Aiming For A* Live Sessions Friday at 5pm

Name: ____

Proof Exam Questions

Topic Test and Revision

Date:

Time: 45 Minutes

Total marks available: 30

Total marks achieved: ______

Calculator Allowed

Mathvault.io

Worked Solutions and Official Mark Scheme

Video Solutions Walkthough

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Questions

Q1.

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

$(x-4)^2 = x^2 - 8x + 16$

$-16 \qquad -16$

$(x-4)^2 - 16 = x^2 - 8x$

$\therefore \quad (x-4)^2 - 16 + 17$

$(x-4)^2 + 1$

$(x-4)^2 \geq 0 \quad , \quad \therefore (x-4)^2 + 1 \geq 1$ hence always greater than zero.

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

let the number $= n$

$(n+3)^2 > n^2$

$n^2 + 6n + 9 > n^2$

$6n + 9 > 0$

$6n > -9$

$n > -\frac{9}{6}$

$n > -\frac{3}{2}$ $\therefore$ statement is sometimes true

(2)

(Total for question = 5 marks)

(**Q02 8MA0/01, June 2018**)

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Q2.

Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8.

Natural numbers can either be odd or even.

let odd numbers $= 2k+1$

$(2k+1)^3+2$

$(2k+1)(4k^2+4k+1)+2$

$8k^3+8k^2+2k+4k^2+4k+1+2$

$8k^3+12k^2+6k+3$

$\underbrace{2(4k^3+6k^2+3k)}_{\text{Even}}+3$

Even number $+ 3 =$ odd number

An odd number cannot be divisible by 8.

let even numbers $= 2k$

$(2k)^3+2$

$8k^3+2$

$8\left[k^3+\frac{2}{8}\right]$

$8\left[k^3+\frac{1}{4}\right]$

$k^3+\frac{1}{4}$ is not a whole number

hence $8k^3+2$ is not divisible by 8

This means given $n \in \mathbb{N}$, n^3+2 is not divisible by 8.

(Total for question = 4 marks)

(**Q15 8MA0/01, June 2019**)

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Q3.

(i) A student states

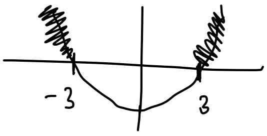
"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

$x^2 > 9$

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$



$x < -3 \quad x > 3$

This statement is not true because x could also be less than -3.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

$n(n^2 + 3n + 2)$

$n(n+1)(n+2)$

This is the general form for the product of three consecutive integers.

$\therefore$ Within three consecutive integers, a multiple of 2 and a multiple of 3 must always exist thus $n^3 + 3n^2 + 2n$ is always divisible by 6 for all integers n.

(3)

(Total for question = 4 marks)

(**Q14 8MA0/01, June 2022**)

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Q4.

In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

Try $q = 8$, $p = 3$

$8^3 - 3^3$

$512 - 27$

485

$485 \div 5 = 97$ hence at $q = 8$ and $p = 3$, $q^3 - p^3$ is a multiple of 5.

(1)

Statement 2: When p and q are consecutive even integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

If 1st even integer , $p = 2n$

Then 2nd even integer , $q = 2n+2$

$(2n+2)^3 - (2n)^3$

$(2[n+1])^3 - 8n^3$

$8[(n+1)(n^2+2n+1)] - 8n^3$

$8[n^3+2n^2+n+n^2+2n+1] - 8n^3$

$8[n^3+3n^2+3n+1] - 8n^3$

$8[n^3+3n^2+3n+1-n^3]$

$8[3n^2+3n+1]$

Thus, $q^3 - p^3$ is a multiple of 8

(4)

(Total for question = 5 marks)

(**Q17 8MA0/01, June 2023**)

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Q5.

Prove, using algebra, that

$$n^2 + 5n$$

is even for all $n \in \mathbb{N}$

If n is even, $n = 2k$

$(2k)^2 + 5(2k)$

$4k^2 + 10k$

$2(2k^2 + 5k)$

multiple of 2 hence even

If n is odd, $n = 2k+1$

$(2k+1)^2 + 5(2k+1)$

$4k^2 + 4k + 1 + 10k + 5$

$4k^2 + 14k + 6$

$2(2k^2 + 7k + 3)$

multiple of 2 hence even.

$n^2 + 5n$ is thus even for all $n \in \mathbb{N}$

(Total for question = 4 marks)

(**Q14 8MA0/01, June 2024**)

Q6.

$$\text{“}n^3 + 4n \text{ is prime for } n \in \mathbb{N}\text{”} \qquad \text{(I)}$$

(a) Determine whether statement (I) is always true, sometimes true or never true.

You must fully justify your answer.

At $n = 1$, $1^3 + 4(1) = 5$ which is prime

At $n = 2$, $2^3 + 4(2) = 16$ which is not prime

This statement is sometimes true.

(2)

$$\text{“}n^3 + 5n \text{ is prime for } n \in \mathbb{N}\text{”} \qquad \text{(II)}$$

(b) Determine whether statement (II) is always true, sometimes true or never true.

You must fully justify your answer.

At $n=1$, $1^3 + 5(1) = 6$

All natural numbers are multiples of 1 so if at $n = 1$, $n^3 + 5n$ is not prime, $n^3 + 5n$ will not be prime for any value of n.

$n(n^2 + 5)$ will always be a product of two other numbers not equal to 1

The statement is never true.

(2)

(Total for question = 4 marks)

(**Q15 8MA0/01, June 2025**)