OCR

Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Wednesday 8 November 2017 – Morning

Time allowed: 1 hour 30 minutes

You may use:
- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

First name	
Last name	

Centre number						Candidate number				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Use the formula $s = ut + \frac{1}{2}at^2$.

(a) Calculate s when $u = 5$, $t = 10$ and $a = 3$.

$$s = (5)(10) + \frac{1}{2}(3)(10)^2$$

$$= 200$$

(a) $s =$ 200 **[2]**

(b) Make a the subject of the formula.

$$s = ut + \frac{1}{2}at^2$$

$$-ut \qquad -ut$$

$$s - ut = \frac{1}{2}at^2$$

$$\times 2 \qquad \times 2$$

$$2(s-ut) = at^2$$

$$\div t^2 \qquad \div t^2$$

$$\frac{2(s-ut)}{t^2} = a$$

(b) $a = \frac{2(s-ut)}{t^2}$ **[2]**

2 Carla runs every 3 days.
She swims every Thursday.
On Thursday 9 November, Carla both runs and swims.

What will be the next date on which she both runs and swims?

Run = 3 days

Swims = 7 days

LCM = 21

9 Nov + 21 days = 30 Nov

30 November **[3]**

3 A shop records the time taken by its customers to complete a purchase on its website.
The results from one day are summarised in this table.

Time taken (t minutes)	Number of customers	midpoints	midpoints x no. of customers
$0 < t \leqslant 3$	6	1.5	9
$3 < t \leqslant 6$	10	4.5	45
$6 < t \leqslant 9$	6	7.5	45
$9 < t \leqslant 12$	2	10.5	21
$12 < t \leqslant 15$	1	13.5	13.5
Total	25		133.5

(a) Calculate an estimate of the mean time taken.

Mean = 133.5 ÷ 25

= 5.34

(a) ......5.34...... minutes **[4]**

(b) Explain why it is not possible to use the information from this table to calculate the **exact** value of the mean time taken.

Exact times for each customer are not recorded.

.... **[1]**

4 Jeat is growing carrots from seed in his garden.
He plants 28 carrot seeds but only 12 grow.

Jeat says

The probability of one of my carrot seeds growing is $\frac{3}{7}$.

(a) Use Jeat's result to show that he is correct. **[1]**

$$\frac{12}{28} \begin{matrix} \div 4 \\ \div 4 \end{matrix} = \frac{3}{7}$$

(b) A farmer uses this probability to calculate how many carrot seeds he should plant to grow 10 000 carrots.

How many seeds should he plant?

$$10,000 \div \frac{3}{7} = 23333.\dot{3}$$

$$\approx 23,333$$

(b) ....23,333........ seeds **[2]**

(c) Explain why it may not be sensible for the farmer to use Jeat's experimental probability to calculate the number of seeds he should plant.

The growing conditions on the farm may be different to Jeat's garden.

.... **[1]**

5 A company makes sweets.
The sweets are put into packets.

Here are some facts.

1.47×10^7 sweets are made every day	3.5×10^5 packets of sweets are produced every day

(a) Calculate the mean number of sweets in one packet.

$(1.47 \times 10^7) \div (3.5 \times 10^5) = 42$

(a) ....42.......... **[2]**

(b) Sweets are made on 288 days each year.

Calculate the number of sweets made each year.
Give your answer in standard form.

$(1.47 \times 10^7) \times 288 = 4233600000$
$= 4.2336 \times 10^9$

(b) ....4.2336×10^9.......... **[3]**

(c) The company has 152 machines making the sweets.
Each machine operates for 15 hours each day.

(i) Calculate the number of sweets made by one machine each hour.
Give your answer as an ordinary number correct to the nearest 10.

In one hour: $1.47 \times 10^7 \div 15 = 980{,}000$ sweets

One machine $= 980{,}000 \div 152$
$= 6447.368421$
≈ 6450

(c)(i) ....6450.......... **[3]**

(ii) State one assumption you have made in part **(c)(i)**.

Machines are running at the same rate.

.... **[1]**

6 **(a)** Two bags each contain only red counters and yellow counters.

In Bag A, the ratio of red counters to yellow counters is 1 : 4. Red = $\frac{1}{5}$

In Bag B, $\frac{1}{4}$ of the counters are red.

(i) Sharon says

The proportion of the counters that are red is the same in both bags.

Explain why Sharon is not correct.

In Bag A, $\frac{1}{5}$ are red.

[1]

(ii) The number of counters in the two bags is the same.

Complete the table below to show how many counters of each colour could be in the bags.

Bag A = $\frac{1}{5}$ red Bag B = $\frac{1}{4}$

LCm of 5 and 4 = 20

Bag A = $\frac{1}{5} \times 20 = 4$ red

$20 - 4 = 16$ yellow

Bag B = $\frac{1}{4} \times 20 = 5$ red

$20 - 5 = 15$ yellow

	Red counters	Yellow counters
Bag A	4	16
Bag B	5	15

[3]

(b) In another bag, Bag C, the ratio of red counters to yellow counters is 3 : 4.
If 3 of the red counters are removed from Bag C, the ratio of red counters to yellow counters is 3 : 5.

How many **yellow** counters are in Bag C?

R : Y → R : Y

3 : 4 → 3 : 5

×5 ×5 ×4 ×4

15 : 20 → 12 : 20

(b) ....20........ **[3]**

7 Gustavo invests £520 for 6 years in a bank account paying simple interest.
At the end of 6 years, the amount in the bank account is £629.20.

Calculate the annual rate of interest.

Interest gained : £629.20 − £520.00
= £109.20

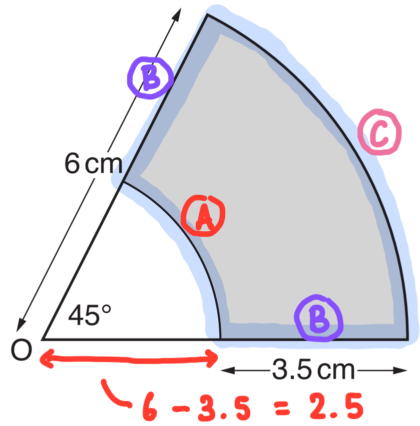
Interest per year = £109.20 ÷ 6
= £18.20

Interest rate = $\frac{18.2}{520} \times 100$

= 3.5

........3.5........ % **[4]**

8 The design below is made from two sectors of circles, centre O.



Arc length
$= \frac{\theta}{360} \times 2\pi r$

Calculate the perimeter of the shaded part.
Give your answer correct to 3 significant figures.

A $\frac{45}{360} \times 2\pi(2.5) = \frac{5}{8}\pi$ cm

B $2 \times 3.5 = 7$ cm

C $\frac{45}{360} \times 2\pi(6) = \frac{3}{2}\pi$ cm

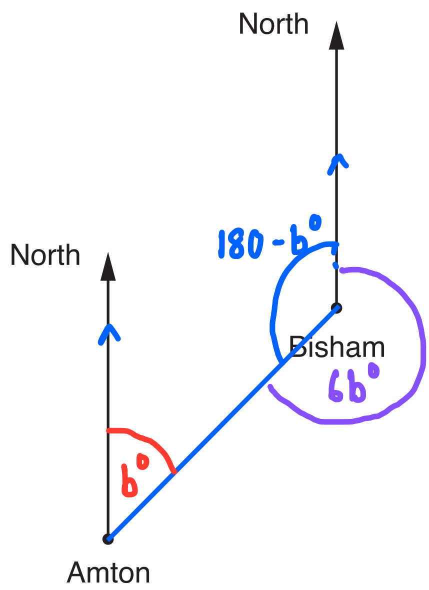
Perimeter $= \frac{5}{8}\pi + 7 + \frac{3}{2}\pi$

$= 13.67588439$

≈ 13.7 cm

.......................13.7......... cm **[5]**

9 The diagram shows the positions of two towns, Amton and Bisham.



Not to scale

The bearing of Bisham from Amton is $b°$.
The bearing of Amton from Bisham is $6b°$.

Calculate the 3-figure bearing of Amton from Bisham.

$180 - b + 6b = 360°$

$180 + 5b = 360$

$-180 \qquad -180$

$5b = 180$

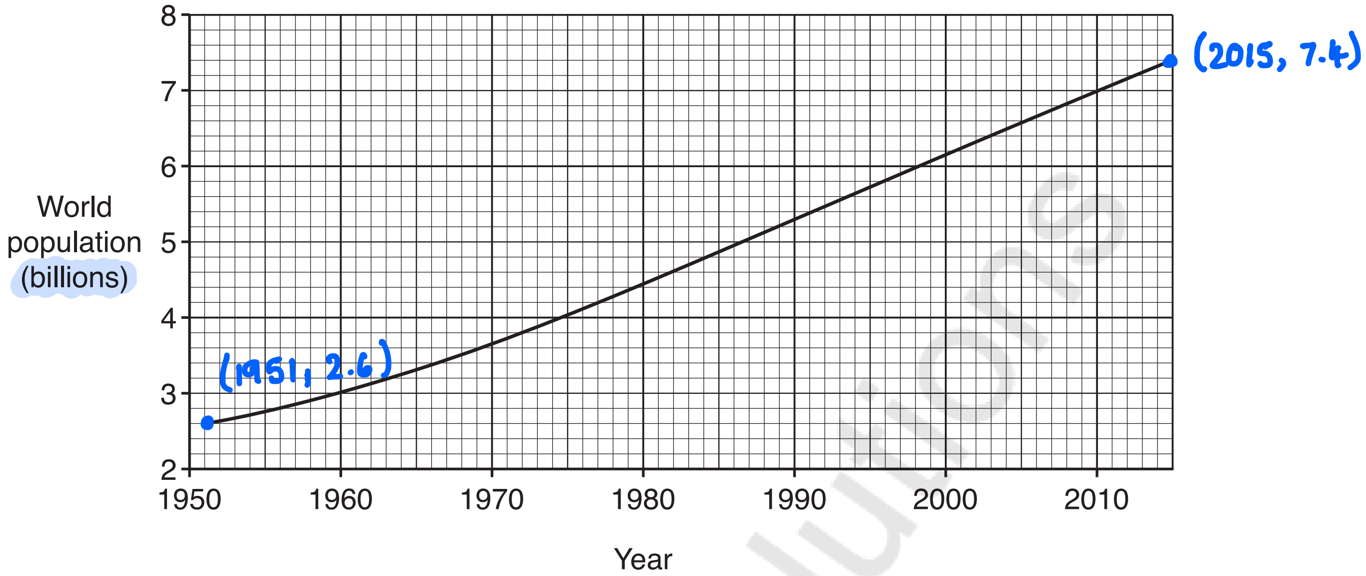
$\div 5 \qquad \div 5$

$b = 36$

$6b = 6 \times 36$
$= 216°$

........................216........° **[4]**

10 This graph shows the world population, in billions, between 1951 and 2015.



gradient

Use the graph to estimate the average rate of growth of the world population between 1951 and 2015.
Give suitable units for your answer.

$(1951, 2.6)$ $\quad$ $(2015, 7.4)$

$x_1 \quad y_1 \quad\quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7.4 - 2.6}{2015 - 1951}$$

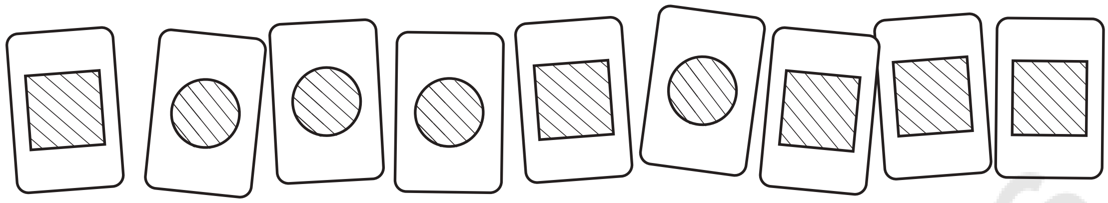
$$= 0.075 \text{ bn}$$

$$= 0.075 \times 1 \times 10^9$$

$$= 75{,}000{,}000 \text{ people per year}$$

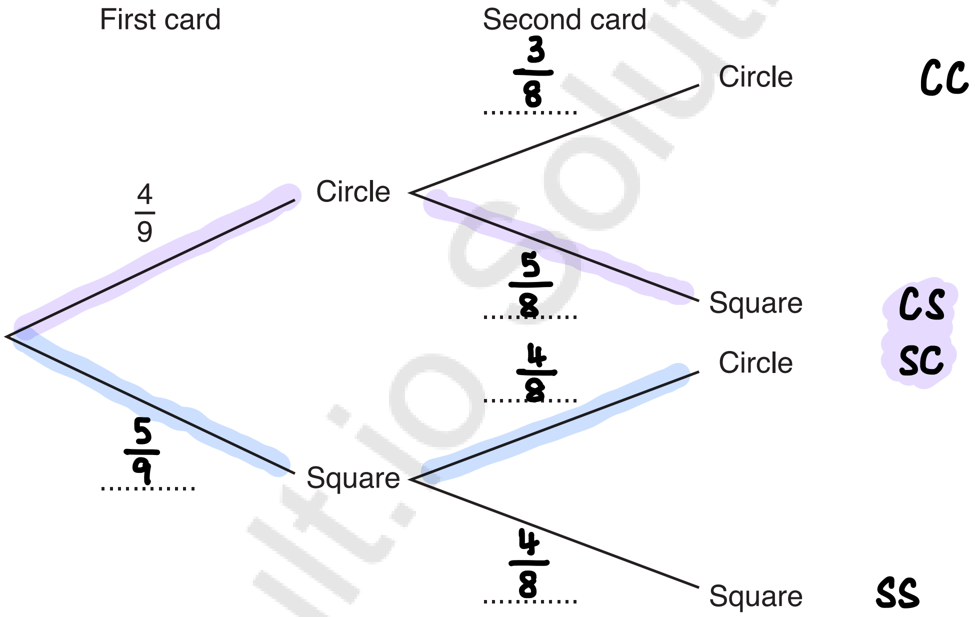
75,000,000 people / year [3]

11 Reuben is playing a matching game with these cards.



He turns the cards over and shuffles them.
Reuben takes a card and keeps it. He then takes a second card.
If the cards are different, he wins the game.

(a) Complete this tree diagram to show the probabilities for each card picked in the game.



[2]

(b) What is the probability that Reuben wins the game?

$p(\text{Circle}, \text{Square}) = \frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$p(\text{Square}, \text{Circle}) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$\frac{20}{72} + \frac{20}{72}$

(b) $\frac{40}{72}$ **[3]**

12 (a) A sequence is defined using this term-to-term rule.

$$u_{n+1} = \sqrt{2u_n + 15}$$

If $u_1 = 5$, find u_2.

$u_2 = \sqrt{2u_1 + 15}$

$= \sqrt{2(5) + 15}$

$= \sqrt{25} = 5$

(a) ....5.......... [1]

(b) Another sequence is defined using this term-to-term rule,

$$u_{n+1} = ku_n + r$$

where k and r are constants.

Given that $u_2 = 41$, $u_3 = 206$ and $u_4 = 1031$, find the value of k and the value of r.

$u_3 = ku_2 + r$

$206 = k(41) + r$

$206 = 41k + r$

$u_4 = ku_3 + r$

$1031 = k(206) + r$

$1031 = 206k + r$

$1031 = 206k + r$

$206 = 41k + r$

$825 = 165k$

$k = \dfrac{825}{165}$

$= 5$

$206 = 41(5) + r$

$206 = 205 + r$

$-205 \qquad -205$

$1 = r$

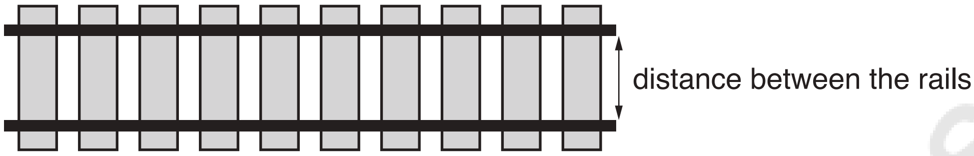
(b) k = ...5..........

r = ...1.......... [5]

13 A model railway is built using the scale 1 : 87.

(a) On the model railway, the distance between the rails is 16.5 mm.

$mm \xrightarrow{\div 10} cm$



$cm \xrightarrow{\div 100} m$

Calculate, in metres, the distance between the rails for a full-size train.

Model : Real

$\times 16.5$ 1 : 87 $\times 16.5$

16.5 mm : 1435.5 mm

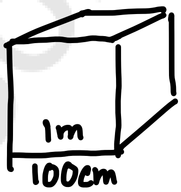
$1435.5 \div 10 = 143.55$ cm

$143.55 \div 100 = 1.4355$ m

(a) ....1.4355.................. metres **[2]**

(b) The volume of a full-size train carriage is 220 m³.
Trevor calculates the volume of a model train carriage to be 334 cm³ correct to 3 significant figures.

Is Trevor's calculation correct?
Show how you decide.



$V = 1m^3$ $\times 10^6$

$V = 1000000 cm^3$

Length 1 : 87 ← SF

$220 \times 1 \times 10^6 = 220,000,000 cm^3$

Volume $1^3 : 87^3$

1 : 658503 ← SF

$\div 658503$

$220,000,000 \div 658503 = 334.0911127$

≈ 334 (3sf)

....Yes, Trevor is correct.. **[3]**

14 The diagram shows a cross placed on a number grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

multiply

L is the product of the left and right numbers of the cross.
T is the product of the top and bottom numbers of the cross.
M is the middle number of the cross.

(a) Show that when $M = 35$, $L - T = 99$. **[2]**

$L = 34 \times 36$
$= 1224$

$T = 25 \times 45$
$= 1125$

$L - T = 1224 - 1125$
$= 99$

(b) Prove that, for any position of the cross on the number grid above, $L - T = 99$. **[5]**

Middle number, $m = x$

Left number $= x - 1$

Right number $= x + 1$

Top number $= x - 10$

Bottom number $= x + 10$

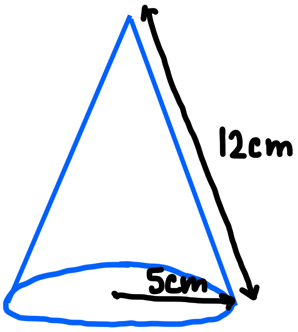
$L = (x-1)(x+1)$
$= x^2 - x + x - 1$
$= x^2 - 1$

$T = (x-10)(x+10)$
$= x^2 - 10x + 10x - 100$
$= x^2 - 100$

$L - T = (\cancel{x^2} - 1) - (\cancel{x^2} - 100)$
$= -1 - -100$
$= -1 + 100$
$= 99$

15 The following formula is for the area, A, of the curved surface area of a cone.
$A = \pi r l$, where r is the radius and l is the slant height of the cone.

Calculate the **total** surface area of a cone with radius 5 cm and slant height 12 cm.



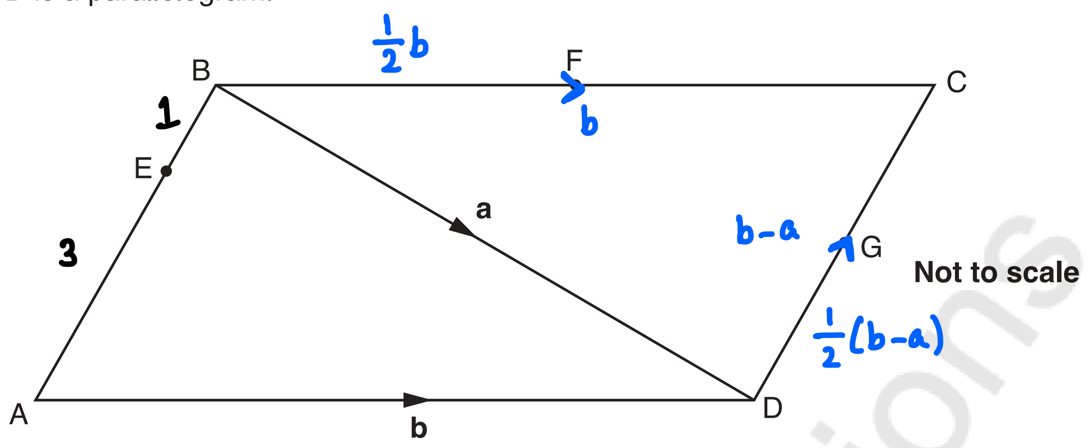
Curved surface area $= \pi r l$
$= \pi(5)(12)$
$= 60\pi$

Area of circle base $= \pi r^2$
$= \pi(5)^2$
$= 25\pi$

Total $= 60\pi + 25\pi$
$= 85\pi$

....85π...... cm^2 **[3]**

16 ABCD is a parallelogram.



$\overrightarrow{BD} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.
F is the midpoint of BC.
G is the midpoint of DC.
AE = 3EB.

(a) Write down simplified expressions in terms of **a** and **b** for

(i) $\overrightarrow{AB}$,

$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$
$= b - a$

(a)(i) .... $b - a$ .... [1]

(ii) $\overrightarrow{EB}$.

$\frac{1}{4}(\overrightarrow{AB}) = \frac{1}{4}(b-a)$

(ii) .... $\frac{1}{4}(b-a)$ .... [1]

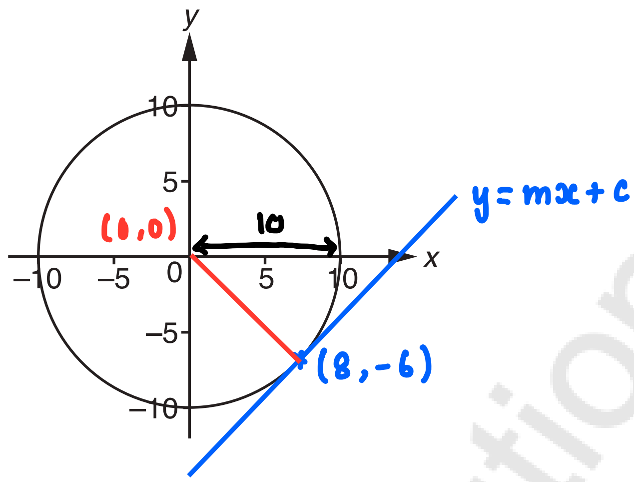
(b) Show that $\overrightarrow{EF} = \frac{1}{4}(3\mathbf{b} - \mathbf{a})$. [2]

$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$
$= \frac{1}{4}(b-a) + \frac{1}{2}b$
$= \frac{1}{4}b - \frac{1}{4}a + \frac{1}{2}b$ $= \frac{3}{4}b - \frac{1}{4}a$
$= \frac{1}{4}(3b - a)$

(c) Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel.

$\overrightarrow{AG} = \overrightarrow{AD} + \overrightarrow{DG}$
$= b + \frac{1}{2}(b-a)$
$= b + \frac{1}{2}b - \frac{1}{2}a$
$= \frac{3}{2}b - \frac{1}{2}a$ which is $2\left(\frac{3}{4}b - \frac{1}{4}a\right)$
$\overrightarrow{AG} = 2\overrightarrow{EF}$ $\therefore$ parallel. [3]

17 The diagram shows a circle, centre the origin.



(a) Write down the equation of the circle.

$x^2+y^2=r^2$

$x^2+y^2=10^2$

(a) .... $x^2+y^2=100$ .... **[1]**

(b) Point P has coordinates (8, –6).
Show that point P lies on the circle. **[2]**

$(8, -6)$
$x \quad y$

$(8)^2+(-6)^2$

$64+36=100$ ∴ lies on the circle

(c) Find the equation of the tangent to the circle at point P.

$m_{radius} = \frac{0--6}{0-8}$

$= \frac{6}{-8}$

$= -\frac{3}{4}$

$m_{tangent} = \frac{4}{3}$

$m = \frac{4}{3}$ $\quad (8, -6)$ (x, y)

$y = mx + c$

$-6 = \frac{4}{3}(8)+c$

$-6 = \frac{32}{3}+c$

$-\frac{50}{3} = c$

(c) .... $y=\frac{4}{3}x-\frac{50}{3}$ .... **[5]**

18 The diagram below shows a 1 cm coordinate grid.

$m = \frac{-2}{1} = -2$

$m = -2$ $\quad (5, 8)$

$y = mx + c$

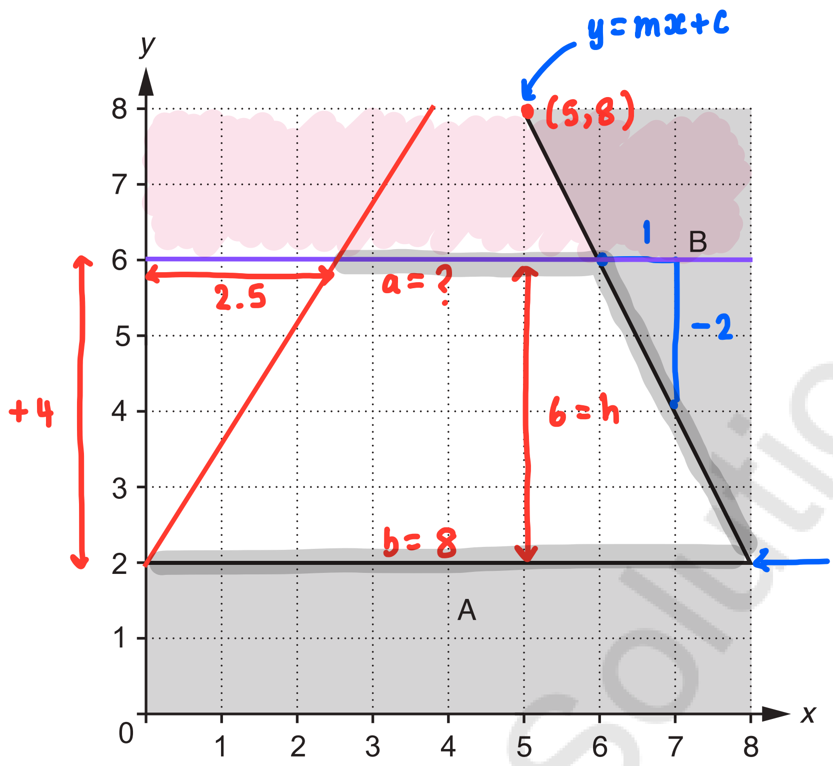
$8 = -2(5) + c$

$8 = -10 + c$

$18 = c$

$y = -2x + 18$

$y \geqslant -2x + 18$



$y = 2$

$y \leqslant 2$

(a) Find an inequality that defines region A and another inequality that defines region B.

(a) Region A: $y \leqslant 2$

Region B: $y \geqslant -2x + 18$ **[4]**

(b) Shade the region on the grid given by the inequality $y \geqslant 6$. **[2]**

(c) A fourth shaded region, given by the inequality

$$y \geqslant kx + 2,$$

is added to the grid.

$y \geqslant mx + c$

k = gradient

$c = 2$

The **unshaded** region now has area 23 cm².

Find the value of k.

Area trapezium $= \frac{1}{2}(a+b) \times h$

$23 = \frac{1}{2}(a+8) \times 4$

$\div 4 \qquad \div 4$

$5.75 = \frac{1}{2}(a+8)$

$\times 2 \qquad \times 2$

$11.5 = a + 8$

$-8 \qquad -8$

$3.5 = a$

$m = \frac{4}{2.5}$

$= \frac{8}{5}$

$k = m = \frac{8}{5}$

(c) k = $\frac{8}{5}$ .... **[5]**

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).