

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Thursday 2 November 2017 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper



First name					
Last name					
Centre number					
Candidate number					

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer all the questions.

1 Donald swims 3 lengths of a swimming pool in 93 seconds.

(a) Use this information to show that he could swim 100 lengths in under 55 minutes. [4]

$$\begin{array}{r}
 3 \text{ lengths} = 93 \text{ seconds} \\
 \div 3 \qquad \qquad \qquad \div 3 \\
 1 \text{ length} = 31 \text{ seconds} \\
 \times 100 \qquad \qquad \qquad \times 100 \\
 100 \text{ lengths} = 3100 \text{ seconds}
 \end{array}$$

$$\begin{array}{r}
 \text{s} \xrightarrow{\div 60} \text{m} \\
 3100 \div 60 = 51.6 \text{ mins}
 \end{array}$$

(b) What assumption did you make in part (a)?

..... He swims at the same rate.
 [1]

(c) Donald tries to swim the 100 lengths in under 55 minutes.

Suggest one reason why he might not achieve this.

..... He will get tired.
 [1]

2 (a) Simplify.

(i) $a^6 \div a^2$

$$a^{6-2}$$

(a)(i) a^4 [1]

(ii) $(b^5)^3$

$$b^{5 \times 3}$$

(ii) b^{15} [1]

(b) Factorise.

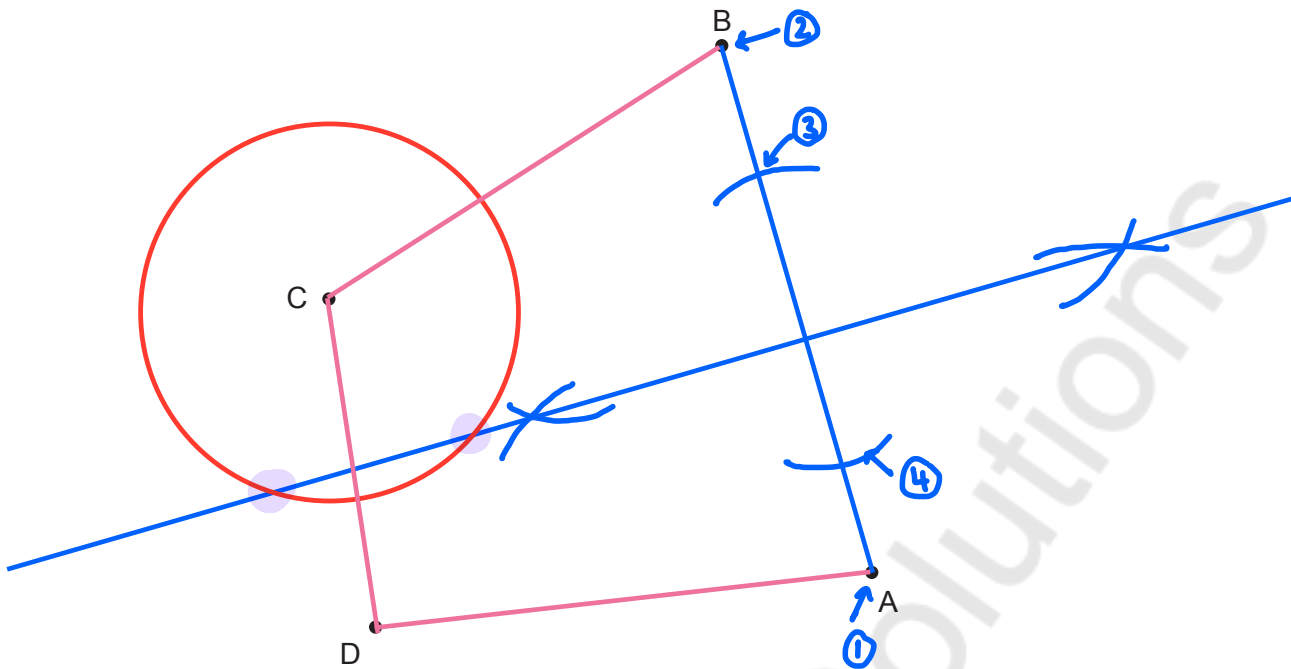
$$6x - x^2$$

$$\text{HCF} = x$$

$$x(6 - x)$$

(b) $x(6 - x)$ [1]

3 Four points A, B, C and D are shown on the scale diagram below.



Scale: 1 cm represents 5 m

$\div 5$

(a) On the diagram, construct and mark the two points that are

- the same distance from A and B **perpendicular bisector** and
- 15m from C. **$15\text{ m} \div 5 = 3\text{ cm radius}$**

Show all your construction lines.

[5]

(b) The points A, B, C and D represent the four corners of Monty's garden. His garden is bounded by four straight fences A to B, B to C, C to D and D to A.

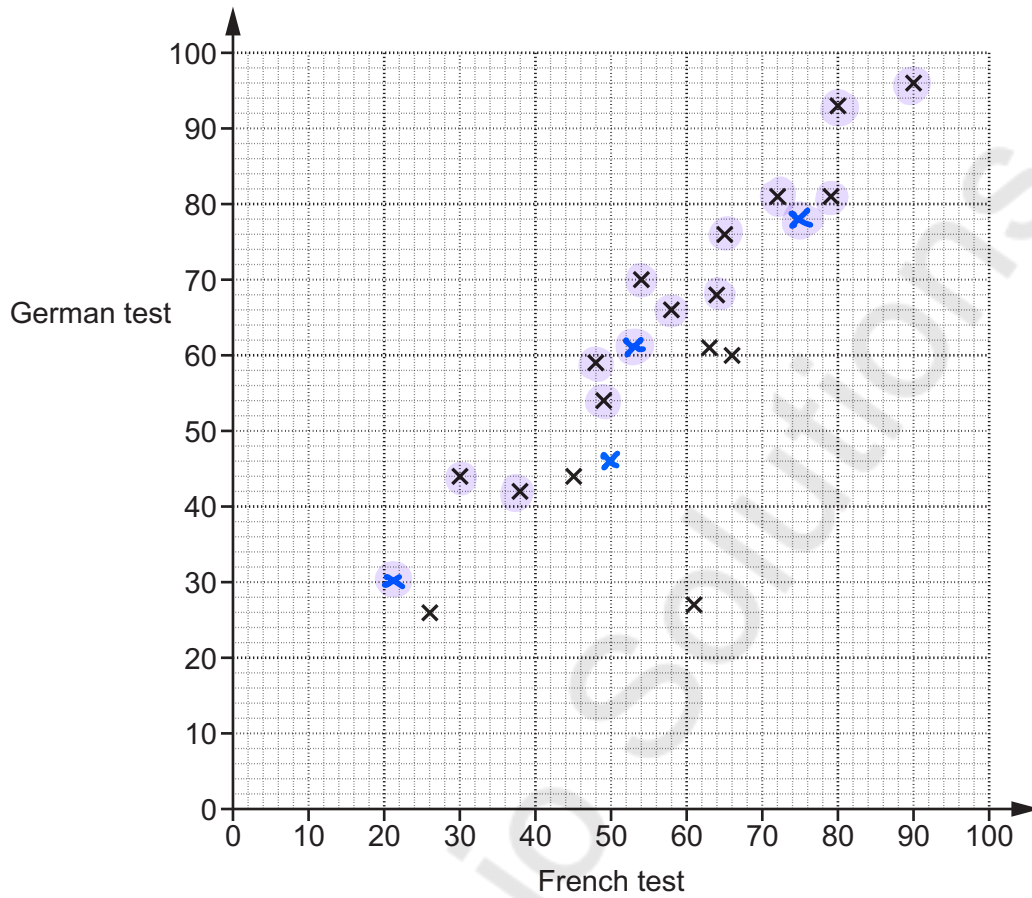
Monty wants to plant a tree in his garden at a place that satisfies the two conditions in part (a).

Explain why there is only one position where Monty can plant his tree.

..... One of the points is not in his garden.

[1]

- 4 The scatter diagram shows the results of 17 students in their French test and their German test. Both tests are out of 100.



- (a) Here are the results of another 4 students.

French	21	75	48	53
German	30	78	46	61

Plot these results on the scatter diagram.

[2]

- (b) Describe the type and strength of the correlation shown in this diagram.

(b) **Strong positive** [2]

- (c) Work out the percentage of the students whose German result was **higher** than their French result.

$$\frac{15}{21} \times 100 = 71.428571$$

(c) **71.4** % [4]

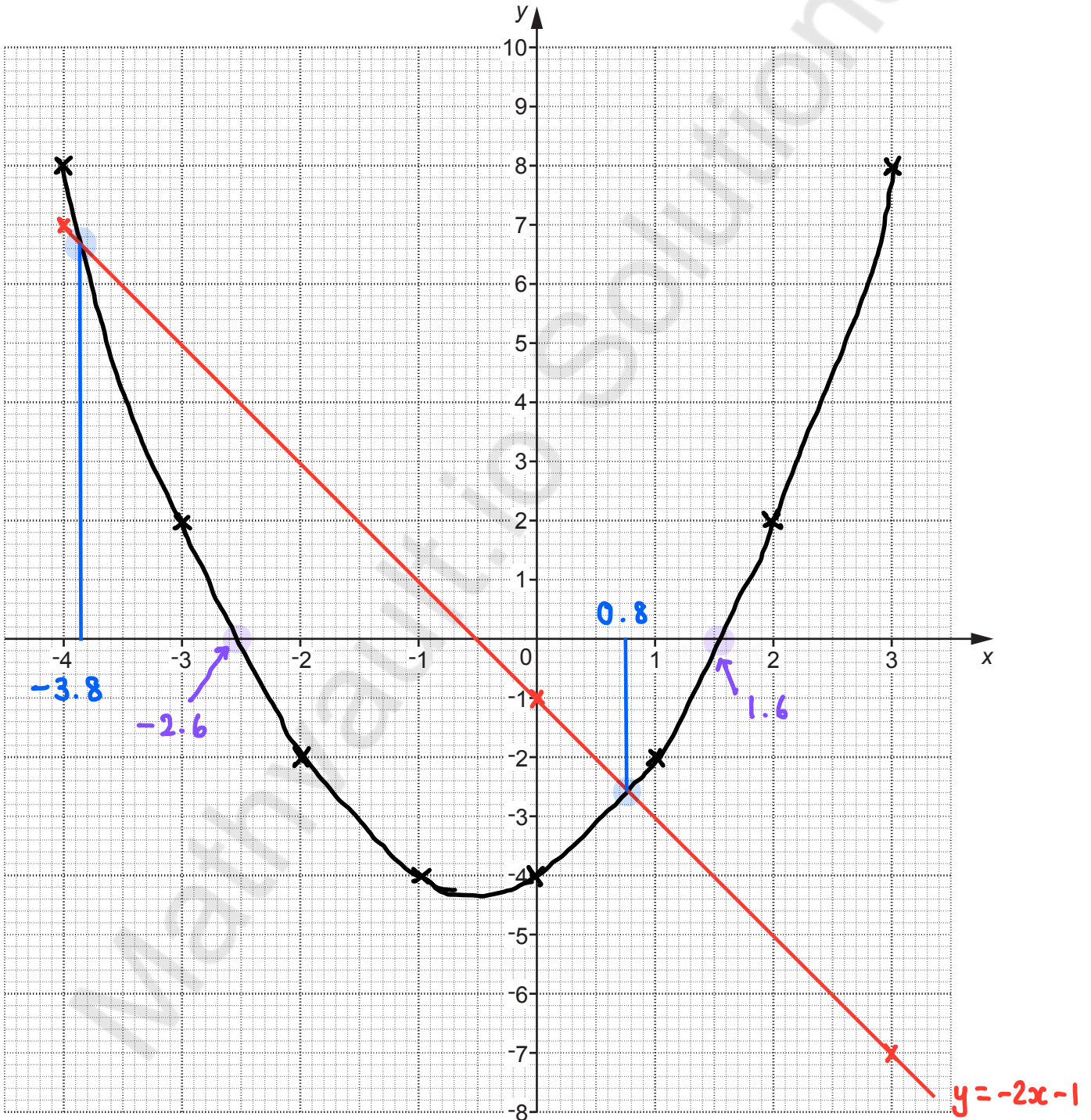
- 5 (a) Complete this table for $y = x^2 + x - 4$.

x	-4	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2	8

$y = (-2)^2 + (-2) - 4$
 $y = (-4)^2 + (-4) - 4$
 $y = (1)^2 + (1) - 4$
 $y = (3)^2 + (3) - 4$

[2]

- (b) Draw the graph of $y = x^2 + x - 4$ for $-4 \leq x \leq 3$.



[3]

(c) Use your graph to solve $x^2 + x - 4 = 0$.

$$\leftarrow y = 0 \text{ - x-axis}$$

(c) $x = \dots -2.6 \dots$ or $x = \dots 1.6 \dots$ [2]

(d) On the same grid, draw the graph of $y = -2x - 1$ for $-4 \leq x \leq 3$.
You may use the table if you wish.

x	-4	0	3
y	7	-1	-7

$$\leftarrow y = -2(3) - 1$$

$$\leftarrow y = -2(0) - 1$$

[3]

(e) Use your graphs to solve the equation $x^2 + x - 4 = -2x - 1$.

(e) $x = \dots -3.8 \dots$ or $x = \dots 0.8 \dots$ [2]

- 6 Maria mixes white paint and red paint in the ratio 2 : 3.
She makes a total of 15 litres of paint.

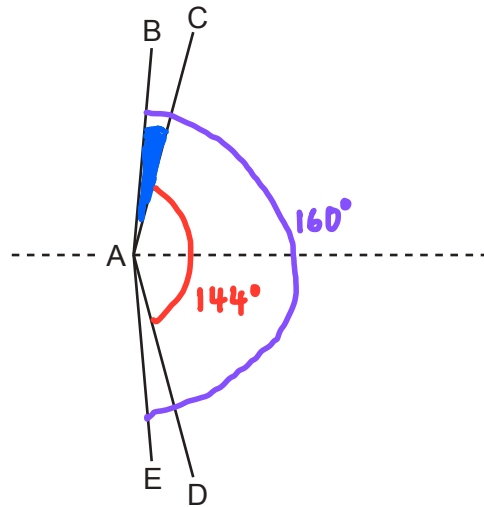
How much more red paint does she need to add to the mixture so that the ratio of white paint to red paint becomes 1 : 5?

$W : R$		$W : R$
$2 : 3$	$2+3 = 5 \text{ parts}$	$1 : 5$
$\times 3 \quad \times 3$	$15L \div 5 = 3L$	$\times 6 \quad \times 6$
$6L : 9L$		$6L : 30L$

$$30L - 9L = 21L$$

..... **21** litres [4]

- 7 Angle BAE is part of a regular 18-sided polygon.
 Angle CAD is part of a regular 10-sided polygon.
 The dashed line through A is a line of symmetry of both polygons.



Not to scale

Work out angle BAC.

18 - sided polygon

Angle BAE

$$\begin{aligned} \text{Exterior} &= \frac{360}{n} \\ &= \frac{360}{18} \\ &= 20^\circ \end{aligned}$$

$$\begin{aligned} \text{Interior} &= 180^\circ - 20^\circ \\ &= 160^\circ \\ &\quad \swarrow \\ &\quad \text{BAE} \end{aligned}$$

10 - sided polygon

Angle CAD

$$\begin{aligned} \text{Exterior} &= \frac{360}{10} \\ &= 36^\circ \end{aligned}$$

$$\begin{aligned} \text{Interior} &= 180^\circ - 36^\circ \\ &= 144^\circ \end{aligned}$$

$$\begin{aligned} \text{Angle BAC} &= (160^\circ - 144^\circ) \div 2 \\ &= 16^\circ \div 2 \\ &= 8^\circ \end{aligned}$$

..... 8 [5]

- 8 Diners choose one starter and one main from the options given in the table below. Vegetarian dishes are indicated with a (v).

Starter	Main
Cheese salad (v)	Steak and chips
Prawn cocktail	Fish and chips
Mozzarella sticks (v)	Tomato pizza (v)
	Pork chops
	Nut cutlet (v)

- (a) Work out the fraction of all the meal combinations which have at least one vegetarian option.

$$\begin{aligned} \text{Total meal combinations} &= 3 \times 5 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{At least 1 veg option} &= 2 \times 5 + 2 \\ &= 12 \end{aligned}$$

(a) $\frac{12}{15}$ [3]

- (b) Diners also choose one of 6 dessert options.

How many different three-course meal combinations are available?

$$3 \times 5 \times 6$$

(b) 90 [2]

9 Here are the results of a survey of 437 people in a town.

- 62 males speak Spanish.
- 153 females do not speak Spanish.
- 280 people do not speak Spanish.

Jeff says

At least 2 out of every 5 females in the town can speak Spanish.

Is he correct?

Show clearly how you reached your decision.

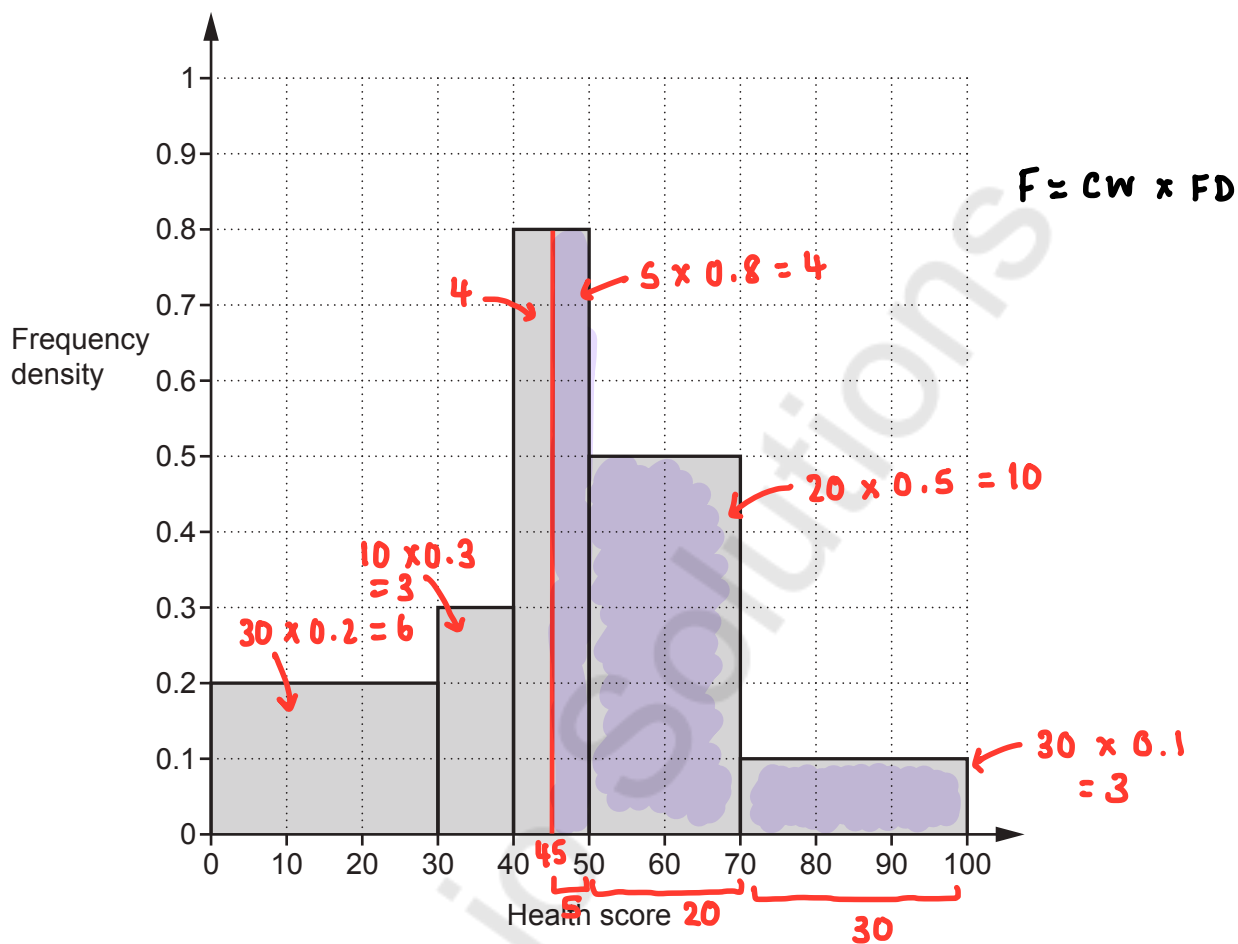
	S	S'	Total
m	62	127	189
F	95	153	248
Total	157	280	437

$$\begin{aligned} \text{Spanish-speaking females} &= \frac{95}{248} & \frac{2}{5} &= 0.4 \\ &= 0.383\dots \end{aligned}$$

No. $0.383\dots < 0.4$

[6]

10 The histogram summarises a health score for a group of people.



(a) Estimate the fraction of the group who had a score of 45 or more.

$$\frac{4 + 10 + 3}{6 + 3 + 4 + 4 + 10 + 3} = \frac{17}{30}$$

(a) $\frac{17}{30}$ [4]

(b) What assumption did you make in answering part (a)?

They were evenly spread out in the 40-50 class. [1]

- 11 y is inversely proportional to x^2 and $y = 5$ when $x = 4$.

Find a formula linking x and y .

$$y \propto \frac{1}{x^2}$$

$$y = \frac{k}{x^2} \quad \rightarrow \quad y = \frac{80}{x^2}$$

$$5 = \frac{k}{4^2}$$

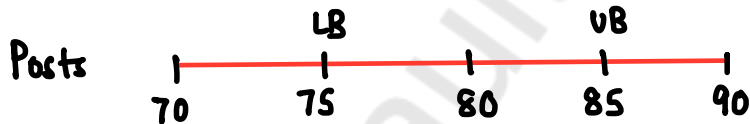
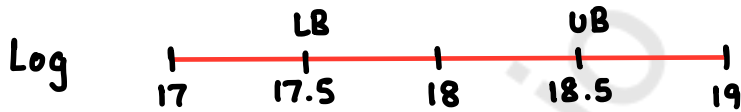
$$80 = k$$

$$y = \frac{80}{x^2}$$

[3]

- 12 A log is 18m long, correct to the nearest metre.
It is to be cut into fence posts which must be 80cm long, correct to the nearest 10 centimetres.

What is the largest number of fence posts that can possibly be cut from this log?



$$\frac{\text{Log}_{UB}}{\text{Posts}_{LB}} = \frac{18.5\text{m}}{75\text{cm}} \times 100 = \frac{1850\text{cm}}{75\text{cm}} = 24.6 \approx 24$$

24

[4]

13 (a) Solve.

$$x^2 - 6x + 15 = 3x - 5$$

$$-3x + 5 \quad -3x + 5$$

$$x^2 - 9x + 20 = 0$$

$$(x - 4)(x - 5) = 0$$

$$x - 4 = 0$$

$$+4 \quad +4$$

$$x = 4$$

$$x - 5 = 0$$

$$+5 \quad +5$$

$$x = 5$$

$$\frac{-4}{-4} \times \frac{-5}{-5} = 20$$

$$\frac{-4}{-4} + \frac{-5}{-5} = -9$$

(a) $x = \underline{4}$ or $x = \underline{5}$ [4]

(b) Expand and simplify.

$$(2x - 1)(x + 5)(3x - 2)$$

$$(2x - 1)(x + 5)$$

x	2x	-1
x	2x ²	-x
+5	+10x	-5

$$2x^2 + 9x - 5$$

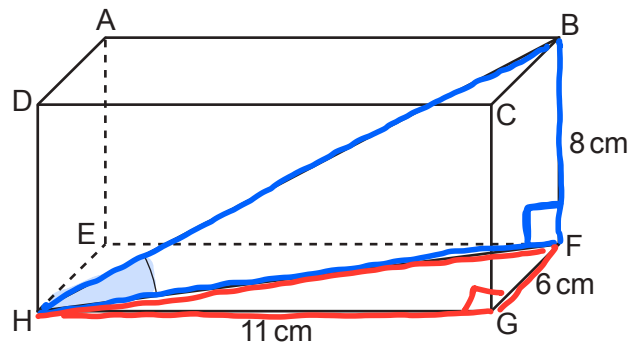
$$(2x^2 + 9x - 5)(3x - 2)$$

x	2x ²	+9x	-5
3x	6x ³	+27x ²	-15x
-2	-4x ²	-18x	+10

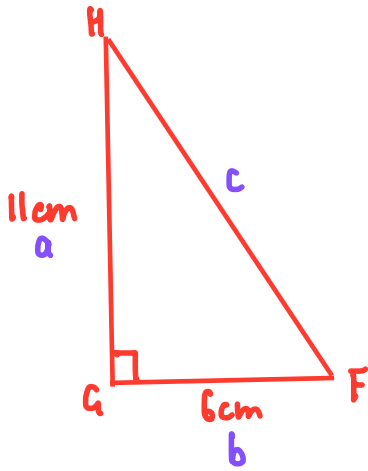
$$6x^3 + 23x^2 - 33x + 10$$

(b) $6x^3 + 23x^2 - 33x + 10$ [4]

14 The diagram shows a cuboid ABCDEFGH.



Calculate angle BHF.



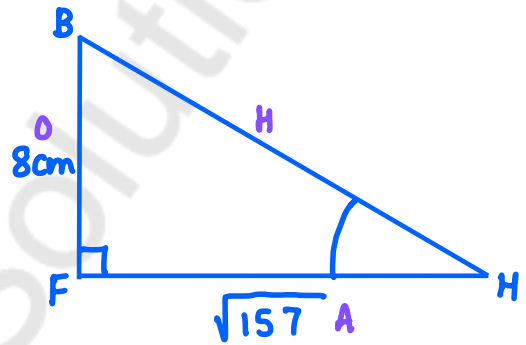
$$a^2 + b^2 = c^2$$

$$11^2 + 6^2 = HF^2$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$HF = \sqrt{11^2 + 6^2}$$

$$= \sqrt{157}$$



$$S^{\circ} H \quad C^A H \quad T^{\circ} A^{\checkmark}$$

$$\downarrow$$

$$\tan \theta = \frac{O}{A}$$

$$\tan (BHF) = \frac{8}{\sqrt{157}}$$

$$BHF = \tan^{-1} \left(\frac{8}{\sqrt{157}} \right) = 32.5569\dots$$

..... 32.6 ° [5]

15 Show that $\frac{4+2\sqrt{5}}{\sqrt{5}-1}$ can be simplified to $\frac{3\sqrt{5}+7}{2}$.

[4]

$$\frac{4+2\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(4+2\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$$

$2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10$

$$= \frac{4\sqrt{5} + 4 + 10 + 2\sqrt{5}}{5 + \cancel{\sqrt{5}} - \cancel{\sqrt{5}} - 1}$$

$$\frac{6\sqrt{5} + 14}{4} \div 2 = \frac{3\sqrt{5} + 7}{2}$$

16 (a) Write $x^2 - 6x + 20$ in the form $(x-a)^2 + b$.

$$(x-3)^2 + 11$$

$$\begin{aligned} (x-3)(x-3) \\ x^2 - 6x + 9 \\ + 11 \\ \hline + 20 \end{aligned}$$

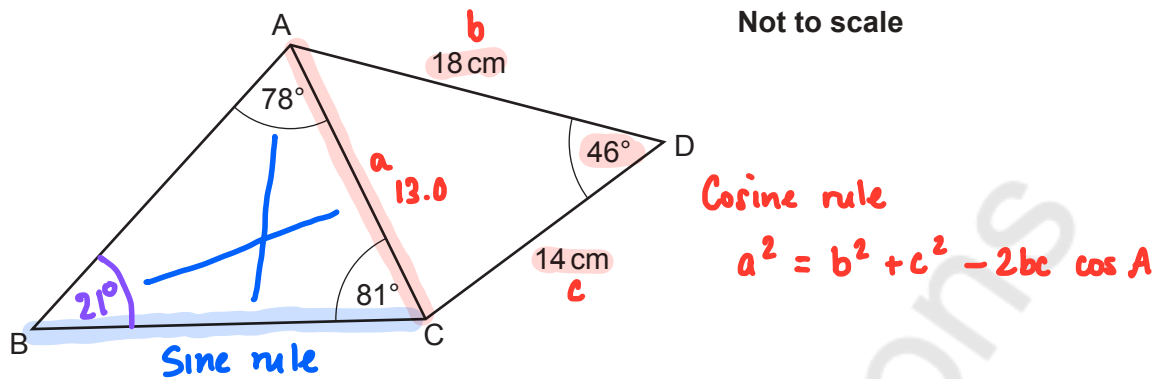
(a) $(x-3)^2 + 11$ [3]

(b) Write down the turning point of the graph of $y = x^2 - 6x + 20$.

(b) (..... 3 , 11 ) [2]

$$\begin{aligned} x-3 &= 0 \\ x &= 3 \end{aligned}$$

17 ABC and ACD are triangles.



(a) Show that $AC = 13.0$ cm, correct to 3 significant figures.

[4]

$$AC^2 = 18^2 + 14^2 - 2 \times 18 \times 14 \times \cos 46$$

$$AC^2 = 169.8921813$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$AC = 13.0342695$$

$$\approx 13.0$$

(b) Calculate BC.

$$\begin{aligned} \angle ABC &= 180 - 78 - 81 \\ &= 21^\circ \end{aligned}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{BC}{\sin 78} = \frac{13.0}{\sin 21}$$

$$\times \sin 78$$

$$\times \sin 78$$

$$BC = \frac{13.0 \times \sin 78}{\sin 21}$$

(b) **35.5** cm [3]

$$= 35.4828 \dots \approx 35.5$$

- 18 Solve this equation algebraically.
Give your solutions correct to 2 decimal places.

$$3x^2 + 5x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$a = 3 \quad b = 5 \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$$

$$x = 0.1804604217 \quad \text{or} \quad x = -1.847127088$$

$$\approx 0.18 \quad (2dp) \quad \approx -1.85 \quad (2dp)$$

$$x = \dots \mathbf{0.18} \dots \text{or } x = \dots \mathbf{-1.85} \dots [4]$$

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

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