Proof Exam Questions Mark Scheme

Topic Test and Revision

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Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2 \geqslant 0 \Rightarrow (x-4)^2+1 \geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2 \geqslant 0, (x-4)^2+1 \geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

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$x^2-8x+17$

$=(x-4)^2+1 \geqslant 1$ as $(x-4)^2 \geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2 \geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0

correct but not explained

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1

correct and explained

$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$ scores M1 A0 (no explanation) A0

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as +ve x^2 etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{dy}{dx} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the student just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value.
For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ×)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true.
For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true **and** states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x \le -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

(**Q02 8MA0/01, June 2018**)

Q2.

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions.
  So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8} = \frac{1}{8}n^3 + \frac{3}{8}n^2 + \frac{3}{8}n + \frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so $n^3 + 2$ is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so $n^3 + 2$ cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"$n^3 + 2$ is odd and cannot be divided by 8 **exactly**"
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so $n^3 + 2$ is not divisible by 8"
"if n^3 is a multiple of 8 then $n^3 + 2$ cannot be divisible by 8"
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8, $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=\underline{\underline{8k^3+12k^2+6k+3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3+12k^2+6k+3}}$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes
Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$ Some students will use $2k-1$ for odd numbers There is no requirement to change the variable. They may use $2n$ and $2n\pm1$ Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!) Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

(**Q15 8MA0/01, June 2019**)

Q3.

Question	Scheme	Marks	AOs
(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n(n^2 + 3n + 2) = n(n+1)(n+2)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving

- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review

(**Q14 8MA0/01, June 2022**)

Q4.

Question	Scheme	Marks	AOs
(a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state "not true".
e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.
It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$
Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = ...$ condoning slips but must lead to a quadratic.
Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$
May be subtracted the wrong way round for this mark as below.
$(2n)^3 - (2n+2)^3 = ...$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$
or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.
Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ or, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:
If the even numbers are set as n and $n+2$ there must be sufficient work seen before marks can be awarded.
e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8\left(3k^2 + 3k + 1\right)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:
If they just use any two even numbers, e.g., $2a$ and $2b$, **or** $2m$ and $2n+2$ then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8\left(a^3 - b^3\right)$ would imply this mark.

A1: $= 8\left(a^3 - b^3\right)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers

(Q17 8MA0/01, June 2023)

Q5.

Question	Scheme	Marks	AOs
	Sets up the proof by exploring when $n=2k$ or $n=2k+1$ e.g. $(2k)^2+5(2k)=...k^2+...k$ **or** $(2k+1)^2+5(2k+1)=...k^2+...k+...$	M1	1.1b
	e.g. $4k^2+10k$ **or** $4k^2+14k+6$ and shows or gives a reason why the expression is even (see notes)	A1	2.2a
	Explores when $n=2k$ **and** $n=2k+1$ eg $(2k)^2+5(2k)=...k^2+...k$ **and** $(2k+1)^2+5(2k+1)=...k^2+...k+...$	dM1	2.1
	e.g. $4k^2+10k$ **and** $4k^2+14k+6$ and shows or gives a reason why both of the expressions are even (see notes) hence n^2+5n is even for all $n(\in\mathbb{N})$ (or equivalent)	A1*	2.4
			(4 marks)

Notes
Main scheme algebraic method using e.g. $n=2k$ and $n=2k\pm1$ **You will need to look at both cases and mark the one which is fully correct first.** **Allow a different variable to k and may be different letters for odd and even.** **Condone use of n as a variable for the first three marks.** **There should be no errors in the algebra for the A marks but allow e.g. invisible brackets to be "recovered".**
M1: Sets up the proof by exploring when n is odd or even e.g. $n=2k$ or $n=2k+1$ (or equivalent), and either expands and achieves a quadratic expression (which may be unsimplified) or allow to factorise e.g. $2k(2k+5)$ or e.g. $(2k+2)(2k+7)$ Condone slips. e.g. $2k(2k+5)=2k^2+10k$ or slips when collecting terms.
A1: Correct quadratic expression (which may be unsimplified) for n^2+5n for either odds or evens and shows or gives a reason why the expression is even. They must have fully multiplied out or the quadratic expression must be factorised completely. e.g. $4k^2+10k=2(2k^2+5k)$ (which is even) e.g. $4k^2+14k+6=2(2k^2+7k+3)$ (which is even) e.g. $\frac{4k^2+10k}{2}=2k^2+5k$ (hence even) e.g. "2 is a factor of both terms", "all divisible by 2" (so even) If a reason is given as well as an algebraic expression it must be correct e.g. $4k^2+10k=2(2k^2+5k)$ so even as can be multiplied by 2 can score M1A1 but $\frac{4k^2+10k}{2}=2k^2+5k$ so it can be divided by 2 so even is M1A0 (needs to say divisible by 2) **Do not isw if they simplify their quadratic incorrectly.** Note that they do not have to state that the expression is even if they conclude for all cases at the end.
dM1: Explores when n is odd and when n is even leading to two quadratic expressions (may be factorised) for when $n=2k$ and $n=2k+1$ (or equivalent) (see first M1 for guidance)

A1*: Requires

- correct quadratic expression for n^2+5n for both odds and evens
- shows or gives a reason for each why the expressions are even (see first A1 for guidance)
- makes a concluding overall statement. "Hence n^2+5n is even for all $n(\in\mathbb{N})$" (or equivalent).

Note that if they have stated for each separate case that the expression is even then allow minimal statements of "hence proven", "statement proved", "QED", tick
Do not isw this mark if they simplify their quadratic incorrectly.

	n^2+5n
$2k-3$	$4k^2-2k-6$
$2k-2$	$4k^2+2k-6$
$2k-1$	$4k^2+6k-4$
$2k$	$4k^2+10k$
$2k+1$	$4k^2+14k+6$
$2k+2$	$4k^2+18k+14$
$2k+3$	$4k^2+22k+24$

Alternative methods:
Algebraic with logic example

e.g. $n^2+5n=n(n+5)$
When n is odd then $n+5$ is even so odd x even is even
When n is even then $n+5$ is odd so even x odd is even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n=k$, substitutes $n=k+1$ into n^2+5n, multiplies out the brackets and attempts to simplify to a quadratic expression (which may be unsimplified)
e.g. k^2+7k+6 Condone arithmetical slips

A1: $\left(\text{f}(k+1)=3k^2+3k+1+6(k+1)=\right)$ $k^2+5k+2k+6=\text{f}(k)+2(k+3)$
which is even + even = even

dM1: Attempts to substitute $n=1\Rightarrow$ $1^2+5\times1=6$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n=1$
- if it is true for $n=k$ then it is true for $n=k+1$
- therefore it is true for all $n(\in\mathbb{N})$

Solutions via just logic (no algebraic manipulation) scores 0 marks.
e.g.
If n is odd, then n^2+5n is $odd^2+odd\times odd=odd+odd=even$
If n is even, then n^2+5n is $even^2+odd\times even=even+even=even$

(**Q14 8MA0/01, June 2024**)

Q6.

Question	Scheme	Marks	AOs
(a)	n^3+4n		
	Attempts n^3+4n for any 2 natural numbers.	M1	1.1b
	$1^3+4\times1=5$ prime and e.g. $2^3+4\times2=16$ not prime $\therefore$ sometimes true.	A1	2.4
		(2)	

Condone the use of n for e.g. k for both marks.
All methods require attempting n^3+4n when $n=1$ for full marks.

Way 1:

M1: Attempts n^3+4n for any 2 natural numbers.

A1: Requires:

- obtains $n^3+4n=5$ when $n=1$ and states "prime" or "true" or ✓
- correct evaluation for any other natural number and states "not prime" or "composite" or "not true" or ×
- states "sometimes true"

You can ignore any other incorrectly evaluated examples or e.g. use of negative numbers as long as these conditions are met.

Way 2 (factorisation):

M1: Attempts to factorise $n^3+4n=n\left(n^2+4\right)$

Allow for $n^3+4n=n\left(n^2+\ldots\right)$ or $n^3+4n=n\left(\ldots+4\right)$

A1: Requires:

- uses $n=1$ and obtains $n^3+4n=5$ and states "prime" or "true"
- correct factorisation and states $n^3+4n=n\left(n^2+4\right)$ is "composite" or "not prime" or "not true" (when $n>1$)
- states "sometimes true"

Way 3 (odd/even):

M1: Attempts to substitute $n=2k$ oe **or** $n=2k+1$ oe

A1: Requires:

- uses $n=1$ and obtains $n^3+4n=5$. Here the value for $n=1$ might be found using $k=1$ for $n=2k-1$ or $k=0$ for $n=2k+1$ and states "prime" or "true"
- correctly factorises any one correct form for odd or even e.g. $8k^3+8k=8k\left(k^2+1\right)$ and concludes "not prime" or "composite" or "not true" (only one form needed here)
- states "sometimes true"

Question	Scheme	Marks	AOs
(b)	n^3+5n		
	$n^3+5n=n\left(n^2+5\right)$	M1	3.1a
	Since $1^3+5\times1=6$, n^3+5n is not prime for $n=1$ For all other n, $n^3+5n=n\left(n^2+5\right)$ is not prime as it is the product of two other numbers not equal to 1. Hence never true.	A1	2.4
		(2)	
			(4 marks)

Notes:
Condone the use of n for e.g. k for both marks. **Way 1 (Factorisation):** **M1:** Attempts to factorise $n^3 + 5n = n(n^2 + 5)$ Allow $n^3 + 5n = n(n^2 + ...)$ or $n^3 + 5n = n(... + 5)$ **A1:** Requires: • Correct factorisation $n^3 + 5n = n(n^2 + 5)$ • Substitution of $n = 1$: $1^3 + 5 \times 1 = 6$ **or** states $n^3 + 5n \neq 2$ oe e.g. $n^3 + 5n > 2$ oe • "Never true" **Way 2 (Odd/Even):** **M1:** Attempts to substitute $n = 2k$ **and** either $n = 2k + 1$ or $n = 2k - 1$ oe **A1:** Requires: • Correctly factorising both even and odd forms e.g. $8k^3 + 10k = 2k(4k^2 + 5)$ **and** e.g. $(2k+1)^3 + 5(2k+1) = (2k+1)((2k+1)^2 + 5)$ or e.g. $2(2k+1)(2k^2 + 2k + 3)$ (in this part both cases are needed) • Substitution of $n = 1$: $1^3 + 5 \times 1 = 6$, or e.g. $k = 1$ for $n = 2k - 1$ or $k = 0$ for $n = 2k + 1$ **or** states $n^3 + 5n \neq 2$ oe e.g. $n^3 + 5n > 2$ oe e.g. $2k(4k^2 + 5)$ and $2(2k+1)(2k^2 + 2k + 3)$ are > 2 or $\neq 2$ • "Never true" **Way 3 (Odd/Even via logic):** **M1:** Considers $n^3 + 5n$ with "odds" and "evens" e.g. If n is **odd** then $n^3 + 5n$ = odd + odd = even or e.g. $n^3 + 5n = n(n^2 + 5)$ = odd(odd + odd) = odd×even = even If n is **even** then $n^3 + 5n$ = even + even = even or e.g. $n^3 + 5n = n(n^2 + 5)$ = even(even + odd) = even×odd = even **A1:** Requires: • Fully correct argument for both odds and evens • A full justification of any assumed results e.g. n odd $\Rightarrow n^3$ is odd via algebra or e.g. n odd means n^3 = odd×odd×odd = odd • When $n = 1$, $n^3 + 5n = 6$ so $n^3 + 5n \neq 2$ • "Never true"

(**Q15 8MA0/01, June 2025**)