

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Monday 6 November 2017 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- Geometrical instruments
- Tracing paper

Do not use:

- A calculator



First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS

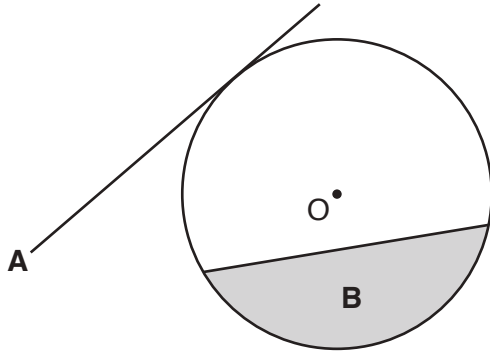
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document consists of **16** pages.

Answer **all** the questions.

- 1 The diagram shows a circle, centre O.



Write down the mathematical name of

- (a) line A,

(a) **tangent** [1]

- (b) shaded region B.

(b) **segment** [1]

- 2 (a) Write the next term in each of these sequences.

(i) 1 1 2 3 5 8

5 + 8

(a)(i) **13** [1]

(ii) 2 4 8 16 32 64

64 × 2

(ii) **128** [1]

- (b) Write an expression for the n th term of the sequence below.

18 15 12 9 6
+3 **-3** **-3** **-3**

$-3n + 18$

(b) **$-3n + 18$** [2]

3 Andrew is thinking of a number.

- It is between 1 and 150.
- It is one more than a square number.
- It is three less than a cube number.
- It is not a prime number.

What is Andrew's number?

You must show all your reasoning.

Cubes: 1 8 27 64 125 216
 -3 -2 5 24 61 122 213

Squares: 1 4 9 16 25 36 49 64 81 100 121 144
 +1 2 5 10 17 26 37 50 65 82 101 122 145
 prime

.....122..... [4]

4 (a) Factorise.

$$\sqrt{x^2 - 43^2}$$

$$(x + 43)(x - 43)$$

(a)(x + 43)(x - 43)..... [1]

(b) Calculate.

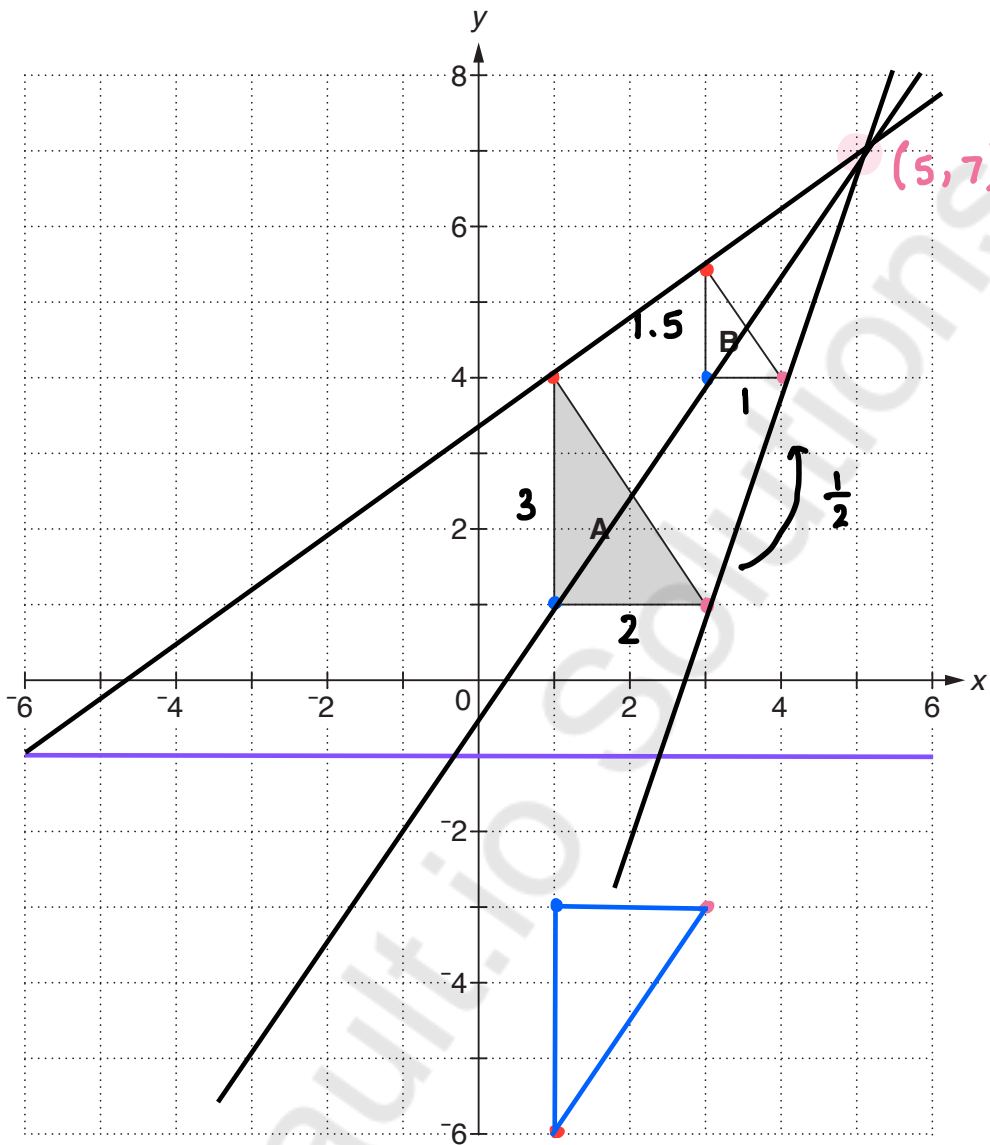
$$\sqrt{57^2 - 43^2}$$

$$(57 + 43)(57 - 43)$$

$$(100)(14) = 1400$$

(b)1400..... [2]

5 Here is a coordinate grid.

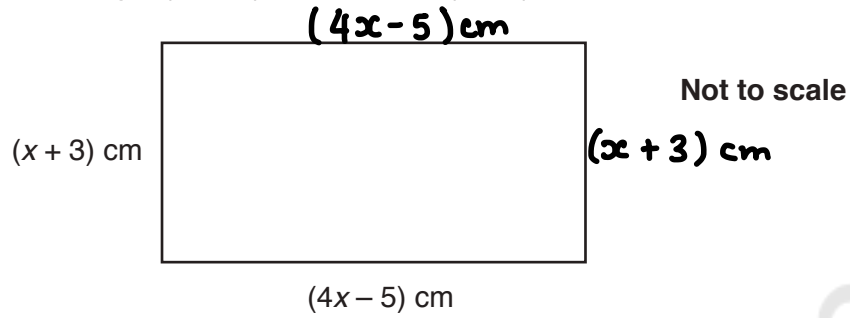


(a) Draw the image of triangle A after a reflection in the line $y = -1$. [2]

(b) Describe fully the **single** transformation that maps triangle A onto triangle B.
 Enlargement, scale factor = $\frac{1}{2}$, centre (5, 7) [3]

(c) Complete this statement.
 A rotation of 180° around (0, 0) has the same effect as an enlargement by
 scale factor -1 with centre of enlargement (....., 0.....). [2]

- 6 This rectangle has length $(4x - 5)$ cm and width $(x + 3)$ cm.



The perimeter of the rectangle is 46 cm.

Calculate the area of the rectangle.

$P =$ add all sides

$$46 = x + 3 + x + 3 + 4x - 5 + 4x - 5$$

$$46 = 10x - 4$$

$$+4 \qquad +4$$

$$50 = 10x$$

$$\div 10 \qquad \div 10$$

$$5 = x$$

$$\text{Area} = l \times w$$

$$l = 4(5) - 5$$

$$= 15$$

$$w = (5) + 3$$

$$= 8$$

..... 120 cm^2 [5]

$$\text{Area} = 15 \times 8$$

$$= 120 \text{ cm}^2$$

- 7 Naomi is given a 10% pay decrease.
Her new wage is £252 per week. ← 90%.

What would be her weekly wage if, instead, she had received a 10% pay increase?

$$\begin{array}{l} 90\% = £252 \\ \div 9 \quad \quad \quad \div 9 \\ 10\% = £28 \\ \times 10 \quad \quad \quad \times 10 \\ 100\% = £280 \end{array}$$

$$9 \overline{) 252} \quad \leftarrow 110\%$$

$$\begin{aligned} 110\% &= 100\% + 10\% \\ &= £280 + £28 \\ &= £308 \end{aligned}$$

£ 308 [5]

- 8 The angles in a triangle are in the ratio 1 : 2 : 3.

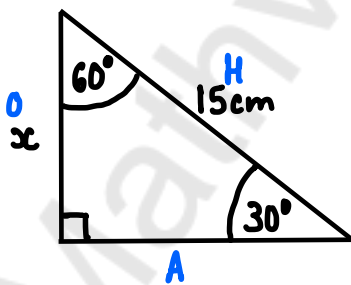
$$180^\circ$$

- (a) Show that the triangle is a right-angled triangle. [2]

$$\begin{aligned} 1 + 2 + 3 &= 6 \text{ parts} \\ 180^\circ \div 6 &= 30^\circ \\ 30 \times 1 &= 30^\circ \\ 30 \times 2 &= 60^\circ \\ 30 \times 3 &= 90^\circ \leftarrow \text{right angle.} \end{aligned}$$

- (b) The hypotenuse of the triangle is 15 cm long.

Calculate the length of the shortest side in the triangle.



SOHCAHTOA
 \downarrow
 $\sin \theta = \frac{O}{H}$

$$\sin 30 = \frac{x}{15}$$

$$\times 15 \quad \quad \times 15$$

$$15 \times \sin 30 = x$$

$$15 \times \frac{1}{2} = x \quad (b) \dots\dots\dots 7.5 \dots\dots \text{cm [4]}$$

$$x = \frac{15}{2} \text{ or } 7.5$$

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0

$$\sin 30 = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

- 9 There is a total of 250 men, women and children on a train.
The ratio of men to women is 4 : 5.
The ratio of women to children is 10 : 7.

How many men are on the train?

$$\begin{array}{l} m : w \\ 4 : 5 \\ \times 2 \quad \times 2 \end{array}$$

$$8 : 10 \quad 10 : 7$$

$$\begin{array}{l} m : w : c \\ 8 : 10 : 7 \end{array}$$

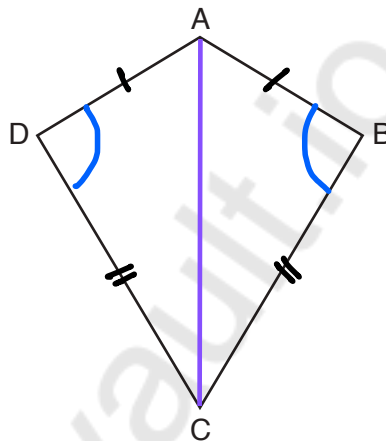
$$8 + 10 + 7 = 25 \text{ parts}$$

$$250 \div 25 = 10$$

$$\begin{array}{l} \text{Men} = 10 \times 8 \\ = 80 \end{array}$$

.....80..... [4]

- 10 ABCD is a quadrilateral.
AD = AB and CD = CB.



Not to scale

Prove that angle ADC is equal to angle ABC.

..... AD = AB (given)

..... CD = CB (given)

..... AC = AC (common / shared)

..... Congruent - SSS

..... Angle ADC = angle ABC

[4]

- 11 Amelia buys a new car.
The expected future value of this car, £ V , is given by

$$V = 16000 \times 0.75^t$$

where t is the age of the car in complete years.

- (a) (i) Write down the value of the car when new.

$$16000 \times 0.75^0 \quad \text{(i) } \text{£ } \dots 16,000 \dots \quad [1]$$

$$16000 \times 1$$

- (ii) Write down the annual percentage decrease in the expected value of the car.

$$0.75 \times 100 = 75\%.$$

$$100\% - 75\% = 25\%.$$

(ii) $\dots 25 \dots$ % [1]

- (iii) Show that the expected value of the car when 2 years old is £9000. [2]

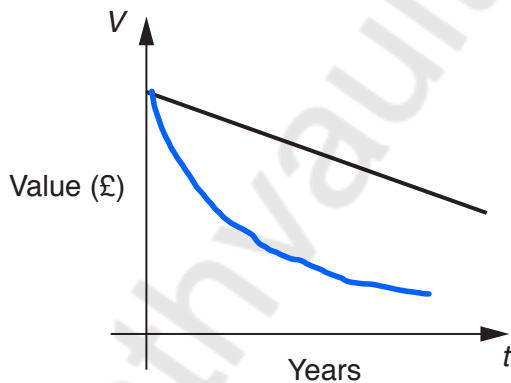
$$t = 2 \quad \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$V = 16000 \times 0.75^2$$

$$= \frac{16000}{1000} \times \frac{9}{16}$$

$$= 1000 \times 9 = 9000$$

- (b) Amelia sketches a graph to show the expected value of her car as it gets older.



Explain how you know that Amelia's graph is incorrect.

Equation does not give a straight line. [1]

- (c) Amelia assumes that her car will have no value at all after 20 years. $16,000 \times 0.75^{20} > 0$

Explain why her assumption is mathematically incorrect.

If you calculate a value for a 20-year-old car, it is greater than 0. [1]

- 12 (a) Write $\frac{5}{6}$ as a recurring decimal.

$$5 \div 6$$

$$\begin{array}{r} 0.833 \\ 6 \overline{) 5.5000} \\ \underline{6} \\ 50 \\ \underline{48} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

(a) $0.8\dot{3}$ [2]

- (b) Convert $0.12\dot{6}$ to a fraction.
Give your answer in its lowest terms.

$$x = 0.12666\dots$$

$$10x = 1.2666\dots$$

$$100x = 12.666\dots$$

$$1000x = 126.666\dots$$

$$1000x - 100x = 900x$$

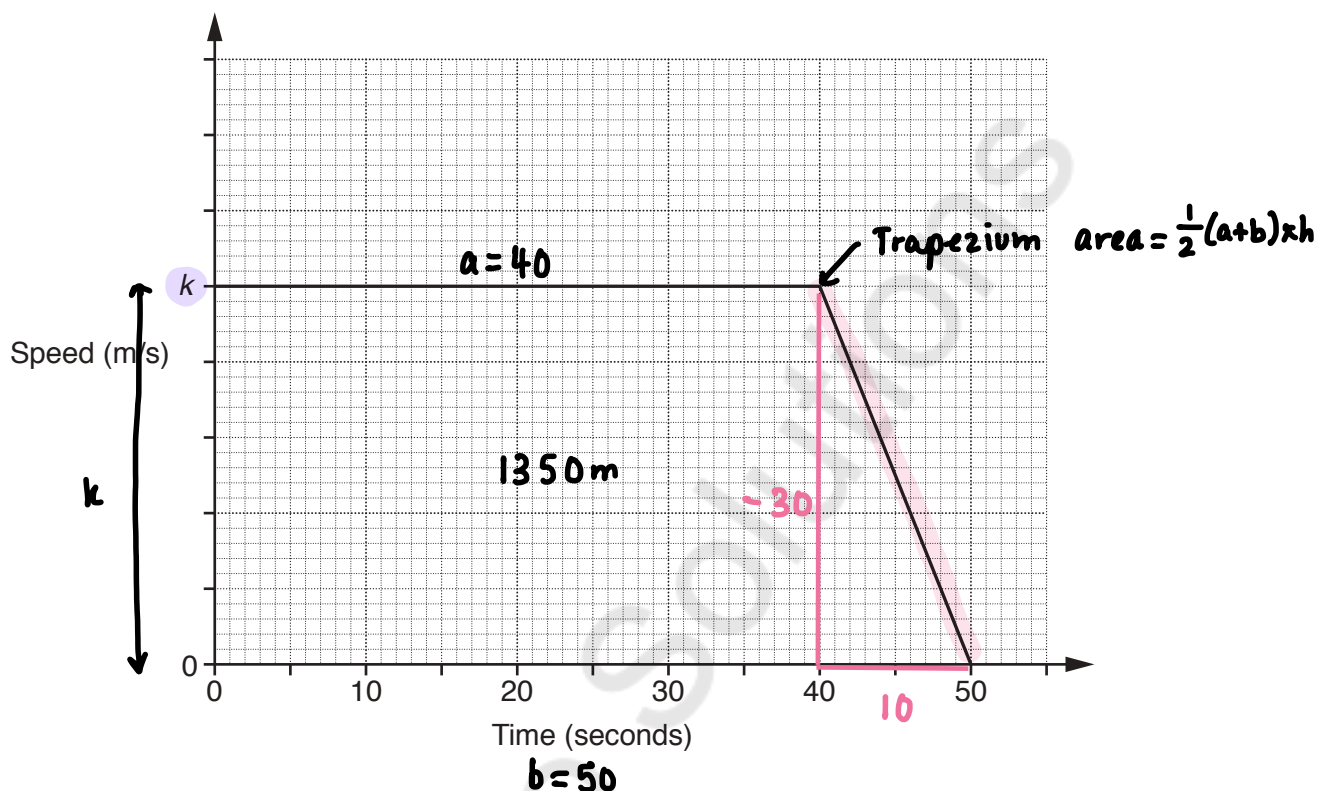
$$126.\dot{6} - 12.\dot{6} = 114$$

$$\begin{array}{l} 900x = 114 \\ \div 900 \qquad \qquad \div 900 \end{array}$$

(b) $\frac{19}{150}$ [3]

$$x = \frac{114}{900} \begin{array}{l} \div 2 \\ = \\ \div 2 \end{array} = \frac{57}{450} \begin{array}{l} \div 3 \\ \div 3 \end{array} = \frac{19}{150}$$

- 13 The graph shows information about the speed of a vehicle during the final 50 seconds of a journey. At the start of the 50 seconds the speed is k metres per second. The distance travelled during the 50 seconds is 1.35 kilometres.



- (a) Work out the average speed of the vehicle during the 50 seconds. Give your answer in metres per second.

$$S = \frac{D}{T} \quad D = 1.35 \text{ km} \times 1000 = 1350 \text{ m}$$

$$T = 50 \text{ s}$$

$$S = \frac{1350}{50} = \frac{135}{5} = 27 \quad \text{(a) } \dots\dots\dots 27 \dots\dots \text{ m/s [2]}$$

- (b) Work out the value of k .

$$1350 = \frac{1}{2}(40 + 50) \times k$$

$$1350 = 45k$$

$$\div 45 \quad \div 45$$

$$30 = k$$

$$45 \overline{) 1350} \quad \begin{array}{r} 30 \\ 45 \times 30 = 1350 \end{array}$$

(b) $k = \dots 30 \dots\dots\dots$ [5]

- (c) (i) Calculate the gradient of the graph in the final 10 seconds of the journey.

$$\frac{-30}{10} = -3$$

(c)(i) -3 [1]

- (ii) Describe what this gradient represents.

..... Deceleration in m/s^2

..... [2]

- 14 Adam has 10 sweets in a bag.

5 are cherry sweets, 4 are lemon sweets and 1 is an orange sweet.

Adam chooses a sweet at random from the bag and eats it. 10 sweets

He then takes another sweet at random from the bag and eats it. 9 sweets

- (a) Adam says

The probability that I choose two cherry sweets is $\frac{25}{100}$.

He is incorrect. Explain his error.

$$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$$

..... [2]

- (b) Find the probability that the two sweets he chooses have different flavours.

$$\begin{aligned} C \ \& \ O &= \frac{5}{10} \times \frac{1}{9} = \frac{5}{90} \\ O \ \& \ C &= \frac{1}{10} \times \frac{5}{9} = \frac{5}{90} \end{aligned} \quad + \quad = \quad \frac{10}{90}$$

$$\begin{aligned} L \ \& \ C &= \frac{4}{10} \times \frac{5}{9} = \frac{20}{90} \\ C \ \& \ L &= \frac{5}{10} \times \frac{4}{9} = \frac{20}{90} \end{aligned} \quad + \quad = \quad \frac{40}{90}$$

$$\begin{aligned} L \ \& \ O &= \frac{4}{10} \times \frac{1}{9} = \frac{4}{90} \\ O \ \& \ L &= \frac{1}{10} \times \frac{4}{9} = \frac{4}{90} \end{aligned} \quad + \quad = \quad \frac{8}{90}$$

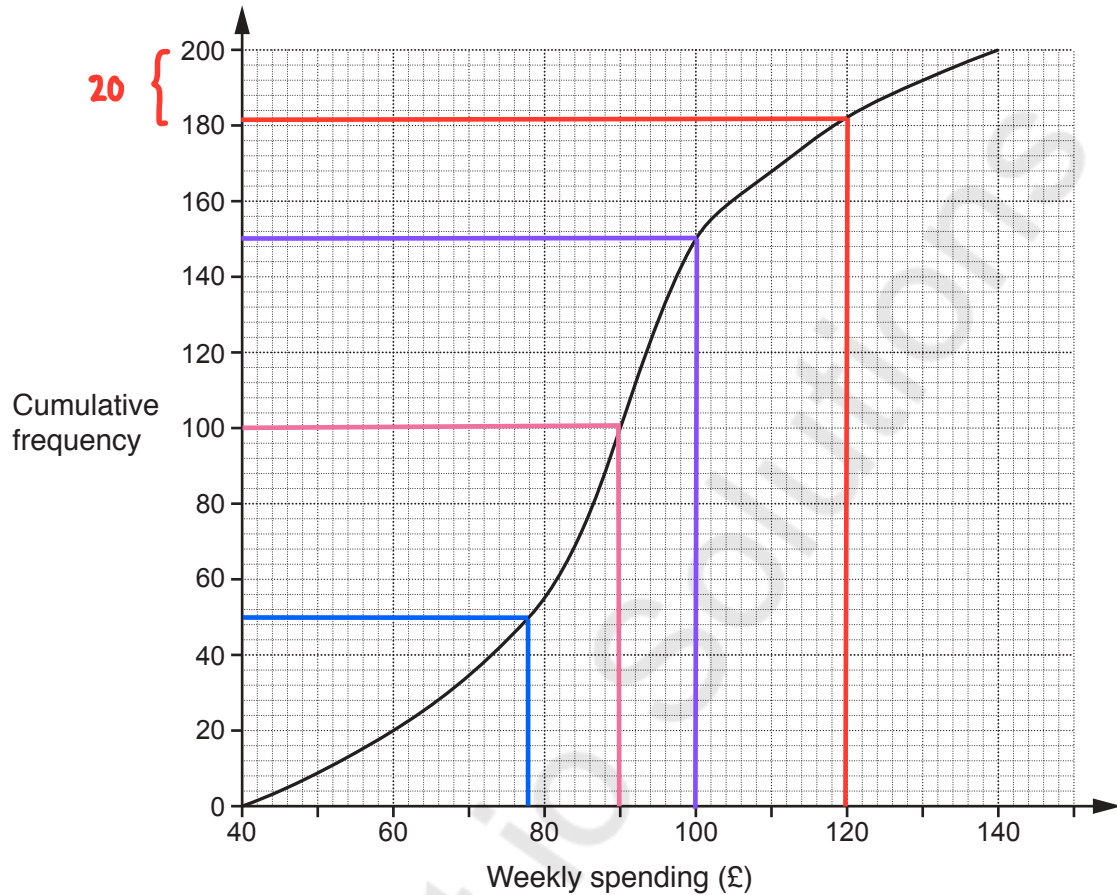
$$\frac{10}{90} + \frac{40}{90} + \frac{8}{90}$$

$$\frac{58}{90}$$

(b) [4]

- 15 Iqrah carries out a survey of 200 families in the **north** of England on their weekly spending on food.

The cumulative frequency diagram summarises the results.



(a) Find

- (i) the median,

$$200 \div 2 = 100$$

(a)(i) £ 90 [1]

- (ii) the interquartile range.

$$IQR = UQ - LQ$$

$$UQ = \frac{3}{4} \text{ of } 200$$

$$= 150$$

$$= \text{£}100$$

$$LQ = \frac{1}{4} \text{ of } 200$$

$$= 50$$

$$= \text{£}78$$

(ii) £ 22 [2]

$$IQR = \text{£}100 - \text{£}78$$

$$= \text{£}22$$

$$15\% \text{ of } 200 = 30$$

$$10\% = 20$$

$$5\% = 10$$

(b) Iqrah says

15% of these families spent over £120.

Is her statement correct?

State the evidence you have used in making your decision.

..... No, 15% = 30, and 18-20 families spent over £120.

[2]

(c) In a survey of 200 families in the **south** of England, the median weekly amount spent on food was £84 and the interquartile range was £28.

Make two comparisons between the weekly amounts spent on food in the north of England and the south of England.

State the evidence you have used in making your comparisons.

1 Families in the south spent less on average as their median was lower [2]

2 Families in the south were more spread in their spending as the IQR was larger. [2]

16 (a) Write $\sqrt{12} + \sqrt{75}$ in the form $k\sqrt{3}$.

$$\begin{aligned}\sqrt{12} &= \sqrt{4} \times \sqrt{3} \\ &= 2 \times \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

$$\begin{aligned}\sqrt{75} &= \sqrt{25} \times \sqrt{3} \\ &= 5 \times \sqrt{3} \\ &= 5\sqrt{3}\end{aligned}$$

$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

(a) $7\sqrt{3}$ [3]

(b) Work out.

$$16^{-\frac{3}{4}}$$

$$x^{-a} = \frac{1}{x^a}$$

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$$

$$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$$

$$16^{\frac{3}{4}} = (\sqrt[4]{16})^3$$

$$= 2^3$$

$$= 8$$

(b) $\frac{1}{8}$ [3]

17 Solve the inequality.

$$x^2 - 5x - 6 \leq 0$$

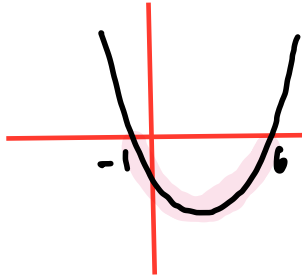
$$\underline{-6} \times \underline{+1} = -6$$

$$(x - 6)(x + 1) = 0$$

$$\underline{-6} + \underline{+1} = -5$$

$$\begin{array}{r} x - 6 = 0 \\ +6 \quad +6 \end{array} \quad \begin{array}{r} x + 1 = 0 \\ -1 \quad -1 \end{array}$$

$$x = 6 \quad x = -1$$



$$\dots\dots\dots -1 \leq x \leq 6 \dots\dots\dots [4]$$

18 Prove that the difference between two consecutive square numbers is always odd. [4]

Square numbers : x^2 , $(x + 1)^2$

Difference : $(x + 1)^2 - x^2$

$$(x + 1)(x + 1) - x^2$$

$$x^2 + 2x + 1 - x^2$$

$$= 2x + 1$$

$2x + 1$ is always odd.

19 Solve these simultaneous equations algebraically.

$$\begin{aligned} y &= 2x^2 - 7x + 4 \\ y &= 4x - 1 \end{aligned}$$

$$\begin{aligned} 2x^2 - 7x + 4 &= 4x - 1 \\ -4x + 1 &\quad -4x + 1 \end{aligned}$$

$$2x^2 - 11x + 5 = 0$$

$$2x^2 - x \quad | \quad -10x + 5$$

$$x(2x-1) - 5(2x-1)$$

$$(2x-1)(x-5) = 0$$

$$\begin{aligned} 2x-1 &= 0 & x-5 &= 0 \\ +1 & +1 & +5 & +5 \end{aligned}$$

$$\begin{aligned} 2x &= 1 \\ \div 2 & & \div 2 & \\ x &= \frac{1}{2} \end{aligned}$$

$$x = 5$$

$$2 \times 5 = 10$$

$$\underline{-1} \times \underline{-10} = 10$$

$$\underline{-1} + \underline{-10} = -11$$

$$y = 4x - 1$$

$$\begin{aligned} y &= 4\left(\frac{1}{2}\right) - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= 4(5) - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

$$x = \dots \frac{1}{2} \dots \quad y = \dots 1 \dots$$

$$x = \dots 5 \dots \quad y = \dots 19 \dots \quad [6]$$

END OF QUESTION PAPER

