

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Thursday 24 May 2018 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



First name				
Last name				
Centre number				
Candidate number				

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

- 1 (a) The ratio 2 centimetres to 5 metres can be written in the form $1 : n$.

Find the value of n .

$$m \xrightarrow{\times 100} cm$$

$$5 \times 100 = 500cm$$

$$\begin{array}{ccc} 2cm : 500cm & & \\ \div 2 & & \div 2 \\ 1 & : & 250 \end{array}$$

(a) $n = \underline{250}$ [2]

- (b) Jay, Sheila and Harry share £7200 in the ratio $1 : 2 : 5$.

How much does Harry receive?

$$J : S : H$$

$$1 : 2 : 5$$

$$\text{Total} = 8 \text{ parts}$$

$$7200 \div 8 = \pounds 900$$

$$\text{Harry} = 5 \times \pounds 900$$

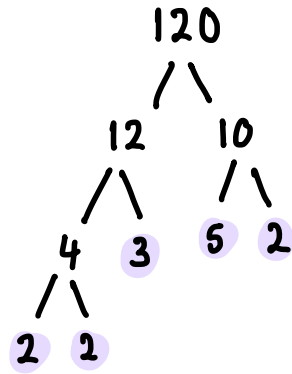
(b) $\pounds \underline{4500}$ [2]

- 2 Given that $y^{18} \div y^6 = y^k$, find the value of k .

$$\begin{aligned} y^{18} \div y^6 &= y^{18-6} \\ &= y^{12} \end{aligned}$$

$k = \underline{12}$ [1]

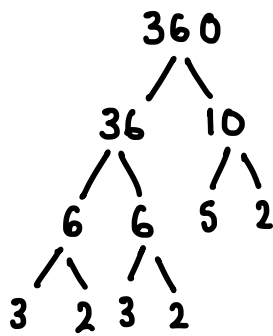
- 3 (a) (i) Write 120 as a product of its prime factors.



(a)(i) $2 \times 2 \times 2 \times 3 \times 5$ [3]

- (ii) The lowest common multiple (LCM) of x and 120 is 360.

Find the smallest possible value of x .



$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$x = 3^2$$

$$= 9$$

(ii) 9 [2]

- (b) Two numbers, A and B , are written as a product of prime factors.

$$A = 2^4 \times 3^2 \times 7^2$$

$$B = 2^3 \times 3 \times 5 \times 7$$

Find the highest common factor (HCF) of A and B .

$$A = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$$

$$B = 2 \times 2 \times 2 \times 3 \times 5 \times 7$$

$$\text{HCF} = 2^3 \times 3 \times 7$$

$$= 168$$

(b) 168 [2]

- 4 Lee wishes to find out if there is a relationship between a person's age and the time it takes them to complete a puzzle.

Lee decides to conduct an experiment.

She asks 12 people to complete the puzzle.

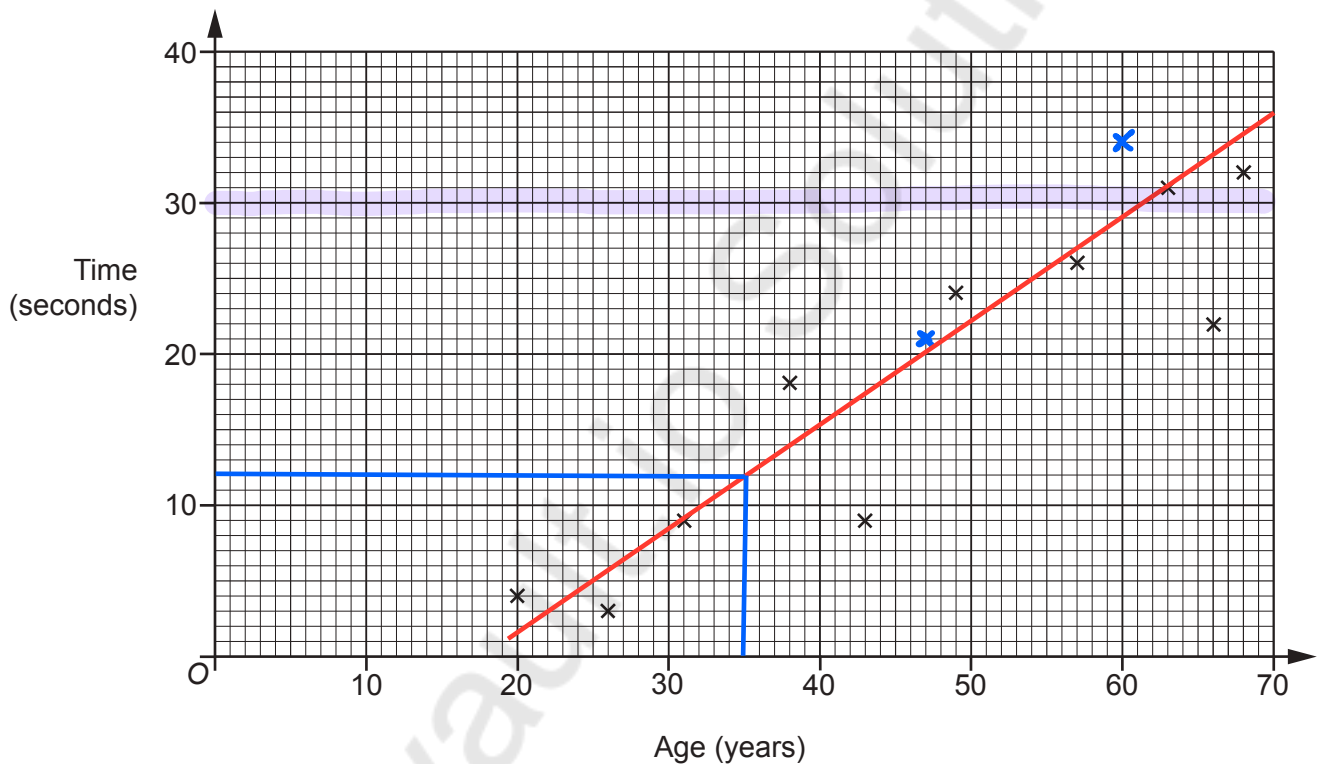
She records each person's age and the time taken to complete the puzzle.

- (a) Make one criticism of Lee's method.

..... **Sample size is small.**

[1]

- (b) This scatter diagram shows the results for ten of the people in Lee's experiment.



Here are the other two results.

Age (years)	47	60
Time (seconds)	21	34

Plot these results on the scatter diagram.

[2]

- (c) What type of correlation is shown in the scatter diagram?

(c) **Positive** [1]

- (d) Estimate the time it would take a person aged 35 to complete the puzzle.

Show your working to justify your answer.

(d) **12 seconds** [2]

- (e) Lee says that at least 80% of the 12 people completed the puzzle in under 30 seconds.

Is Lee correct?

Show working to support your answer.

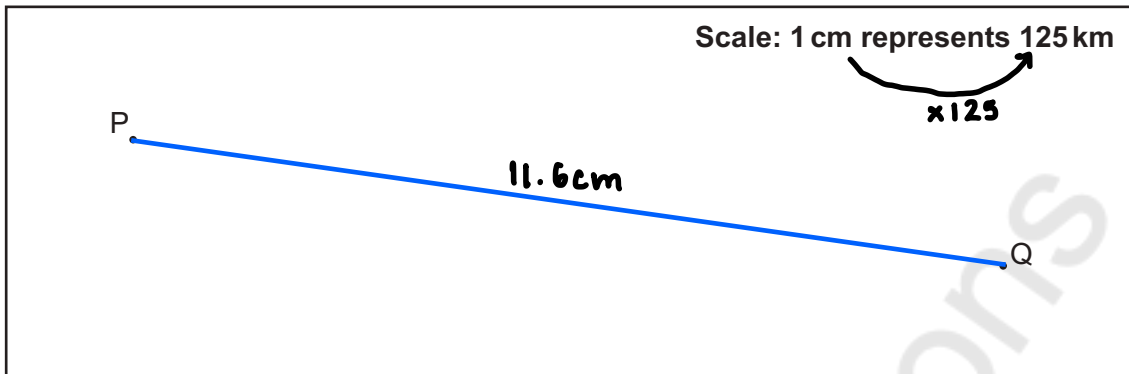
$$\frac{9}{12} \times 100 = 75\%$$

$$75\% < 80\%$$

No.

[3]

- 5 The scale diagram below shows two cities, P and Q.



A plane departs from P at 09 47 and arrives at Q at 12 07.

- (a) Work out the average speed, in kilometres per hour, of the plane.

$$\begin{aligned} \text{Distance} &= 11.6 \times 125 \\ &= 1450 \text{ km} \end{aligned}$$

$$\text{Time} = 2.\dot{3} \text{ h}$$

$$09 \ 47 \quad + 13 \text{ mins} \quad 10 \ 00 \quad + 2\text{h } 7 \text{ mins} \quad 12 \ 07$$

$$\begin{aligned} 13 \text{ mins} + 2\text{h } 7 \text{ mins} &= 2\text{h } \underline{20 \text{ mins}} && \text{mins} \xrightarrow{\div 60} \text{h} \\ &20 \div 60 = 0.\dot{3} \text{ h} \\ &= 2.\dot{3} \text{ h} \end{aligned}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

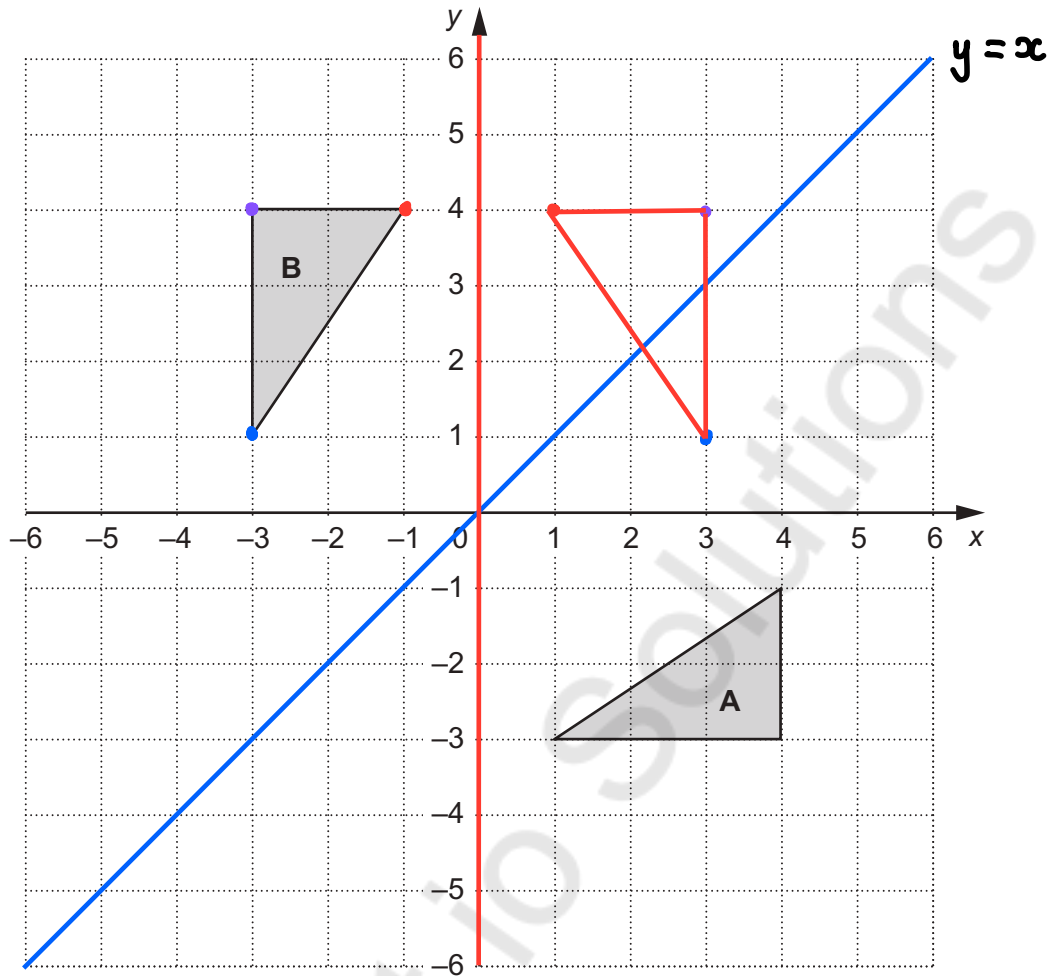
$$= \frac{1450}{2.\dot{3}} = 621.428\dots$$

(a) 621.4 km/h [5]

- (b) Give one reason why your answer may be inaccurate.

..... The plane may not have flown in a straight line. [1]

6 Triangles **A** and **B** are drawn on a coordinate grid.



(a) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

..... Reflection in the line $y = x$ [2]

(b) Triangle **A** can also be mapped onto triangle **B** using a combination of two transformations:

- a transformation **T**, followed by
- a reflection in the line $x = 0$.

Describe fully transformation **T**.

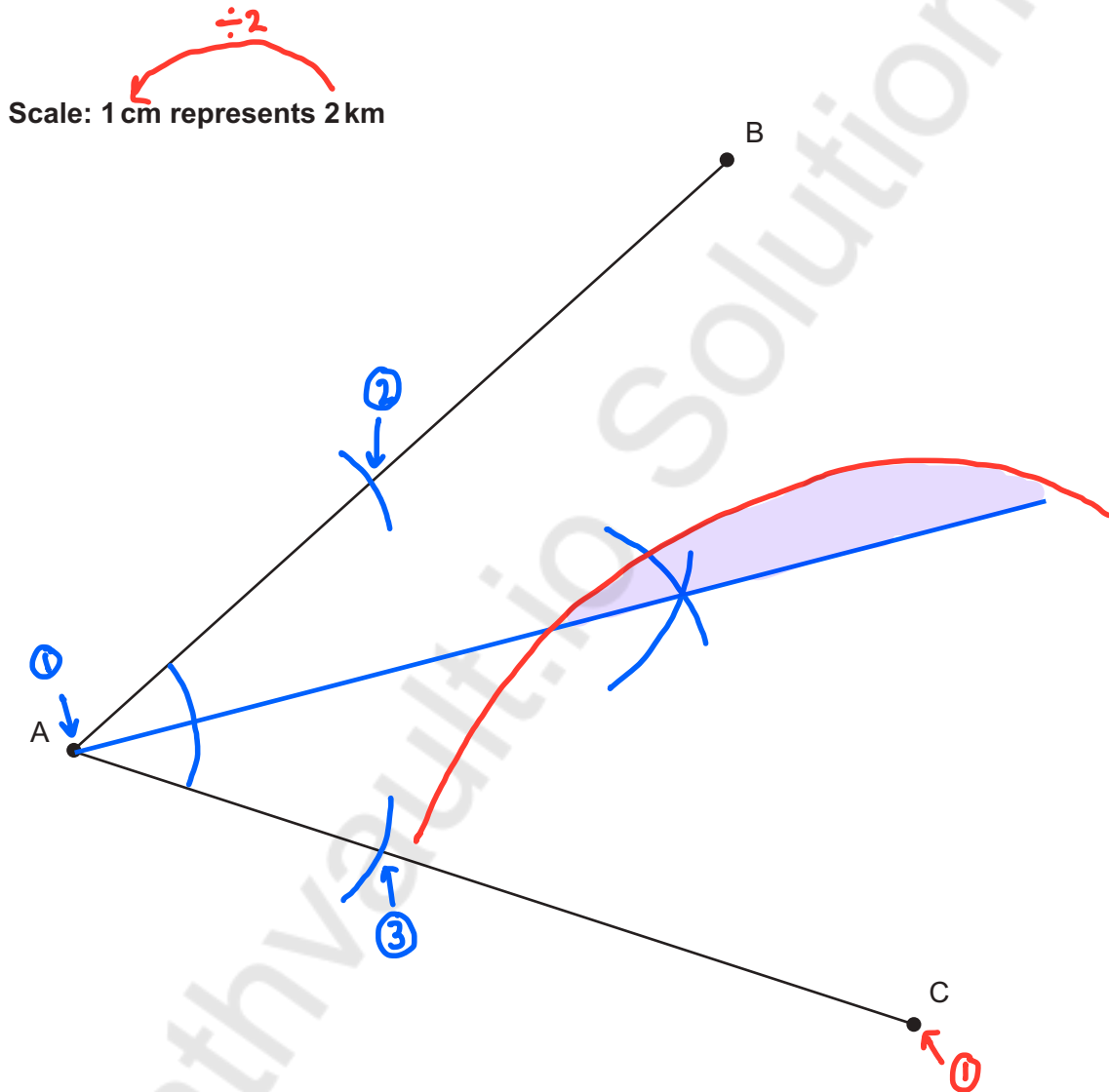
..... Rotation 90° clockwise, about $(0, 0)$ [4]

- 7 The scale diagram below shows towns, A, B and C.
Line AB represents the road from A to B and line AC represents the road from A to C.

A shopping centre is to be built so that it is

- nearer to the road from A to B than the road from A to C, **angle bisector**
- less than 14 km from town C.

- (a) Using construction, shade the region where the shopping centre could be built.
Show all your construction lines.

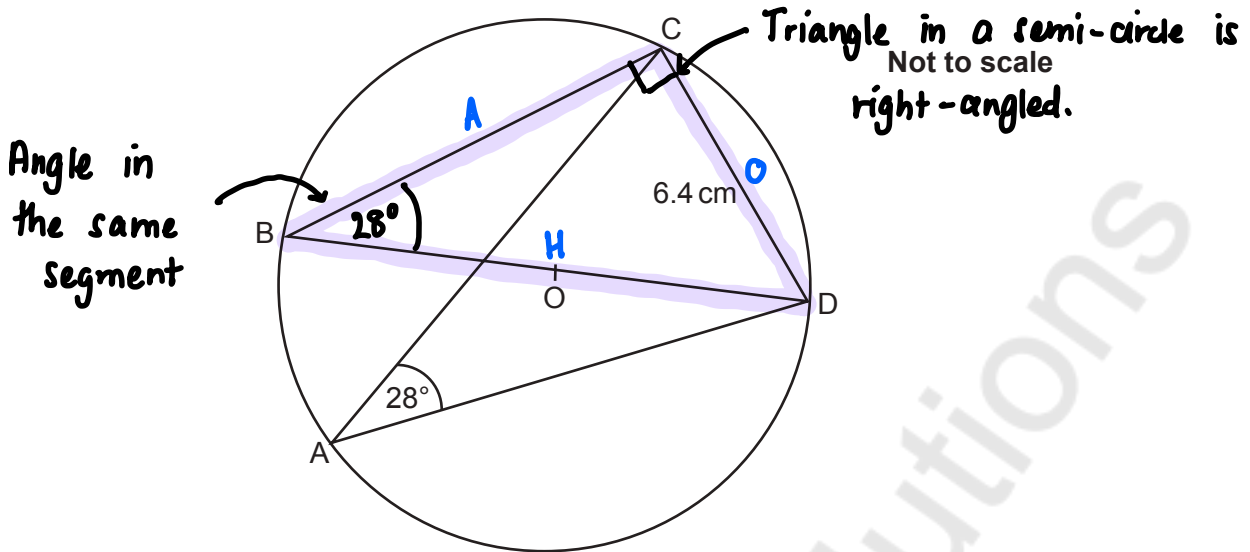


- (b) Explain why the region found in part (a) may not be an appropriate site for the shopping centre.

Roads are not straight.

[1]

8 A, B, C and D are points on the circumference of a circle, centre O.



Angle CAD = 28° and CD = 6.4 cm.
BD is a diameter of the circle.

Calculate the area of the circle.

$$\rightarrow A = \pi r^2$$

$$\begin{array}{c} \text{S} \quad \text{O} \quad \text{H} \quad \text{C} \quad \text{A} \quad \text{H} \quad \text{T} \quad \text{O} \quad \text{A} \\ \downarrow \\ \sin \theta = \frac{O}{H} \end{array}$$

$$\sin(28) = \frac{6.4}{BD}$$

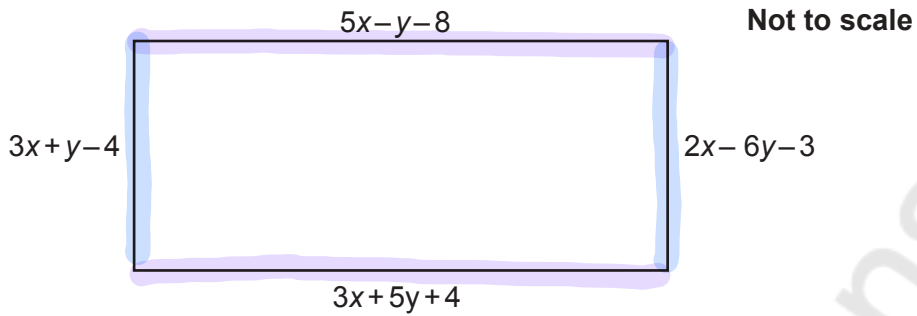
$$\begin{aligned} BD &= 6.4 \div \sin(28) \\ &= 13.6323486 \text{ cm} \\ &[\text{Diameter}] \end{aligned}$$

$$\begin{aligned} \therefore \text{Radius} &= \frac{1}{2} \times 13.632... \\ &= 6.816174298 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \pi (6.816...)^2 \\ &= 145.9591237 \\ &\approx 146 \text{ cm}^2 \end{aligned}$$

..... 146 cm² [5]

- 9 The dimensions, in centimetres, of this rectangle are shown as algebraic expressions.



Work out the length and width of the rectangle.

$$\begin{array}{r} 5x - y - 8 = 3x + 5y + 4 \\ -3x - 5y + 8 \quad -3x - 5y + 8 \end{array}$$

$$2x - 6y = 12$$

$$\begin{array}{r} 3x + y - 4 = 2x - 6y - 3 \\ -2x + 6y + 4 \quad -2x + 6y + 4 \end{array}$$

$$x + 7y = 1$$

$$\begin{array}{r} 2x - 6y = 12 \\ x + 7y = 1 \quad \times 2 \end{array}$$

$$\begin{array}{r} 2x - 6y = 12 \\ 2x + 14y = 2 \\ \hline -20y = 10 \\ \div -20 \quad \div -20 \\ y = -0.5 \end{array}$$

$$\begin{array}{r} x + 7y = 1 \\ x + 7(-0.5) = 1 \\ x - 3.5 = 1 \\ + 3.5 \quad + 3.5 \\ x = 4.5 \end{array}$$

$$\begin{aligned} \text{Length} &= 5x - y - 8 \\ &= 5(4.5) - (-0.5) - 8 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{Width} &= 3x + y - 4 \\ &= 3(4.5) + (-0.5) - 4 \\ &= 9 \end{aligned}$$

length = **15** cm

width = **9** cm
[6]

- 10 60% of the people in a town are males.
 20% of the males are left-handed.
 21.6% of all the people are left-handed.

Work out the percentage of the people who are not male who are left-handed.

	m	m'	Total
RH	48	30.4	78.4
LH	12	9.6	21.6
Total	60	40	100 (Assumed)

% of people m', LH

$$\frac{9.6}{40} \times 100 = 24$$

..... 24 % [5]

- 11 y is directly proportional to the square of x .

Find the percentage increase in y when x is increased by 15%.

$$y \propto x^2$$

$$y = kx^2$$

$$y = k(1.15)^2$$

$$y = 1.3225k$$

$$100\% + 15\% = 115\%$$

$$115\% \div 100 = 1.15$$

$$1.3225 \times 100 = 132.25\%$$

$$132.25\% - 100\% = 32.25\%$$

..... **32.25** % [4]

- 12 The value of a car, £ V , is given by

$$V = 16500 \times 0.82^n$$

where n is the number of years after it is bought from new.

- (a) Write down the value of the car when new.

$$16500 \times 0.82^0$$

$$16500 \times 1$$

(a) £ **16500** [1]

- (b) Write down the annual percentage decrease in the value of the car.

$$0.82 \times 100 = 82\%$$

$$100\% - 82\% = 18\%$$

(b) **18** % [1]

- (c) Show that the value of the car after 4 years is less than half its value when new. [2]

$$n = 4$$

$$V = 16500 \times 0.82^4$$

$$= 7460.01$$

$$\text{New} = \pounds 16500$$

$$16500 \div 2 = \pounds 8250$$

$$\pounds 7460.01 < \pounds 8250$$

13 A menu has

- 6 starters
- 10 main dishes
- 7 desserts.

(a) A three-course meal consists of a starter, a main dish and a dessert.

How many different three-course meals are possible?

$$6 \times 10 \times 7 = 420$$

(a) **420** [2]

(b) A two-course meal consists either of a starter with a main dish, a starter with a dessert or a main dish with a dessert.

Show that there are 172 possible different two-course meals.

[3]

Starter & Main

Starter & Dessert

Main & Dessert

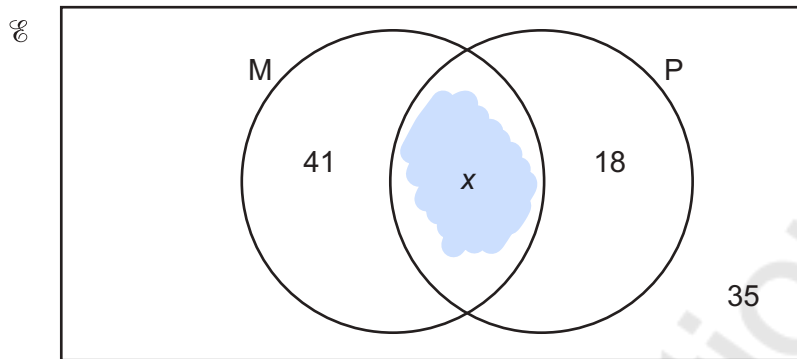
$$6 \times 10 = 60$$

$$6 \times 7 = 42$$

$$10 \times 7 = 70$$

$$\begin{aligned} \text{Total : } & 60 + 42 + 70 \\ & = 172 \end{aligned}$$

- 14 The Venn diagram shows the number of students studying Mathematics (M) and the number of students studying Physics (P) in a college. 35 students do not study either subject.



- (a) The total number of students is 121.

Find the value of x .

$$41 + 18 + 35 = 94$$

$$121 - 94 = 27$$

(a) $x = \dots 27 \dots$ [1]

- (b) One of the 121 students is selected at random.

Find the probability that this student studies Mathematics, given that they study Physics.

$$27 \text{ out of } (18 + 27)$$

$$27 \text{ out of } 45$$

(b) $\dots \frac{27}{45} \dots$ [2]

- 15 (a) Write $x^2 - 8x + 25$ in the form $(x - a)^2 + b$.

$$(x - 4)^2 + 9$$

$$\begin{aligned} (x-4)^2 &= (x-4)(x-4) \\ &= x^2 - 8x + 16 \\ &\quad + 9 \\ &= x^2 - 8x + 25 \end{aligned}$$

(a) $(x-4)^2 + 9$ [3]

- (b) Write down the coordinates of the turning point of the graph of $y = x^2 - 8x + 25$.

$$(x-4)^2 + 9$$

(b) (4, 9) [2]

- (c) Hence describe the single transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 25$.

↳ turning point (4, 9)

(0, 0) → (4, 9)

↳ turning point (0, 0)

Translation by the vector $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$.

..... [2]

$$\begin{pmatrix} \longleftrightarrow \\ \updownarrow \end{pmatrix}$$

16 Solve by factorisation.

$$3x^2 + 11x - 20 = 0$$

$$3x^2 - 4x + 15x - 20$$

$$x(3x - 4) + 5(3x - 4)$$

$$(3x - 4)(x + 5) = 0$$

$$3x - 4 = 0 \quad \text{or} \quad x + 5 = 0$$

$$+4 \quad +4 \quad \quad \quad -5 \quad -5$$

$$3x = 4$$

$$\div 3 \quad \quad \div 3$$

$$x = \frac{4}{3}$$

$$x = -5$$

$$3x - 20 = -60$$

$$\underline{-4} \times \underline{15} = -60$$

$$\underline{-4} + \underline{15} = 11$$

60

1 60

2 30

3 10

4 15

5 12

6 10

$$x = \frac{4}{3} \quad \text{or} \quad x = -5 \quad [3]$$

17 For each graph below, select its possible equation from this list.

$$y = \frac{1}{x}$$

$$y = \cos x$$

$$y = x^2$$

$$y = \left(\frac{1}{2}\right)^x$$

$$y = 2^x$$

$$y = \sin x$$

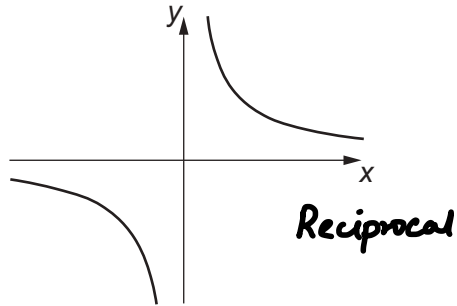
$$y = 2^{-x}$$

$$y = \tan x$$

$$y = x^3$$

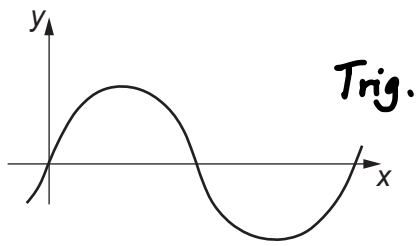
$$y = \frac{1}{x^2}$$

(a)



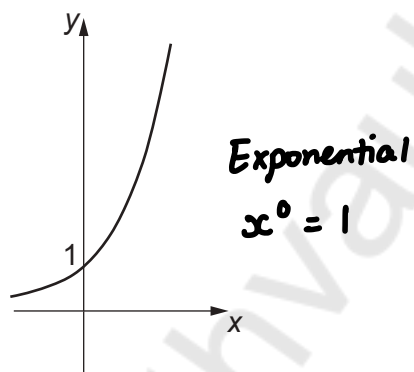
(a) $y = \frac{1}{x}$

(b)



(b) $y = \sin x$

(c)



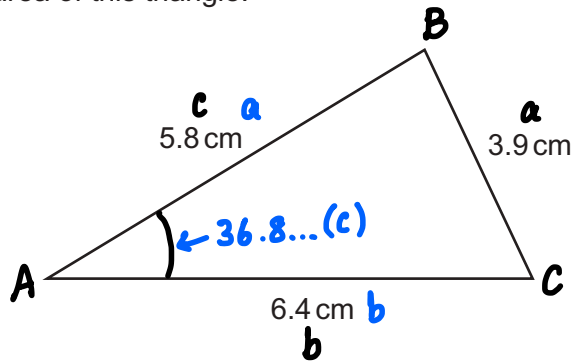
(c) $y = 2^x$

[3]

$$A = \frac{1}{2} ab \sin C$$

18

18 Calculate the area of this triangle.



Not to scale

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{6.4^2 + 5.8^2 - 3.9^2}{2(6.4)(5.8)}$$

$$\cos A = 0.79997306...$$

$$A = \cos^{-1}(0.79997306...)$$

$$A = 36.87247012$$

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 5.8 \times 6.4 \times \sin(36.87...)$$

$$= 11.13666664$$

$$\approx 11.1 \text{ cm}^2$$

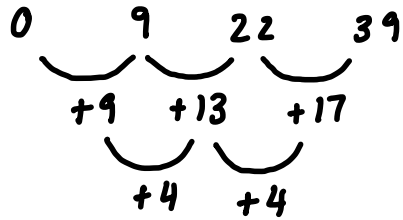
..... 11.1 cm² [6]

19 Here are the first four terms of a quadratic sequence.

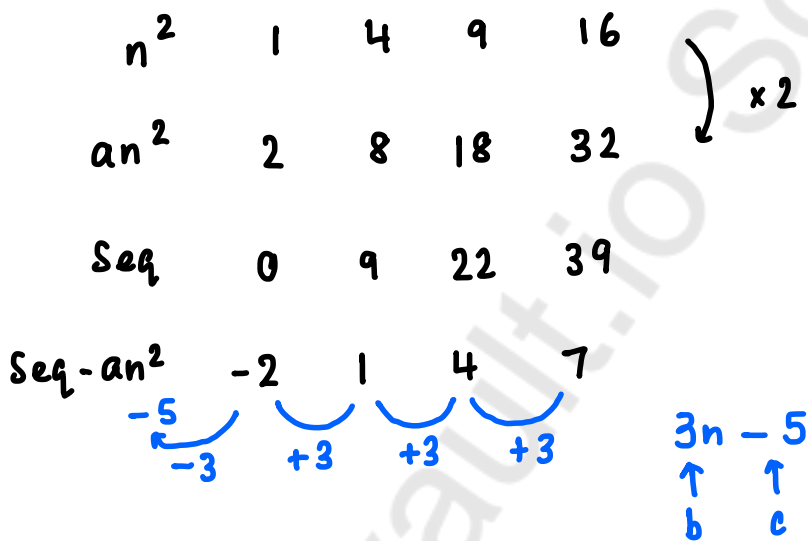
0 9 22 39

The n th term can be written as $an^2 + bn + c$.

Find the values of a , b and c .



$$a = 4 \div 2 = 2$$



$a = \underline{2}$

$b = \underline{3}$

$c = \underline{-5}$

[4]

20 Solve this equation, giving your answers correct to 1 decimal place.

$$\frac{5}{x+2} + \frac{3}{x-3} = 2$$

$$\frac{5(x-3) + 3(x+2)}{(x+2)(x-3)} = 2$$

$$5x - 15 + 3x + 6 \equiv 8x - 9$$

$$\frac{8x - 9}{(x+2)(x-3)} = 2$$

$$x(x+2)(x-3) \quad x(x+2)(x-3)$$

$$8x - 9 = 2(x+2)(x-3)$$

$$8x - 9 = 2(x^2 - 3x + 2x - 6)$$

$$8x - 9 = 2(x^2 - x - 6)$$

$$8x - 9 = 2x^2 - 2x - 12$$

$$-8x + 9 \quad -8x + 9$$

$$0 = 2x^2 - 10x - 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2 \quad b = -10 \quad c = -3$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-3)}}{2(2)}$$

$$x = 5.283882181$$

OR

$$x = -0.2838821814$$

$$x = \dots 5.3 \dots \text{ or } x = \dots -0.3 \dots [6]$$

END OF QUESTION PAPER

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