OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Tuesday 12 June 2018 – Morning

Time allowed: 1 hours 30 minutes

7048300902

You may use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

J560006

First name									
Last name									
Centre number						Candidate number				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Ping chooses four numbers.

most common → mode

highest − lowest → range

The mode of these four numbers is 8, the range is 7 and the mean is 11.

Find Ping's four numbers.

8 8 13 15

7 = highest − 8

+8 +8

15 = highest

mean

$\frac{8 + 8 + ? + 15}{4} = 11$

$\frac{31 + ?}{4} = 11$

×4 ×4

31 + ? = 44

−31 −31

? = 13

8 , 8 , 13 , 15 [3]

2 A box contains only red, blue and green pens.
The ratio of red pens to blue pens is 5 : 9.
The ratio of blue pens to green pens is 1 : 4.

Calculate the percentage of pens that are blue.

R : B B : G

5 : 9 1 : 4

×9 ×9

5 : 9 9 : 36

R : B : G

5 : 9 : 36

$\text{Blue} = \frac{9}{5+9+36}$

$= \frac{9}{50} = \frac{18}{100} = 18\%$

18 % [4]

3 Asha worked out $\dfrac{326.8 \times (6.94 - 3.4)}{59.4}$.

She got an answer of 19.5, correct to 3 significant figures.

Write each number correct to 1 significant figure to decide if Asha's answer is reasonable.

$326.8 \approx 300$

$6.94 \approx 7$

$3.4 \approx 3$

$59.4 \approx 60$

$\dfrac{300 \times (7-3)}{60}$

$\dfrac{300 \times 4}{60} = 20$

Yes. It is close to 20.

.....

..... **[3]**

4 **(a)** Show that $a^5 \times (a^3)^2$ can be expressed as a^{11}. **[2]**

$a^{3\times 2} = a^6$

$a^5 \times a^6 = a^{5+6}$

$= a^{11}$

(b) Write $\dfrac{1}{125} \times 25^9$ as a power of 5.

$125 = 5^3$

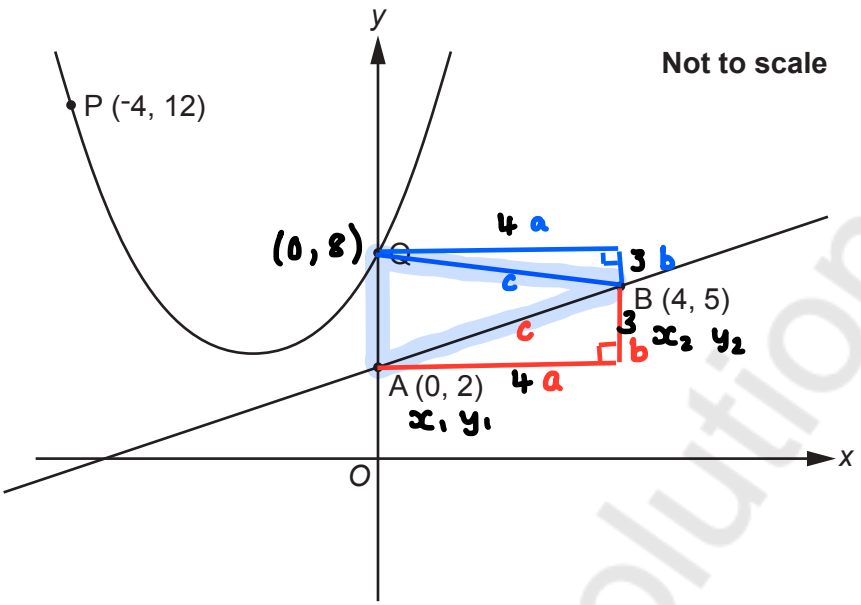
$25 = 5^2$

$\dfrac{1}{5^3} \times (5^2)^9$

$5^{-3} \times 5^{18} = 5^{-3+18}$

(b) 5^{15} ..... **[3]**

5 The diagram shows a straight line that passes through points A and B, and a curve that passes through points P and Q.



(a) Find the equation of the straight line.

$y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5 - 2}{4 - 0}$

$= \frac{3}{4}$

$c = (0, 2)$

$= 2$

(a) $y = \frac{3}{4}x + 2$ [3]

(b) The equation of the curve is $y = x^2 + kx + 8$.

Find the value of k.

$(-4, 12)$

$x \quad y$

$12 = (-4)^2 + k(-4) + 8$

$12 = 16 - 4k + 8$

$12 = 24 - 4k$

$-24 \quad -24$

$-12 = -4k$

$\div -4 \qquad \div -4$

$3 = k$

(b) $k =$ 3 [3]

(c) Diann draws line BQ.
She says

Triangle ABQ is isosceles.

Is Diann correct?
You must show all your working.

$a^2 + b^2 = c^2$

$4^2 + 3^2 = AB^2$

$\sqrt{\quad}\qquad\sqrt{\quad}$

$\sqrt{4^2+3^2} = AB$

$5 = AB$

$a^2 + b^2 = c^2$

$4^2 + 3^2 = BQ^2$

$\sqrt{\quad}\qquad\sqrt{\quad}$

$\sqrt{4^2+3^2} = BQ$

$5 = BQ$

Yes. AB = BQ = 5 ..... **[4]**

6 y is inversely proportional to x.
$y = 0.04$ when $x = 80$.

Find the value of y when $x = 32$.

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$

$0.04 = \frac{k}{80}$

$\times 80 \qquad \times 80$

$3.2 = k$

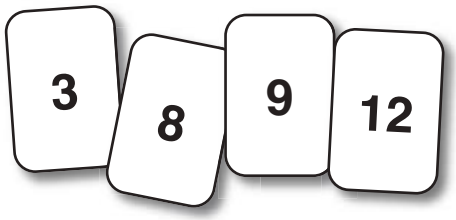
$y = \frac{3.2}{x}$

$y = \frac{3.2}{32}$

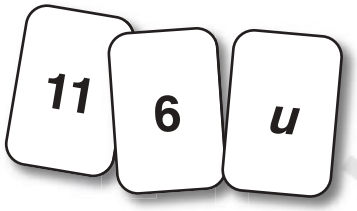
$= \frac{1}{10}$ or 0.1

$y =$ 0.1 ..... **[3]**

7 Edsel has four number cards.



Sharon has three number cards.
u represents a number that Sharon knows.



Edsel and Sharon each pick one of their cards at random.
They calculate the **difference** between the numbers on their cards.
This is their sample space.

		Edsel			
		3	8	9	12
Sharon	6	3	2	3	6
	11	8	3	2	1
	u	11	6	r	t

Work out the values of r and t.

$u = 11 + 3$
$= 14$

$r = 14 - 9$
$= 5$

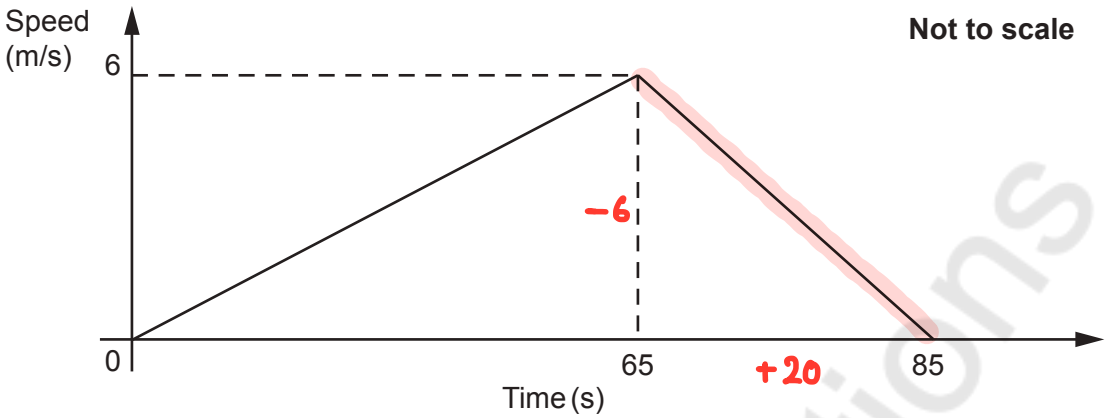
$t = 14 - 12$
$= 2$

$r =$ 5

$t =$ 2

[4]

8 The graph shows the speed of a tram as it travels from the library to the town hall.



(a) Calculate the deceleration of the tram as it approaches the town hall.

$\frac{-6}{20} = \frac{-3}{10} = -0.3$

(a) ..... 0.3 ..... m/s^2 **[2]**

(b) Calculate the distance travelled by the tram between the library and the town hall.

↘ Area under graph

$A = \frac{1}{2}bh \qquad b = 85 \qquad h = 6$

$A = \frac{1}{2} \times 85 \times 6$

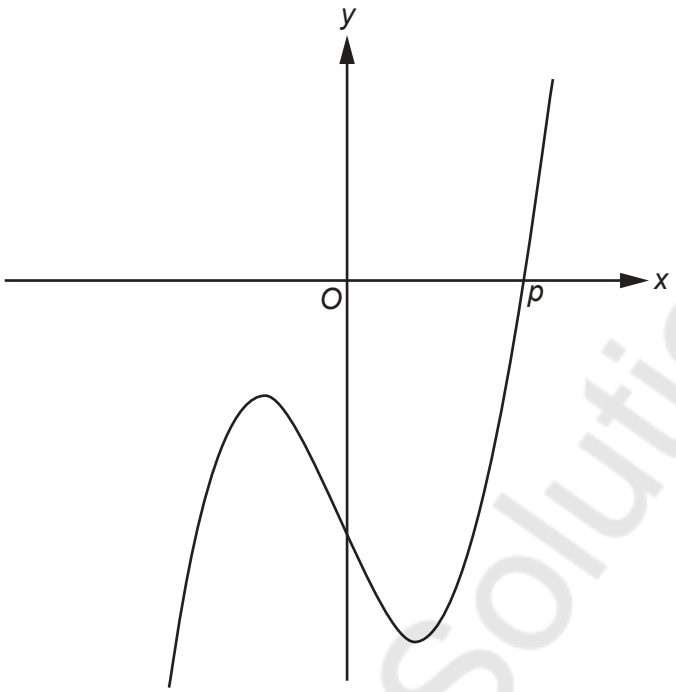
(b) ..... 255 ..... m **[3]**

(c) What was the maximum speed of the tram as it travelled between the library and the town hall?
Give your answer in **kilometres per hour**.

$\frac{6\text{m}}{1\text{s}} \overset{\times 60}{\underset{\times 60}{=}} \frac{360\text{m}}{1\text{min}} \overset{\times 60}{\underset{\times 60}{=}} \frac{21600\text{m}}{1\text{ hour}} \overset{\div 1000}{=} \frac{21.6\text{ km}}{1\text{ hour}}$

(c) ..... 21.6 ..... km/h **[4]**

9 The graph of $y = x^3 - 7x - 12$ is shown below.
The root of the equation $x^3 - 7x - 12 = 0$ is p.



(a) Calculate y when $x = 3$.

$y = (3)^3 - 7(3) - 12$
$= -6$

(a) $y =$ -6 **[1]**

(b) Show that $3 < p < 4$. **[2]**

When $x = 3$ $y = -6$

When $x = 4$ $y = (4)^3 - 7(4) - 12$
$= 24$

Change of sign, so p lies between 3 and 4.

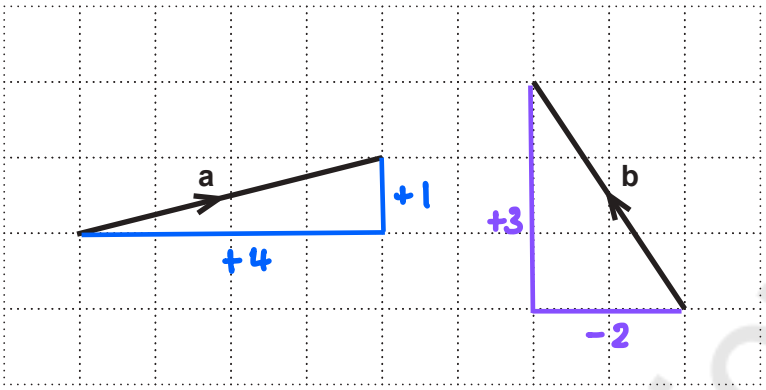
(c) Find a smaller interval that contains the value of p.
You must show calculations to support your answer.

$x = 3.5$ $y = (3.5)^3 - 7(3.5) - 12$
$= 6.4$

$x = 3.1$ $y = (3.1)^3 - 7(3.1) - 12$
$= -3.9$

(c) $3.1 < p < 3.5$ **[3]**

10 Two vectors, **a** and **b**, are shown on the 1 centimetre grid below.



Show that the vector **a** + 2**b** has length 7 cm.
You may use the grid below.

$a = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

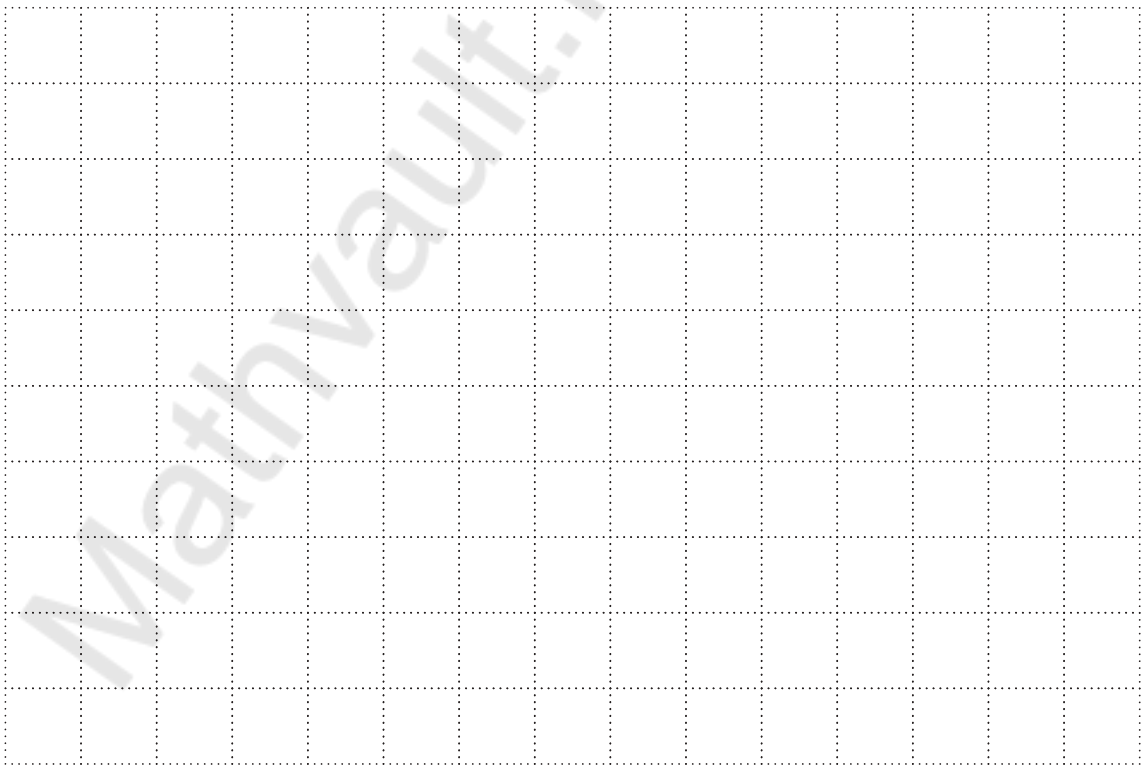
$2b = 2 \times \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

$a + 2b$

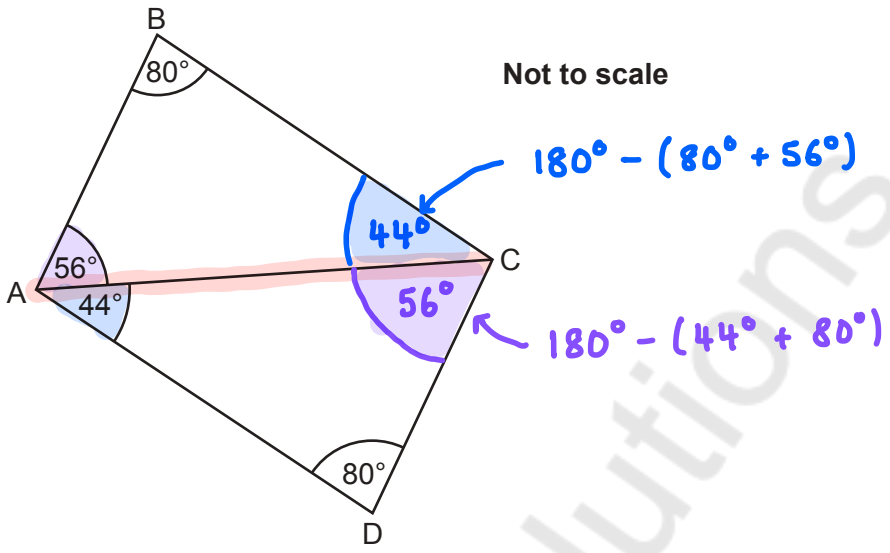
$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

$\therefore$ 7cm



[3]

11 The diagram below shows two triangles.



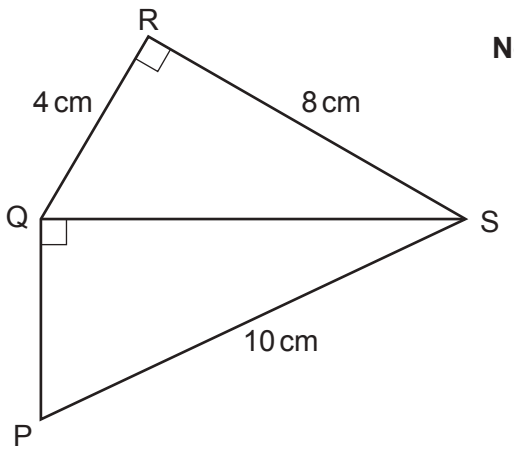
Prove that triangle ABC is congruent to triangle ACD.

- AC is common
- Angle DAC = Angle BCA = 44°
- Angle BAC = Angle ACD = 56°

Congruent – ASA

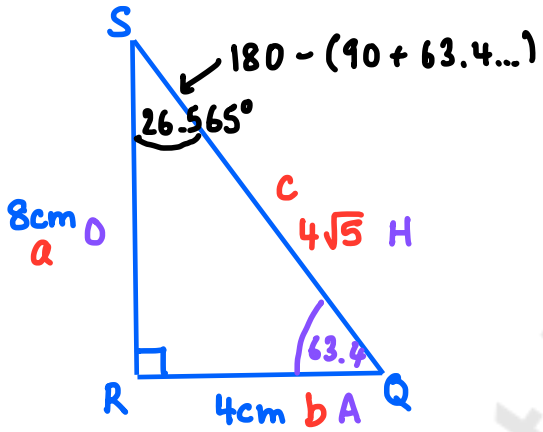
[4]

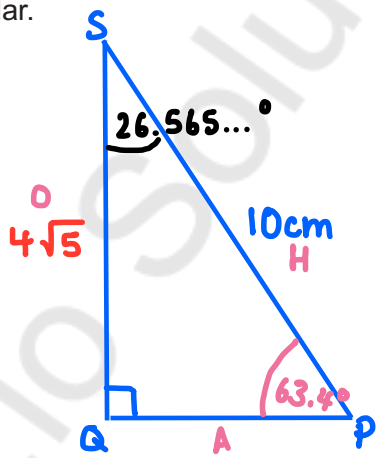
12 The diagram below shows two right-angled triangles.



Not to scale

Prove that triangles PQS and QRS are similar.





$a^2 + b^2 = c^2$

$8^2 + 4^2 = SQ^2$

$\sqrt{\quad}$ $\sqrt{\quad}$

$\sqrt{8^2+4^2} = SQ$

$4\sqrt{5} = SQ$

SOH CAH TOA

$\tan Q = \frac{8}{4}$

$Q = \tan^{-1}(2)$

$= 63.434...°$

$\sin P = \frac{4\sqrt{5}}{10}$

$P = \sin^{-1}\left(\frac{4\sqrt{5}}{10}\right)$

$= 63.434...°$

Angle SRQ = angle SQP = 90°

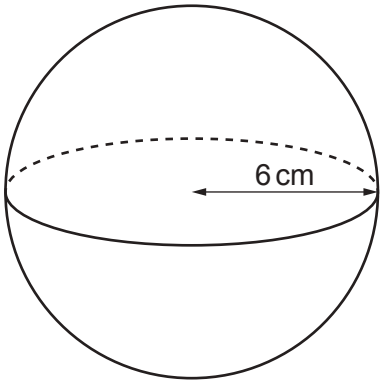
Angle SQR = angle SPQ = 63.4°

Angle RSQ = angle QSP = 26.6°

∴ triangles are similar

[5]

13 (a) Calculate the volume of a sphere with radius 6 cm.



[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

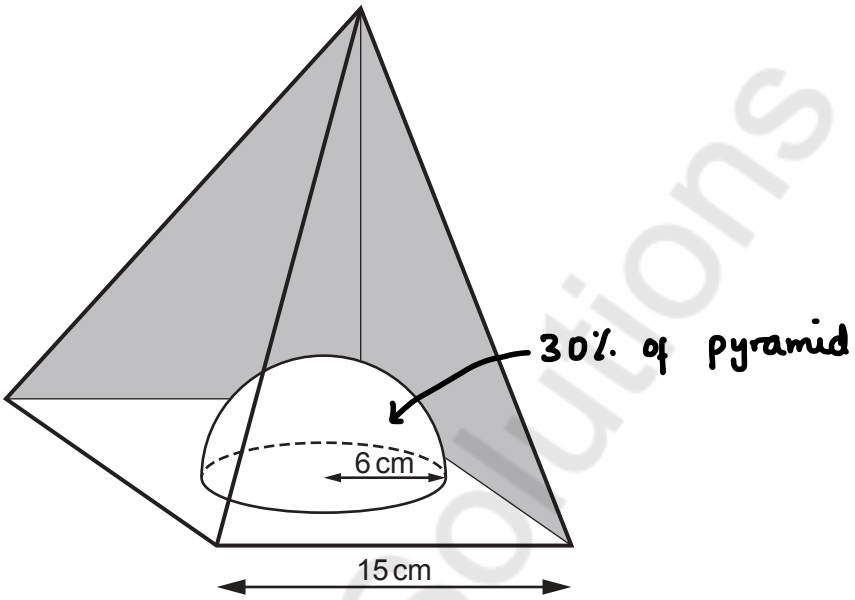
$V = \frac{4}{3}\pi(6)^3$

$= 288\pi$

$= 904.8 \text{ cm}^3$

(a) .....904.8..... cm^3 **[2]**

(b) An ornament is made from a solid glass square-based pyramid.
The base has side length 15 cm.
A hemisphere with radius 6 cm is cut out of the base of the pyramid.
This reduces the volume of glass contained in the ornament by 30%.



Calculate the perpendicular height of the pyramid.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.

A hemisphere is half a sphere.]

↳ square (15 × 15)

Volume of hemisphere = $\frac{288\pi}{2}$ = 144π cm^3

144π = 30% of volume of pyramid

30% ÷ 100 = 0.3

$144\pi = 0.3 \times \frac{1}{3} \times 15^2 \times h$

$144\pi = \frac{45}{2}h$

× 2 × 2

$288\pi = 45h$

÷ 45 ÷ 45

$\frac{32}{5}\pi = h$

(b) ..... 20.1 ..... cm **[5]**

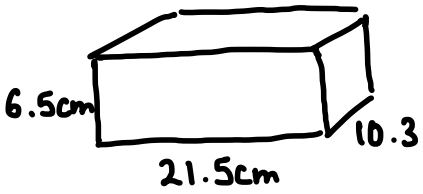
14 (a) Standard bricks have dimensions 21.5 cm by 10.3 cm by 6.5 cm, correct to 1 decimal place.

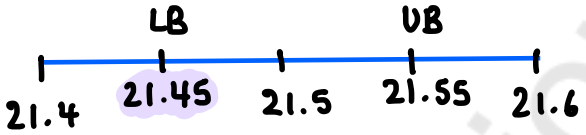
A house is built using 4663 standard bricks.

Joslin says

Placed end to end, the bricks from the house would definitely reach over 1 km.

Show that Joslin's statement is correct. **[4]**





$21.45 \times 4663 = 100021.35$ cm

$\div 100$

$= 1000.2135$ m

$\div 1000$

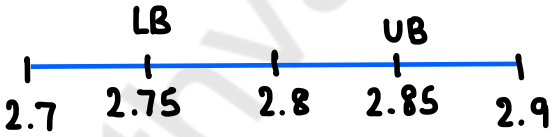
$= 1.0002135$ km > 1km

(b) A standard brick should weigh 2.8 kg, correct to 1 decimal place.
A truck can carry a maximum load of 20 tonnes.

(i) Calculate the maximum number of standard bricks that the truck should be able to carry.

tonnes $\xrightarrow{\times 1000}$ kg

$20 \times 1000 = 20{,}000$ kg



$$\text{Max bricks} = \frac{20{,}000}{2.85}$$

$= 7017.5...$

≈ 7017

(b)(i) 7017 ..... **[3]**

(ii) Explain why your answer to **(b)(i)** may not be possible to achieve.

The truck may not have enough room.

..... **[1]**

15 Ratna invests £1200 for 2 years in a bank account paying r % per year compound interest.
At the end of 2 years, the amount in the bank account is £1379.02.

Calculate r.

Compound Interest:

Final amount = investment × multiplier^{n} ← no. of years

$$1379.02 = 1200 \times \left(\frac{100 + r}{100}\right)^2$$

$\div 1200$ $\quad$ $\div 1200$

$$1.149183333 = \left(\frac{100 + r}{100}\right)^2$$

$\sqrt{}$ $\quad$ $\sqrt{}$

$$1.071999689 = \frac{100 + r}{100}$$

$\times 100$ $\quad$ $\times 100$

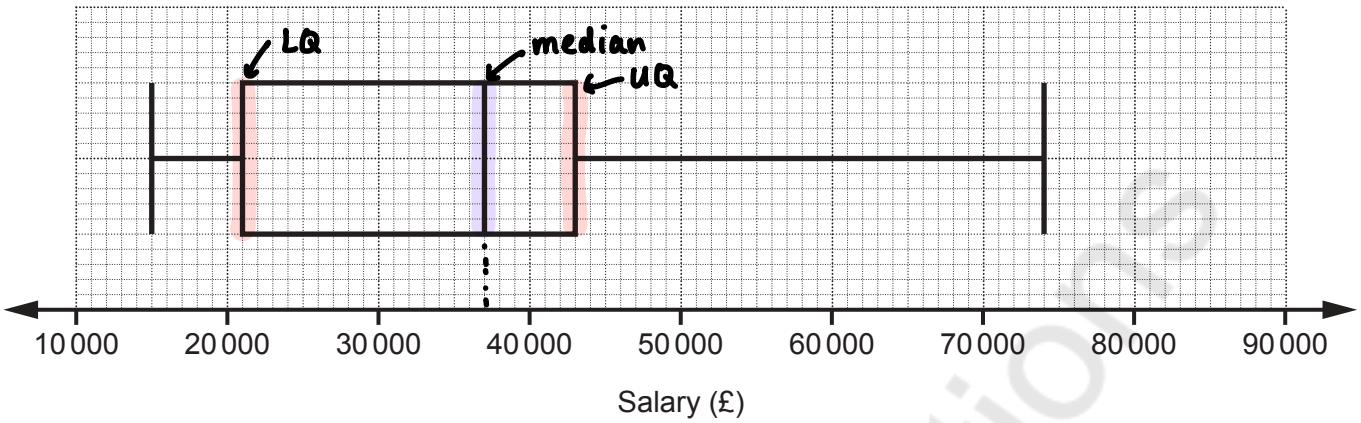
$$107.1999689 = 100 + r$$

-100 $\quad$ -100

$7.1999689 = r$

$r =$ 7.20 **[4]**

16 The box plot shows the distribution of the salaries for the workers at Bexbridge Biscuits.



(a) State the median salary.

(a) £ 37,000 **[1]**

(b) Find the interquartile range.

IQR = UQ - LQ

= 43,000 - 21,000

= 22,000

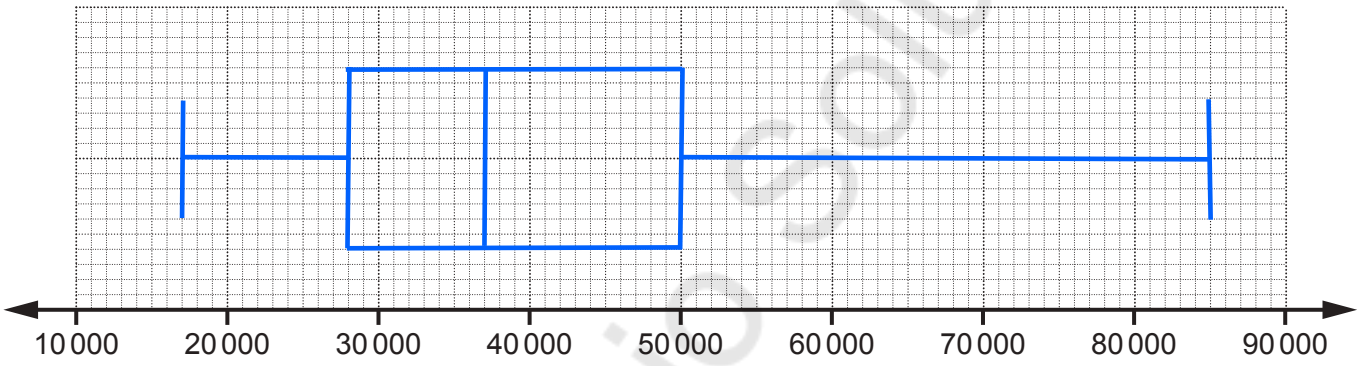
(b) £ 22,000 **[2]**

(c) The following salary information is true for workers at Camford Cookies.

- The highest paid worker earns £85 000. ← greatest
- The lowest paid worker earns 20% of the salary of the highest paid worker. ← 0.2 x 85000 = 17,000
- 25% of the workers earn more than £50 000. ← UQ
- 25% of the workers earn less than £28 000. ← LQ
- The median salary is £37 000.

Draw a box plot to show the salaries of the workers at Camford Cookies.

IQR = 50,000 − 28,000
= 22,000



[3]

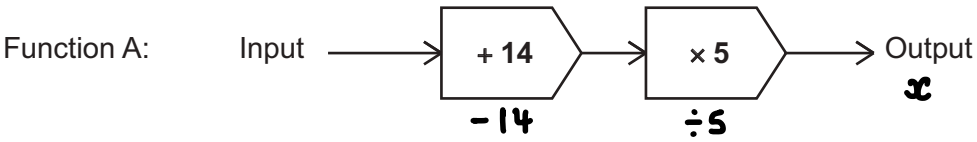
(d) Make two different comparisons between the distribution of the salaries at Bexbridge Biscuits and the salaries at Camford Cookies.

1: The interquartile range is the same for both

2: The median salaries are the same.

[2]

17 Here is a function.



(a) The **output** of function A is x.

Write an algebraic expression, in terms of x, for the input of function A.

$x \div 5 = \frac{x}{5}$

$\frac{x}{5} - 14$ (input)

(a) $\frac{x}{5} - 14$ **[2]**

(b) A number, k, is put into function A.
The output is also k.

Find the value of k.

$5(k + 14) = k$

$5k + 70 = k$

$-k \qquad -k$

$4k + 70 = 0$

$-70 \quad -70$

$4k = -70$

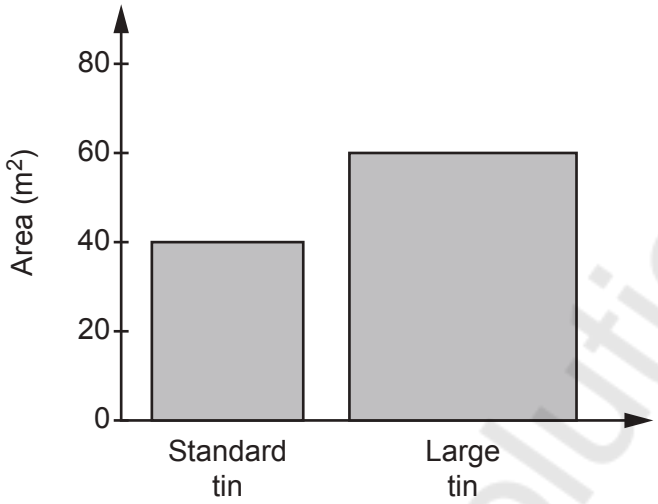
$\div 4 \qquad \div 4$

$k = -17.5$

(b) $k =$ -17.5 **[3]**

18 Percy sells paint in standard tins and large tins.
The standard tin covers 40 m^2 and the large tin covers 60 m^2.

(a) Percy publishes this chart showing the area that can be covered with each tin of paint.



Explain why the chart is misleading.

Bars are of different widths.

[1]

(b) The standard tin and the large tin are mathematically similar.
The **volume** of the large tin is 50% more than the volume of the standard tin.
Both tins are cylinders.
The radius of the standard tin is 10 cm.

Calculate the radius of the large tin.

	Standard	Large	SF
L	10 cm	$10 \times \sqrt[3]{1.5}$ $= 11.447...$	$\sqrt[3]{1.5}$
A			
V	1	1.5	1.5

$\sqrt[3]{}$

(b) 11.4 cm **[4]**

19 Show that $\frac{2x^2+13x+20}{2x^2+x-10}$ simplifies to $\frac{x+a}{x-b}$ where a and b are integers. **[4]**

$2x^2 + 13x + 20$ $\quad 2 \times 20 = 40$

$5 \times 8 = 40$

$5 + 8 = 13$

$2x^2 + 5x + 8x + 20$

$x(2x+5) + 4(2x+5)$

$(2x+5)(x+4)$

$2x^2 + x - 10$ $\quad 2 \times -10 = -20$

$5 \times -4 = -20$

$5 + -4 = +1$

$2x^2 + 5x - 4x - 10$

$x(2x+5) - 2(2x+5)$

$(2x+5)(x-2)$

$$\frac{(2x+5)(x+4)}{(2x+5)(x-2)} = \frac{x+4}{x-2}$$

END OF QUESTION PAPER