ and substituting in $x = 0$

Allow this mark if they use the letters A and B e.g. $\frac{dy}{dx} = \frac{...}{x+B} = \frac{...}{0+B}$ or allow a "made up" A and B.

or

Attempts to find $\frac{dy}{dx}$ when $t = 2$ by finding $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{5}}{2\times 2+6} \left(= \frac{6}{50} = \frac{3}{25}\right)$

For the derivative look for $\frac{dy}{dx} = \frac{\left(\frac{...}{t+3}\right)}{at+b}$ oe e.g. $\left(\frac{...}{t+3}\right) \times \frac{1}{at+b}$ $a, b \neq 0$

NOTE if candidates find $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} = \frac{6(2t+6)}{t+3} = 12$ we will give BOD that $t = 2$ has been used unless there is clear evidence that $t = 2$ has not been used.

dM1: Attempts to find the equation of the tangent. Score for sight of $y - "3\ln 25" = "\frac{3}{25}"(x\{-0\})$ or if they use $y = mx + c$ they must proceed as far as $c = ...$ **It is dependent on the previous method mark.** Must have numeric A and B now.

A1: $25y - 3x = 150\ln 5$ or any integer multiple of this equation in the form $ax + by = c\ln 5$

(Q09 9MA0/02, June 2023)

Q29.

Question	Scheme	Marks	AOs
(a)	$x = 4, y = 2 \Rightarrow t = -1$	B1	2.2a
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -3t^2 \times \frac{1}{2(t+3)}$	M1	1.1b
	$\frac{dy}{dx} = -3(-1)^2 \times \frac{1}{2(-1+3)} = -\frac{3}{4}$	M1	1.1b
	$\Rightarrow y - 2 = -\frac{3}{4}(x-4)$ or $\Rightarrow y = -\frac{3}{4}x + c \to 2 = -\frac{3}{4}\times 4 + c \Rightarrow c...$	ddM1	2.1
	$y - 2 = -\frac{3}{4}(x-4) \Rightarrow 4y - 8 = -3x + 12$ or $c = 5 \Rightarrow y = -\frac{3}{4}x + 5$ $\Rightarrow 3x + 4y = 20$*	A1*	1.1b
		(5)	
(b)	Maximum height is 9m	B1	3.4
		(1)	
			(6 marks)

Notes	
(a)	**<u>If parametric differentiation is not used in part (a) (e.g. uses Cartesian form) then only the B mark is available but see alternative below.</u>**
B1:	Uses the given Cartesian coordinates to deduce the correct value for t. If more than one value for t e.g. $t=-5$ is given and $t=-1$ is not "selected" score B0 but if just $t=-1$ is used subsequently allow recovery and score B1
M1:	Attempts to use $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ or equivalent with their differentiated equations. There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0. Both parameters must be "changed". Condone confusion with the variables e.g. referring to $\frac{dy}{dt}$ as $\frac{dy}{dx}$ if the intention is clear. This may be implied by e.g. $\frac{dy}{dt}=-3t^2,\ \frac{dx}{dt}=2(t+3), t=-1, \frac{dy}{dt}=-3, \frac{dx}{dt}=4 \Rightarrow \frac{dy}{dx}=-\frac{3}{4}$
M1:	Uses their numerical value of t (not 4) in their $\frac{dy}{dx}$ to obtain a value. Condone attempts with different values of t e.g. $t=-1$ and $t=-5$
ddM1:	Applies a correct straight line method with their value of $\frac{dy}{dx}$ which has come from an attempt to use parametric differentiation with their value of t (not 4) and with $x = 4$ and $y = 2$ correctly placed. An attempt at the equation of the normal is M0. If using $y = mx + c$ they must reach as far as $c = \dots$ **Depends on both previous M marks.**
A1*:	Correct equation as printed with no errors but condone $4y+3x=20$* Allow equivalents e.g. $20=4y+3x$* or $3x+4y=20$* This is a printed answer so there must be at least one intermediate step as shown in the main scheme.

Alternative for (a) using parametric differentiation but avoids the need for a value for t:

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=-3t^2\times\frac{1}{2(t+3)}$$

$$-3t^2\times\frac{1}{2(t+3)}=-3(1-y)^{\frac{2}{3}}\times\frac{1}{2\sqrt{x}}=-3(1-2)^{\frac{2}{3}}\times\frac{1}{2\sqrt{4}}=-\frac{3}{4}$$

or

$$-3t^2\times\frac{1}{2(t+3)}=-3\left(\sqrt{x}-3\right)^2\times\frac{1}{2\sqrt{x}}=-3(2-3)^2\times\frac{1}{2\sqrt{4}}=-\frac{3}{4}$$

or

$$-3t^2\times\frac{1}{2(t+3)}=-3(1-y)^{\frac{2}{3}}\times\frac{1}{2\left((1-y)^{\frac{1}{3}}+3\right)}=-3(1-2)^2\times\frac{1}{2\times 2}=-\frac{3}{4}$$

$$\Rightarrow y-2=-\frac{3}{4}(x-4)\Rightarrow 3x+4y=20*$$

B1: **Either** a correct expression for $\frac{dy}{dx}$ in terms of x and/or y following a correct $\frac{dy}{dx}$ in terms of t **or** for $t=-1$ seen anywhere.

M1: Attempts to use $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ with their differentiated equations.

There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0

Condone confusion with the variables e.g. referring to $\frac{dy}{dt}$ as $\frac{dy}{dx}$ if the intention is clear.

This may be implied by e.g. $\frac{dy}{dt}=-3t^2$, $\frac{dx}{dt}=2(t+3), t=-1$, $\frac{dy}{dt}=-3, \frac{dx}{dt}=4\Rightarrow\frac{dy}{dx}=-\frac{3}{4}$

M1: Attempts to express their $\frac{dy}{dx}$ which is in terms of t, in terms of x and/or y **and** uses $x=4$ and $y=2$ correctly placed in an attempt to find the gradient of the tangent.

ddM1: Applies a correct straight line method with their value of $\frac{dy}{dx}$ which has come from an attempt to use parametric differentiation with their gradient and with $x=4$ and $y=2$ correctly placed.

If using $y=mx+c$ they must reach as far as $c=\ldots$

Depends on both previous M marks.

A1*: Correct equation as printed with no errors.

This is a printed answer so there must be at least one intermediate step as shown in the main scheme.

(b)

B1: 9m or equivalent **including correct units**. Accept e.g. 9 metres, 900cm etc.

(**Q10 9MA0/02, June 2024**)

Q30.

Question	Scheme	Marks	AOs
(a)	Attempts $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{4\sec^2 t\tan t}{2\sec^2 t}(=2\tan t)$	M1 A1	1.1b 1.1b
	At $t=\dfrac{\pi}{4}$, $\dfrac{dy}{dx}=2, x=3, y=7$	M1	2.1
	Attempts equation of normal $y-7=-\dfrac{1}{2}(x-3)$	M1	1.1b
	$y=-\dfrac{1}{2}x+\dfrac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t=1+\tan^2 t\Rightarrow \dfrac{y-3}{2}=1+\left(\dfrac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y-3=2+\dfrac{(x-1)^2}{2}\Rightarrow y=\dfrac{1}{2}(x-1)^2+5$ *	A1*	2.1
		(2)	

	(b) Alternative 1:		
	$y=\dfrac{1}{2}(x-1)^2+5=\dfrac{1}{2}(2\tan t+1-1)^2+5$ $=\dfrac{1}{2}4\tan^2 t+5=2\left(\sec^2 t-1\right)+5$	M1	3.1a
	$=2\sec^2 t+3=y$*	A1	2.1
	(b) Alternative 2:		
	$x=2\tan t+1\Rightarrow t=\tan^{-1}\left(\dfrac{x-1}{2}\right)\Rightarrow y=2\sec^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)+3$ $\Rightarrow y=2\left(1+\tan^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)\right)+3$	M1	3.1a
	$\Rightarrow y=2\left(1+\left(\dfrac{x-1}{2}\right)^2\right)+3=\dfrac{1}{2}(x-1)^2+5$*	A1	2.1
	(b) Alternative 3:		
	$\dfrac{dy}{dx}=2\tan t=x-1\Rightarrow y=\int(x-1)\,dx=\dfrac{x^2}{2}-x+c$ $(3,7)\to 7=\dfrac{3^2}{2}-3+c\Rightarrow c=\dfrac{11}{2}$	M1	3.1a
	$\dfrac{x^2}{2}-x+\dfrac{11}{2}=\dfrac{1}{2}\left(x^2-2x\right)+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2-\dfrac{1}{2}+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2+5$*	A1	2.1

(c)	**Attempts the lower limit for k:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=...$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for k: $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1, 7), y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=...$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)
Notes:			

(a) <u>**Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form**</u>

M1: For the key step of attempting $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt}=2\sec^2 t,\ \frac{dy}{dt}=4\sec^2 t\tan t,\ t=\frac{\pi}{4} \Rightarrow \frac{dx}{dt}=4,\ \frac{dy}{dt}=8 \Rightarrow \frac{dy}{dx}=2$

A1: $\frac{dy}{dx}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}(=2\tan x)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of x, y and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x=3,\ y=7,\ \frac{dy}{dx}=2\left(\text{or their }\frac{dy}{dx}\text{ at } t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

M1: For a correct attempt at the normal equation using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y=mx+c$ they must reach as far as a value for c using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ all correctly placed.

A1*: Proceeds with a clear argument to the given answer with no errors.

(b)

M1: Attempts to use $\sec^2 t = 1 + \tan^2 t$ oe to obtain an equation involving y and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right) + 3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right) + 3$ for M1 and then

$y = \frac{1}{2}(x-1)^2 + 5$* for A1

A1*: Proceeds with a clear argument to the given answer with no errors

Alternative 1:

M1: Uses the given result, substitutes for x and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the y parameter and makes a (minimal) conclusion e.g. " $= y$" QED, hence proven etc.

Alternative 2:

M1: Uses the x parameter to obtain t in terms of arctan, substitutes into y and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the given answer with no errors

Alternative 3:

M1: Uses $\frac{dy}{dx}$ from part (a) to express $\frac{dy}{dx}$ in terms of x, integrates and uses (3, 7) to find "c" to reach a Cartesian equation.

A1: Proceeds with a clear argument to the given answer with no errors

Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)

(c)

M1: A full attempt to find the **lower** limit for k.

$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11-2k) = 0 \Rightarrow b^2 - 4ac = 1 - 4(11-2k) = 0 \Rightarrow k = ...$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac ... 0$

e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for k.

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using calculus for lower limit:

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{dy}{dx} = x - 1,\ x - 1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2} - 1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{dy}{dx} =$ "a linear expression in x", sets $= -\frac{1}{2}$, solves a linear equation to find x and then substitutes into the given result in (b) to find y and then uses $y = -\frac{1}{2}x + k$ to find a value for k. **A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using parameters for lower limit:

$$y=-\frac{1}{2}x+k \Rightarrow 2\sec^2 t+3=-\frac{1}{2}(2\tan t+1)+k$$

$$\Rightarrow 2\left(1+\tan^2 t\right)+3=-\frac{1}{2}(2\tan t+1)+k \Rightarrow 2\tan^2 t+\tan t+5.5-k=0$$

$$b^2-4ac=0 \Rightarrow 1-4\times 2(5.5-k)=0 \Rightarrow k=\frac{43}{8}$$

Score **M1** for substituting parametric form of x and y into $y=-\frac{1}{2}x+k$, uses $\sec^2 t=1+\tan^2 t$ rearranges to 3TQ form and attempts $b^2-4ac...0$ or e.g. $b^2-4ac>0$ or $b^2-4ac<0$ correctly to find a value for k.

A1: $k=\frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k>\frac{43}{8}$, $k<\frac{43}{8}$

M1: A full attempt to find the **upper** limit for k. This requires an attempt to find the value of x and the value of y using $t=-\frac{\pi}{4}$, the substitution of these values into $y=-\frac{1}{2}x+k$ and solves for k.

A1: $k=\frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for k: $\frac{43}{8}<k\leqslant\frac{13}{2}$

Allow equivalent notation e.g. $\left(k\leqslant\frac{13}{2} \text{ and } k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cap k>\frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k\leqslant\frac{13}{2},\ k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cup k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2} \text{ or } k>\frac{43}{8}\right)$ and do not allow if in terms of x.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.

(**Q16 9MA0/02, June 2022**)

Q31.

Question	Scheme	Marks	AOs
(a)	$\text{gf}(e^2) = \text{g}(3) = \dfrac{4\times3+3}{2\times3+1}$	M1	1.1b
	$=\dfrac{15}{7}$	A1	1.1b
		(2)	
(b)	$\text{g}'(x) = \dfrac{4(2x+1)-2(4x+3)}{(2x+1)^2}$	M1	3.1a
	$\text{g}'(x) = \dfrac{-2}{(2x+1)^2}$	A1	1.1b
	Requires • Correct $\text{g}'(x)$ • Statement $\text{g}'(x) < 0$ oe • Conclusion e.g. "proven", "true", "decreasing"	A1	2.1
		(3)	
(c)	Achieves either boundary $\frac{3}{2}\ln 2$ or $\frac{3}{2}\ln 3$	M1	1.1b
	$\frac{3}{2}\ln 2 < \text{fg} < \frac{3}{2}\ln 3$ or e.g. $\frac{3}{2}\ln 2 < y < \frac{3}{2}\ln 3$	A1	2.5
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts to apply the operations in the correct order. E.g. substitutes $\frac{3}{2}\ln e^2$ or 3 into g

A1: Achieves $\frac{15}{7}$ or exact equivalent.

(b)

M1: Attempts the quotient rule (or product rule) to obtain the correct form or divides to reach the form $A + \dfrac{B}{2x+1}$ where A and B are positive constants.

A1: $\text{g}'(x) = \dfrac{-2}{(2x+1)^2}$ OR $\text{g}(x) = 2 + \dfrac{1}{(2x+1)}$

A1: See scheme. For the alternative method look for

- Correct $\text{g}(x) = 2 + \dfrac{1}{(2x+1)}$
- Correct statement. As x increases (or $x \to \infty$) $\dfrac{1}{(2x+1)}$ decreases (or $\to 0$) so $\text{g}(x)$ is a decreasing function

(c)

M1: Achieves either exact boundary $\frac{3}{2}\ln 2$ or $\frac{3}{2}\ln 3$

A1: Correct range written with correct notation and with strict inequalities

Q32.

Question	Scheme	Marks	AOs
(a)	$\frac{dV}{dh}=200$ oe e.g. $\frac{dh}{dV}=\frac{1}{200}$	B1	1.1b
	$\left(\frac{dh}{dt}=\right)\frac{dh}{dV}\times\frac{dV}{dt}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow ...h^{\frac{3}{2}}=\lambda t\{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times0+c\Rightarrow c=1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times8+\text{"}1.152\text{"}\Rightarrow\lambda=0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
	(b) Alternative:		
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\frac{dt}{dh}=\frac{\sqrt{h}}{\lambda}\Rightarrow t=...h^{\frac{3}{2}}(+c)$	M1	1.1b
	$t=\frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	A1	1.1b
	$0=\frac{2(1.44)^{\frac{3}{2}}}{3\lambda}+c$ and $8=\frac{2(3.24)^{\frac{3}{2}}}{3\lambda}+c$ $\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ or $c=...\left(-\frac{64}{19}\right)$	dM1	3.4
	$\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ and $c=...\left(-\frac{64}{19}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
(c)	$5^{\frac{3}{2}}=0.513t+1.728\Rightarrow t=...$	M1	3.4
	$(t=)$ awrt18.4 min	A1	3.2a
		(2)	
			(10 marks)

Notes

(a)

B1: For $\frac{dV}{dh} = 200$ stated or used – may be implied by their chain rule attempt

M1: Requires:

- $\frac{dV}{dh} = p,\ \ p > 1$
- $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ (or a suitable letter for k, which may be λ, but must **not** be a number)
- **application** of the correct chain rule $\left(\frac{dh}{dt} = \right)\frac{dh}{dV} \times \frac{dV}{dt}$ or any equivalent with $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or $\pm\frac{1}{k\sqrt{h}}$ and their $\frac{dV}{dh}$ correctly placed. So $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{200k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores M0 as $\frac{dh}{dV}$ is incorrectly placed.

A1*: A rigorous argument with all steps shown and simplifies to achieve $\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$ with no errors.

Do not allow the use of λ for both constants. Allow use of e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ for full marks.

e.g. $\frac{dV}{dh} = 200,\ \ \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{\lambda}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1M1A0* *unless* e.g. "let $\lambda = \frac{\lambda}{200}$ " seen.

Allow correct work leading to e.g. $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} \to \frac{\lambda}{\sqrt{h}}$ or $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ so $\lambda = \frac{k}{200}$

There must be an attempt to link the $\frac{dh}{dt}$ with the $\frac{\lambda}{\sqrt{h}}$ which may be missing an = sign.

Allow an argument with $\frac{dV}{dt} = -\frac{k}{\sqrt{h}}$ e.g. $\frac{dV}{dh} = 200,\ \ \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times -\frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$

Withhold this A mark if there are notational errors e.g. $\frac{dV}{dt} = 200,\ \ \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1(implied)M1A0*

(b) Note that some candidates may work with e.g. $\lambda = \frac{k}{200}$ or e.g. $\lambda = 200k$ which is acceptable.

Candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ then only the first 2 method marks are available (see note below). Condone use of other variables if the intention is clear but the final answer must be in terms of h and t.

M1: Separates the variables and integrates to obtain an equation of the form $..h^{\frac{3}{2}} = \lambda t\{+c\}$ oe

The constant of integration is not needed for this mark.

A1: $\frac{2}{3}h^{\frac{3}{2}} = \lambda t(+c)$ oe. The constant of integration is not needed for this mark.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Substitutes $t = 0$ and $h = 1.44$ and attempts to find c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find "c" as long as they are using $t = 0$ and $h = 1.44$

May be implied by their value of c.

ddM1: Substitutes $t = 8$ and $h = 3.24$ and their c and attempts to find λ. Do not be concerned with the "processing" to find λ as long as they are using $t = 8$ and $h = 3.24$.

It is dependent on both previous method marks.

A1: Correct equation in the correct form from correct work. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Note candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ can use either $t = 0$ and $h = 1.44$ **or** $t = 8$ and $h = 3.24$ to find their constant of integration.

(b)Alternative:

M1: Finds the reciprocal of both sides and integrates to obtain an equation of the form $t = ...h^{\frac{3}{2}}(+c)$

A1: $t = \frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe. The constant of integration is not needed for this mark.

dM1: Substitutes $t = 0$ and $h = 1.44$ **and** substitutes $t = 8$ and $h = 3.24$ **and** attempts to find λ **or** c.
It is dependent on the previous method mark.
Do not be concerned with the "processing" to find λ or c as long as they are using $t = 0$ and $h = 1.44$ and $t = 8$ and $h = 3.24$ and reach a value for λ or c. May be implied by their value(s).

ddM1: Complete attempt to find λ **and** c. **It is dependent on both previous method marks.**
Do not be concerned with the "processing" to find λ and c as long as they are using $t = 0$ and $h = 1.44$ and $t = 8$ and $h = 3.24$.

A1: Correct equation in the correct form. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$
Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

<u>Special Case:</u>

Some candidates are using the given equation in part (b) to find the value of A and the value of B using the given conditions. May score a maximum of 00110. This should be marked as follows:

M0A0: (No attempt to integrate)
M1: Substitutes $t = 0$ and $h = 1.44$ to find a value for B
dM1: Substitutes $t = 8$ and $h = 3.24$ with their value of B to find a value for A
A0: Since they have not used the given model.
(Allow full recovery in (c) if this equation is correct)

(c)

M1: Attempts to substitute $h = 5$ into their equation which must be of the form $h^{\frac{3}{2}} = At + B$ or possibly a rearranged equation e.g. $h^{\frac{1}{2}} = \sqrt[3]{At + B}$ with values of A and B leading to a value for t.
Do not be concerned about the processing as long as they use $h = 5$ and obtain a value for t even if t is negative.

A1: Awrt 18.4 minutes **following a correct equation in (b)**.
The units are required but allow e.g. min, mins, but not just 18.4 and not m (which means metres)
Allow e.g. 18 minutes 25 seconds or 18 mins 26 secs
Note this may follow A0 in part (b) as they may have rearranged incorrectly in (b) but use a correct equation in (c) e.g. $\frac{2}{3}h^{\frac{3}{2}} = \frac{171}{500}t + \frac{144}{125}$, $h^{\frac{1}{2}} = \sqrt[3]{\frac{513}{1000}t + \frac{216}{125}}$ or may come from the special case.
Apply isw following a correct time and units, e.g., 18.4 followed by 18 mins.

(**Q11 9MA0/02, June 2023**)

Q33.

Question	Scheme	Marks	AOs
(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x) =\right) \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \dfrac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x} \pm ...) = ...e^{3x} \pm ...$	M1	1.1b
	$\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \dfrac{2e^3-4}{e^3+4} = 1.5017756...$	M1	1.1b
	$x_2 =$ awrt 1.502	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \dfrac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
			(13 marks)

Notes
(a)
M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.
A1: $k\left(xe^x + e^x\right)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)
B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right)=\right)\frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1:	Attempts to use the quotient rule. It is dependent on the previous method mark. Score for achieving an expression of the form $(\mathrm{f}'(x)=)\ \dfrac{(\mathrm{e}^{3x}-2)^{\frac{1}{2}}("7"x\mathrm{e}^x+"7"\mathrm{e}^x)-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\times"7"x\mathrm{e}^x}{\mathrm{e}^{3x}-2}$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1) If it is clear that the quotient rule has been applied the wrong way round then score M0. Alternatively, applies the product rule. Score for achieving an expression of the form $(\mathrm{f}'(x)=)\ (\mathrm{e}^{3x}-2)^{-\frac{1}{2}}("7"x\mathrm{e}^x+"7"\mathrm{e}^x)-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\times"7"x\mathrm{e}^x$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1) Do not condone invisible brackets.
A1:	$(\mathrm{f}'(x)=)\ \dfrac{7\mathrm{e}^x\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression. **You may need to check to see if (a) is continued after other parts for evidence of this.** Condone the lack of $\mathrm{f}'(x)=$ on the left hand side or allow the use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(7x\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)=7\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^x\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$

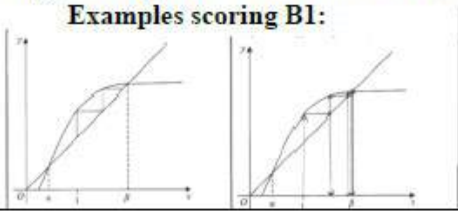
$$\Rightarrow\frac{\left(7\mathrm{e}^x+7x\mathrm{e}^x\right)(\mathrm{e}^{3x}-2)+7x\mathrm{e}^x\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}=\frac{7\mathrm{e}^x\left(\mathrm{e}^{3x}-2+x\mathrm{e}^{3x}-2x-\frac{3}{2}x\mathrm{e}^{3x}\right)}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}\Rightarrow\frac{7\mathrm{e}^x\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$$

M1:	Attempts the product rule on $x\mathrm{e}^x\to...x\mathrm{e}^x\pm...\mathrm{e}^x$ which may be seen within the expression $...\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\pm..x\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+...$ simplified or unsimplified.
A1:	$k\left(x\mathrm{e}^x+\mathrm{e}^x\right)$ which may be seen within the expression $k\left(\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+x\mathrm{e}^x(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)+...$ simplified or unsimplified.
B1:	$\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression....$+k\left(x\mathrm{e}^x\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.
dM1:	A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.
A1:	As above in main scheme notes.
(b)	**Note that if they do not have values $A=-4$, $B=-4$ in (a) (which may be seen later) then maximum score is M1A0***
M1:	Sets their $\mathrm{e}^{3x}(2-x)-4x-4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of "-4" and "-4"
A1*:	Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1}=\dfrac{2\mathrm{e}^{3x_n}-4}{\mathrm{e}^{3x_n}+4}$ then isw

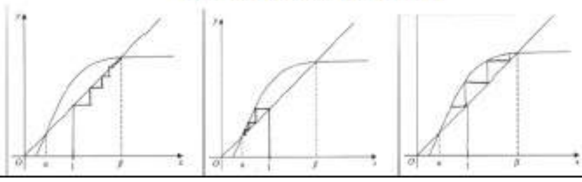
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

  (If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)
- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

  (If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$", "so rounds to 0.432". Do not allow "hence root"

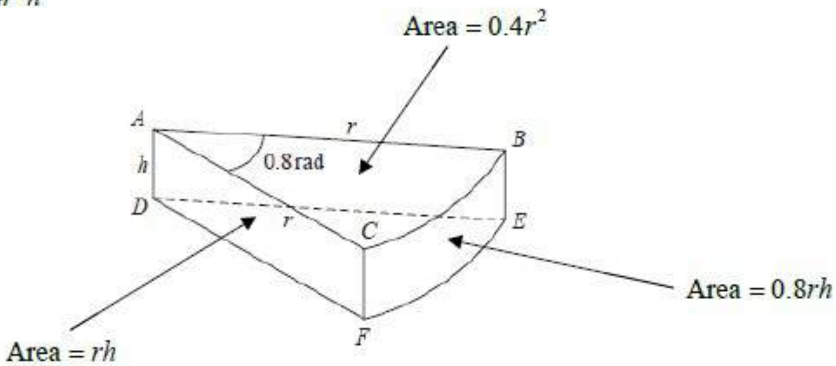
(**Q15 9MA0/01, June 2023**)

Q34.

Question	Scheme	Marks	AOs
(a)	Sets up an allowable equation using volume $=240$ E.g. $\frac{1}{2}r^2\times 0.8h=240\Rightarrow h=\frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h=\frac{600}{r^2}$ into $(S=)\frac{1}{2}r^2\times 0.8+\frac{1}{2}r^2\times 0.8+2rh+0.8rh$	dM1	3.4
	$S=0.8r^2+2.8rh=0.8r^2+2.8\times\frac{600}{r}=0.8r^2+\frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right)=1.6r-\frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr}=0\Rightarrow r^3=1050$ $r=$ awrt 10.2	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right)=1.6+\frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2}=1.6+\frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2}=5>0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)

Notes:

Volume $=0.4r^2h$



Total surface area $=2rh+0.8r^2+0.8rh$

(a)

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = ...$ or $rh = ...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = ...$ or $rh = ...$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n$, $m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r =$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} =$ awrt $5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r =$ awrt 10 and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $\underline{r > 0}$, so minimum value of S. For consistency it is also dependent upon having achieved $r =$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark

Q35.

Question	Scheme	Marks	AOs
	$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$; as $h \to 0, \frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h)-\cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
			(5 marks)

Notes for Question	
B1:	Gives the correct fraction such as $\frac{\cos(\theta+h)-\cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta)-\cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h)-\cos\theta}{(\theta+h)-\theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of *curve* is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{d}{d\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $f'(\theta)$ where $f(\theta) = \cos\theta$ or $\frac{dy}{d\theta}$ where $y = \cos\theta$ used in place of $\frac{d}{d\theta}(\cos\theta)$

Notes for Question Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{d}{d\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{d}{d\theta} = ...$ • Defining $f(x) = \cos\theta$ and applying $f'(x) = ...$ • $\frac{d}{dx}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{d}{d\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{d}{d\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h \to 0,\ \sin h \to h,\ \cos h \to 1$ to give e.g. $\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \lim_{h\to 0}\left(-(1)\sin\theta + 0\cos\theta\right) = -\sin\theta$. So for not removing $\lim_{h\to 0}$ when the limit was taken is final A0
Note:	<u>**Alternative Method:**</u> Considers $\frac{\cos(\theta+h) - \cos(\theta-h)}{(\theta+h)-(\theta-h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

(**Q09 9MA0/02, June 2018**)

Q36.

Question	Scheme	Marks	AOs
(a)	Attempts $\text{f}(5) = -18$ and $\text{f}(6) = (+)\,4$	M1	1.1b
	States change of sign, function continuous so root between	A1	2.1
		(2)	
(b)	$p = \ln 2$ (so $\text{f}'(x) = 2^x \ln 2 - 10$)	B1	1.2
		(1)	
(c)	Attempts $x_1 = 6 - \dfrac{4}{64\ln 2 - 10}$	M1	1.1b
	Awrt 5.88	A1	1.1b
		(2)	
(d)	Sets $\text{f}'(x) = 2^x \ln 2 - 10 = 0 \Rightarrow 2^x = \dfrac{10}{\ln 2} \Rightarrow x = \ldots$	M1	1.1b
	$\Rightarrow x =$ awrt 3.85	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts both $\text{f}(5) = -18$ and $\text{f}(6) = (+)\,4$ with at least one correct.

A1: Completes the argument.
Requires

- both values correct
- gives full reason including "change of sign" (o.e) and "continuity"
- a minimal conclusion, e.g. root, tick, α lies in the interval $[5, 6]$

Accept equivalent statements for sign change e.g. $\text{f}(6) > 0$, $\text{f}(5) < 0$ e.g. $\text{f}(5) \times \text{f}(6) < 0$, $\text{f}(5) < 0 < \text{f}(6)$, "one negative one positive", "there is a change of sign"

A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(b)

B1: $p = \ln 2$ or $\text{f}'(x) = 2^x \ln 2 - 10$ Condone $p = \log 2$ unless $p = \log_{10} 2$ is implied by subsequent work.

(c)

M1: Attempts $x_1 = 6 - \dfrac{\text{f}(6)}{\text{f}'(6)}$ following through on their f (6) and f′(6)

May be implied by their value if no working is shown.

A1: Awrt 5.88

(d)

M1: Sets $p \times 2^x - 10 = 0$ with their positive numerical value of p (which may be 1) and uses correct processing to find a value for x e.g. $2^x = \dfrac{10}{p} \Rightarrow x = \log_2 \dfrac{10}{p}$ or $2^x = \dfrac{10}{p} \Rightarrow x = \dfrac{\log \frac{10}{p}}{\log 2}$ or $2^x = \dfrac{10}{p} \Rightarrow x = \dfrac{\ln \frac{10}{p}}{\ln 2}$

A1: awrt 3.85

Q37.

Question	Scheme	Marks	AOs
(a)	$2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=...$	M1	1.1b
	$54-81+15+k=0 \Rightarrow k=12$* or $-12+k=0 \Rightarrow k=12$*	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3-9(3)^2+5(3)+12=0$	M1	1.1b
	$54-81+15+12=0$ Hence $k=12$ *	A1*	1.1b
		(2)	
(b)	$\int(2x^3-9x^2+5x+12)\,dx ... x^4 \pm ... x^3 \pm ... x^2 \pm ... x \pm ...$	M1	3.1a
	$\frac{1}{2}(3)^4-3(3)^3+\frac{5}{2}(3)^2+12(3)+c=-10 \Rightarrow c=...$	dM1	1.1b
	$(0, -28)$	A1	2.2a
		(3)	
			(5 marks)
Notes			

(a) **Mark (a) and (b) together**

M1: Substitutes $x=3$ completely into the given derivative, sets $=0$ and solves for k.

e.g., $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=...$

May be implied by e.g. $54-81+15+k=0 \Rightarrow k=...$ with at least 2 correctly evaluated powers.

A1*: Obtains $k=12$ with no errors seen and sufficient working shown. As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow -12+k=0 \Rightarrow k=12$ **or** $54-81+15+k=0 \Rightarrow k=12$ **or** $2\times27-9\times9+5(3)+k=0 \Rightarrow k=12$

But $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=12$ scores M1A0 for lack of working

Note that some are just writing the expression for $\frac{dy}{dx}$, then write "sub in $x=3$" but don't actually show 3 substituted in and then go on to write $-12+k=0$ leading to $k=12$ scores M0A0*.

Alternative:

M1: Substitutes $x=3$ and $k=12$ into the given derivative and attempts to evaluate

A1*: Correct work to obtain an answer of 0 with a (minimal) conclusion e.g., tick, hence proven etc.

As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+12=54-81+15+12=0$ ✓

(b)

M1: Attempts to integrate. Evidence can be taken for integrating to obtain at least 2 from:

$2x^3 \to ...x^4$ or $-9x^2 \to ...x^3$ or $5x \to ...x^2$ or $12 \to ...x$ where ... are constants

dM1: Substitutes $x=3$ into their integrated expression that includes a constant of integration, sets this equal to ±10 and proceeds to find their constant. **Depends on the previous mark.**

If the substitution is not shown this mark may be implied by their value for c or by their equation e.g., $18+c=\pm10$

A1: $(0, -28)$ Condone -28 or $y=-28$ but not just $c=-28$. There must be no other values or points.

Condone $(-28, 0)$ following $y=-28$

<u>Beware of circular arguments which avoid doing part (a) e.g.</u>

Integration is used on the given derivative to give y in terms of x, k and c

$(3, -10)$ is substituted to give $3k+c=8$

Part (b) is then done first using $k=12$ to find $c=-28$

This is then substituted into $3k + c = 8$ to give $k = 12$
This scores (a) M0A0 (b) M1dM1A1 (if −28 is identified as the intercept)

Alternative for part (a) using algebraic division:

$$\begin{array}{r} 2x^2 - 3x \quad - 4 \\ x-3\overline{)2x^3 - 9x^2 + 5x + k} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ -3x^2 + 5x \quad \\ \underline{-3x^2 + 9x} \quad \\ -4x + k \\ \underline{-4x + 12} \\ k - 12 \text{ (or 0)} \end{array}$$

leading to $k - 12 = 0$ and then $k = 12$.

M1: Attempts to divide the given cubic by $(x-3)$ and proceeds as far as a remainder set = 0. Requires at least $2x^2 \pm 3x$.

A1*: Obtains $k = 12$ with no errors seen and sufficient working. Their algebraic division needs to be correct but allow them to have either $k - 12$ or 0 as their "remainder". If their remainder is given in their working as 0 they may proceed directly to $k = 12$.

(**Q05 9MA0/02, June 2023**)

Q38.

Question	Scheme		Marks	AOs
(a)	$(\mathrm{f}'(x)=)\ \dfrac{\lambda x(1+x^2)^2-\mu x(1-x^2)(1+x^2)}{(1+x^2)^4}$		M1	1.1b
	$=\dfrac{-2x(1+x^2)^2-4x(1-x^2)(1+x^2)}{(1+x^2)^4}$		A1	1.1b
	e.g. $\dfrac{-2x(1+x^2)-4x(1-x^2)}{(1+x^2)^3}=0$ $\Rightarrow (1+x^2)+2(1-x^2)=0$ $\Rightarrow 3-x^2=0$	or e.g. $=\dfrac{-2x(1+x^2)\left[(1+x^2)+2(1-x^2)\right]}{(1+x^2)^4}$ $=\dfrac{-2x(1+x^2)(3-x^2)}{(1+x^2)^4}$	dM1	3.1a
	e.g. $3-x^2=0$ or $2x(1+x^2)(x^2-3)=0$ $\Rightarrow x=...\Rightarrow y=...$		ddM1	2.1
	$\left(-\sqrt{3},-\dfrac{1}{8}\right)$		A1	2.3
			(5)	

Notes	
(a)	**There may be other valid ways to differentiate or to solve the resulting equation.**
M1:	Attempts the quotient rule to achieve $\dfrac{\pm\lambda x\left(1+x^2\right)^2 \pm \mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$ but condone $\dfrac{\pm\lambda x\left(1+x^2\right)^2 \pm \mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^2}$ provided an incorrect quotient rule is not seen. Alternatively, attempts the product rule on $\text{f}(x)=\left(1-x^2\right)\left(1+x^2\right)^{-2}$ to achieve $\pm\lambda x\left(1+x^2\right)^{-2} \pm \mu x\left(1-x^2\right)\left(1+x^2\right)^{-3}$ There may be attempts at splitting the numerator to $\text{f}(x)=\left(1+x^2\right)^{-2}-x^2\left(1+x^2\right)^{-2}$ which should differentiate to $\pm\lambda x\left(1+x^2\right)^{-3} \pm \mu x\left(1+x^2\right)^{-2} \pm \gamma x^3\left(1+x^2\right)^{-3}$ In **all** cases, do not penalise signs of λ, μ and/or γ unless an incorrect quotient rule or an incorrect product rule is stated. Any occurrences of $\left(1+x^2\right)^2$ may be replaced with $1+2x^2+x^4$ There is no need for a LHS e.g. $\text{f}'(x)=$ to be present so ignore an incorrect LHS. Invisible brackets may be recovered/implied by later work.
A1:	Correct differentiation. May be unsimplified. Ignore absence of (or incorrect) LHS. The correct derivative using the quotient rule is $=\dfrac{-2x\left(1+x^2\right)^2-4x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$ o.e. using the product rule is $-2x\left(1+x^2\right)^{-2}-4x\left(1-x^2\right)\left(1+x^2\right)^{-3}$ o.e. and splitting the numerator is $-4x\left(1+x^2\right)^{-3}-2x\left(1+x^2\right)^{-2}+4x^3\left(1+x^2\right)^{-3}$ o.e. You may need to check carefully for equivalent derivatives and ISW after a correct expression is seen. Note that some candidates may set = 0 at the start and may omit the denominator as a result – send to review if you are unsure in such cases.
dM1:	Attempts to reduce the expression to a suitable form so that the roots can be found, either by cancelling a factor of $\left(1+x^2\right)$ and simplifying the other brackets e.g. $\dfrac{-2x\left(1+x^2\right)-4x\left(1-x^2\right)}{\left(1+x^2\right)^3} \rightarrow$ e.g. $\dfrac{-6x+2x^3}{\left(1+x^2\right)^3}$ or $\dfrac{2x\left(x^2-3\right)}{\left(1+x^2\right)^3}$ or $2x\left(x^2-3\right)\ (=0)$ or by attempting to take x, $\left(1+x^2\right)$ or $\pm 2x\left(1+x^2\right)$ out as a factor and simplify the other brackets. If they only take out a factor of x then they must have $x\left(\pm Ax^4 \pm Bx^2 \pm C\right)$

e.g. $\dfrac{-2x(1+x^2)\left[(1+x^2)+2(1-x^2)\right]}{(1+x^2)^4} \to$ e.g. $\dfrac{-2x(1+x^2)(3-x^2)}{(1+x^2)^4}$ or $\dfrac{x(2x^4-4x^2-6)}{(1+x^2)^4}$

Depends on the first M mark.
Note: they might attempt to expand the brackets first, or set = 0 and multiply through by $(1+x^2)^k$ and/or divide through by x or $2x$ at any stage.

Do not be concerned if any factors of $(1+x^2)$, x or 2 go "missing" during their work.

The = 0 does not explicitly need to be seen.

Look for $\pm\dfrac{2x(1+x^2)(Ax^2+B)}{(1+x^2)^k}$ or $\pm\dfrac{(1+x^2)(Ax^3+Bx)}{(1+x^2)^k}$ or $\pm\dfrac{x(Ax^4+Bx^2+C)}{(1+x^2)^k}$ where

- k may be 0, 1, 2, 3 or 4
- A and B (and C if present) are non-zero
- any of $2x$ or $(1+x^2)$ in the numerator and/or $(1+x^2)^k$ in the denominator may be absent

ddM1: Attempts to solve their numerator set = 0 using a valid non-calculator method **and** uses a solution for x to find a corresponding value for y.
The substitution may be implied by their value of y but, if not, the substitution must be seen.

- For either Ax^2+B or Ax^3+Bx, we require $A \neq B$ and $A \times B < 0$ From either expression they can write down their $x = \pm\sqrt{\dfrac{-B}{A}}$ without working.
- For Ax^4+Bx^2+C they must show a valid non-calculator method for solving the quartic, treating it as a quadratic in x^2 and using the usual rules for solving a quadratic algebraically, e.g. $a = x^2 \to 2a^2-4a-6(=0) \to (a+1)(2a-6)(=0) \to x = \pm\sqrt{3}$. They must reach a value for x (and not just x^2) as well as finding a value for y.

Dependent on both previous method marks. They must use a value for x that is not −1, 0 or 1.

A1: Deduces the correct **exact** coordinates for $P\left(-\sqrt{3}, -\dfrac{1}{8}\right)$ o.e. $\left(-\sqrt{3}, -0.125\right)$

Requires all the previous marks to have been scored.

Condone $x = -\sqrt{3}$, $y = -0.125$ or e.g. $-\sqrt{3}, -\dfrac{1}{8}$

If there is more than one pair of coordinates given, then the correct coordinates must be clearly selected or any others clearly rejected.

Ignore any mistakes that occur if they multiply out the denominator $(1+x^2)^4$ after differentiation.

Question	Scheme		Marks	AOs
(b)	$x=-1 \Rightarrow \alpha = -\dfrac{\pi}{4}$ and $x=1 \Rightarrow \beta = \dfrac{\pi}{4}$		B1	2.2a
	$\dfrac{dx}{d\theta} = \sec^2\theta$		B1	1.1b
	$\displaystyle\int \frac{1-x^2}{(1+x^2)^2}\,dx = \int \frac{1-\tan^2\theta}{(1+\tan^2\theta)^2}\text{"}\sec^2\theta\text{"}\,d\theta$ or $= \displaystyle\int \frac{1-\tan^2\theta}{\sec^4\theta}\text{"}\sec^2\theta\text{"}\,d\theta$		M1	1.1b
	$=\int(1-\tan^2\theta)\cos^2\theta\,d\theta$ $=\displaystyle\int\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta\,d\theta$ $=\int(\cos^2\theta-\sin^2\theta)\,d\theta$	$=\displaystyle\int\frac{2-1-\tan^2\theta}{\sec^2\theta}\,d\theta$ $=\displaystyle\int\left(\frac{2}{\sec^2\theta}-\frac{1+\tan^2\theta}{\sec^2\theta}\right)d\theta$ $=\int(2\cos^2\theta-1)\,d\theta$	dM1	3.1a
	$=\int\cos 2\theta\,d\theta$ *		A1*	2.1
			(5)	

Notes
(b) **Mark parts (b) and (c) together.**
B1: Deduces the correct limits for the integral in θ which may be seen separately as side working or within their integral work. This mark cannot be scored working in degrees. Allow $\dfrac{3\pi}{4}$ instead of $-\dfrac{\pi}{4}$ but not decimal approximations.
B1: $\dfrac{dx}{d\theta}=\sec^2\theta$ or equivalent e.g. $\dfrac{dx}{d\theta}=1+x^2$ or $\dfrac{\cos^2\theta--\sin^2\theta}{\cos^2\theta}$ (coming from $x=\dfrac{\sin\theta}{\cos\theta}$) but must be correct, so not e.g. $\dfrac{dx}{d\theta}=\sec^2 x$ unless recovered.
M1: Makes a complete attempt at using the substitution $x=\tan\theta$ Requires: • An attempt at using $\dfrac{dx}{d\theta}$ to replace dx with dθ either way round, so if $\dfrac{dx}{d\theta}=g(\theta)$ allow either $dx = g(\theta)\{d\theta\}$ **or** $dx=\dfrac{1}{g(\theta)}\{d\theta\}$. $\dfrac{dx}{d\theta}$ must be a function of θ and not a constant.   • All terms in x replaced with $\tan\theta$ (or $1+x^2$ with $\sec^2\theta$)   Condone the absence of dθ but dx must no longer be present.   Condone if they fail to square the denominator or e.g. a slip in missing a θ Use of notation such as $d(\tan\theta)$ is correct but does not score the M1 until replaced with $\sec^2\theta\{d\theta\}$ or their derivative of $\tan\theta$ (which must not be a constant multiple of $\tan\theta$).
dM1: Uses trigonometric identities e.g. $\pm1\pm\tan^2\theta=\pm\sec^2\theta$, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\sec\theta=\dfrac{1}{\cos\theta}$, $\pm\cos^2\theta\pm\sin^2\theta=\pm1$ to simplify the integral to an expression of the form $\pm a\sin^2\theta\pm b\cos^2\theta\pm c$ where one of a, b or c may be 0.   The algebra should essentially be correct but condone e.g. sign slips or errors collecting terms.

	Alternatively, e.g. $\int \frac{1-\tan^2\theta}{1+\tan^2\theta} \mathrm{d}\theta = \int \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \mathrm{d}\theta = \int \frac{\cos^2\theta - \sin^2\theta}{1} \mathrm{d}\theta$ They must have replaced $\mathrm{d}x$ with $\mathrm{d}\theta$ the correct way round for this mark, so if $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \mathrm{g}(\theta)$ then they must have $\mathrm{d}x = \mathrm{g}(\theta)\{\mathrm{d}\theta\}$ and **not** $\mathrm{d}x = \frac{1}{\mathrm{g}(\theta)}\{\mathrm{d}\theta\}$ It is acceptable to replace e.g. $\tan^2\theta\cos^2\theta$ with $\sin^2\theta$ Dependent on the previous method mark.
A1*:	cso Arrives at $\int \cos 2\theta \, \mathrm{d}\theta$ with sufficient working shown and no incorrect work ignoring limits. The final line must be fully correct, including the integral sign and $\mathrm{d}\theta$ (ignoring limits). Note that this may be seen in part (c) and may score the mark. All trigonometric identities must be fully correct. Condone one or two slips in a missing θ but not frequent omissions. Condone missing integral signs in their intermediate work, but it must be present on the final line. $\mathrm{d}x$ must be replaced with $\mathrm{d}\theta$ at some stage before the final line but does not need to be present in every intermediate line of working. Condone notational errors throughout e.g. $\sin\theta^2$ provided they are recovered. This mark is independent from any work to do with limits, i.e., B0B1M1dM1A1* is possible.

Question	Scheme	Marks	AOs
(c)	$\int \cos 2\theta \, d\theta = \frac{1}{2}\sin 2\theta$	M1	1.1b
	$\left[\frac{1}{2}\sin 2\theta\right]_{"-\frac{\pi}{4}"}^{"\frac{\pi}{4}"} = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right) - \frac{1}{2}\sin\left(2\times -\frac{\pi}{4}\right)$	dM1	1.1b
	$=1$	A1	2.1
		(3)	
		(13 marks)	

Notes

(c) **Mark parts (b) and (c) together.**

M1: $\int \cos 2\theta \, d\theta \to \pm\frac{1}{2}\sin 2\theta \; \{+c\}$

dM1: Substitutes their changed limits (not 1 and −1) into $\pm\frac{1}{2}\sin 2\theta$ and subtracts either way round.

May be implied e.g. $\frac{1}{2}+\frac{1}{2}$ provided they have the correct limits.

Dependent on the previous method mark.

A1: Area of 1 found following correct work. This mark requires clear substitution of the correct limits into $\frac{1}{2}\sin 2\theta$ the correct way round. If they have the limits the wrong way round and achieve an answer of −1 they cannot just make the answer positive for this mark.

They may use limits from 0 to $\frac{\pi}{4}$ and multiply their result by 2.

$\frac{3\pi}{4}$ may be used in place of $-\frac{\pi}{4}$ as the lower limit which is acceptable.

Condone any spurious integral symbol accompanying $\frac{1}{2}\sin 2\theta$

The substitution may be implied by e.g. $\frac{1}{2}--\frac{1}{2}$ or $\frac{1}{2}\sin\left(\frac{\pi}{2}\right)-\frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$ but must follow sight of $\frac{1}{2}\sin 2\theta$

Condone use of limits −45 and 45 (degrees) the correct way round for full marks in part (c).
Note that use of a calculator on the original integral will give the correct answer. An answer of 1 scores no marks without evidence of scoring both method marks above including the substitution.

(**Q15 9MA0/02, June 2025**)

Q39.

Question	Scheme	Marks	AOs
(a)	Either $x \le -1$ or $2 \le x \le 5$	M1	2.2a
	Both $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$ o.e.	A1	2.5
		(2)	
(b)	States $(y =)\ \alpha(x+1)^2(x-5)^2$ or $(\mathrm{f}(x) =)\ \alpha(x+1)^2(x-5)^2$	M1	1.1b
	Substitutes $(0, -75)$ into $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α	dM1	3.1a
	$y = -3(x+1)^2(x-5)^2$ o.e.	A1	2.1
		(3)	
(c)	Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2 \Rightarrow y = (-243)$	M1	2.1
	$0 < k < 243$	A1ft	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Either

- $x \le -1$ o.e. e.g. $-1 \ge x$
- $2 \le x \le 5$ o.e.

but condone use of strict inequalities anywhere for this mark.
e.g. $2 < x < 5$ or $2 < x \le 5$ or $2 \le x < 5$ May also write e.g. $x < 5$ and $x > 2$ which scores M1 but not "$x < 5$ or $x > 2$"

Allow interval notation such as e.g. $[2, 5]$ or $(-\infty, -1]$ or condone e.g. $(2, 5)$

Ignore incorrect inequality statements not related to the one which is valid.
e.g. "$2 \leqslant x < 5$ and $x > -1$" which scores M1 for the first inequality.

A1: Requires $\{\ \}$ and $\cup$

$\{x : x \le -1\} \cup \{x : 2 \le x \le 5\}$ or $\{x \mid x \le -1\} \cup \{x \mid 2 \le x \le 5\}$ either way round

but condone $\{x \le -1\} \cup \{2 \le x \le 5\}$, $\{x \le -1 \cup 2 \le x \le 5\}$.

Allow e.g. $\{x : x \le -1\} \cup \{x : 2 \le x \cap x \le 5\}$

Use of $\cap$ to join the two separate regions is A0
It is acceptable (but not required) to mention $\mathbb{R}$
e.g. $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$

Condone use of a lower limit written as e.g. $\{x : -\infty \le x \le -1\} \cup \{x : 2 \le x \le 5\}$

(b) Note a correct equation written down scores all 3 marks.
A correct expression but missing e.g. $y = ...$ or $f(x) = ...$ scores M1dM1A0

M1: Forms the equation of the form $(y=)\ \alpha(x+1)^2(x-5)^2$. Condone $\alpha = 1$

Award for sight of $\alpha(x+1)^2(x-5)^2$ even with $\alpha = 1$ i.e. $(x+1)^2(x-5)^2$

dM1: Substitutes $(0, -75)$ into the form $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α. It is dependent on the previous method mark.

A1: $y = -3(x+1)^2(x-5)^2$ o.e. (e.g. $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$)

isw after a correct answer. Condone $f(x) = -3(x+1)^2(x-5)^2$ but not $C = -3(x+1)^2(x-5)^2$

A correct equation scores all 3 marks. Allow if seen in (c)
isw if they attempt to multiply out.

Alternative I part (b):
Using the form $y = ax^4 + bx^3 + cx^2 + dx + e$, then setting up and solving simultaneous equations.
There are various versions of this but can be marked similarly.

M1: Sets e equal to -75 (may just be seen in their equation) and forms **three correct** different equations in a, b, c and d which may be unsimplified.

Note that the form $y = ax^4 + bx^3 + cx^2 + dx + e$ is M0 until e is set equal to -75

There are 5 equations that can be formed, only 3 are necessary for this mark.
Do not condone slips.

Using $(-1, 0)$ $\Rightarrow 0 = a - b + c - d - 75$ o.e.

Using $(5, 0)$ $\Rightarrow 0 = 625a + 125b + 25c + 5d - 75$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 2$ $\Rightarrow 0 = 32a + 12b + 4c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = -1$ $\Rightarrow 0 = -4a + 3b - 2c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 5$ $\Rightarrow 0 = 500a + 75b + 10c + d$ o.e.

dM1: Forms **four correct** different equations and solves to find values for a, b, c and d. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

dM1: Forms **four correct** different equations and solves to find values for a, b, c and d. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=-3x^4+24x^3-18x^2-120x-75$ o.e. isw if they attempt to factorise but withhold this mark if they e.g. divide all terms by 3.
Condone $f(x)=...$ but not $C=...$
A correct equation scores all 3 marks. Allow if seen in (c)

Alternative II part (b): Uses the form $y=(x+1)(x-5)(ax^2+bx+c)$

M1: Substitutes $x=0,\ y=-75$ $-75=-5c\Rightarrow c=15$, multiplies out, differentiates
$\Rightarrow \frac{dy}{dx}=(2x-4)(ax^2+bx+15)+(x^2-4x-5)(2ax+b)$
and forms **a correct equation** in a and b which may be unsimplified.
Using $\frac{dy}{dx}=0$ at $x=2$ $\Rightarrow 0=4a+b$ o.e.
Using $\frac{dy}{dx}=0$ at $x=-1$ $\Rightarrow 0=a-b+15$ o.e.
Using $\frac{dy}{dx}=0$ at $x=5$ $\Rightarrow 0=5a+b+3=0$ o.e.

dM1: Forms **two correct** different equations and solves to find values for a and b. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=(x+1)(x-5)(-3x^2+12x+15)$ o.e. isw if they attempt to multiply out or factorise
Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

Alternative III part (b): Uses $\frac{dy}{dx}=\beta(x+1)(x-2)(x-5)$ (β may be 1) and integrates.

M1: Integrates $\left(\frac{dy}{dx}=\right)\beta(x+1)(x-2)(x-5)$ to $(y=)\ \beta\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+k\right)$ and forms **one correct** equation using either $(0,-75)$: $-75=\beta k$ (allow $-75=k$)
$(-1,0)$: $0=\beta\left(\frac{1}{4}+2+\frac{3}{2}-10+k\right)$ $(5,0)$: $0=\beta\left(\frac{625}{4}-250+\frac{75}{2}+50+k\right)$

dM1: Forms a different equation using one of $(0,-75)$, $(-1,0)$, $(5,0)$ and solves to find values for β and k. You do not need to be concerned by the process of solving . A calculator can be used to solve the equations.

A1: $y=-12\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+\frac{25}{4}\right)$ o.e. isw if they attempt to multiply out or factorise
Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

(c)

M1: Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2$ (must be a quartic in any form) and proceeds to find a value for y. Sight of their $\pm y$ (or ± 243) scores M1.
You may need to check this on your calculator if only a value is seen.

A1ft: $0 < k < 243$ o.e. ft on their negative y value at $x = 2$.
Allow use of set notation, interval notation and allow e.g. $k < 243,\ k > 0$ but do not allow OR or $\cup$. Do not accept $0 \le k \le 243$ o.e.
If there are multiple attempts at describing the region, mark what appears to be their final answer.
This mark can only be scored if they have a negative quartic graph function
i.e. $\alpha < 0$ for their $y = \alpha(x+1)^2(x-5)^2$ or $a < 0$ for their $y = ax^4 + bx^3 + cx^2 + dx + e$

(**Q07 9MA0/01, June 2025**)

Q40.

Question	Scheme	Marks	AOs
(a)	$f'(x) = 4e^{-x^2} + (4x-k)\times -2xe^{-x^2}$	M1 A1	1.1b 1.1b
	$f'(1) = 4e^{-1} + (4-k)\times -2e^{-1} = 0$	M1	2.1
	$f'(1) = 4e^{-1} - 8e^{-1} + 2ke^{-1} = 0 \Rightarrow -4e^{-1} + 2ke^{-1} = 0 \Rightarrow k = 2$*	A1*	1.1b
		(4)	
(b)	Turning points are where $f'(x) = e^{-x^2}\{-8x^2 + 4x + 4\} = -4e^{-x^2}(x-1)(2x+1) = 0$ And states that $e^{-x^2} \ne 0$ so only other point is $x = -\frac{1}{2}$	B1	2.1
		(1)	
(c)	Attempts f(1) or $f\left(-\frac{1}{2}\right)$ with $k = 2$	M1	2.1
	One correct "end", either $p < 2e^{-1}$ or $p > -4e^{-\frac{1}{4}}$	A1	1.1b
	$-4e^{-\frac{1}{4}} < p < 2e^{-1},\ p \ne 0$	A1	2.5
		(3)	
			(8 marks)

Notes:

(a)

M1: Correct attempt to differentiate using the product rule (or quotient rule).

Award for $f'(x) = \alpha e^{-x^2} + \beta x(4x-k)e^{-x^2}$ or $f'(x) = \dfrac{\alpha e^{x^2} + \beta x(4x-k)e^{x^2}}{\left(e^{x^2}\right)^2}$

If they expand first it is for $xe^{-x^2} \to \alpha e^{-x^2} + \beta x^2 e^{-x^2}$

A1: Correct differentiation which may be unsimplified.

M1: Sets $f'(1) = 0$ to obtain an equation in k (may just use the numerator for quotient rule).

A1*: Shows that $k = 2$ with sufficient working shown.

(b)

B1: Shows that $x = -\frac{1}{2}$ is the only other turning point. This requires

- A correct $f'(x) = (4x-2) \times -2xe^{-x^2} + 4e^{-x^2}$ using $k = 2$ (may just use the numerator for the quotient rule)
- Some statement alluding to the fact that $e^{-x^2} \neq 0$
- A correct factorisation/solution of the quadratic factor
- A minimal conclusion

Attempts to just verify that $f'(-\frac{1}{2}) = 0$ score B0

(c)

M1: For the key step in finding one of the limits.

A1: One correct end. Allow any equivalent expressions and allow decimals here:

NB $-4e^{-\frac{1}{4}} = -3.1152...$, $2e^{-1} = 0.73575...$

Condone use of non-strict inequalities for this mark.

A1: For subtle use of inequalities in realising that $p = 0$ is not included due to the nature of the curve.

$-4e^{-\frac{1}{4}} < p < 2e^{-1}, p \neq 0$ or equivalent such as $-4e^{-\frac{1}{4}} < p < 0, 0 < p < 2e^{-1}$

Allow $-4e^{-\frac{1}{4}} < p < 0$ or $0 < p < 2e^{-1}$ and condone $-4e^{-\frac{1}{4}} < p < 0$ and $0 < p < 2e^{-1}$

Allow $-4e^{-\frac{1}{4}} < p < 0 \cup 0 < p < 2e^{-1}$ but not $-4e^{-\frac{1}{4}} < p < 0 \cap 0 < p < 2e^{-1}$

(Q10 9MA0/02/M, June 2025)

Q41.

Question	Scheme	Marks	AOs
(a)	$\mathrm{f}(x)=\dfrac{2x-3}{x^2+4}\Rightarrow \mathrm{f}'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}$ **or** $\mathrm{f}(x)=(2x-3)(x^2+4)^{-1}\Rightarrow \mathrm{f}'(x)=2(x^2+4)^{-1}-2x(2x-3)(x^2+4)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$	A1	1.1b
		(3)	
(b)	$-2x^2+6x+8=0\Rightarrow -2(x+1)(x-4)=0\Rightarrow x=-1,\ 4$	B1ft (M1 on EPEN)	1.1b
	Chooses correct region for their numerator and their critical values $x<-1$ or $x>4$	M1 A1	1.1b 2.2a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{P(x^2+4)-Qx(2x-3)}{(x^2+4)^2}$ $P,Q>0$

condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$).

Condone, e.g. $\{\mathrm{f}'(x)=\}\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)}$ provided an incorrect formula is not quoted.

May also see the product rule applied to $(2x-3)(x^2+4)^{-1}$ to obtain an expression of the form $\{\mathrm{f}'(x)=\}P(x^2+4)^{-1}-Qx(2x-3)(x^2+4)^{-2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$)

A1: Fully correct differentiation in any form with correct bracketing which may be implied by subsequent working.

A1: $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ or simplified equivalent, e.g. numerator terms in a different order.

Allow recovery from "invisible" brackets earlier and apply isw once a correct answer is seen. Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\mathrm{f}'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}\rightarrow\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ for the final mark. The denominator $(x^2+4)^2$ may go "missing" on an intermediate line provided it is present in their initial derivative **and** recovered in the final answer. Allow recovery from incorrect expansion of the denominator.

The $\mathrm{f}'(x)=$ must appear at some point but allow e.g. $"\dfrac{\mathrm{d}y}{\mathrm{d}x}="$

Note that just e.g. $\mathrm{f}'(x)=\dfrac{-2(x^2-3x-4)}{(x^2+4)^2}$ without sight of a correct derivative in the correct form scores A0.

(b) **Note:** it is possible to score B0M1A1 in this question due to the demand to "use algebra".
Note: if their numerator from (a) is not a 3 term quadratic then no marks can be scored in (b).

B1ft: Uses algebra to solve their $ax^2+bx+c...0$ with $a,b,c \neq 0$ where ... is any equality or inequality, finding the correct, real critical values for their 3TQ.
The ... 0 may be implied by their method.
They must show their working for this mark, so expect to see factorisation, substitution into the correct quadratic formula or completing the square.
Correct values for their quadratic do **not** imply this mark.
Approaches via factorisation must have completely correct factorisation, e.g.

$-2x^2+6x+8\{=0\} \Rightarrow -2(x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(4-x)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow x^2-3x-4\{=0\} \Rightarrow (x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B0ft

$-2x^2+6x+8\{=0\} \Rightarrow (x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B0ft

M1: Selects the "correct" region for their critical values and their a from part (a). Must be x not f(x)
CVs may have been found using a calculator and may be implied if they are correct for their 3TQ. CVs may be incorrect due to errors in their calculations (but not errors in their method).

- For $a<0$ and roots $\alpha<\beta$ they need e.g. $x<\alpha$, $x>\beta$ (or e.g. $x \leqslant \alpha$ or $x \geqslant \beta$)
- For $a>0$ and roots $\alpha<\beta$ they need e.g. $\alpha<x<\beta$ (or e.g. $x \geqslant \alpha,\ x \leqslant \beta$)

Do not be overly concerned about their use of $=$, $>$, $<$ in reference to their $-2x^2+6x+8...0$ for this mark or for the A1.
Indicating the region on a sketch is not sufficient. Allow , / or / and / $\cup$ / $\cap$ for the M1.
If they have complex roots (or they use the discriminant to find there are no real roots) then they can score this mark for concluding:

- if $a<0$, "all values for x (have f decreasing)" or "f is always decreasing" or $x \in \mathrm{R}$
- if $a>0$, "no values for x (have f decreasing)" or "f is never decreasing"

A1: Correct solution $x<-1$ or $x>4$ (allow $x \leqslant -1$ or $x \geqslant 4$) coming from the correct numerator.
Do not isw if they go on to select e.g. $x>4$ or combine incorrectly to $4<x<-1$
Allow full marks to be scored in (b) from an incorrect denominator (but it must be positive for all x), e.g. from $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x+4)^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{4x^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{x^2+16}$

Examples: Just "$x<-1$ or $x>4$" stated scores B0M1A1

$-2x^2+6x+8\{=0\} \Rightarrow -2(x+1)(x-4)=0 \Rightarrow x<-1, x>4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\} \Rightarrow x^2-3x-4\{=0\} \Rightarrow (x+1)(x-4)=0 \Rightarrow x \leqslant -1,\ x \geqslant 4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(x-4) \Rightarrow x<-1,\ x>4$ scores B0ftM1A1

$-2x^2+6x+8<0 \Rightarrow x^2-3x-4<0 \Rightarrow (x+1)(x-4)<0 \Rightarrow x<-1, x>4$ scores B1ftM1A1 (as this has correct factorisation shown, the region follows from $a<0$ (M1) and we condone reference to $x^2-3x-4<0$ as part of their working to find critical values (A1).)

Acceptable notation: allow a ",", "or", "and" or "$\cup$" to link the two regions, which may also be in set notation. e.g. $x<-1$ or $x>4$; $x \leqslant -1$, $x \geqslant 4$; $x<-1$ and $x>4$
$x \leqslant -1 \cup x \geqslant 4$; $\{x : x<-1 \cup x>4\}$; $\{x \in \square : x \leqslant -1\} \cup \{x \in \square : x \geqslant 4\}$;
$x \in (-\infty,-1) \cup (4,\infty)$; $(-\infty,-1] \cup [4,\infty)$ etc.
Do not accept $4<x<-1$ or use of the $\cap$ symbol e.g. $(-\infty,-1] \cap [4,\infty)$ for the final mark, but they may be condoned for the M1. Note also that $[-\infty,-1] \cup [4,\infty]$ scores A0.

(Q05 9MA0/01, June 2024)

Q42.

Question	Scheme	Marks	AOs
(a)	$\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k}\Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}\Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y=\mathrm{f}(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1	2.2a
	Applies $b^2-4ac\ (\geqslant)\ 0$ with $a=12, b=-8, c=3k$	M1	2.1
	$0<k\leqslant\frac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)\mathrm{e}^{3x}-\beta x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$, $\alpha\beta>0$

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $\mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}+\beta x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ $\alpha,\beta 0<0$ condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

A1: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

A1: Obtains $\mathrm{f}'(x)=\left(12x^2-8x+3k\right)\mathrm{g}(x)$ where $\mathrm{g}(x)=\dfrac{\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $\mathrm{g}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

The "$f'(x) =$" must appear at some point but allow e.g. "$\frac{dy}{dx} =$"

(b) Note that B0M1A1 is not possible in (b)

B1: Deduces that if $y = f(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root. There is no requirement to formally state $\frac{e^{3x}}{(4x^2+k)^2} > 0$

This may be implied by an attempt at $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac > 0$ condoning slips.

M1: Attempts $b^2 - 4ac ... 0$ with $a = 12,\ b = -8,\ c = 3k$ where ... is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2 - 8x + 3k = 0 \Rightarrow x^2 - \frac{2}{3}x + \frac{1}{4}k = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{1}{9} - \frac{1}{4}k$ leading to $\frac{1}{9} - \frac{1}{4}k \geqslant 0$

A1: $0 < k \leqslant \frac{4}{9}$ but condone $k \leqslant \frac{4}{9}$ and condone $0 \leqslant k \leqslant \frac{4}{9}$

Must be in terms of *k* not *x* so do not allow e.g. $0 < x \leqslant \frac{4}{9}$ but condone $\left(0, \frac{4}{9}\right]$ or $\left[0, \frac{4}{9}\right]$

(**Q12 9MA0/02, June 2022**)

Q43.

Question	Scheme	Marks	AOs
(a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a) May be seen in the question so watch out.

B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.

(b)

M1: Differentiates to a form $ke^{0.05t}, k > 0, k \neq 220$. Do not be too concerned about the lhs.

M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.

The left hand side must have implied differentiation. E.g. Rate = , $N', \frac{dN_b}{dt}, \frac{dN}{dt}$ or even $\frac{dy}{dx}$

A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}, \frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11e^{0.05\times10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\mathrm{e}^{0.05t}+35=800\mathrm{e}^{-0.05t}$ o.e but condone slips

It is also possible to set $\frac{N-45}{220}=\left(\mathrm{e}^{0.05t}=\right)\frac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(\mathrm{e}^{0.05t}\right)^2+35\mathrm{e}^{0.05t}-800=0$ or $220\mathrm{e}^{0.1t}+35\mathrm{e}^{0.05t}-800=0$ but allow with terms in different order such as $220\mathrm{e}^{0.1t}+35\mathrm{e}^{0.05t}=800$

FYI the equation in N is $N^2-55N-175550=0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $\mathrm{e}^{0.05t}$ followed by the use of lns.
If the answers to the quadratic just appear (from a calculator) you will need to check.
Accuracy should be to 3sf.
You may see different variables used such as x

$x=\mathrm{e}^{0.05t}$, $220\mathrm{e}^{0.1t}+35\mathrm{e}^{0.05t}=800 \Rightarrow 220x^2+35x=800 \Rightarrow x=1.82... \Rightarrow t=20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $\mathrm{e}^{0.05t}=1.82.. \Rightarrow t=12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45+220\mathrm{e}^{0.05t}=10+800\mathrm{e}^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

(**Q10 9MA0/01, June 2022**)

Q44.

Question	Scheme	Marks	AOs
(a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{\mathrm{d}v}{\mathrm{d}t}=\ln(t+1)\times -0.4+\frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{\mathrm{d}v}{\mathrm{d}t}=0 \Rightarrow \frac{(10-0.4t)}{(t+1)}=0.4\ln(t+1)$ **and** then makes progress towards making "t" the subject (See notes for this)	dM1	1.1b
	$t=\frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t=\frac{26}{1+\ln(t+1)}-1$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2=\frac{26}{1+\ln 8}-1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
			(8 marks)

Notes:

(a)

B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)

M1: Attempts to use the product rule in an attempt to differentiate $v=(10-0.4t)\ln(t+1)$

Look for $(10-0.4t)\times\frac{1}{(t+1)}\pm k\ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu'-uv'$ it is M0

You will see attempts from $v=10\ln(t+1)-0.4t\ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1}\pm\frac{bt}{t+1}\pm c\ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v', $\frac{\mathrm{d}y}{\mathrm{d}x}$ or even $=0$

$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)=\ln(t+1)\times -0.4+\frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)=\frac{10}{t+1}-\frac{0.4t}{(t+1)}-0.4\ln(t+1)$

dM1: Score for setting their $dV/dt = 0$ (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring $\ln(t + 1)$.

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow \ln(t+1) = \frac{25-t}{t+1}$$
$$\Rightarrow \ln(t+1) = -1+\frac{26}{t+1}$$

E.g 2

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow 0.4t\ln(t+1)+0.4\ln(t+1) = 10-0.4t$$
$$\Rightarrow 0.4t\left(1+\ln(t+1)\right) = 10-0.4\ln(t+1)$$

A1*: Correctly proceeds to the given answer of $t = \frac{26}{1+\ln(t+1)}-1$ showing all key steps.

The key steps must include

- use of $\frac{dv}{dt}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ or $\frac{26}{t+1}-1 = \ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2 = \frac{26}{1+\ln 8}-1$ which may be implied by awrt 7.44

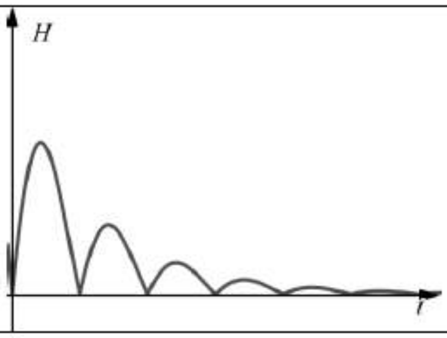
A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c)(ii)

A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 s Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

(**Q08 9MA0/01, June 2022**)

Q45.

Question	Scheme	Marks	AOs
12 (a)	$\mathrm{f}(x) = 10\mathrm{e}^{-0.25x}\sin x$		
	$\Rightarrow \mathrm{f}'(x) = -2.5\mathrm{e}^{-0.25x}\sin x + 10\mathrm{e}^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x) = 0 \Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}}\sin x + 10\cancel{\mathrm{e}^{-0.25x}}\cos x = 0$	M1	2.1
	$\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5} \Rightarrow \tan x = 4$ *	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \left\|10\mathrm{e}^{-0.25\times 4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...\mathrm{e}^{-0.25x}\sin x \pm ...\mathrm{e}^{-0.25x}\cos x$ M1

Sight of $vdu - udv$ however is M0

A1: $\mathrm{f}'(x) = -2.5\mathrm{e}^{-0.25x}\sin x + 10\mathrm{e}^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $\mathrm{f}'(x) = 0$, factorising/ cancelling out the $\mathrm{e}^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x = \arctan 4$ into $\mathrm{f}'(x)$ to score this mark.

A1*: Shows the steps $\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4$ *. $\dfrac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt $76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change"

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

(Q12 9MA0/01, June 2019)

Q46.

Question	Scheme	Marks	AOs
	$\dfrac{2(x+h)^2 - 2x^2}{h} = \ldots$	M1	2.1
	$\dfrac{2(x+h)^2 - 2x^2}{h} = \dfrac{4xh + 2h^2}{h}$	A1	1.1b
	$\dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{4xh + 2h^2}{h} = \lim_{h \to 0}(4x + 2h) = 4x$ *	A1*	2.5
		(3)	
			(3 marks)
Notes:			

Throughout the question allow the use of δx for h or any other letter e.g. α if used consistently.
If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2 = x^2 + h^2$.

Note that $\frac{2(x-h)^2 - 2x^2}{-h} = ...$ is also a possible approach.

A1: Reaches a correct fraction oe with the x^2 terms cancelled out.

E.g. $\frac{4xh + 2h^2}{h}$, $\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x^2}}{h}$, $4x + 2h$

A1*: Completes the process by applying a limiting argument and deduces that $\frac{dy}{dx} = 4x$ with no errors seen. The "$\frac{dy}{dx} =$" doesn't have to appear but there must be something equivalent e.g. "$f'(x) =$" or "Gradient =" which can appear anywhere in their working. If $f'(x)$ is used then there is no requirement to see f (x) defined first. Condone e.g. $\frac{dy}{dx} \to 4x$ or $f'(x) \to 4x$.

Condone missing brackets so allow e.g. $\frac{dy}{dx} = \lim_{h\to 0} \frac{4xh + 2h^2}{h} = \lim_{h\to 0} 4x + 2h = 4x$

Do not allow $h = 0$ if there is never a reference to $h \to 0$

e.g. $\frac{dy}{dx} = \lim_{h\to 0} \frac{4xh + 2h^2}{h} = \lim_{h\to 0} 4x + 2(0) = 4x$ is acceptable

but e.g. $\frac{dy}{dx} = \frac{4xh + 2h^2}{h} = 4x + 2h = 4x + 2(0) = 4x$ is not if there is no $h \to 0$ seen.

The $h \to 0$ does not need to be present throughout the proof e.g. on every line.

They must reach $4x + 2h$ at the end and not $\frac{4xh + 2h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

(**Q04 9MA0/02, June 2022**)

Q47.

Question	Scheme	Marks	AOs
(a)	$\left(f'(x)=\right)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $f'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4)>0$, $f(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $f(5)=-1.212\ldots$ and $f'(5)=-6.204\ldots$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
			(7 marks)
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $f'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $f'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $f'(x)=\ldots$ or $\frac{dy}{dx}=\ldots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x.

Look for

- $f'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\ a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4\ldots$ or $11.1\ldots$ (or $82.8\ldots$ or $637\ldots$. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. $14.011\ldots$

Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.

Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)

Accept equivalent statements for $f(4)>0$, $f(5)<0$ e.g. $f(4)\times f(5)<0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = −5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0

(**Q06 9MA0/02, June 2022**)

Q48.

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = ...$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx} = Ax + B + Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4} = ...$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{dy}{dx} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x = 4 \Rightarrow \frac{d^2y}{dx^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation, a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

(Q02 9MA0/01, June 2018)

Q49.

Question	Scheme	Marks	AOs
(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x\Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2\to Ay\frac{dy}{dx}$ or $2xy\to 2x\frac{dy}{dx}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy\to -2x\frac{dy}{dx}+2y$ unless you see evidence that they have used the incorrect law $vu'-uv'$

A1: Fully correct derivative $2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

Allow attempts where candidates write $2x\,dx-2x\,dy-2y\,dx+6y\,dy=0$

but watch for students who write $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

M1: For a valid attempt at making $\frac{dy}{dx}$ the subject. with two terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$

Look for $(...\pm...)\frac{dy}{dx}=......$ It is implied by $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$

This cannot be scored from attempts such as $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y$ which only has one correct term.

A1*: $\frac{dy}{dx}=\frac{y-x}{3y-x}$ with no errors or omissions.

The previous line $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y-x=0$ oe

M1: Attempts to find either the x or y coordinates of P and Q by solving their $y=\frac{1}{3}x$ with $x^2-2xy+3y^2=50$ simultaneously. Allow for finding a quadratic equation in x or y and solving to find at least one value for x or y.

This may be awarded when candidates make the numerator = 0 ie using $y=x$

A1: $\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the y coordinate from their x (or vice versa)
This may also be scored following the numerator being set to 0 ie using $y=x$

A1: Deduces that $P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be $x=...$ $y=...$

(c)

B1ft: Explains that this is where $\frac{dy}{dx}=0$ and so you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution (or larger solution).

Allow a follow through for candidates who mix up parts (b) and (c)

Alternatively candidates could complete the square $(x-y)^2+2y^2=50$ and state that y would reach a maximum value when $x=y$ and choose the positive solution from $2y^2=50$

(**Q09 9MA0/01, June 2018**)

Q50.

Question	Scheme	Marks	AOs
	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

(**Q05 9MA0/01, June 2018**)