Aiming For A* Live Sessions Friday at 5pm

Name: ____

Differentiation Exam Questions

Topic Test and Revision

Date:

Time: 500

Total marks available: 384

Total marks achieved: ______

Video Solutions Walkthough

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Worked Solutions and Official Mark Scheme

Calculator Allowed

Mathvault.io

Video Solutions Walkthough

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Questions

Q1.

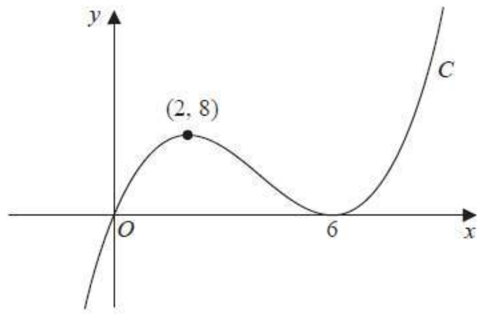

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at (2, 8)
- has a minimum turning point at (6, 0)

(a) Write down the set of values of x for which

$$f'(x) < 0$$

$2 < x < 6$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

At $y = 8$, two points of intersection.

At $y = 0$, two points of intersection.

$\therefore$ For one point of intersection, $k > 8$ or $k < 0$

In set notation $\{k : k > 8\} \cup \{k : k < 0\}$

(2)

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(c) Find the equation of *C*. You may leave your answer in factorised form.

This Cubic has repeated roots at $x = 6$ $\therefore (x-6)^2$ is a factor of $f(x)$

$x = 0$ is also a root

$\therefore f(x) = ax(x-6)^2$

$f(2) = 8$

$\therefore 8 = a(2)[2-6]^2$

$8 = 2a(-4)^2$

$8 = 32a$

$a = \frac{1}{4}$

$\therefore f(x) = \frac{1}{4}x[x-6]^2$

(3)

(Total for question = 6 marks)

(**Q06 9MA0/01, June 2022**)

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Q2.

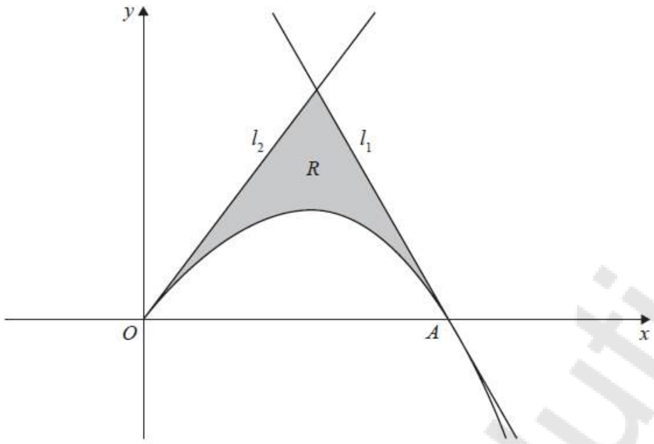

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the *x*-axis at the point *A*.

(a) Verify that the *x* coordinate of *A* is 4

At A, $y = 8(4) - (4)^{5/2}$

$y = 32 - 32$

$y = 0$ Thus, A is $(4, 0)$

(1)

The line l_1 is the tangent to the curve at *A*.

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

$y = 8x - x^{5/2}$

$\frac{dy}{dx} = 8 - \frac{5}{2}x^{3/2}$

$m_{l_1} = 8 - \frac{5}{2}(4)^{3/2} = -12$

$y - y_1 = m(x - x_1)$

$y - 0 = -12(x - 4)$

$y = -12x + 48$

$12x + y = 48$

(3)

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The line l_2 has equation $y = 8x$

The region R, shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R.

To find point where l_1 and l_2 meet.

$$12x + 8x = 48$$

$$20x = 48$$

$$x = 2.4$$

$$\therefore y = 8 \times 2.4 = 19.2$$

$$\text{Area of triangle} = \frac{4 \times 19.2}{2} = 38.4$$

$$\text{Area under curve} = \int_0^4 8x - x^{5/2} \, dx$$

$$\left[\frac{8x^2}{2} - \frac{x^{7/2}}{7/2} \right]_0^4$$

$$\left[4x^2 - \frac{2}{7} x^{7/2} \right]_0^4$$

$$\left[4(4)^2 - \frac{2}{7}(4)^{7/2} \right] - \left[4(0) - \frac{2}{7}(0)^{7/2} \right]$$

$$\text{Area under curve} = \frac{192}{7}$$

$$R = 38.4 - \frac{192}{7}$$

$$R = \frac{384}{35}$$

(5)

(Total for question = 9 marks)

(**Q10 9MA0/01, June 2024**)

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Q3.

$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

$$y = \sin x$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{\sin(x+h) - \sin x}{h} \right)$$

$$\lim_{h \to 0} \left(\frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \right)$$

$$\lim_{h \to 0} \left(\frac{\sin h \cos x}{h} + \frac{\sin x \cos h - \sin x}{h} \right)$$

$$\lim_{h \to 0} \left[\frac{\sin h}{h} (\cos x) + \frac{(\cos h - 1)}{h} (\sin x) \right]$$

$$\frac{dy}{dx} = 1 (\cos x) + 0 (\sin x)$$

$$\frac{dy}{dx} = \cos x$$

(Total for question = 5 marks)

(Q12 9MA0/01, June 2023)

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Q4.

Given that $y = x^2$, use differentiation from first principles to show that $\frac{dy}{dx} = 2x$

$$y = x^2$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{(x+h)^2 - x^2}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{x^2 + 2xh + h^2 - x^2}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} 2x + h$$

$$\frac{dy}{dx} = 2x$$

(Total for question = 3 marks)

(Q04 9MA0/01, June 2024)

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Q5.

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

$$\left(\text{You may assume that as } h \to 0,\ \frac{\sin h}{h} \to 1 \text{ and } \frac{\cos h - 1}{h} \to 0\right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left[\frac{\sin(\theta + h) - \sin\theta}{h}\right]$$

$$\lim_{h \to 0} \left[\frac{\sin\theta\cos h + \cos\theta\sin h - \sin\theta}{h}\right]$$

$$\lim_{h \to 0} \left[\frac{\sin\theta\cos h - \sin\theta + \cos\theta\sin h}{h}\right]$$

$$\lim_{h \to 0} \left(\sin\theta\left[\frac{\cos h - 1}{h}\right] + \cos\theta\left[\frac{\sin h}{h}\right]\right)$$

$$\frac{dy}{dx} = (\sin\theta) \times 0 + (\cos\theta) \times 1$$

$$\frac{dy}{dx} = \cos\theta$$

(Total for question = 5 marks)

(**Q11 9MA0/02/M, June 2025**)

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Q6.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

$f'(x) = 3x^2 + 4x - 8$

$f''(x) = 6x + 4$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

i) $6x + 4 = 0$

$6x = -4$

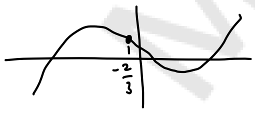
$x = \frac{-4}{6}$

$x = -\frac{2}{3}$

ii) ← This is Concave Shape.

Point of inflection at $x = -\frac{2}{3}$

Rough Sketch of $f(x)$



Based on Sketch $f(x)$ is Concave when $x < -\frac{2}{3}$

(2)

(Total for question = 4 marks)

(**Q01 9MA0/02, June 2023**)

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Q7.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{(x+1)^n}$ where *A* and *n* are constants to be found.

$$u = 5x^2 + 10x \qquad v = (x+1)^2$$

$$\frac{du}{dx} = 10x + 10 \qquad \frac{dv}{dx} = 2(x+1)$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{(x+1)^2(10x+10) - (5x^2+10x)(2(x+1))}{[(x+1)^2]^2}$$

$$\frac{dy}{dx} = \frac{(x+1)^2(10(x+1)) - 2(5x^2+10x)(x+1)}{(x+1)^4}$$

$$\frac{dy}{dx} = \frac{10(x+1)(x+1) - (10x^2+20x)}{(x+1)^3}$$

$$\frac{dy}{dx} = \frac{10(x^2+2x+1) - 10x^2 - 20x}{(x+1)^3}$$

$$\frac{dy}{dx} = \frac{10x^2+20x+10-10x^2-20x}{(x+1)^3} = \frac{10}{(x+1)^3}$$

$\therefore \ A = 10$

$n = 3$

(4)

(b) Hence deduce the range of values for *x* for which $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$

$$\frac{10}{(x+1)^3} < 0 \quad \text{if} \quad (x+1)^3 < 0$$

$$x + 1 < 0$$

$$\underline{\underline{x < -1}}$$

(1)

(Total for question = 5 marks)

(**Q03 9MA0/01, June 2019**)

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Q8.

$$y = 4x^3 - 7x^2 + 5x - 10$$

(a) Find in simplest form

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

i) $\frac{dy}{dx} = 12x^2 - 14x + 5$

ii) $\frac{d^2y}{dx^2} = 24x - 14$

(3)

(b) Hence find the exact value of *x* when $\frac{d^2y}{dx^2} = 0$

$0 = 24x - 14$

$14 = 24x$

$\frac{14}{24} = x$

$\frac{7}{12} = x$

(2)

(Total for question = 5 marks)

(Q01 9MA0/02, June 2024)

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Q9.

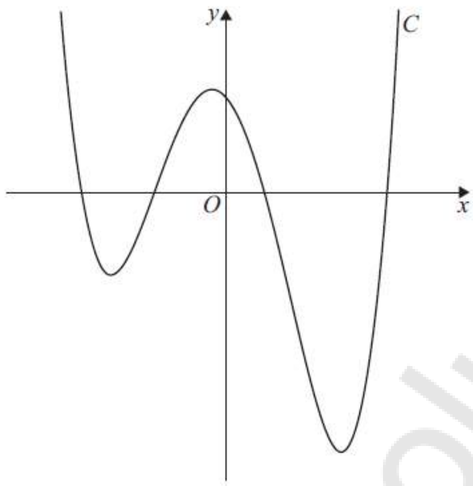

Figure 2

$$f(x) = x^4 + \frac{1}{3}x^3 - 8x^2 + ax + \frac{17}{3}$$

where *a* is a constant.

Figure 2 shows a sketch of the curve *C* with equation $y = f(x)$

Given that *C* has a local maximum at $x = -\frac{1}{4}$

(a) show that $a = -4$

$f'(x) = 4x^3 + x^2 - 16x + a$

$f'(-\frac{1}{4}) = 0$

$f''(x) = 12x^2 + 2x - 16$

$\therefore \; 0 = 4(-\frac{1}{4})^3 + (-\frac{1}{4})^2 - 16(-\frac{1}{4}) + a$

$0 = -\frac{1}{16} + \frac{1}{16} + 4 + a$

$a = -4$

(4)

(b) find the exact *y* coordinate of the local maximum.

$$f(x) = x^4 + \frac{1}{3}x^3 - 8x^2 - 4x + \frac{17}{3}$$

$$y = f\left(-\frac{1}{4}\right) = \left(-\frac{1}{4}\right)^4 + \frac{1}{3}\left(-\frac{1}{4}\right)^3 - 8\left(-\frac{1}{4}\right)^2 - 4\left(-\frac{1}{4}\right) + \frac{17}{3}$$

$$y = \frac{4735}{768}$$

(1)

The equation f $(x) = k$, where k is a constant, has 4 distinct solutions.

(c) Using algebra and showing all stages of your working, find the range of values of k.

Give the answer using set notation.

(Solutions relying on calculator technology are not acceptable.)

$$f'(x) = 4x^3 + x^2 - 16x - 4$$

If $x = -\frac{1}{4}$ is a stationary point, $4x + 1$ is a factor of $f'(x)$

$$\begin{array}{r l}
 & x^2 + 0x - 4 \\
4x + 1 & \overline{)\,4x^3 + x^2 - 16x - 4} \\
 & -(4x^3 + x^2) \\
 & \quad 0 - 16x \\
 & \quad -(0 + 0) \\
 & \qquad -16x - 4 \\
 & \qquad -(-16x - 4) \\
 & \qquad\quad 0 \quad 0
\end{array}$$

$$f'(x) = (4x+1)(x^2-4)$$

$$f'(x) = (4x+1)(x+2)(x-2)$$

At stationary point, $f'(x) = 0$

$$0 = (4x+1)(x+2)(x-2)$$

$x = -\frac{1}{4}$ local maximum $\quad x = -2$ local minimum $\quad x = 2$ local minimum

$$f(-2) = (-2)^4 + \frac{(-2)^3}{3} - 8(-2)^2 - 4(-2)^2 + \frac{17}{3} = -5$$

(4)

$$\therefore \quad -5 < k < \frac{4735}{768}$$

$$\left\{ k : -5 < k < \frac{4735}{768} \right\}$$

(Total for question = 9 marks)

(**Q08 9MA0/02, June 2025**)

Q10.

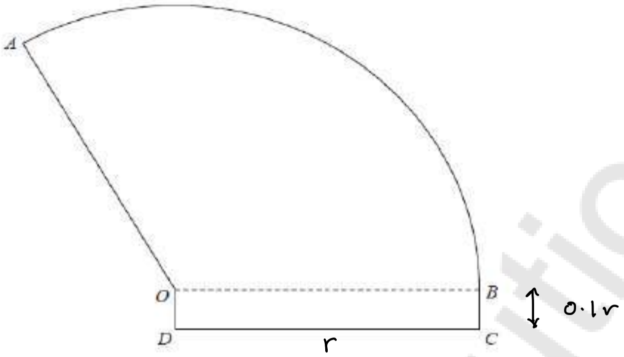

Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle *AOB* joined to a rectangle *OBCD*.

Given that

- the radius of the sector is *r* metres and angle AOB is θ radians
- the length and width of the rectangle are *r* metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is 240 m^2

(a) show that the perimeter of the stage, *P* metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

$P = r + 0.1r + 0.1r + r + \text{arc length}$

$\text{arc length} = \theta r$

$\therefore P = 2.2r + \theta r$

$\text{Area of sector} = \frac{1}{2}r^2\theta$

$\text{Total area} = \text{Area of Sector} + \text{area of rectangle}$

$240 = \frac{r^2\theta}{2} + r(0.1r)$

$240 = \frac{r^2\theta}{2} + 0.1r^2$

$240 - 0.1r^2 = \frac{r^2\theta}{2}$

$480 - 0.2r^2 = r^2\theta$

$\frac{480}{r^2} - 0.2 = \theta$

$\therefore P = 2.2r + r\left[\frac{480}{r^2} - 0.2\right]$

$P = 2.2r + \frac{480}{r} - 0.2r \implies P = 2r + \frac{480}{r}$

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

$$P = 2r + 480r^{-1}$$

$$\frac{dP}{dr} = 2 - 480r^{-2}$$

$$\frac{dP}{dr} = 2 - \frac{480}{r^2}$$

Stationary value at $\frac{dP}{dr} = 0$ $\quad \therefore \quad 0 = 2 - \frac{480}{r^2}$

$$\frac{480}{r^2} = 2$$

$$480 = 2r^2$$

$$240 = r^2$$

$$\sqrt{240} = r$$

$$r = 4\sqrt{15} \text{ m}$$

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

$$\frac{d^2P}{dr^2} = 960r^{-3}$$

$$\frac{d^2P}{dr^2} = \frac{960}{r^3}$$

At $r = 4\sqrt{15}$, $\frac{d^2P}{dr^2} = \frac{960}{(4\sqrt{15})^3} \approx 0.2582$ (4 d.p.)

$$\frac{d^2P}{dr^2} > 0$$

$\therefore$ minimum perimeter at $r = 4\sqrt{15}$ m

(2)

(Total for question = 9 marks)

(**Q15 9MA0/01, June 2025**)

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Q11.

The curve *C*, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve *C* passes through the origin *O*

(a) Find the value of $\frac{dy}{dx}$ at the origin.

$\frac{dx}{dy} = 4 \times 2 \times \cos 2y$

$\frac{dx}{dy} = 8\cos 2y$

$\therefore \quad \frac{dy}{dx} = \frac{1}{8\cos 2y}$

At $(0,0)$, $\frac{dy}{dx} = \frac{1}{8\cos 2(0)}$

$\frac{dy}{dx} = \frac{1}{8 \times 1} = \frac{1}{8}$

(2)

(b) (i) Use the small angle approximation for sin 2*y* to find an equation linking *x* and *y* for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

i) $\sin 2y \approx 2y$

$\therefore x = 4(2y)$

$x \approx 8y$

ii) $x = 8y$

$y = \frac{1}{8}x$

The answer in (a) is the gradient of the line in bi)

(2)

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(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

$$\frac{dy}{dx} = \frac{1}{8\cos 2y}$$

$$x = 4\sin 2y$$

$$\frac{x}{4} = \sin 2y$$

$$\frac{x^2}{16} = \sin^2 2y$$

$$\sin^2 2y + \cos^2 2y = 1$$

$$\cos 2y = \sqrt{1 - \sin^2 2y}$$

$$8\cos 2y = 8\sqrt{1 - \sin^2 2y}$$

$$8\cos 2y = 8\sqrt{1 - \frac{x^2}{16}}$$

$$8\cos 2y = 8\sqrt{\frac{1}{16}(16 - x^2)}$$

$$8\cos 2y = 8\sqrt{\frac{1}{16}} \times \sqrt{16 - x^2}$$

$$8\cos 2y = 8 \times \frac{1}{4} \times \sqrt{16 - x^2}$$

$$8\cos 2y = 2\sqrt{16 - x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{16 - x^2}}$$

(3)

(Total for question = 7 marks)

(**Q14 9MA0/01, June 2019**)

Q12.

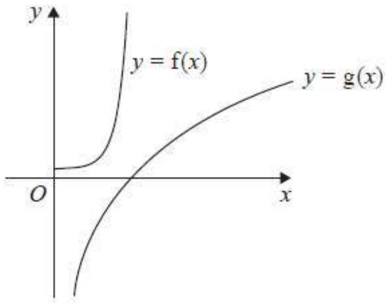

Figure 1

Figure 1 shows a sketch of the curves with equations $y = \text{f}(x)$ and $y = \text{g}(x)$ where

$$\text{f}(x) = e^{4x^2-1} \qquad x > 0$$
$$\text{g}(x) = 8\ln x \qquad x > 0$$

(a) Find

(i) $\text{f}'(x)$
(ii) $\text{g}'(x)$

i) $f'(x)$: $y = e^{4x^2-1}$

$u = 4x^2 - 1 \qquad y = e^u$

$\dfrac{du}{dx} = 8x \qquad \dfrac{dy}{du} = e^u$

$f'(x) = \dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$

$f'(x) = 8x \times e^u$

$f'(x) = 8xe^u$

$\underline{\underline{f'(x) = 8xe^{4x^2-1}}}$

ii) $g'(x) = \underline{\underline{\dfrac{8}{x}}}$

(2)

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Given that f'(x) = g'(x) at x = α

(b) show that α satisfies the equation

$$4x^2 + 2\ln x - 1 = 0$$

$$8xe^{4x^2-1} = \frac{8}{x}$$

$$8x^2e^{4x^2-1} = 8$$

$$x^2e^{4x^2-1} = 1$$

$$\ln(x^2e^{4x^2-1}) = \ln 1$$

$$\ln x^2 + \ln e^{4x^2-1} = 0$$

$$2\ln x + 4x^2 - 1 = 0$$

$$4x^2 + 2\ln x - 1 = 0$$

shown.

(2)

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2\ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

(i) the value of x^2
(ii) the value of α

i) $x_1 = 0.6$

$$x_2 = \sqrt{\frac{1 - 2\ln 0.6}{4}}$$

$x_2 = 0.7109$ (4 d.p.)

ii) $$x_3 = \sqrt{\frac{1 - 2\ln 0.7109}{4}}$$

$x_3 = 0.6485329086$

⋮

$\alpha = x_{30} = 0.6706416243$

$\alpha = 0.6706$ (4 d.p.)

(3)

(Total for question = 7 marks)

(**Q06 9MA0/02, June 2024**)

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Q13.

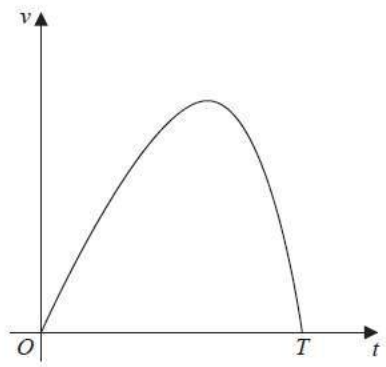

Figure 2

A racing car is driven along a straight road.

Figure 2 shows a graph of the speed of the car as it travels along the road.

The car starts from rest and is driven for T seconds before stopping.

The speed of the car is modelled by the equation

$$v = 15t - te^{0.2t} \qquad 0 \leq t \leq T$$

where t seconds is the time after the car starts to move.

According to the model,

(a) find the value of T, giving your answer to one decimal place,

At $v = 0$, $0 = 15t - te^{0.2t}$

$0 = t(15 - e^{0.2t})$

$t = 0$ $\quad 15 - e^{0.2t} = 0$

$15 = e^{0.2t}$

$\ln 15 = \ln e^{0.2t}$

$\ln 15 = 0.2t \ln e$

$\ln 15 = 0.2t$

$5 \ln 15 = t$

$\therefore T = 5 \ln 15 \approx \underline{\underline{13.5}}$ seconds (1 d.p.)

(2)

(b) show that the maximum speed of the car occurs when

$$t = 5\ln\left(\frac{75}{t+5}\right)$$

$v = 15t - te^{0.2t}$

$\frac{dv}{dt} = 15 - [0.2te^{0.2t} + e^{0.2t}]$

$\frac{dv}{dt} = 15 - 0.2te^{0.2t} - e^{0.2t}$

Maximum occurs at $\frac{dv}{dt} = 0$

$\therefore\ 0 = 15 - 0.2te^{0.2t} - e^{0.2t}$

$0.2te^{0.2t} + e^{0.2t} = 15$

$te^{0.2t} + 5e^{0.2t} = 75$

$e^{0.2t}[t + 5] = 75$

$e^{0.2t} = \frac{75}{t+5}$

$\ln e^{0.2t} = \ln\left|\frac{75}{t+5}\right|$

$0.2t \ln e = \ln\left|\frac{75}{t+5}\right|$

$0.2t = \ln\left|\frac{75}{t+5}\right|$

$t = 5\ln\left|\frac{75}{t+5}\right|$

Shown.

$y = te^{0.2t}$

$u = t \quad v = e^{0.2t}$

$u' = 1 \quad v' = 0.2e^{0.2t}$

$y' = uv' + vu'$

$y' = t(0.2e^{0.2t}) + e^{0.2t}(1)$

$y' = 0.2te^{0.2t} + e^{0.2t}$

(4)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{75}{t_n + 5}\right) \quad \text{with } t_1 = 8$$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

i) $t_1 = 8$

$t_2 = 5\ln\left|\frac{75}{8+5}\right|$

$t_2 = 5\ln\left|\frac{75}{13}\right|$

$t_3 = 5\ln\left|\frac{75}{5\ln\left|\frac{75}{13}\right| + 5}\right|$

$t_3 = 8.478$ (3 d.p.)

ii) $t_4 = 8.582283212$

$\vdots$

$t_{30} = 8.554023401$ seconds

(3)

(Total for question = 9 marks)

(**Q09 9MA0/01, June 2025**)

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Q14.

The value of a car, £V, is modelled by the equation

$$V = 1500 + Ae^{-kt}$$

where A and k are positive constants and t is the age of the car in years.

Given that

- the initial value of the car was £20 000
- the value of the car was £12 000 when it was 2.5 years old

(a) find a complete equation for the model, giving the exact value of A and the value of k to 3 significant figures.

At $t = 0$, $V = 20{,}000$

$20000 = 1500 + Ae^{-k(0)}$

$18500 = A$

$\therefore V = 1500 + 18500e^{-kt}$

$12000 = 1500 + 18500e^{-2.5k}$

$10500 = 18500e^{-2.5k}$

$\frac{21}{37} = e^{-2.5k}$

$\frac{21}{37} = \frac{1}{e^{2.5k}}$

$e^{2.5k} = \frac{37}{21}$

$\ln e^{2.5k} = \ln\left|\frac{37}{21}\right|$

$2.5k \ln e = \ln\left|\frac{37}{21}\right|$

$2.5k = \ln\left|\frac{37}{21}\right|$

$k = \frac{2}{5}\ln\left(\frac{37}{21}\right)$

$k \simeq 0.227$ (3 s.f.)

$V = 1500 + 18500e^{-0.227t}$

(4)

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(b) Show that the rate of change in the value of the car can be expressed in the form

$$-k(V - 1500)$$

$V = 1500 + 18500e^{-0.227t}$

$\frac{dv}{dt} = 18500(-0.227)e^{-0.227t}$

$\frac{dv}{dt} = -0.227\left[18500e^{-0.227t}\right]$

$\frac{dv}{dt} = -0.227\left[V - 1500\right]$

$V = 1500 + 18500e^{-0.227t}$

$V - 1500 = 18500e^{-0.227t}$

(3)

(c) State a limitation of this model.

The model suggests that the value of the car will never go below £1500

(1)

(Total for question = 8 marks)

(**Q11 9MA0/01, June 2025**)

Q15.

A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.

(You should define the variables that you use.)

$$\frac{dr}{dt} \propto \frac{1}{r^2}$$

let r = radius of spherical mint in mm

let t = time in minutes from when the mint is in mouth.

$$\frac{dr}{dt} = \frac{k}{r^2}$$

$$r^2\,dr = k\,dt$$

$$\int r^2\,dr = \int k\,dt$$

$$\frac{r^3}{3} = kt + c$$

At $t=0$, $r=5$

$$\frac{5^3}{3} = k(0) + c$$

$$\frac{125}{3} = c$$

At $t=4$, $r=3$

$$\frac{3^3}{3} = k(4) + c$$

$$9 = 4k + \frac{125}{3}$$

$$-\frac{98}{3} = 4k$$

$$-\frac{49}{6} = k$$

$$\therefore \quad \frac{r^3}{3} = -\frac{49}{6}t + \frac{125}{3}$$

$$r^3 = -\frac{49}{2}t + 125$$

(5)

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(b) Hence find the total time taken for the mint to completely dissolve.

Give your answer in minutes and seconds to the nearest second.

$$r^3 = -\frac{49}{2}t + 125$$

mint totally dissolved at $r = 0$

$$0 = -\frac{49}{2}t + 125$$

$$\frac{49}{2}t = 125$$

$$49t = 250$$

$$t = \frac{250}{49} \text{ minutes}$$

$$t \simeq 5.102040816 \text{ minutes}$$

$$0.102040816 \times 60 \approx 6.122 \text{ seconds} \approx 6 \text{ seconds}$$

$$\therefore \quad t = 5 \text{ minutes } 6 \text{ seconds}$$

(2)

(c) Suggest a limitation of the model.

The model does not consider the temperature in the mouth.

(1)

(Total for question = 8 marks)

(**Q10 9MA0/02, June 2018**)

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Q16.

A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

At $t=0$, $N = \dfrac{900}{3+7e^{-0.25(0)}}$

$N = \dfrac{900}{3+7} = 90$

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

$N = 900(3+7e^{-0.25t})^{-1}$

$N = 900u^{-1}$ $\quad u = 3+7e^{-0.25t}$

$\dfrac{dN}{du} = -900u^{-2}$ $\quad \dfrac{du}{dt} = -\frac{7}{4}e^{-0.25t}$

$\dfrac{dN}{du} = \dfrac{-900}{u^2}$

$\dfrac{dN}{dt} = \dfrac{dN}{du} \times \dfrac{du}{dt}$

$\dfrac{dN}{dt} = -\dfrac{900}{u^2} \times -\frac{7}{4}e^{-0.25t}$

$\dfrac{dN}{dt} = \dfrac{-900}{(3+7e^{-0.25t})^2} \times -\frac{7}{4}e^{-0.25t}$

$\dfrac{dN}{dt} = \dfrac{900}{3+7e^{-0.25t}} \times \dfrac{-1}{3+7e^{-0.25t}} \times -\frac{7}{4}e^{-0.25t}$

$\dfrac{dN}{dt} = N \times -\dfrac{N}{900} \times -\frac{7}{4}e^{-0.25t}$

$\dfrac{dN}{dt} = N \times \dfrac{-N}{900} \times -\frac{1}{4}\left(\dfrac{900}{N} - 3\right)$

$\dfrac{dN}{dt} = \dfrac{N^2}{3600}\left(\dfrac{900-3N}{N}\right)$

$\dfrac{dN}{dt} = \dfrac{N(900-3N)}{3600}$

$\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$

Shown.

$N = \dfrac{900}{3+7e^{-0.25t}}$

$-\dfrac{N}{900} = \dfrac{-1}{3+7e^{-0.25t}}$ ⊛

$3+7e^{-0.25t} = \dfrac{900}{N}$

$7e^{-0.25t} = \dfrac{900}{N} - 3$

$\dfrac{-7e^{-0.25t}}{4} = -\frac{1}{4}\left(\dfrac{900}{N} - 3\right)$ ⊛

(4)

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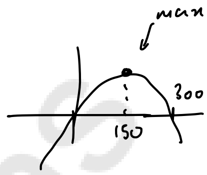
The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

$\frac{dN}{dt} = \frac{N(300-N)}{1200}$ take the form of a quadratic



Max occurs at $N = 150$

$\therefore \quad 150 = \frac{900}{3+7e^{-0.25t}}$

$1 = \frac{6}{3+7e^{-0.25t}}$

$3+7e^{-0.25t} = 6$

$7e^{-0.25t} = 3$

$e^{-0.25t} = \frac{3}{7}$

$e^{0.25t} = \frac{7}{3}$

$\ln e^{0.25t} = \ln\frac{7}{3}$

$0.25t \ln e = \ln\frac{7}{3}$

$0.25t = \ln\frac{7}{3}$

$t = 4\ln\frac{7}{3}$ months

$t = \underline{\underline{3.39 \text{ months}}}$ (3 s.f.)

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

P occurs as $t \to \infty$

As $t \to \infty$, $7e^{-0.25t} \to 0$

$\therefore \quad P = \frac{900}{3} = \underline{\underline{300}}$

(1)

(Total for question = 10 marks)

(**Q14 9MA0/02, June 2018**)

Q17.

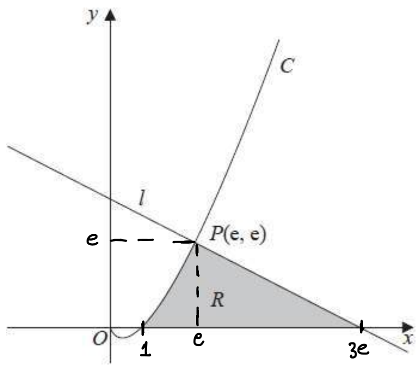


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x \ln x$, $x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

$y = x\ln x$

At $y = 0$

$0 = x \ln x$

$\therefore x = 0 \quad \ln x = 0$

$x = 1$

$\int_0^1 x \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln x \quad dv = x\,dx$

$\frac{du}{dx} = \frac{1}{x} \quad \int dv = \int x\,dx$

$\therefore du = \frac{dx}{x} \quad v = \frac{x^2}{2}$

$\int_1^e x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{dx}{x}$

$\int_1^e x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2}\, dx$

$\int_1^e x \ln x \, dx = \left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^e$

$\int_1^e x \ln x \, dx = \left(\frac{e^2}{2} \ln e - \frac{e^2}{4}\right) - \left(\frac{1^2}{2} \ln 1 - \frac{1^2}{4}\right)$

$\int_1^e x \ln x \, dx = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4}$

$y = x \ln x$

$y' = uv' + vu'$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x$

$\frac{dy}{dx} = 1 + \ln x$

At $x = e$ $\quad m_T = 1 + \ln e$

$m_T = 1 + 1 = 2$

$m_N = -\frac{1}{2} \quad$ At (e, e)

Equation of Normal

$y - e = -\frac{1}{2}(x - e)$

At $y = 0$

$-e = -\frac{1}{2}(x - e)$

$-2e = -x + e$

$x = 2e + e$

$x = 3e$

Area of triangle

base $= 3e - e = 2e$

height $= e$

Area of $\Delta = \frac{2e \times e}{2}$

Area of $\Delta = e^2$

Area of $R = e^2 + \frac{e^2}{4} + \frac{1}{4}$

Area of $R = \frac{5}{4}e^2 + \frac{1}{4}$

(10)

(Total for question = 10 marks)

(**Q13 9MA0/02, June 2018**)

Q18.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants *A*, *B* and *C*.

$$\frac{1+11x-6x^2}{x-2x^2-3+6x} = \frac{-6x^2+11x+1}{-2x^2+7x-3}$$

$$\begin{array}{r|l} & 3 \\ \hline -2x^2+7x-3 & -6x^2+11x+1 \\ & -(-6x^2+14x-9) \\ \hline & -3x+10 \end{array}$$

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{10-3x}{(x-3)(1-2x)}$$

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{B}{x-3} + \frac{C}{1-2x}$$

$$1+11x-6x^2 \equiv 3(x-3)(1-2x) + B(1-2x) + C(x-3)$$

At $x = 3$

$$1+11(3)-6(3)^2 = B(1-2(3))$$

$$-20 = -5B$$

$$4 = B$$

At $x = ½$

$$1+11(\tfrac{1}{2})-6(\tfrac{1}{2})^2 = C(\tfrac{1}{2}-3)$$

$$5 = -2.5C$$

$$-2 = C$$

$$\therefore \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{4}{x-3} - \frac{2}{1-2x}$$

$A = 3,\ B = 4,\ C = -2$

(4)

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$$f(x) = \frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that f (x) is a decreasing function.

$$f(x) = 3 + \frac{4}{x-3} - \frac{2}{1-2x}$$

$$f'(x) = \frac{-4}{(x-3)^2} - \frac{4}{(1-2x)^2}$$

$(x-3)^2 > 0$ for $x > 3$

$(1-2x)^2 > 0$ for $x > 3$

$\frac{-4}{(x-3)^2} < 0$ for $x > 3$ $\quad \frac{-4}{(1-2x)^2} < 0$ for $x > 3$

$$\frac{-4}{(x-3)^2} - \frac{4}{(1-2x)^2} < 0$$

Thus, $f'(x) < 0$ for $x > 3$

which means, $f(x)$ is a decreasing function.

(3)

(Total for question = 7 marks)

(**Q11 9MA0/02, June 2018**)

Video Solutions Walkthough

Q19.

Water flows at a constant rate into a large container.

There is a tap at the bottom of the container.

At time *t* hours after the tap was opened

- the volume of water in the container is V m^3
- water is flowing into the container at a constant rate of 0.45 m^3 per hour
- water is leaving the container through the tap at a rate of $0.3V$ m^3 per hour

(a) Show that

$$20\frac{dV}{dt} = 9 - 6V$$

$$\frac{dV}{dt} = 0.45 - 0.3V$$

$$\frac{dV}{dt} = \frac{45}{100} - \frac{3V}{10}$$

$$100\frac{dV}{dt} = 45 - 30V$$

$$20\frac{dV}{dt} = 9 - 6V$$

(2)

Given that when the tap was opened, there was 0.25 m^3 of water in the container,

(b) solve the differential equation to show that

$$V = P - Qe^{-kt}$$

where P, Q and k are positive constants to be found.

$$20\frac{dV}{dt} = 9 - 6V$$

$$20\,dV = (9-6V)\,dt$$

$$\frac{20}{9-6V}\,dV = dt \qquad \times \frac{-3}{10}$$

$$\times \frac{-3}{10} \qquad \frac{-6}{9-6V}\,dV = -0.3\,dt$$

$$\int \frac{-6}{9-6V}\,dV = \int -0.3\,dt$$

$$\ln|9-6V| = -0.3t + C$$

At $t=0$, $V=0.25$

$$\ln|9-6(0.25)| = -0.3(0) + C$$

$$\ln|7.5| = C$$

$$\therefore \ln|9-6V| = -0.3t + \ln|7.5|$$

$$e^{\ln|9-6V|} = e^{-0.3t + \ln|7.5|}$$

$$9-6V = \left(e^{-0.3t}\right)\left(e^{\ln|7.5|}\right)$$

$$9-6V = 7.5e^{-0.3t}$$

$$-6V = -9 + 7.5e^{-0.3t}$$

$$V = \frac{-9}{-6} + \frac{7.5}{-6}e^{-0.3t}$$

$$V = 1.5 - 1.25e^{-0.3t}$$

$\therefore P = 1.5 \quad Q = 1.25 \quad k = 0.3$

(5)

Given that

- the capacity of the container is 2 m^3
- the tap remains open
- the water continues to flow into the tank at the same rate

(c) determine whether the container will ever become full, giving a reason for your answer.

As $t \to \infty$, $V \to 1.5\,m^3$

$\therefore$ The container will never become full.

$1.5 < 2$

(2)

(Total for question = 9 marks)

(**Q10 9MA0/02, June 2025**)

Q20.

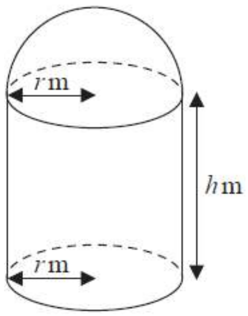

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

Curved surface area $= 2\pi r h$

Base area $= \pi r^2$

hemisphere surface area $= 4\pi r^2 \div 2 = 2\pi r^2$

Total surface area $= 2\pi r h + \pi r^2 + 2\pi r^2$

Total surface area $= 2\pi r h + 3\pi r^2$

Total surface area $= 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + 3\pi r^2$

Total Surface area $= \frac{12}{r} - \frac{4\pi r^2}{3} + 3\pi r^2$

Total Surface area $= \frac{12}{r} + \frac{5}{3}\pi r^2$

Volume of cylinder $= \pi r^2 h$

volume of hemisphere $= \frac{4}{3}\pi r^3 \div 2 = \frac{2}{3}\pi r^3$

Total Volume $= \pi r^2 h + \frac{2}{3}\pi r^3$

$6 = \pi r^2 h + \frac{2}{3}\pi r^3$

$18 = 3\pi r^2 h + 2\pi r^3$

$3\pi r^2 h = 18 - 2\pi r^3$

$h = \frac{18}{3\pi r^2} - \frac{2\pi r^3}{3\pi r^2}$

$h = \frac{6}{\pi r^2} - \frac{2}{3}r$

Shown.

(4)

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The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

$$A = 12r^{-1} + \frac{5}{3}\pi r^2$$

$$\frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$$

when A is minimum, $\frac{dA}{dr} = 0$

$$0 = -\frac{12}{r^2} + \frac{10}{3}\pi r$$

$$\frac{12}{r^2} = \frac{10}{3}\pi r$$

$$r^2 \times 3 \times \frac{12}{r^2} = \frac{10}{3}\pi r \times r^2 \times 3$$

$$36 = 10\pi r^3$$

$$\frac{3.6}{\pi} = r^3$$

A_{min} occurs at $r = \left(\frac{3.6}{\pi}\right)^{1/3} \approx 1.05\text{ m}$ (3 s.f.)

$$\frac{d^2A}{dr^2} = 24r^{-3} + \frac{10}{3}\pi$$

At $r = \left(\frac{3.6}{\pi}\right)^{1/3}$

$$\frac{d^2A}{dr^2} = 24\left[\left(\frac{3.6}{\pi}\right)^{1/3}\right]^{-3} + \frac{10}{3}\pi$$

$$\frac{d^2A}{dr^2} = 24\left[\frac{\pi}{3.6}\right] + \frac{10}{3}\pi = 10\pi$$

$\frac{d^2A}{dr^2} > 0 \therefore$ At $r = 1.05\text{ m}$ local minimum occurs (A_{min})

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

$$A_{min} = 12(1.05)^{-1} + \frac{5}{3}\pi(1.05)^2$$

$$A_{min} = 17.20105\ldots$$

$A_{min} \approx 17\text{ m}^2$ to nearest integer.

(2)

(Total for question = 10 marks)

(**Q13 9MA0/02, June 2019**)

Video Solutions Walkthough

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Q21.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

A new type of car is released for sale.

The total number of this type of car sold, N, in a particular region, t months after the cars were released for sale, is modelled by the equation

$$N = 5000 - 5000e^{-0.075t} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the total number of cars sold in the first 3 months.

At $t=3$, $N = 5000 - 5000e^{-0.075 \times 3}$

$N = 1007$ cars

(2)

Given that $N = 3000$ when $t = T$

(b) find the value of T giving the answer to 2 decimal places.

$3000 = 5000 - 5000e^{-0.075T}$

$-2000 = -5000e^{-0.075T}$

$0.4 = e^{-0.075T}$

$\ln 0.4 = \ln e^{-0.075T}$

$\ln 0.4 = -0.075T \ln e$

$\ln 0.4 = -0.075T$

$\therefore T = \dfrac{\ln 0.4}{-0.075}$

$T \approx 12.22$ months (2d.p.)

(3)

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(c) Find the rate of increase in the total number of cars sold when $t = 3$, giving the answer to 3 significant figures.

$$\frac{dN}{dt} = -5000 \times -0.075 \times e^{-0.075t}$$

$$\frac{dN}{dt} = 375e^{-0.075t}$$

At $t = 3$, $\frac{dN}{dt} = 375e^{-0.075(3)}$

$\frac{dN}{dt} \approx 299$ cars per month

(2)

After a marketing campaign, the total number of cars sold is expected to rise and have an upper limit of 6500

(d) Using this information, suggest **one** refinement to the model.

change the equation of the model from $N = 5000 - 5000e^{-0.075t}$ to $N = 6500 - 6500e^{-0.075t}$

(1)

(Total for question = 8 marks)

(**Q09 9MA0/02, June 2025**)

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Q22.

The curve C has equation

$$(x + y)^3 = 3x^2 - 3y - 2$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

$$(x+y)(x^2+2xy+y^2) = 3x^2 - 3y - 2$$

$$x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 = 3x^2 - 3y - 2$$

$$x^3 + 3x^2y + 3xy^2 + y^3 = 3x^2 - 3y - 2$$

$$3x^2 + 6xy + 3x^2\frac{dy}{dx} + 3y^2 + 6xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 6x - 3\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(3x^2 + 6xy + 3y^2 + 3\right) = 6x - 3x^2 - 6xy - 3y^2$$

$$\frac{dy}{dx} = \frac{6x - 3x^2 - 6xy - 3y^2}{3x^2 + 6xy + 3y^2 + 3}$$

(5)

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The point $P(1, 0)$ lies on C.

(b) Show that the normal to C at P has equation

$$y = -2x + 2$$

At $P(1,0)$, $x = 1$, $y = 0$

$$\therefore \ m_T = \frac{dy}{dx} = \frac{6(1) - 3(1)^2 - 6(1)(0) - 3(0)}{3(1)^2 + 6(1)(0) + 3(0)^2 + 3}$$

$$m_T = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \ m_N = -2$$

$$y - 0 = -2(x - 1)$$

$$y = -2x + 2 \quad \text{shown.}$$

(2)

(c) Prove that the normal to C at P does **not** meet C again.

You should use algebra for your proof and make your reasoning clear.

$(x+y)^3 = 3x^2 - 3y - 2 \longrightarrow$ Curve C

$y = -2x + 2 \longrightarrow$ Normal to C at $P(1,0)$

$$(x - 2x + 2)^3 = 3x^2 - 3(-2x + 2) - 2$$

$$(2 - x)^3 = 3x^2 + 6x - 6 - 2$$

$$(2-x)(4 - 4x + x^2) = 3x^2 + 6x - 8$$

$$8 - 8x + 2x^2 - 4x + 4x^2 - x^3 = 3x^2 + 6x - 8$$

$$0 = x^3 + 3x^2 - 4x^2 - 2x^2 + 6x + 4x + 8x - 8 - 8$$

$$0 = x^3 - 3x^2 + 18x - 16$$

$P(1,0)$ is a point of intersection

Thus $x - 1$ is a factor of $x^3 - 3x^2 + 18x - 16$

$$\begin{array}{r|l} & x^2 - 2x + 16 \\ \hline x - 1 & x^3 - 3x^2 + 18x - 16 \\ & -(x^3 - x^2) \\ \hline & \quad -2x^2 + 18x \\ & \quad -(-2x^2 + 2x) \\ \hline & \qquad 16x - 16 \\ & \qquad 16x - 16 \\ \hline & \qquad 0 \quad\ \ 0 \end{array}$$

$$0 = (x - 1)(x^2 - 2x + 16)$$

$$(-2)^2 - 4(1)(16) = -60$$

$$-60 < 0$$

Thus $x^2 - 2x + 16$ has no real roots.

This means to normal to C at $P(1,0)$ will not meet C again.

(5)

(Total for question = 12 marks)

(**Q15 9MA0/02, June 2024**)

Aiming For A* Live Sessions Friday at 5pm

Q23.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

$$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x + 6y) = -2y - 3x^2$$

$$\frac{dy}{dx} = \frac{-2y - 3x^2}{2x + 6y}$$

(4)

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The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

$$\frac{dy}{dx} = \frac{-2y - 3x^2}{2x + 6y}$$

At $P(-2,5)$

$$m_T = \frac{-2(5) - 3(-2)^2}{2(-2) + 6(5)}$$

$$m_T = -\frac{11}{13}$$

$$\therefore m_N = \frac{13}{11}$$

$$y - 5 = \frac{13}{11}(x - (-2))$$

$$11y - 55 = 13x + 26$$

$$0 = 13x - 11y + 55 + 26$$

$$13x - 11y + 81 = 0$$

$$a = 13 \quad b = -11 \quad c = 81$$

(3)

(Total for question = 7 marks)

(Q07 9MA0/02, June 2023)

Video Solutions Walkthough

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Q24.

$$f(x) = x + \tan\left(\frac{1}{2}x\right) \qquad \pi < x < \frac{3\pi}{2}$$

Given that the equation f(x) = 0 has a single root α

(a) show that α lies in the interval [3.6, 3.7]

$f(3.6) = 3.6 + \tan\left(\frac{3.6}{2}\right)$

$f(3.6) = -0.6862616746$ (negative)

$f(3.7) = 3.7 + \tan\left(\frac{3.7}{2}\right)$

$f(3.7) = 0.2119404113$ (positive)

α lies between 3.6 and 3.7 because there is a sign change for f(x) and f(x) is a continuous fuction in the interval [3.6, 3.7]

(2)

(b) Find f'(x)

$f'(x) = 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$

(2)

(c) Using 3.7 as a first approximation for α, apply the Newton–Raphson method once to obtain a second approximation for α. Give your answer to 3 decimal places.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

if $x_1 = 3.7$

$x_2 = 3.7 - \frac{0.2119404113}{1 + \frac{1}{2}\sec^2\left(\frac{3.7}{2}\right)}$

$x_2 = 3.672$ (3 d.p.)

(2)

(Total for question = 6 marks)

(**Q03 9MA0/01, June 2024**)

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Q25.

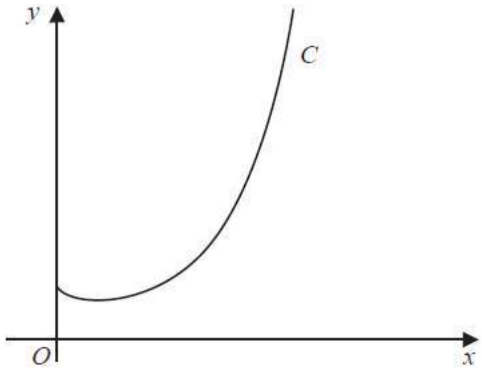

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

using implicit differentiation

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + 1 \ln x$$

$$\frac{dy}{dx} = y + y \ln x$$

Turning point at $\frac{dy}{dx} = 0$

$$\therefore \quad 0 = y + y \ln x$$

$$0 = y(1 + \ln x)$$

$y = 0$ $\quad 1 + \ln x = 0$

$x^x = 0$ $\quad \ln x = -1$

No real roots $x > 0$ $\quad e^{\ln x} = e^{-1}$

$$x = e^{-1}$$

$$x \approx 0.368 \ (3 \text{ s.f.})$$

(5)

Video Solutions Walkthough

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The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

At $x = 1.5$, $y = 1.5^{1.5} \approx 1.837$ (3 d.p.)

At $x = 1.6$, $y = 1.6^{1.6} \approx 2.121$ (3 d.p.)

$f(1.5) < 2 < f(1.6)$

Thus $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

$x_2 = 2(1.5)^{1-1.5}$

$x_2 = 1.63299\ldots$

$x_3 = 2(1.63299\ldots)^{1-1.63299\ldots}$

$x_3 = 1.466\ldots$

$x_4 = 2(1.466\ldots)^{1-1.466\ldots}$

$x_4 \approx 1.673$ (3 d.p.)

(2)

(d) describe the long-term behaviour of x_n

x_n is divergent

(2)

(Total for question = 11 marks)

(**Q11 9MA0/02, June 2019**)

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Q26.

The curve C has parametric equations

$$x = \frac{t-1}{2} \qquad y = 5(t+2)^4 \qquad t \in \mathbb{R}$$

The point P with x coordinate -3 lies on C.

(a) Find the y coordinate of P.

$$-3 = \frac{t-1}{2} \qquad y = 5(-5+2)^4$$
$$-6 = t-1 \qquad y = 5(-3)^4$$
$$-5 = t \qquad y = 405$$

(2)

(b) Find a Cartesian equation for C, giving the answer in the form $y = \mathrm{f}(x)$

$$x = \frac{t-1}{2}$$
$$2x = t-1$$
$$2x+3 = t+2$$
$$y = 5(2x+3)^4$$

(2)

(c) Hence, or otherwise, find the gradient of C at the point P.

$$y = 5(2x+3)^4$$
$$\frac{dy}{dx} = 5 \times 4 \times 2(2x+3)^3$$
$$\frac{dy}{dx} = 40(2x+3)^3$$

At $x = -3$

$$\frac{dy}{dx} = 40(2(-3)+3)^3$$

(3)

$$m_e = -1080$$

(Total for question = 7 marks)

(Q05 9MA0/02, June 2025)

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Q27.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

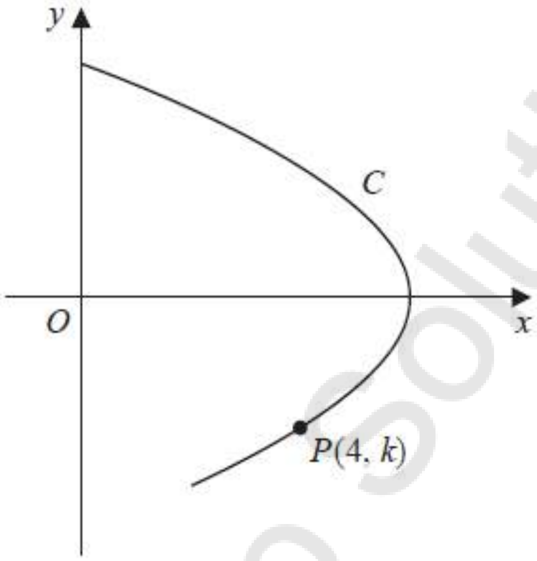

Figure 6

Figure 6 shows a sketch of the curve *C* with parametric equations

$$x = 2 + 4\cos t \qquad y = 3t + 2\sin t \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{2\pi}{3}$$

The point *P*(4, *k*), where *k* is a negative constant, lies on the curve.

(a) Find the exact value of *k*

$4 = 2 + 4\cos t$

$2 = 4\cos t$

$\frac{1}{2} = \cos t$

$t = \frac{\pi}{3}, -\frac{\pi}{3}$

$k = 3\left(\frac{\pi}{3}\right) + 2\sin\left(\frac{\pi}{3}\right)$ or $k = 3\left(-\frac{\pi}{3}\right) + 2\sin\left(-\frac{\pi}{3}\right)$

$k = \pi + \sqrt{3}$ or $k = -\pi - \sqrt{3}$

reject

(2)

Video Solutions Walkthough

The point Q lies on the curve.

Given that the gradient of the curve at Q is $-\frac{3}{2}$

(b) find the value of t at Q, giving your answer to 3 significant figures.

$x = 2 + 4\cos t \qquad y = 3t + 2\sin t$

$\frac{dx}{dt} = -4\sin t \qquad \frac{dy}{dt} = 3 + 2\cos t$

$\frac{dt}{dx} = \frac{1}{-4\sin t}$

$\frac{dy}{dx} = \frac{3 + 2\cos t}{-4\sin t}$

$-\frac{3}{2} = \frac{3 + 2\cos t}{-4\sin t}$

$6\sin t = 3 + 2\cos t$

$6\sin t - 2\cos t = 3$

$6(\sqrt{1 - \cos^2 t}) = 3 + 2\cos t$

$36[1 - \cos^2 t] = 9 + 12\cos t + 4\cos^2 t$

$36 - 36\cos^2 t = 9 + 12\cos t + 4\cos^2 t$

$0 = -36 + 9 + 12\cos t + 36\cos^2 t + 4\cos^2 t$

$0 = -27 + 12\cos t + 40\cos^2 t$

$0 = -27 + 12Y + 40Y^2$

$Y = \frac{-3 + 3\sqrt{31}}{20} \qquad Y = \frac{-3 - 3\sqrt{31}}{20}$

$\therefore t = \cos^{-1}\left[\frac{-3 + 3\sqrt{31}}{20}\right] \qquad t = \cos^{-1}\left[\frac{-3 - 3\sqrt{31}}{20}\right]$

$t \approx 0.816$ (3 s.f.) $\qquad t \approx 2.97$ (3 s.f)

reject

(6)

(Total for question = 8 marks)

(Q16 9MA0/02/M, June 2025)

Aiming For A* Live Sessions Friday at 5pm

Q28.

The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6\ln(t+3) \qquad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x+B) \qquad x > -B$$

where A and B are integers to be found.

$x = (t+3)^2 - 9 - 16$

$x = (t+3)^2 - 25$

$x + 25 = (t+3)^2$

$(x+25)^{1/2} = t+3$

$y = 6\ln(t+3)$

$y = 6\ln(x+25)^{1/2}$

$y = 3\ln(x+25)$

$\therefore A = 3 \quad B = 25$

(3)

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The curve C cuts the y-axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c \ln 5$$

where a, b and c are integers to be found.

$P(0, y)$ At $x = 0$, $y = 3\ln(0+25)$

$y = 3\ln 25$

$y = 3\ln(x+25)$

$$\frac{dy}{dx} = \frac{3}{x+25}$$

At $x=0$, $m_T = \frac{3}{25}$, $y - 3\ln 25 = \frac{3}{25}(x-0)$

$25y - 75\ln 25 = 3x$

$3x - 25y = -75\ln 25$

$3x - 25y = -75\ln 5^2$

$3x - 25y = -150\ln 5$

$a = 3$ $b = -25$ $c = -150$

(4)

(Total for question = 7 marks)

(**Q09 9MA0/02, June 2023**)

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Q29.

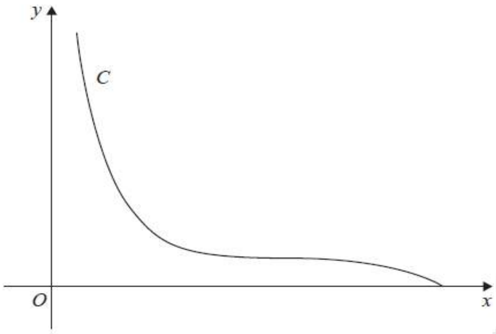

Figure 4

Figure 4 shows a sketch of the curve *C* with parametric equations

$$x = (t+3)^2 \qquad y = 1 - t^3 \qquad -2 \leqslant t \leqslant 1$$

The point *P* with coordinates (4, 2) lies on *C*.

(a) Using parametric differentiation, show that the tangent to *C* at *P* has equation

$$3x + 4y = 20$$

$\frac{dx}{dt} = 2(t+3) \qquad \frac{dy}{dt} = -3t^2 \qquad 4 = (t+3)^2 \qquad 2 = 1 - t^3$

$2 = t + 3 \qquad 1 = -t^3$

$\frac{dy}{dx} = -3t^2 \times \frac{1}{2t+6} = \frac{-3t^2}{2t+6} \qquad -1 = t \qquad -1 = t^3$

$-1 = t$

At $P(4, 2)$, $t = -1$, $m_T = \frac{dy}{dx} = \frac{-3(-1)^2}{2(-1)+6} = -\frac{3}{4}$

$y - 2 = -\frac{3}{4}(x - 4)$

$4y - 8 = -3x + 12$

$3x + 4y = 20$ Shown.

(5)

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The curve C is used to model the profile of a slide at a water park.

Units are in metres, with y being the height of the slide above water level.

(b) Find, according to the model, the greatest height of the slide above water level.

H_{max} at $t = -2$, $y = 1 - (-2)^3$

$y = 1 + 8$

$y = 9$

$H_{max} = 9\text{m}$

(1)

(Total for question = 6 marks)

(**Q10 9MA0/02, June 2024**)

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Q30.

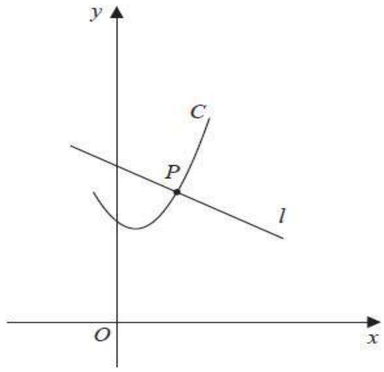

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$x = 2\tan t + 1$ $\quad\quad$ $y = 2\sec^2 t + 3$ $\quad\quad$ $\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$

The line *l* is the normal to *C* at the point *P* where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for *l* is

$y = -0.5x + 8.5$

At $t = \frac{\pi}{4}$

$x = 2\tan\left(\frac{\pi}{4}\right) + 1$ $\quad\quad$ $y = 2\sec^2\left(\frac{\pi}{4}\right) + 3$

$x = 3$ $\quad\quad$ $y = 7$

$P(3, 7)$

$x = 2\tan t + 1$

$\frac{x-1}{2} = \tan t$

$y = 2\sec^2 t + 3$

$y = 2\tan^2 t + 5$

$y = 2\left[\frac{x-1}{2}\right]^2 + 5$

$y = 2\left[\frac{(x-1)^2}{4}\right] + 5$

$y = \frac{(x-1)^2}{2} + 5$

$\sin^2 t + \cos^2 t = 1$

$\tan^2 t + 1 = \sec^2 t$

$2\tan^2 t + 2 = 2\sec^2 t$

$2\tan^2 t + 5 = 2\sec^2 t + 3$

$\frac{dy}{dx} = 2 \times \frac{(x-1)}{2}$

$\frac{dy}{dx} = x - 1$

At $x = 3$, $\quad m_T = \frac{dy}{dx} = 3 - 1 = 2$

$\therefore m_N = -\frac{1}{2}$

$y - 7 = -\frac{1}{2}(x - 3)$

$y = -\frac{1}{2}x + \frac{3}{2} + 7$

$y = -0.5x + 8.5$ $\quad$ shown.

(5)

(b) Show that all points on C satisfy the equation

$y = 0.5(x - 1)^2 + 5$

$y = \frac{(x-1)^2}{2} + 5$ done in part (a)

(2)

The straight line with equation

$y = -0.5x + k$ where k is a constant

intersects C at two distinct points.

(c) Find the range of possible values for k.

$-0.5x + k = \frac{(x-1)^2}{2} + 5$

$-x + 2k = (x-1)^2 + 10$

$-x + 2k = x^2 - 2x + 1 + 10$

$0 = x^2 - x + 11 - 2k$

if two distinct roots, $b^2 - 4ac > 0$

$\therefore (-1)^2 - 4(1)(11 - 2k) > 0$

$1 - 44 + 8k > 0$

$8k > 43$

$k > \frac{43}{8}$

At $y = 7$, $7 = \frac{(x-1)^2}{2} + 5$

$14 = (x-1)^2 + 10$

$4 = (x-1)^2$

$2 = x - 1$ $\quad -2 = x - 1$

$3 = x$ $\quad -1 = x$

$y = -0.5x + k$

At $(-1, 7)$, $7 = -0.5(-1) + k$

$7 = 0.5 + k$

$6.5 = k$

(5)

$\therefore \frac{43}{8} < k \leq 6.5$

(Total for question = 12 marks)

(**Q16 9MA0/02, June 2022**)

Aiming For A* Live Sessions Friday at 5pm

Q31.

The functions f and g are defined by

$$f(x) = \frac{3}{2}\ln x \qquad x > 0$$

$$g(x) = \frac{4x+3}{2x+1} \qquad x > 0$$

(a) Find gf(e^2) writing your answer in simplest form.

$f(e^2) = \frac{3}{2}\ln e^2 = 3$

$g(3) = \frac{4(3)+3}{2(3)+1} = \frac{15}{7}$

$gf(e^2) = \frac{15}{7}$

(2)

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(b) Prove that g is a decreasing function.

$$g(x) = \frac{4x+3}{2x+1} \quad x > 0$$

$g'(x)$: $u = 4x+3$ $\quad v = 2x+1$
$u' = 4$ $\quad v' = 2$

$$g'(x) = \frac{4(2x+1) - 2(4x+3)}{(2x+1)^2}$$

$$g'(x) = \frac{8x+4-8x-6}{(2x+1)^2}$$

$$g'(x) = \frac{-2}{(2x+1)^2}$$

$(2x+1)^2 > 0$ for $x > 0$ hence always positive

$\frac{-2}{(2x+1)^2} < 0$ for $x > 0$ $\quad \frac{\text{negative}}{\text{positive}} = \text{negative}$ hence always negative.

Thus $g(x)$ is a decreasing function because $g'(x) < 0$ for $x > 0$

(3)

(c) Find the range of the function fg

$f(x) = \frac{3}{2}\ln x$ $\qquad g(x) = \frac{4x+3}{2x+1}$

Domain : $x > 0$ $\qquad$ Domain : $x > 0$

Range : $0 < f(x)$ $\qquad$ Range : $2 < g(x) < 3$

$$fg(x) = \frac{3}{2}\ln\left|\frac{4x+3}{2x+1}\right|$$

Range : $\frac{3}{2}\ln 2 < fg(x) < \frac{3}{2}\ln 3$

(2)

(Total for question = 7 marks)

(**Q07 9MA0/02/M, June 2025**)

Video Solutions Walkthough

Q32.

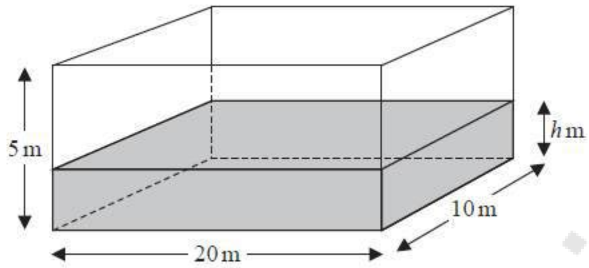

Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

$V = 20 \times 10 \times h$

$V = 200h$

$\frac{dV}{dh} = 200 \quad , \quad \frac{dh}{dV} = \frac{1}{200}$

$\frac{dV}{dt} \propto \frac{1}{\sqrt{h}}$

$\therefore \frac{dV}{dt} = \frac{K}{\sqrt{h}}$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$\frac{dh}{dt} = \frac{K}{\sqrt{h}} \times \frac{1}{200}$

$\frac{dh}{dt} = \frac{0.005K}{\sqrt{h}}$

$\lambda = 0.005K \quad , \quad \frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking *h* with *t*, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where *A* and *B* are constants to be found.

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

$$\sqrt{h}\, dh = \lambda\, dt$$

$$\int h^{1/2}\, dh = \int \lambda\, dt$$

$$\frac{2}{3} h^{3/2} = \lambda t + c$$

At $t = 0$, $h = 1.44$

$$\frac{2}{3}(1.44)^{1.5} = \lambda(0) + c$$

$$1.152 = C$$

$$\therefore \frac{2}{3} h^{1.5} = \lambda t + 1.152$$

At $t = 8$, $h = 3.24$

$$\frac{2}{3}(3.24)^{1.5} = 8\lambda + 1.152$$

$$\lambda = \frac{1}{8}\left[\frac{2}{3}(3.24)^{1.5} - 1.152\right]$$

$$\lambda = 0.342$$

$$\therefore \frac{2}{3} h^{3/2} = 0.342t + 1.152$$

$$h^{3/2} = 0.513t + 1.728$$

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

Tank full at $h = 5$, $5^{1.5} = 0.513t + 1.728$

$$t = \frac{1}{0.513}\left(5^{1.5} - 1.728\right)$$

$$t \approx 18.4 \text{ minutes } (3 \text{ s.f.})$$

(2)

(Total for question = 10 marks)

(**Q11 9MA0/02, June 2023**)

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Q33.

A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{7xe^{x}}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{7e^{x}(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

$$f'(x): \quad u = 7xe^{x} \qquad v = (e^{3x} - 2)^{1/2}$$

$$u' = 7[1(e^{x}) + xe^{x}] \qquad v' = \tfrac{1}{2}(3e^{3x})(e^{3x} - 2)^{-1/2}$$

$$v' = \frac{1.5e^{3x}}{\sqrt{e^{3x} - 2}}$$

$$f'(x) = \frac{7e^{x}(1 + x)\sqrt{e^{3x} - 2} - \dfrac{10.5xe^{4x}}{\sqrt{e^{3x} - 2}}}{e^{3x} - 2} \times \frac{(e^{3x} - 2)^{1/2}}{(e^{3x} - 2)^{1/2}}$$

$$f'(x) = \frac{7e^{x}(1 + x)(e^{3x} - 2) - 10.5xe^{4x}}{(e^{3x} - 2)^{3/2}} \times \frac{2}{2}$$

$$f'(x) = \frac{14e^{x}(1 + x)(e^{3x} - 2) - 21xe^{4x}}{2(e^{3x} - 2)^{3/2}}$$

$$f'(x) = \frac{7e^{x}[2[1 + x][e^{3x} - 2] - 3xe^{3x}]}{2(e^{3x} - 2)^{3/2}}$$

$$f'(x) = \frac{7e^{x}[2(e^{3x} - 2 + xe^{3x} - 2x) - 3xe^{3x}]}{2(e^{3x} - 2)^{3/2}}$$

$$f'(x) = \frac{7e^{x}[2e^{3x} - 4 + 2xe^{3x} - 4x - 3xe^{3x}]}{2(e^{3x} - 2)^{3/2}}$$

$$f'(x) = \frac{7e^{x}[2e^{3x} - xe^{3x} - 4x - 4]}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

$$f'(x) = \frac{7e^{x}[e^{3x}(2 - x) - 4x - 4]}{2(e^{3x} - 2)^{3/2}} \qquad \text{shown.}$$

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

Turning points at $f'(x) = 0$

$$\frac{7e^{x}\left(e^{3x}[2-x] - 4x - 4\right)}{2(e^{3x} - 2)^{3/2}} = 0$$

$$7e^{x}\left(e^{3x}(2-x) - 4x - 4\right) = 0$$

$7e^{x} = 0$ $\quad$ $e^{3x}(2-x) - 4x - 4 = 0$

$$2e^{3x} - xe^{3x} - 4x - 4 = 0$$

$$2e^{3x} - 4 = xe^{3x} + 4x$$

$$2e^{3x} - 4 = x(e^{3x} + 4)$$

$$\frac{2e^{3x} - 4}{e^{3x} + 4} = x$$

shown.

(2)

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots *α* and *β* where *β* > *α*

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for *α* and *β*

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation *y* = *x*

Using the Diagram below

(c) draw a staircase diagram to show that the iteration formula starting with $x^1 = 1$ can be used to find an approximation for β

(1)

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Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

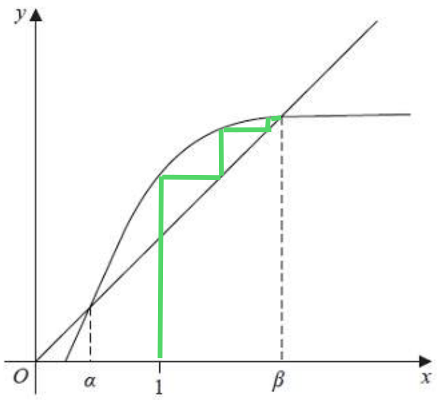
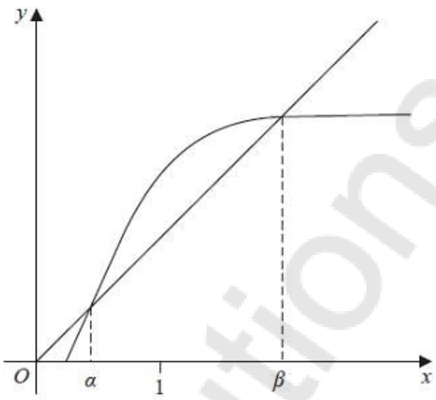


Diagram 1



copy of Diagram 1

Use the iteration formula with $x^1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x^2

(ii) the value of β

i) $x_2 = \dfrac{2e^3 - 4}{e^3 + 4}$

$x_2 = 1.5017756$

$x_2 \approx 1.502$ (3 d.p.)

ii) $\vdots$

$x_{50} \approx 1.968$

$\therefore \beta = 1.968$ (3 d.p.)

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

$0.4315 < \alpha < 0.4325$

where α is the root.

$x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$

$0 = \dfrac{2e^{3x} - 4}{e^{3x} + 4} - x$

$\therefore h(x) = \dfrac{2e^{3x} - 4}{e^{3x} + 4} - x$

$h(0.4315) = -0.000297$

$h(0.4325) = 0.000947$

There is a sign change and $h(x)$ is continuous within the interval

hence $\alpha = 0.432$ (3 d.p.)

(2)

(Total for question = 13 marks)

(Q15 9MA0/01, June 2023)

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Q34.

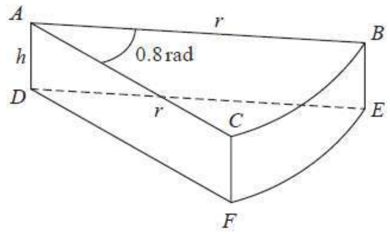

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle BAC = 0.8 radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is 240 cm^3

(a) show that the surface area of the toy, S cm^2, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

Top = base = $\frac{0.8r^2}{2} = 0.4r^2$ cm^2

left = right = rh cm^2

$\overset{\frown}{BC} = 0.8r$

BEFC = $0.8r \times h = 0.8rh$

$S = 0.4r^2 + 0.4r^2 + rh + rh + 0.8rh$

$S = 0.8r^2 + 2.8rh$

$S = 0.8r^2 + 2.8r\left[\frac{600}{r^2}\right]$

$S = 0.8r^2 + \frac{1680}{r}$

shown.

V = base area × height

$V = 0.4r^2 \times h$

$240 = 0.4r^2h$

$\frac{600}{r^2} = h$

(4)

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Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

$S = 0.8r^2 + 1680r^{-1}$

$\frac{dS}{dr} = 1.6r - 1680r^{-2}$

$\frac{dS}{dr} = 0$ at stationary point

$\therefore \quad \frac{1680}{r^2} = 1.6r$

$\frac{1680}{1.6} = r^3$

$\left(\frac{1680}{1.6}\right)^{1/3} = r$

$r \simeq 10.2$ cm (3 s.f.)

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

$\frac{d^2S}{dr^2} = 1.6 + 3360r^{-3}$

At $r = 10.2$

$\frac{d^2S}{dr^2} = 1.6 + 3360\left[\left(\frac{1680}{1.6}\right)^{1/3}\right]^{-3}$

$\frac{d^2S}{dr^2} = 4.8$

$4.8 > 0 \quad \therefore$ If $\frac{d^2S}{dr^2} > 0$

$r = 10.2$cm gives a minimum surface area.

(2)

(Total for question = 10 marks)

(**Q15 9MA0/01, June 2022**)

Video Solutions Walkthough

Q35.

Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for cos ($A \pm B$) and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

$$\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0} \frac{(\cos\theta+h) - \cos\theta}{h}$$

$$\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0} \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$$

$$\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0} \frac{\cos\theta\cos h - \cos\theta}{h} - \frac{\sin\theta\sin h}{h}$$

$$\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0} \cos\theta\left[\frac{\cos h - 1}{h}\right] - \sin\theta\left[\frac{\sin h}{h}\right]$$

$$\frac{d}{d\theta}(\cos\theta) = \cos\theta \times 0 - \sin\theta \times 1$$

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

(5)

(Total for question = 5 marks)

(Q09 9MA0/02, June 2018)

Q36.

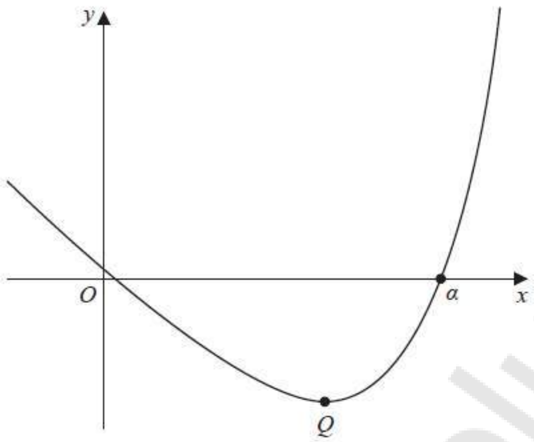

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2^x - 10x$$

The curve crosses the x-axis at $x = \alpha$, $\alpha > 1$, as shown in Figure 2.

(a) Show that α lies in the interval [5, 6]

$f(5) = 2^5 - 10(5) = -18$

$f(6) = 2^6 - 10(6) = 4$

Sign change occurs from $f(5)$ to $f(6)$

The function $f(x)$ is continuous within the interval $[5,6]$

Hence root, α must lie between $[5,6]$.

(2)

Given that $\mathrm{f}'(x) = p \times 2^x - 10$

(b) state the value of the constant p

$y = 2^x$

$\ln y = \ln 2^x$

$\ln y = x \ln 2$

$\frac{1}{y}\frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = y \ln 2$

$\frac{dy}{dx} = 2^x \ln 2$

$\therefore \; p = \ln 2$

$f'(x) = 2^x \ln 2 - 10$

(1)

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(c) Taking $x_0 = 6$ as a first approximation to α, apply the Newton-Raphson method once to f(x) to obtain a second approximation to α

Show your method and give your answer to 3 significant figures.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 6 - \frac{f(6)}{f'(6)}$$

$$x_1 = 6 - \frac{4}{64\ln 2 - 10} = 5.88359.....$$

$$x_2 = 5.88359.... - \frac{f(5.88359...)}{f'(5.88359...)}$$

$$x_2 = 5.87703.....$$

$$x_2 \approx 5.88 \quad (3s.f.)$$

(2)

The curve has a minimum turning point at Q, shown in Figure 2.

(d) Use the answer to part (b) to find the x coordinate of Q

Show your working and give your answer to 3 significant figures.

At Q, $f'(x) = 0$

$$\therefore \quad 2^x \ln 2 - 10 = 0$$

$$2^x \ln 2 = 10$$

$$2^x = \frac{10}{\ln 2}$$

$$\ln 2^x = \ln \left| \frac{10}{\ln 2} \right|$$

$$x \ln 2 = \ln \left| \frac{10}{\ln 2} \right|$$

$$\therefore \quad x = \frac{1}{\ln 2} \left[\ln \left| \frac{10}{\ln 2} \right| \right]$$

$$x \approx 3.85 \quad (3s.f.)$$

(2)

(Total for question = 7 marks)

(**Q04 9MA0/02/M, June 2025**)

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Q37.

The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

At turning point, $\frac{dy}{dx} = 0$, $x = 3$

$0 = 2(3)^3 - 9(3)^2 + 5(3) + k$

$0 = 54 - 81 + 15 + k$

$\therefore k = 12$

(2)

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(b) Hence find the coordinates of the point where *C* crosses the *y*-axis.

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + 12$$

$$\int dy = \int 2x^3 - 9x^2 + 5x + 12 \, dx$$

$$y = \frac{2x^4}{4} - \frac{9x^3}{3} + \frac{5x^2}{2} + 12x + C$$

$$y = \frac{x^4}{2} - 3x^3 + 2.5x^2 + 12x + C$$

$$-10 = \frac{(3)^4}{2} - 3(3)^3 + 2.5(3)^2 - 12(3) + C$$

$$-10 = 18 + C$$

$$C = -28$$

C crosses y-axis at $(0, -28)$

(3)

(Total for question = 5 marks)

(**Q05 9MA0/02, June 2023**)

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Q38.

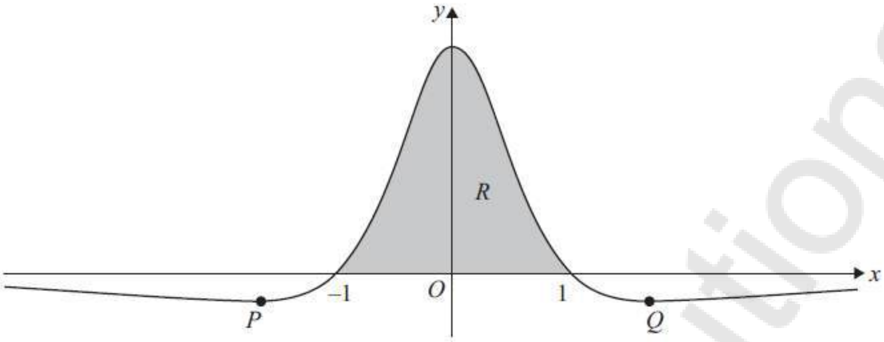

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{\left(1 + x^2\right)^2}$$

The curve

- intersects the x-axis at −1 and 1
- has minimum turning points at P and Q

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of P.

$u = 1 - x^2 \qquad v = (1+x^2)^2$

$u' = -2x \qquad v' = 2 \times 2x\,(1+x^2)$

$v' = 4x(1+x^2)$

$$f'(x) = \frac{-2x(1+x^2)^2 - 4x(1+x^2)(1-x^2)}{(1+x^2)^2}$$

P and Q occur at $f'(x) = 0$

$\therefore\ -2x(1+x^2)^2 - 4x(1+x^2)(1-x^2) = 0$

$-2x(1+x^2)\left[(1+x^2) + 2(1-x^2)\right] = 0$

$-2x(1+x^2) = 0 \qquad (1+x^2) + 2(1-x^2) = 0$

$-2x = 0 \quad 1+x^2 = 0 \qquad 1+x^2 = -2(1-x^2)$

$x = 0 \quad x^2 = -1 \qquad 1 + x^2 = -2 + 2x^2$

Maximum point hence not P or Q; No real roots; $x^2 = 3$

$x = \sqrt{3}$ and $x = -\sqrt{3}$

$$f(-\sqrt{3}) = \frac{1 - (-\sqrt{3})^2}{\left[1 + (-\sqrt{3})^2\right]^2} = \frac{1}{8}$$

$\therefore\ P\left(-\sqrt{3},\ \frac{1}{8}\right)$

(5)

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(b) Using the substitution $x = \tan\theta$ show that

$$\int_{-1}^{1} f(x)\,dx = \int_{\alpha}^{\beta} \cos 2\theta \, d\theta$$

where α and β are constants to be found.

$f(x) = \dfrac{1 - x^2}{(1 + x^2)^2}$

$1 = \tan\theta$, $\theta = \tan^{-1}(1) = \pi/4$

$-1 = \tan\theta$, $\theta = \tan^{-1}(-1) = -\pi/4$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{1 - \tan^2\theta}{(1 + \tan^2\theta)^2} \times \sec^2\theta \, d\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$x = \tan\theta$

$\dfrac{dx}{d\theta} = \sec^2\theta$

$dx = \sec^2\theta \times d\theta$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{1 - \tan^2\theta}{(\sec^2\theta)^2} \times \sec^2\theta \, d\theta$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{1 - \tan^2\theta}{\sec^2\theta} \, d\theta$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{1}{\sec^2\theta} - \dfrac{\tan^2\theta}{\sec^2\theta} \, d\theta$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2\theta - \sin^2\theta \, d\theta$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2\theta \, d\theta$

(5)

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The finite region *R*, shown shaded in Figure 5, is bounded by the *x*-axis and the curve.

(c) Use algebraic integration to find the area of *R*.

$$R = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2\theta \, d\theta$$

$$\left[\frac{\sin 2\theta}{2} \right]_{-\pi/4}^{\pi/4}$$

$$R = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$$

$$R = \underline{\underline{1}}$$

(3)

(Total for question = 13 marks)

(**Q15 9MA0/02, June 2025**)

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Q39.

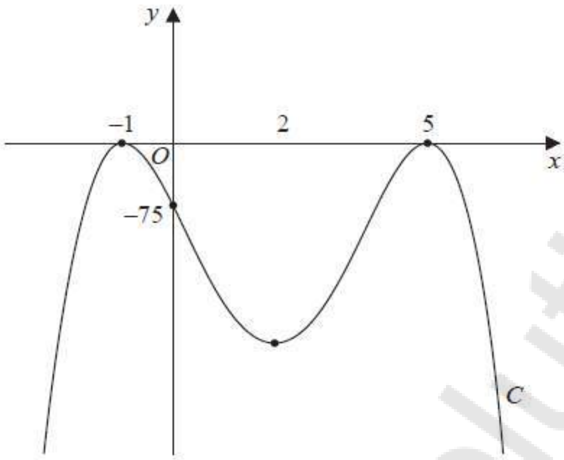

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a quartic expression in x.

The curve

- has maximum turning points at $(-1, 0)$ and $(5, 0)$
- crosses the y-axis at $(0, -75)$
- has a minimum turning point at $x = 2$

(a) Find the set of values of x for which

$$\mathrm{f}'(x) \geq 0$$

writing your answer in set notation.

$f'(x) \geq 0$ when $x \leq -1$ and $2 \leq x \leq 5$

$\{x : x \leq -1\} \cup \{x : 2 \leq x \leq 5\}$

(2)

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(b) Find the equation of C. You may leave your answer in factorised form.

$f(x) = (x+1)^2(x-5)^2$

constant will be 25

To get -75, $f(x)$ must be multiplied by -3

$f(x) = -3(x+1)^2(x-5)^2$

(3)

The curve C_1 has equation $y = f(x) + k$, where k is a constant.

Given that the graph of C_1 intersects the x-axis at exactly four places,

(c) find the range of possible values for k.

$f(2) = -3(2+1)^2(2-5)^2$

$f(2) = -243$

Thus $0 < k < 243$

(2)

(Total for question = 7 marks)

(**Q07 9MA0/01, June 2025**)

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Q40.

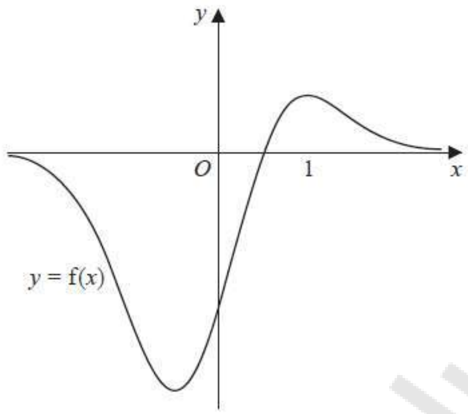

Figure 4

Figure 4 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (4x - k)e^{-x^2} \qquad x \in \mathbb{R}$$

and where k is a positive constant.

Given that the curve has a maximum turning point at $x = 1$

(a) show that $k = 2$

$f'(x)$: $u = 4x - k$ $\quad v = e^{-x^2}$

$u' = 4$ $\quad v' = -2xe^{-x^2}$

$f'(x) = -2xe^{-x^2}[4x - k] + 4e^{-x^2}$

At $x = 1$, $f'(x) = 0$

$0 = -2e^{-1}[4 - k] + 4e^{-1}$

$0 = -2e^{-1}[4 - k - 2]$

$0 = 2 - k$

$k = 2$

(4)

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(b) Hence show that the only other turning point is at $x = -\frac{1}{2}$

$$f'(x) = -2x e^{-x^2}[4x - 2] + 4e^{-x^2}$$

$$0 = -2e^{-x^2}[x(4x-2) - 2]$$

$-2e^{-x^2} = 0$ $\quad$ $4x^2 - 2x - 2 = 0$

No real roots $\quad$ $(x-1)(2x+1) = 0$

$\therefore x = 1 \quad x = -\frac{1}{2}$ $\quad$ hence only other turning point is at $x = -\frac{1}{2}$

(1)

Given that the equation f(x) = p, where p is a constant, has exactly two distinct roots,

(c) find the range of possible values for p

$$f(-\tfrac{1}{2}) = [4(-\tfrac{1}{2}) - 2]e^{-(-\frac{1}{2})^2} = -3.1152....$$

$$f(1) = [4(1) - 2]e^{-(1)^2} = 0.73575...$$

$$-4e^{-\frac{1}{4}} < p < 2e^{-1} \quad p \neq 0$$

(3)

(Total for question = 8 marks)

(**Q10 9MA0/02/M, June 2025**)

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Q41.

The function f is defined by

$$f(x) = \frac{2x - 3}{x^2 + 4} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2 + bx + c}{\left(x^2 + 4\right)^2}$$

where *a*, *b* and *c* are constants to be found.

$u = 2x - 3 \qquad v = x^2 + 4$

$u' = 2 \qquad v' = 2x$

$$f'(x) = \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$$

$$f'(x) = \frac{2x^2 + 8 - 4x^2 + 6x}{(x^2+4)^2}$$

$$f'(x) = \frac{-2x^2 + 6x + 8}{(x^2+4)^2}$$

$a = -2 \quad b = 6 \quad c = 8$

(3)

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(b) Hence, using algebra, find the values of x for which f is decreasing.

You must show each step in your working.

$f'(x) < 0$

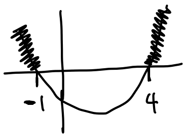
$$\frac{-2x^2+6x+8}{(x^2+4)^2} < 0$$

$-2x^2 + 6x + 8 < 0$

$x^2 - 3x - 4 > 0$

$(x-4)(x+1) > 0$

$x = 4 \quad x = -1$ are critical values



$x < -1 \quad$ or $\quad x > 4$

(3)

(Total for question = 6 marks)

(Q05 9MA0/01, June 2024)

Q42.

The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

$f'(x)$: $u = e^{3x}$ $\quad v = 4x^2 + k$

$u' = 3e^{3x}$ $\quad v' = 8x$

$$f'(x) = \frac{3e^{3x}[4x^2 + k] - 8xe^{3x}}{(4x^2+k)^2}$$

$$f'(x) = (12x^2e^{3x} + 3ke^{3x} - 8xe^{3x})(4x^2+k)^{-2}$$

$$f'(x) = (12x^2 - 8x + 3k)[e^{3x}(4x^2+k)^{-2}]$$

$$g(x) = e^{3x}(4x^2+k)^{-2}$$

(3)

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Given that the curve with equation $y = \mathrm{f}(x)$ has at least one stationary point,

(b) find the range of possible values of k.

$f'(x) = 0$, $0 = 12x^2 - 8x + 3k$

$(-8)^2 - 4(12)(3k) \geqslant 0$

$64 - 144k \geqslant 0$

$64 \geqslant 144k$

$\frac{4}{9} \geqslant k$

(3)

(Total for question = 6 marks)

(**Q12 9MA0/02, June 2022**)

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Q43.

A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b , is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

At $t=0$, $N_b = 45 + 220e^{0.05(0)}$

$N_b = 265$ thousand bees.

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

$N_b = 45 + 220e^{0.05t}$

$\frac{dN_b}{dt} = 11e^{0.05t}$

At $t = 10$

$\frac{dN_b}{dt} = 11e^{0.05(10)}$

$\frac{dN_b}{dt} = 18.1 \simeq 18$ thousand bees per year.

(3)

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The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

when $N_b = N_w$

$$45 + 220e^{0.05t} = 10 + 800e^{-0.05t}$$

$$35 = 800e^{-0.05t} - 220e^{0.05t}$$

$$35e^{0.05t} = 800 - 220(e^{0.05t})^2$$

$$220(e^{0.05t})^2 + 35e^{0.05t} - 800 = 0$$

$$220y^2 + 35y - 800 = 0$$

$$y = 1.829038082 \qquad y = -1.988128991 \text{ reject}$$

$$y = e^{0.05t}$$

$$\ln y = \ln e^{0.05t}$$

$$\ln y = 0.05t \ln e$$

$$\frac{\ln y}{0.05} = t$$

$$T = \frac{\ln 1.829038082}{0.05}$$

$$T \approx 12.08 \text{ years}$$

(4)

(Total for question = 8 marks)

(**Q10 9MA0/01, June 2022**)

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Q44.

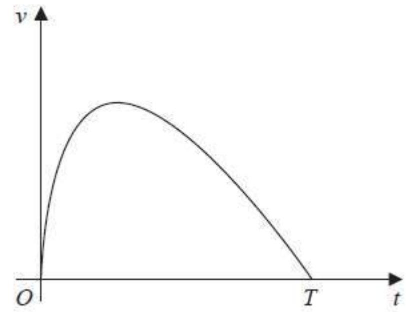


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, v ms^{-1}, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leq t \leq T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

$t = T$ at $v = 0$

$0 = (10 - 0.4t)(\ln|t+1|)$

$10 - 0.4t = 0$

$t = \frac{10}{0.4}$

$\underline{\underline{T = 25 \text{ seconds}}}$

$0 = \ln|t+1|$

$1 = t + 1$

$t = 0$

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

$\frac{dv}{dt}$: $u = 10 - 0.4t$ $\quad v = \ln(t+1)$

$u' = -0.4$ $\quad v' = \frac{1}{t+1}$

$\frac{dv}{dt} = -0.4\ln(t+1) + \frac{10 - 0.4t}{t+1}$

Max Speed at $\frac{dv}{dt} = 0$

$0 = -0.4\ln(t+1) + \frac{10 - 0.4t}{t+1}$

$0.4\ln(t+1) = \frac{10 - 0.4t}{t+1}$

$0.4t\ln(t+1) + 0.4\ln(t+1) = 10 - 0.4t$

$0.4t\ln(t+1) + 0.4t = 10 - 0.4\ln(t+1)$

$0.4t[\ln(t+1) + 1] = 10 - 0.4\ln(t+1)$

$t(\ln(t+1) + 1) = 25 - \ln(t+1)$

$t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$

$t = \frac{25 + 1 - 1 - \ln(t+1)}{1 + \ln(t+1)}$

$t = \frac{26 - [1 + \ln(t+1)]}{1 + \ln(t+1)}$

$t = \frac{26}{1 + \ln(t+1)} - \frac{1 + \ln(t+1)}{1 + \ln(t+1)}$

$t = \frac{26}{1 + \ln(t+1)} - 1$

Shown.

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

i) $t_2 = \frac{26}{1 + \ln(7+1)} - 1 = 7.443\ldots$

$t_3 = \frac{26}{1 + \ln(7.443\ldots + 1)} - 1 \approx 7.298$ (3 d.p.)

ii) $t_4 = 7.344045888$

$\vdots$

$t_{30} = 7.332800321$

time to max speed ≈ 7.33 seconds (3 s.f.)

(3)

(Total for question = 8 marks)

(**Q08 9MA0/01, June 2022**)

Video Solutions Walkthough

Q45.

$$f(x) = 10e^{-0.25x} \sin x, \quad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

$f(x) = 10e^{-0.25x} \sin x$

$u = e^{-0.25x}$ $\quad v = \sin x$

$u' = -0.25e^{-0.25x}$ $\quad v' = \cos x$

$f'(x) = 10\left[e^{-0.25x}\cos x - 0.25e^{-0.25x}\sin x\right]$

At turning point, $f'(x) = 0$

$\therefore \quad 0 = 10\left[e^{-0.25x}\cos x - 0.25e^{-0.25x}\sin x\right]$

$0.25e^{-0.25x}\sin x = e^{-0.25x}\cos x$

$\frac{\sin x}{\cos x} = \frac{1}{0.25}$

$\tan x = 4$ shown.

(4)

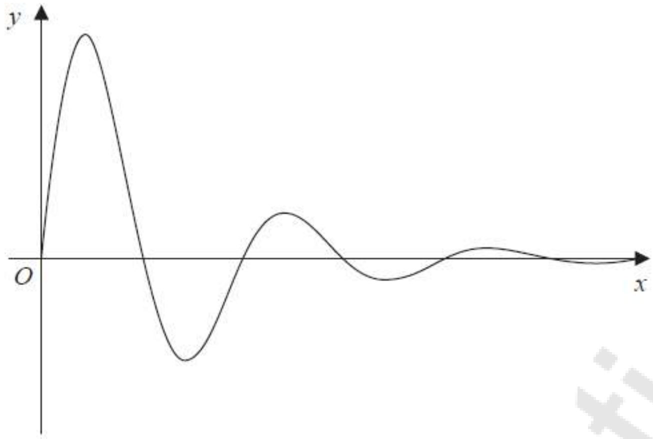


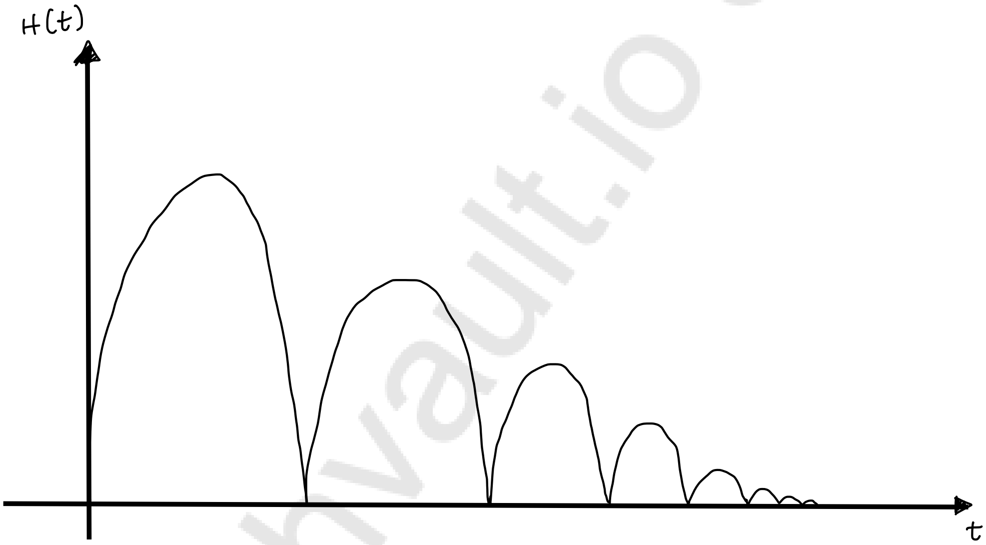
Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t}\sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.



(2)

Aiming For A* Live Sessions Friday at 5pm

The function H(t) is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

At H_{max}, $\frac{dH}{dt} = 0$

$\therefore \tan t = 4$

$t = 1.3258.... + \pi$

$t = 4.46741......$

$H_{max} = H(4.46741...) = |-3.175357...|$

$H_{max} \approx 3.18$ metres (3 s.f.)

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

Time between each bounce should get smaller because the height of each bounce gets lower but the model does not account for this.

(1)

(Total for question = 10 marks)

(**Q12 9MA0/01, June 2019**)

Video Solutions Walkthough

Q46.

Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$\frac{dy}{dx} = 4x$

$$y = 2x^2$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2[x^2 + 2xh + h^2] - 2x^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{4xh + 2h^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{4xh}{h} + \frac{2h^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} 4x + 2h$$

$$\frac{dy}{dx} = 4x$$

(Total for question = 3 marks)

(**Q04 9MA0/02, June 2022**)

Q47.

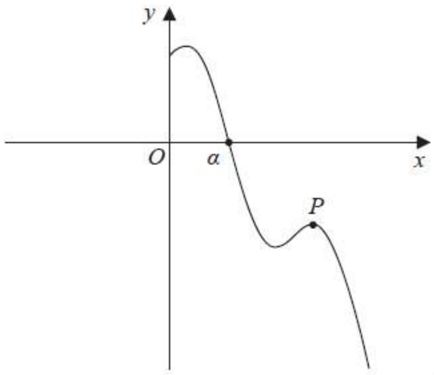


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$\mathrm{f}(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

(a) find the x coordinate of P, giving your answer to 3 significant figures.

$f'(x) = 8 \times \frac{1}{2} \times \cos\left(\frac{1}{2}x\right) - 3$

$0 = 4\cos\left(\frac{1}{2}x\right) - 3$

$\frac{3}{4} = \cos\left(\frac{1}{2}x\right)$

$0.7227\ldots,\ 2\pi - 0.7227,\ 2\pi + 0.7227 = \frac{1}{2}x$

$\therefore\ x = 1.445\ldots,\ 4\pi - 1.445\ldots,\ 4\pi + 1.445\ldots$

$f''(x) = -2\sin\left(\frac{1}{2}x\right)$

$f(1.445\ldots) = 2.29$ hence 1.445 not x coordinate of P. y-coordinate must be negative

$f''(4\pi - 1.445\ldots) = 1.3225\ldots$ $\quad f''(4\pi - 1.445) > 0$ hence local minimum.

$f''(4\pi + 1.445\ldots) = -1.3225\ldots$ $\quad f''(4\pi + 1.445) < 0$ hence local minimum.

$f(4\pi + 1.445) < 0$ Thus $P(14.0, y)$

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, f(4) = 4.274 and f(5) = −1.212

(b) explain why α must lie in the interval [4, 5]

$f(4) > 0$ and $f(5) < 0$ hence a sign change occurs between 4 and 5

The function is continuous within the interval [4,5]

Hence the root, α must lie in the interval [4,5]

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to f(x) to obtain a second approximation to α.
Show your method and give your answer to 3 significant figures.

$$x_1 = 5 - \frac{-1.212}{4\cos(2.5x) - 3}$$

$$x_1 \approx 4.80 \ (3 \text{ s.f.})$$

(2)

(Total for question = 7 marks)

(**Q06 9MA0/02, June 2022**)

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Q48.

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

i) $y = x^2 - 2x - 24x^{1/2}$

$\frac{dy}{dx} = 2x - 2 - 12x^{-1/2}$

ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-3/2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

At $x = 4$, $\frac{dy}{dx} = 2(4) - 2 - 12(4)^{-1/2}$

$\frac{dy}{dx} = 0$ Thus At $x = 4$ C has a stationary point.

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

At $x = 4$, $\frac{d^2y}{dx^2} = 2 + 6[4]^{-3/2}$

$\frac{d^2y}{dx^2} = \frac{11}{4}$

$\frac{d^2y}{dx^2} > 0$ Thus $x = 4$ is a local minimum

(2)

(Total for question = 7 marks)

(**Q02 9MA0/01, June 2018**)

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Q49.

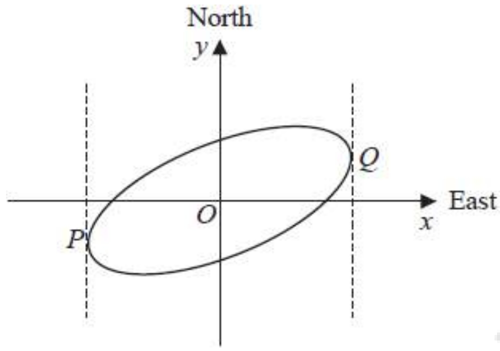

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

$$2x - 2y - 2x\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(6y - 2x) = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y - 2x}{6y - 2x}$$

$$\frac{dy}{dx} = \frac{2(y - x)}{2(3y - x)}$$

$$\frac{dy}{dx} = \frac{y - x}{3y - x}$$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

At Point P gradient $\to \infty$, Thus denominator $\to 0$

$$\frac{dy}{dx} = \frac{y - x}{3y - x}$$

If $3y - x = 0$

$x = 3y$

$(3y)^2 - 2y(3y) + 3y^2 = 50$

$9y^2 - 6y^2 + 3y^2 = 50$

$6y^2 = 50$

$y^2 = \frac{25}{3}$

$y = \frac{5}{\sqrt{3}}$ $\quad y = -\frac{5}{\sqrt{3}}$

reject

$x = \frac{-15}{\sqrt{3}}$

$\therefore P\left(\frac{-15}{\sqrt{3}}, \frac{-5}{\sqrt{3}}\right)$

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

Equate gradient function to zero and solve for x

Thus Solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and select the positive solutions.

(1)

(Total for question = 10 marks)

(**Q09 9MA0/01, June 2018**)

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Q50.

Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where *A* is a rational constant to be found.

$u = 3\sin\theta \qquad v = 2\sin\theta + 2\cos\theta$

$u' = 3\cos\theta \qquad v' = 2\cos\theta - 2\sin\theta$

$$\frac{dy}{d\theta} = \frac{3\cos\theta(2\sin\theta + 2\cos\theta) - 3\sin\theta(2\cos\theta - 2\sin\theta)}{(2\sin\theta + 2\cos\theta)^2}$$

$$\frac{dy}{d\theta} = \frac{6\sin\theta\cos\theta + 6\cos^2\theta - 6\sin\theta\cos\theta + 6\sin^2\theta}{4\sin^2\theta + 8\sin\theta\cos\theta + 4\cos^2\theta}$$

$$\frac{dy}{d\theta} = \frac{6\sin^2\theta + 6\cos^2\theta}{4\sin^2\theta + 4\cos^2\theta + 8\sin\theta\cos\theta}$$

$$\frac{dy}{d\theta} = \frac{6(\sin^2\theta + \cos^2\theta)}{4(\sin^2\theta + \cos^2\theta) + 4(2\sin\theta\cos\theta)}$$

$$\frac{dy}{d\theta} = \frac{6 \times 1}{4 \times 1 + 4 \times \sin 2\theta}$$

$$\frac{dy}{d\theta} = \frac{6}{4 + 4\sin 2\theta}$$

(5)

$$\frac{dy}{d\theta} = \frac{6/4}{1 + \sin 2\theta}$$

(Total for question = 5 marks)

(**Q05 9MA0/01, June 2018**)

$$\frac{dy}{d\theta} = \frac{1.5}{1 + \sin 2\theta} \qquad A = 1.5$$

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