

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Thursday 7 June 2018 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- geometrical instruments
- tracing paper

Do not use:

- a calculator



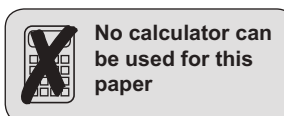
First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document consists of **20** pages.



No calculator can be used for this paper

Answer **all** the questions.

- 1 (a) Calculate.

$$\frac{3}{5} + \frac{5}{8}$$

Give your answer as a mixed number in its simplest form.

$$\text{LCM of } 5 \text{ \& } 8 = 40$$

$$\frac{3}{5} \begin{array}{l} \times 8 \\ \hline = \frac{24}{40} \end{array}$$

$$\frac{24}{40} + \frac{25}{40} = \frac{49}{40} = 1 \frac{9}{40}$$

$$\frac{5}{8} \begin{array}{l} \times 5 \\ \hline = \frac{25}{40} \end{array}$$

(a) $1 \frac{9}{40}$ [3]

- (b) Work out.

$$5 \times 10^4 - 1.6 \times 10^3$$

Give your answer in standard form.

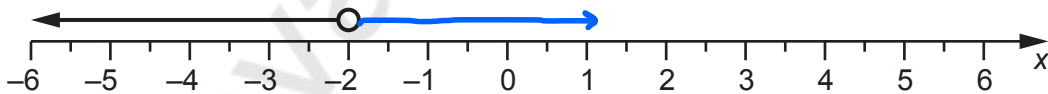
$$5 \times 10^4 - 0.16 \times 10^4$$

$$0.16 \times 10^4$$

$$\begin{array}{r} 5.000 \\ - 0.16 \\ \hline 4.84 \end{array}$$

(b) 4.84×10^4 [3]

- 2 Gemma's solution to the inequality $3x + 1 > -5$ is shown on the number line.



Is Gemma's solution correct?
Explain your reasoning.

$$\begin{array}{r} 3x + 1 > -5 \\ -1 \quad -1 \end{array}$$

$$3x > -6$$

$$\div 3 \qquad \div 3$$

$$x > -2$$

..... No. Arrow points the wrong way. [3]

3 Work out.

(a) $\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

(a) $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$ [1]

(b) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

(b) $\begin{pmatrix} 1 \\ 10 \end{pmatrix}$ [2]

4 Here is the nutritional information for a 110g serving of cereal.

Carbohydrates	99.4g
Proteins	9.5g
Fats	1.1g

Emily says that more than 90% of this serving is carbohydrates.

Is she correct?

Explain your reasoning.

$$\begin{array}{l} 100\% = 110\text{g} \\ \div 10 \qquad \qquad \div 10 \\ 10\% = 11\text{g} \\ \times 9 \qquad \qquad \times 9 \\ 90\% = 99\text{g} \end{array}$$

Yes. $99.4\text{g} > 99\text{g}$

[3]

- 5 The table shows the relative frequencies of the results for a football team after a number of games.

Result of game	won	lost	drew	
Relative frequency	0.2	0.45	0.35	= 1

- (a) Complete the table.

$$\begin{array}{r} 0.20 \\ + 0.45 \\ \hline 0.65 \end{array}$$

$$\begin{array}{r} 0.100 \\ - 0.65 \\ \hline 0.35 \end{array}$$

[2]

- (b) The team lost 10 more games than they won.

How many games did the team play altogether?

$$\begin{array}{r} 0.45 \\ - 0.20 \\ \hline 0.25 \end{array}$$

$$\begin{array}{r} 10 \div 0.25 \\ \times 100 \quad \times 100 \end{array}$$

$$\begin{array}{r} 1000 \div 25 = 40 \\ 25 \overline{)1000} \end{array}$$

(b) **40** [3]

- 6 Jack sent 15% more text messages in March than in February.
Jack sent 460 text messages in March.

How many more texts did Jack send in March than in February?

$$100\% + 15\% = 115\% \text{ in March}$$

$$115\% = 460$$

$$\div 1.15$$

$$\div 1.15$$

$$460 \div 1.15 = 400$$

$$\times 100 \quad \times 100$$

$$46000 \div 115 = 400$$

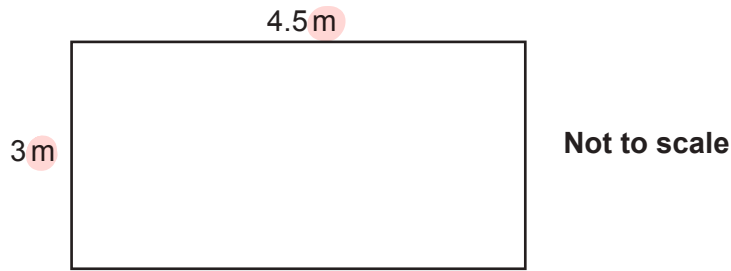
$$\begin{array}{r} 400 \\ 115 \overline{)46000} \end{array}$$

February: $100\% = 400$

$$460 - 400 = 60$$

..... **60** [4]

7 Here is the floor plan of a rectangular room.



Tim buys carpet tiles for this room.

$$\text{cm} \xrightarrow{\div 100} \text{m} \quad 50 \div 100 = 0.5$$

Each tile is a square measuring 50 cm by 50 cm.

$$0.5\text{m} \times 0.5\text{m}$$

The tiles are only sold in packs of ten.

Each pack costs £20.

Tim pays for fitting at a rate of £7.50 per square metre, with any fraction of a square metre rounded up.

Work out the **total** cost of the tiles and fitting.

$$\begin{aligned} \text{Area of floor} &= l \times w \\ &= 3 \times 4.5 \\ &= 13.5\text{m}^2 \end{aligned}$$

$$\begin{array}{r} 4.5 \\ \times 3 \\ \hline 13.5 \end{array}$$

$$\begin{aligned} \text{Area of a tile} &= l \times w \\ &= 0.5 \times 0.5 \\ &\quad \times 10 \quad \times 10 \\ &= 5 \times 5 \\ &= 25 \div 100 \\ &= 0.25\text{m}^2 \end{aligned}$$

$$\begin{aligned} \text{Packs of tiles} &= 54 \div 10 \\ &= 5.4 \\ &\approx 6 \text{ packs} \end{aligned}$$

$$\begin{aligned} \text{Cost of packs} &= 6 \times £20 \\ &= £120 \end{aligned}$$

$$\text{Fitting} = £7.50 \times 14\text{m}^2 \text{ (rounded up)}$$

$$\text{Number of tiles} = \frac{13.5 \times 100}{0.25 \times 100}$$

$$= \frac{1350}{25}$$

$$25 \overline{)1350}$$

$$= 54 \text{ tiles}$$

$$\begin{array}{r} 7.50 \\ \times 14 \\ \hline 30.00 \\ 75.00 \\ \hline £105.00 \end{array}$$

$$\text{Total cost} = £105 + £120$$

$$£ \quad 225 \quad \dots \quad [6]$$

8 Hannah wants to display all the possible outcomes when rolling two fair 6-sided dice.

(a) Give a reason why a tree diagram is not the best method to use.

Too many branches..... [1]

(b) (i) Draw a sample space to display all the possible outcomes. [2]

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
	3	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

(ii) Show that the probability of the scores on the two dice adding to 11 is $\frac{1}{18}$.

$$p(5,6) = \frac{1}{36}$$

$$p(6,5) = \frac{1}{36}$$

$$\frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$$

..... [2]

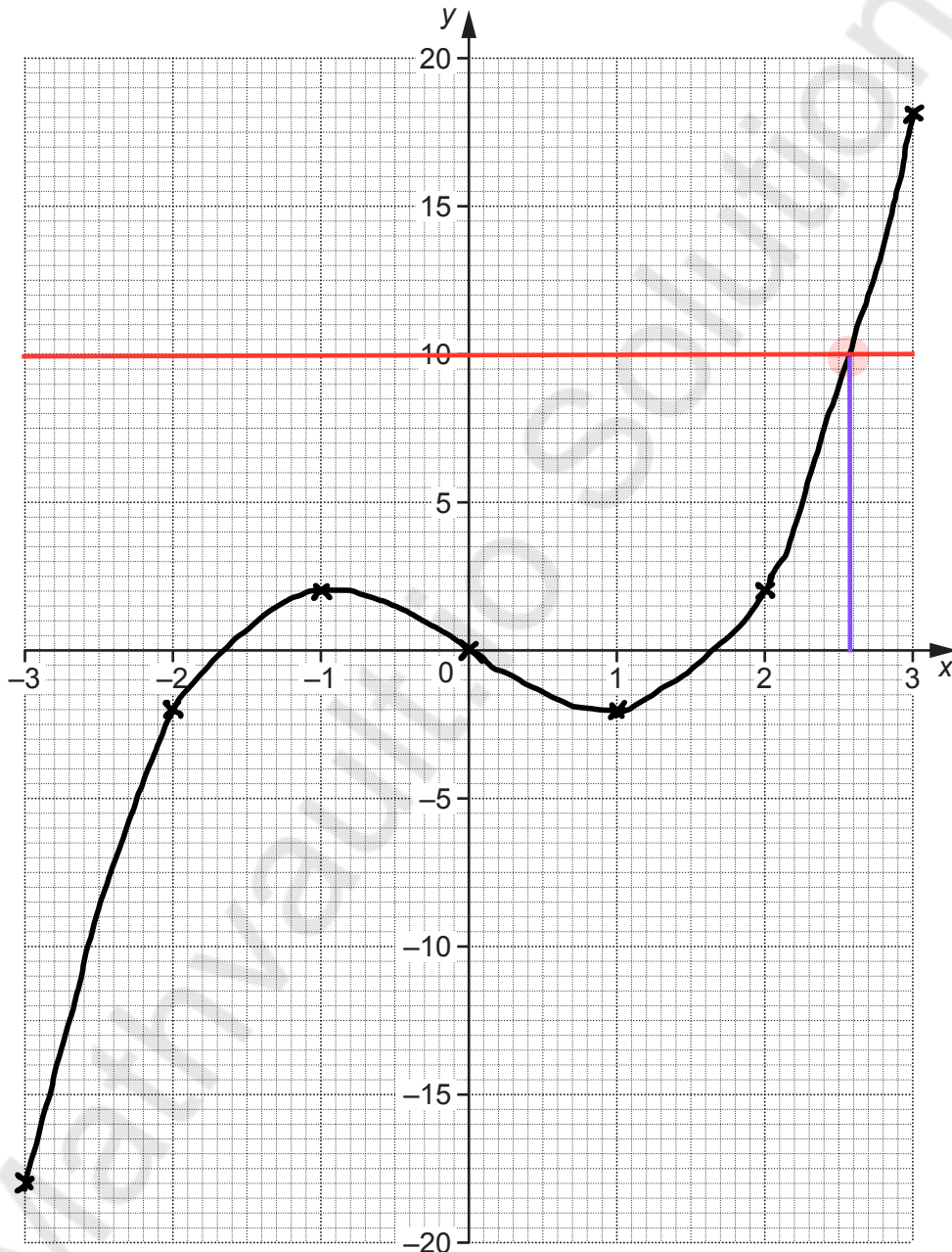
- 9 (a) Complete the table for $y = x^3 - 3x$.

x	-3	-2	-1	0	1	2	3
y	-18	-2	2	0	-2	2	18

$$\begin{aligned} \curvearrowright y &= (-1)^3 - 3(-1) \\ &= -1 + 3 \end{aligned}$$

[1]

- (b) Draw the graph of $y = x^3 - 3x$ for $-3 \leq x \leq 3$.



$$x^3 - 3x = y$$

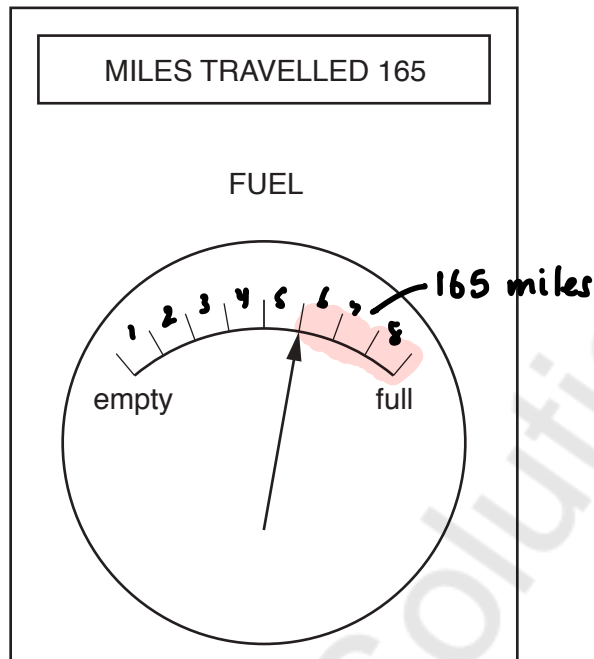
[3]

- (c) Use your graph to solve $x^3 - 3x = 10$.

Draw $y = 10$

- (c) $x = \dots 2.6 \dots$ [1]

- 10 Ifsaw noticed this information on her car's dashboard at the end of her journey. She started her journey with a full tank of fuel and her miles travelled set to zero.



- (a) Work out how far Ifsaw's car can travel on a full tank of fuel.

$$165 \div 3 = 55$$

$$\begin{array}{r} 55 \\ 3 \overline{)165} \end{array}$$

$$8 \times 55$$

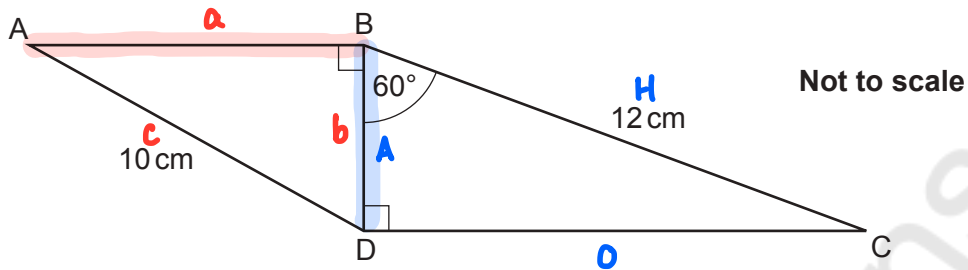
$$\begin{array}{r} 455 \\ \times 8 \\ \hline 440 \end{array}$$

(a) 440 miles [3]

- (b) What assumption have you made when answering part (a)?

..... The rate of fuel consumption remains constant.
 [1]

- 11 The diagram shows two right-angled triangles ABD and BCD, sharing a common side BD. AD = 10 cm, BC = 12 cm and angle DBC = 60°.



Work out the length of AB.

S O H C A ✓ H ✓ T O A

$$\cos \theta = \frac{A}{H}$$

$$\cos 60 = \frac{BD}{12}$$

$$\frac{\sqrt{1}}{2} = \frac{BD}{12}$$

$$\frac{1}{2} = \frac{BD}{12}$$

$$\frac{1}{2} \times 12 = 6$$

$$BD = 6 \text{ cm}$$

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0
				2	

$$a^2 + b^2 = c^2$$

$$AB^2 + 6^2 = 10^2$$

$$AB^2 = 10^2 - 6^2$$

$$AB = \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36}$$

$$= \sqrt{64}$$

$$= 8$$

.....8..... cm [6]

12 Carol says that $64^{-\frac{1}{2}} = \frac{1}{32}$.

Explain her error and give the correct value of $64^{-\frac{1}{2}}$ in the form $\frac{p}{q}$.

..... She divided by 2 and did not square root

..... It should be $\frac{1}{8}$

.....

..... [3]

13 (a) Write $\frac{5}{12}$ as a recurring decimal.

$$5 \div 12$$

$$12 \overline{) 5.50208080}$$

(a) $0.4\dot{1}\dot{6}$ [2]

(b) Convert $0.7\dot{6}$ to a fraction.

$$x = 0.7\dot{6}$$

$$10x = 7.\dot{6}\dot{7}$$

$$100x = 76.\dot{7}\dot{6}$$

$$100x - x = 99x$$

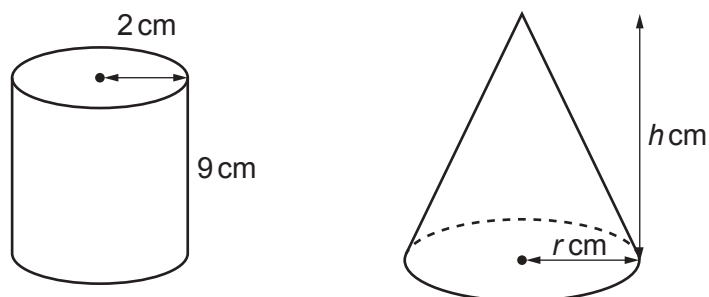
$$76.\dot{7}\dot{6} - 0.7\dot{6} = 76$$

$$99x = 76$$

$$x = \frac{76}{99}$$

(b) $\frac{76}{99}$ [2]

- 14 The diagram shows a cylinder and a cone.



The cylinder has radius 2 cm and height 9 cm.
The cone has radius r cm and height h cm.

The ratio $r : h$ is 1 : 4.

The volume of the cone is **equal to** the volume of the cylinder.

Work out the value of r .

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$$\begin{aligned} \text{Volume cylinder} &= \pi r^2 h & r = 2 & \quad h = 9 \\ &= \pi (2)^2 (9) \\ &= \pi (4)(9) \\ &= 36\pi \end{aligned}$$

$$\begin{aligned} \text{Volume cone} &= \frac{1}{3} \pi r^2 h & r : h \\ & & 1 : 4 \\ & & x : 4x \end{aligned}$$

$$36\pi = \frac{1}{3} \pi (x)^2 (4x)$$

$$36\pi = \frac{1}{3} \pi (4x^3)$$

$$\begin{array}{cc} \times 3 & \times 3 \\ 108\pi = \pi(4x^3) & \end{array}$$

$$\begin{array}{cc} \div \pi & \div \pi \\ 108 = 4x^3 & \end{array}$$

$$\begin{array}{cc} \div 4 & \div 4 \\ 27 = x^3 & \end{array}$$

$$\begin{array}{cc} \sqrt[3]{} & \sqrt[3]{} \\ 3 = x & \end{array}$$

$$3 = x \quad \text{radius} = 3 \quad \text{height} = 12$$

..... 3 [5]

15 n is a positive integer.

Prove that $13n + 3 + (3n - 5)(2n + 3)$ is a multiple of 6.

[4]

$$(3n-5)(2n+3)$$

$$6n^2 + 9n - 10n - 15$$

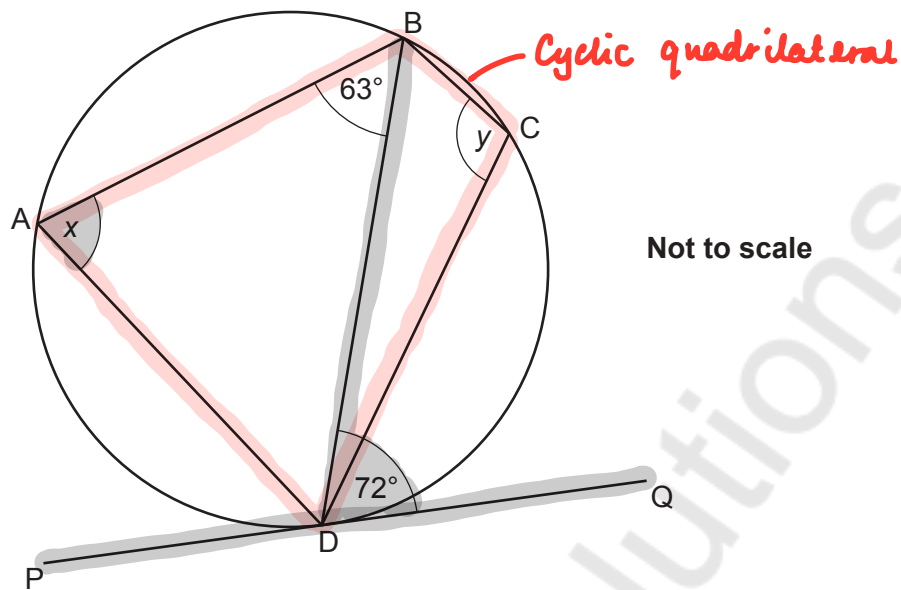
$$6n^2 - n - 15$$

$$13n + 3 + 6n^2 - n - 15$$

$$6n^2 + 12n - 12$$

$$6(n^2 + 2n - 2) \quad \therefore \text{multiple of 6.}$$

16 A, B, C and D are points on the circumference of a circle.



PQ is a tangent to the circle at D.
Angle $BDQ = 72^\circ$ and angle $ABD = 63^\circ$.

- (a) Work out angle x .
Give a reason for your answer.

Angle $x = 72^\circ$ because angles in alternate segments are equal. [2]

- (b) Work out angle y .
Give a reason for your answer.

$$y = 180 - 72 \\ = 108$$

Angle $y = 108^\circ$ because opposite angles in a cyclic quadrilateral sum to 180° . [2]

$$17 \quad (x+a)(x+3)(2x+1) = bx^3 + cx^2 + dx - 12$$

Find the value of a , b , c and d .

$$(x+a)(x+3)$$

x	x	$+a$
x	x^2	$+ax$
$+3$	$+3x$	$+3a$

$$(x^2 + 3x + ax + 3a)(2x + 1)$$

x	x^2	$+3x$	$+ax$	$+3a$
$2x$	$2x^3$	$+6x^2$	$+2ax^2$	$+6ax$
$+1$	x^2	$+3x$	$+ax$	$+3a$

$$2x^3 + 7x^2 + 2ax^2 + 3x + 7ax + 3a$$

$$2x^3 + (7 + 2a)x^2 + (3 + 7a)x + 3a$$

$$bx^3 \quad cx^2 \quad dx \quad -12$$

$$b = 2$$

$$7 + 2a = c$$

$$3 + 7a = d$$

$$\begin{array}{l} 3a = -12 \\ \div 3 \end{array} \quad \div 3$$

$$7 + 2(-4) = c$$

$$3 + 7(-4) = d$$

$$a = -4$$

$$7 - 8 = c$$

$$3 - 28 = d$$

$$-1 = c$$

$$-25 = d$$

$$a = -4$$

$$b = 2$$

$$c = -1$$

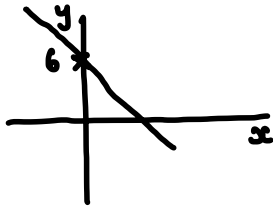
$$d = -25 \quad [4]$$

- 18 (a) A straight line passes through the point (0, 6) and is perpendicular to $y = 4x - 5$.

Find the equation of this line, giving your answer in the form $y = mx + c$.

$$m = 4$$

$$\text{perpendicular } m = -\frac{1}{4}$$



$$c = 6$$

$$y = -\frac{1}{4}x + 6$$

(a) $y = -\frac{1}{4}x + 6$ [3]

- (b) Work out the coordinates of the intersection of the graphs of $y = 4x - 5$ and $y = x^2 - 17$.

$$4x - 5 = x^2 - 17$$

$$-4x + 5 \quad -4x + 5$$

$$0 = x^2 - 4x - 12 \quad \frac{2}{x} \frac{-6}{-6} = -12$$

$$(x + 2)(x - 6) = 0 \quad \frac{2}{2} + \frac{-6}{-6} = -4$$

$$x + 2 = 0 \quad x - 6 = 0$$

$$-2 \quad -2 \quad +6 \quad +6$$

$$x = -2 \quad x = 6$$

$$y = 4x - 5$$

$$y = 4(-2) - 5$$

$$= -8 - 5$$

$$= -13$$

$$(-2, -13)$$

$$y = 4(6) - 5$$

$$= 24 - 5$$

$$= 19$$

$$(6, 19)$$

(b) (..... -2 , -13 )

(..... 6 , 19 ) [6]

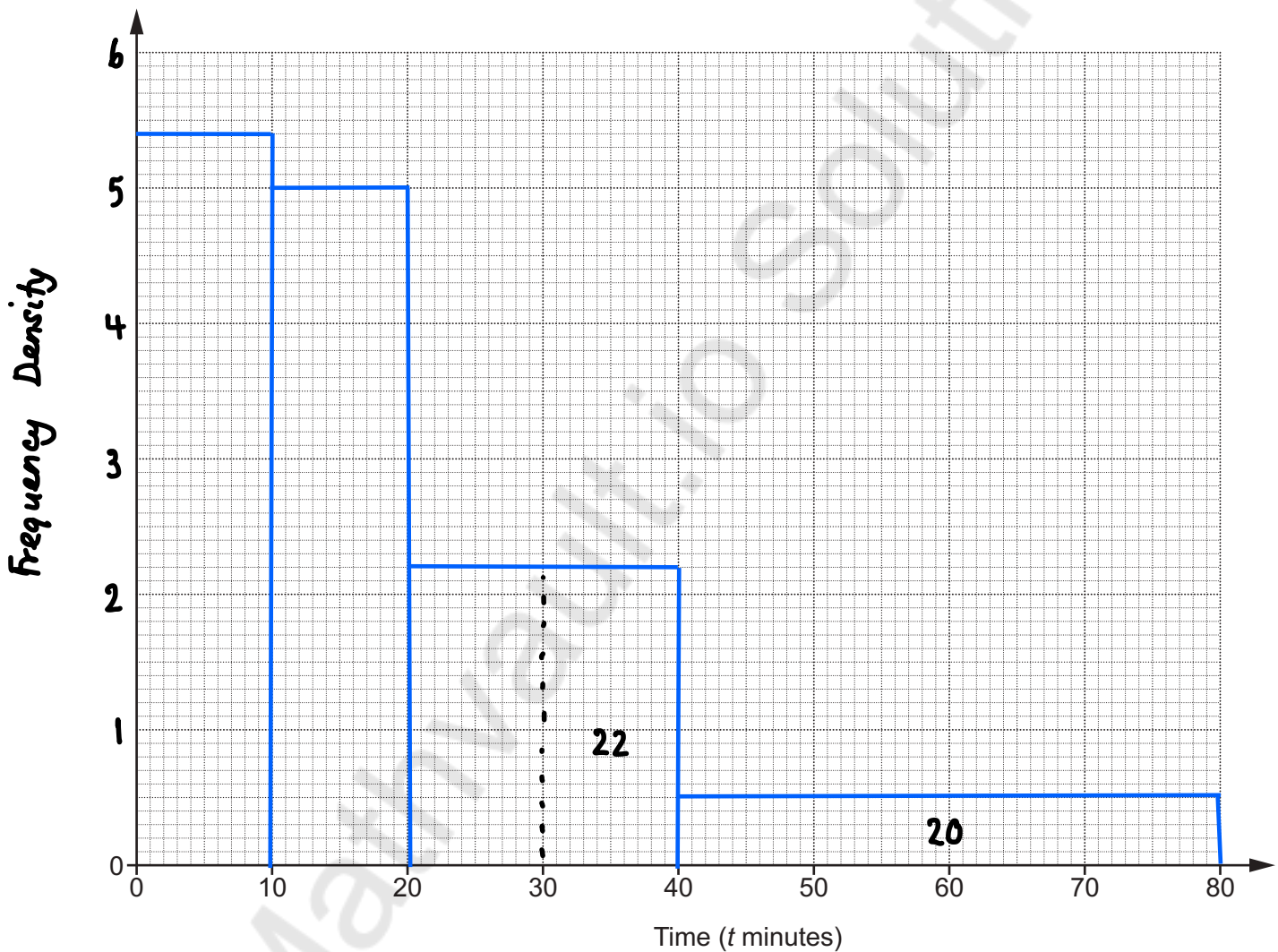
- 19 Ceri records the time taken, t minutes, to travel to school for a sample of 168 students at her Academy.

Time taken (t minutes)	Frequency
$0 < t \leq 10$	54
$10 < t \leq 20$	50
$20 < t \leq 40$	44
$40 < t \leq 80$	20

CW	FD = $F \div CW$
10	$54 \div 10 = 5.4$
10	$50 \div 10 = 5$
20	$44 \div 20 = 2.2$
40	$20 \div 40 = 0.5$

$$\frac{44}{20} = \frac{22}{10} = 2.2$$

- (a) Draw a histogram to represent this information.



[4]

(b) Ceri says

The longest time that any of these students took to travel to school was 80 minutes.

Is she correct?

Give a reason for your answer.

..... *We don't know the exact times.*
 [1]

(c) Ceri also claims that 25% of all of the students at this Academy took more than 30 minutes to travel to school.

(i) Show how Ceri might have worked out her claim. [2]

$$\begin{array}{r}
 25\% \text{ of } 168 \\
 100\% = 168 \\
 \div 4 \\
 25\% = 42
 \end{array}
 \qquad
 \begin{array}{r}
 \frac{1}{2} \text{ of } 44 = 22 \\
 22 + 20 = 42
 \end{array}$$

$$\begin{array}{r}
 42 \\
 \hline
 4 \overline{) 168}
 \end{array}$$

(ii) State one assumption that Ceri has made in making her claim.

..... *The sample is representative of all students*
 *at the school.* [1]

20 In the following equation, n is an integer greater than 1.

$$(\sqrt{2})^n = k\sqrt{2}$$

(a) (i) Find k when $n = 7$.

$$\begin{aligned} (\sqrt{2})^7 &= \underbrace{\sqrt{2} \times \sqrt{2}} \times \underbrace{\sqrt{2} \times \sqrt{2}} \times \underbrace{\sqrt{2} \times \sqrt{2}} \times \sqrt{2} \\ &= 2 \times 2 \times 2 \times \sqrt{2} \\ &= 8\sqrt{2} \end{aligned} \quad \text{(a)(i) } k = \dots 8 \dots [2]$$

(ii) Find n when $k = 64$.

$$\begin{aligned} (\sqrt{2})^n &= 64\sqrt{2} \\ &= 2^6 \sqrt{2} \\ &= (\sqrt{2})^{12} \times \sqrt{2} \\ &= (\sqrt{2})^{13} \end{aligned} \quad \text{(ii) } n = \dots 13 \dots [2]$$

(b) Show that $\frac{14}{3-\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$. [5]

Conjugate = $3 + \sqrt{2}$

$$\frac{14}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$\frac{14(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$\frac{42 + 14\sqrt{2}}{9 + 3\sqrt{2} - 3\sqrt{2} - 2}$$

$$\frac{42 + 14\sqrt{2}}{7}$$

$$= 6 + 2\sqrt{2}$$

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

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