

## GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Thursday 8 November 2018 – Morning

Time allowed: 1 hour 30 minutes



**You may use:**

- geometrical instruments
- tracing paper

**Do not use:**

- a calculator



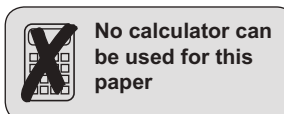
First name				
Last name				
Centre number				
Candidate number				

### INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

### INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document consists of **20** pages.



No calculator can be used for this paper

Answer all the questions.

1 Work out.

(a)  $\sqrt[3]{64} \times 2^{-1}$

$$x^{-a} = \frac{1}{x^a}$$

$$4 \times \frac{1}{2}$$

$$4 \times \frac{1}{2}$$

(a) ..... 2 ..... [2]

(b)  $4.3 \times 10^5 + 3.8 \times 10^4$

Give your answer in standard form.

$$\begin{array}{r} 3.8 \times 10^4 \\ \downarrow \div 10 \quad \downarrow +1 \\ 0.38 \times 10^5 \end{array}$$

$$4.3 \times 10^5 + 0.38 \times 10^5$$

$$\begin{array}{r} 4.30 \\ + 0.38 \\ \hline 4.68 \end{array}$$

(b) .....  $4.68 \times 10^5$  ..... [3]

2 By writing each number correct to 1 significant figure, find an estimate for this calculation.

$$\frac{606.3 \times 0.312}{19.93}$$

$$\downarrow \quad \downarrow$$

$$606.3 \approx 600$$

$$\downarrow \quad \downarrow$$

$$0.312 \approx 0.3$$

$$\uparrow \quad \uparrow$$

$$19.93 \approx 20$$

$$\frac{600 \times 0.3}{20}$$

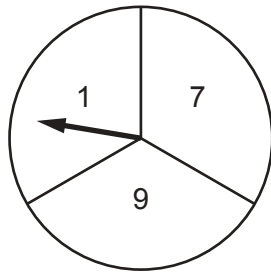
$$\frac{180}{20}$$

$$= 9$$

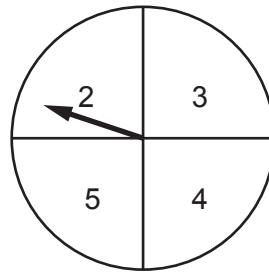
$$\begin{array}{l} 600 \times 0.3 = 180 \\ \uparrow \div 10 \quad \uparrow \div 10 \\ 600 \times 3 = 1800 \end{array}$$

..... 9 ..... [3]

- 3 Geoff has two fair spinners.



Spinner A



Spinner B

He spins both spinners and **multiplies** the numbers on each spinner.

- (a) Complete the table.

		Spinner A		
×		1	7	9
Spinner B	2	2	14	18
	3	3	21	27
	4	4	28	36
	5	5	35	45

[1]

- (b) Geoff wants to work out the probability that the outcome of the multiplication is an even number or a prime number. Here is his working.

The probability the outcome is an even number is  $\frac{6}{12}$ . ✓

The probability the outcome is a prime number is  $\frac{3}{12}$ . ✓

The probability the outcome is an even number or a prime number is  $\frac{6}{12} + \frac{3}{12} = \frac{9}{12}$ .

Geoff is wrong.

Explain his error and give the correct answer.

2 is even and prime, so the probability would be  $\frac{8}{12}$ .

[2]

- 4 A solid metal block has mass 500g and volume 125 cm<sup>3</sup>.

Work out the density of the block.  
Give the units of your answer.

$$D = \frac{m}{V}$$

$$D = \frac{m}{V}$$

$$= \frac{500 \text{ g}}{125 \text{ cm}^3}$$

$$= 4 \text{ g/cm}^3$$

$$4 \text{ g/cm}^3$$

[3]

- 5 The depth of water in a garden pond is 57.8 cm.  
The depth decreases by 0.3 cm per day.

- (a) Assume the depth continues to decrease at the same rate.

After how many days will the depth reach 54.2 cm?

$$\begin{array}{r} 57.8 \\ - 54.2 \\ \hline 03.6 \end{array}$$

$$\text{Days} = 3.6 \div 0.3 = 12$$

$$\equiv 36 \div 3 = 12$$

(a) ..... **12** ..... days [3]

- (b) If the depth of water decreases at a slower rate, what effect will this have on your answer to part (a)?

..... **Answer would be bigger** ..... [1]

- 6 Sally has 30 feet of ribbon.

She cuts strips each of length  $2\frac{2}{5}$  feet from the ribbon.

Sally says

I can cut 13 of these strips from this ribbon.

Is she correct?

Show how you decide.

$$30 \div 2\frac{2}{5}$$

$$\frac{30}{1} \div \frac{12}{5}$$

$$\begin{aligned} \frac{30}{1} \times \frac{5}{12} &= \frac{150}{12} \\ &= 12\frac{6}{12} \\ &= 12\frac{1}{2} \end{aligned}$$

..... No. She can cut 12.5 strips. [4]

- 7 Emily spent £2400 on holiday in 2017.  
This was 20% more than she spent on holiday in 2016.

Calculate the amount she spent on holiday in 2016.

$$2017 = 120\%$$

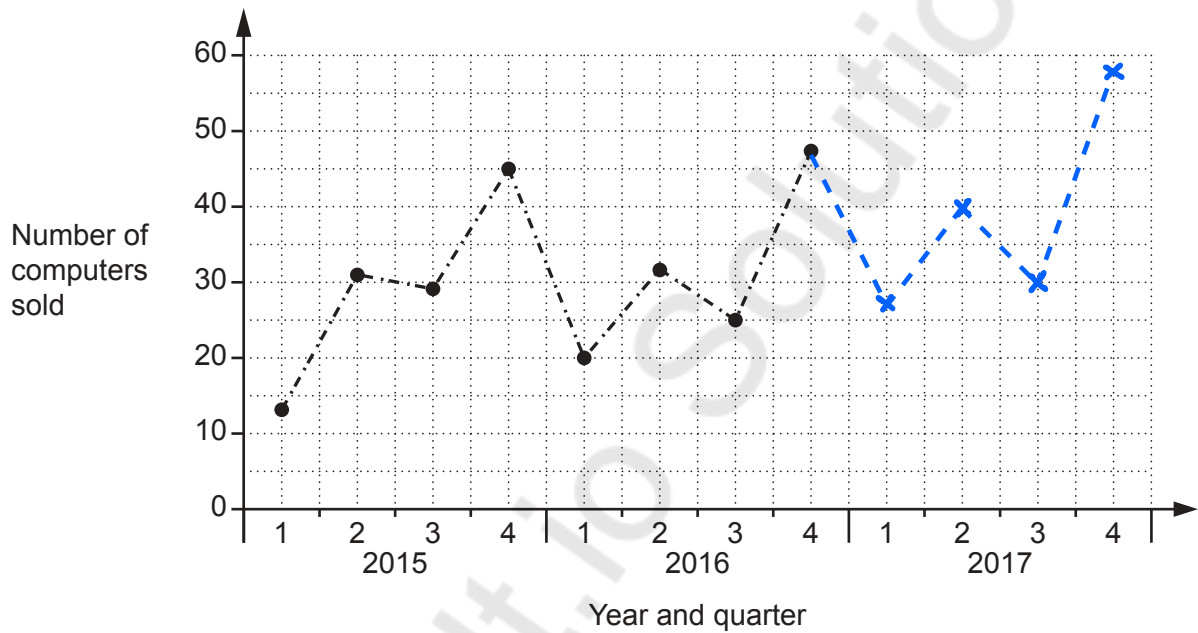
$$\begin{array}{r} 120\% = £2400 \\ \div 12 \qquad \qquad \qquad \div 12 \\ 10\% = £200 \\ \times 10 \qquad \qquad \qquad \times 10 \\ 100\% = £2000 \end{array}$$

£ ..... 2000 ..... [3]

- 8 The table shows the number of computers sold in Tom's shop each quarter from 2015 to 2017.

	2015				2016				2017			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
Number of computers sold	13	31	29	45	20	32	25	47	27	40	30	58

- (a) Complete this graph using the information for 2017.



[2]

- (b) Tom adds the three results for quarter 1 and he adds the three results for quarter 4.  
Tom says

The ratio of the **total** number of computers sold in quarter 1 compared to quarter 4 is 2 : 5.

Is he correct?  
Show your reasoning.

Q<sub>1</sub>

$$13 + 20 + 27 = 60$$

Q<sub>4</sub>

$$45 + 47 + 58 = 150$$

$$Q_1 : Q_4$$

$$60 : 150$$

$$\div 30 \quad \div 30$$

$$2 : 5$$

He is correct.

[2]

- (c) Make two comments about Tom's sales over the period 2015 to 2017.

Comment 1 ..... Number of computers sold is increasing. .....

.....

Comment 2 ..... Sales are weaker in Q1 compared to .....

..... Q4 ..... [2]

- (d) Tom predicts that he will sell more than 60 computers in the 4<sup>th</sup> quarter of 2018.

What assumption has he made?

..... The trend in his sales will continue. .....

..... [1]

- 9 Rearrange this formula to make  $y$  the subject.

$$x = y^2 + 7$$

$$-7 \quad -7$$

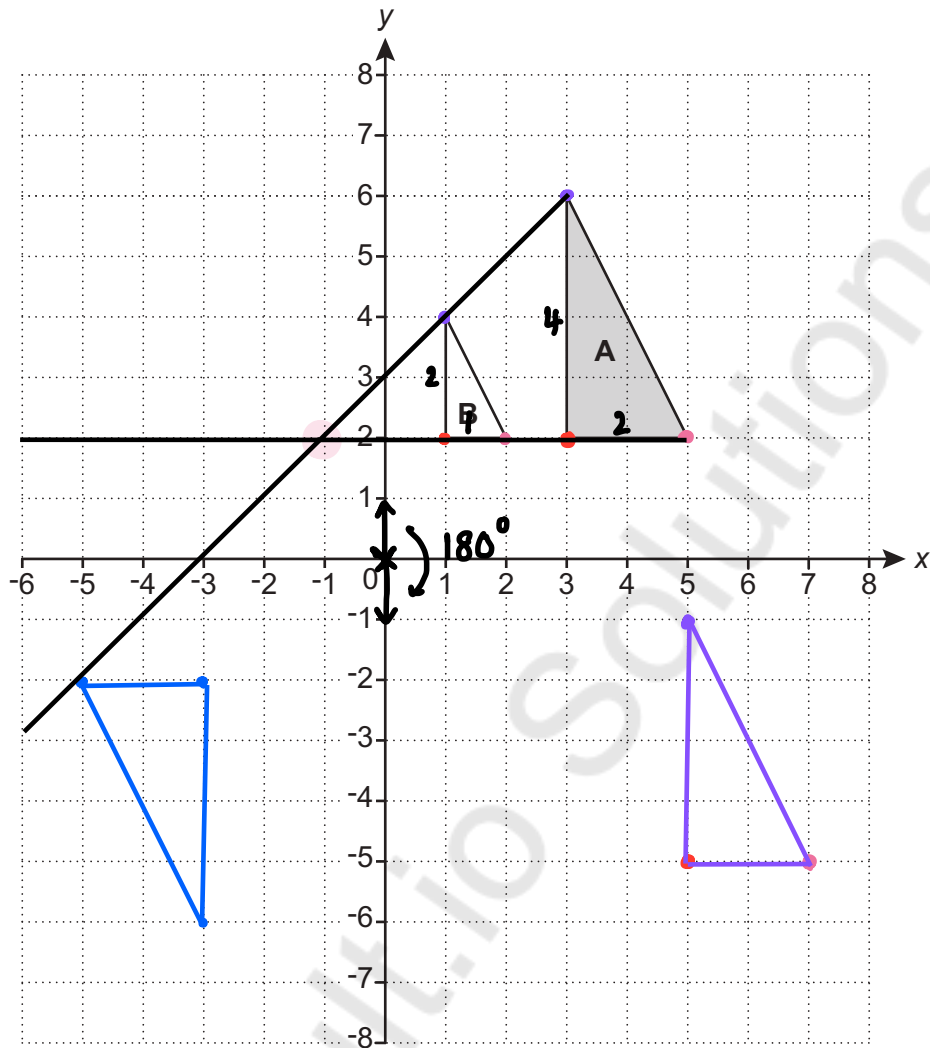
$$x - 7 = y^2$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$\sqrt{x - 7} = y$$

$$..... y = \sqrt{x - 7} ..... [2]$$

10 Triangle A and triangle B are drawn on the coordinate grid.



(a) (i) Draw the image of triangle A after a rotation of  $180^\circ$  about  $(0, 0)$ . [2]

(ii) Draw the image of triangle A after a translation by the vector  $\begin{pmatrix} 2 \\ -7 \end{pmatrix}$ . **2 right**  
**7 down** [2]

(b) Describe fully the **single** transformation that maps triangle A onto triangle B.

..... **Enlargement** ..... by ..... the ..... scale ..... factor .....  $\frac{1}{2}$  ..... centre .....  $(-1, 2)$ .....

..... [3]

- 11 The price of a washing machine is reduced by 20% for a sale. Afterwards, the sale price is increased by 30%.

Joachim says

The washing machine is now 10% more expensive than before the sale.

Explain Joachim's error and work out the correct percentage change in the price of the washing machine from before the sale to after the sale.

Joachim's error is ..... **he has subtracted the two percentages** .....

$$\begin{aligned} 100\% - 20\% &= 80\% \quad \left. \vphantom{100\% - 20\%} \right) \div 100 \\ &= 0.8 \end{aligned}$$

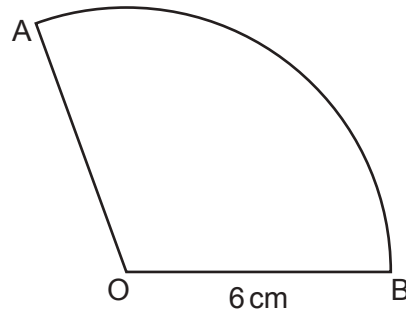
$$\begin{aligned} \text{Increase by } 30\% \\ 100\% + 30\% &= 130\% \quad \left. \vphantom{100\% + 30\%} \right) \div 100 \\ &= 1.3 \end{aligned}$$

$$\begin{aligned} 0.8 \times 1.3 &= 1.04 \quad \left. \vphantom{0.8 \times 1.3} \right) \div 100 \\ 8 \times 13 &= 104 \end{aligned}$$

$$1.04 = 4\% \text{ increase}$$

Correct percentage change is ..... **Increase of 4** ..... % [6]

- 12 AOB is a sector of a circle, centre O and radius 6 cm.  
The length of arc AB is  $5\pi$  cm.



Not to scale

Find the area of the sector.  
Give your answer in terms of  $\pi$ .

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$5\pi = \frac{\theta}{360} \times 2\pi(6)$$

$$5\cancel{\pi} = \frac{\theta}{360} \times 12\pi$$

$$\frac{5}{12} = \frac{\theta}{360}$$

$$\theta = 5 \times 30$$

$$= 150^\circ$$

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{150 \div 3}{360 \div 3} \times \pi(6)^2$$

$$= \frac{5}{12} \times 36\pi$$

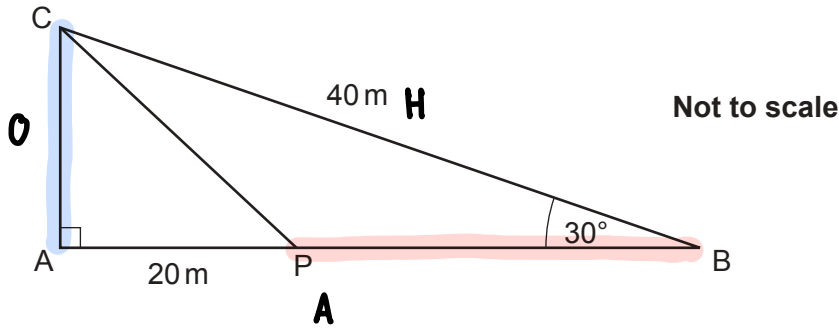
$$= \frac{180\pi}{12}$$

$$= 15\pi$$

$$12 \overline{) 180}$$

.....  $15\pi$  .....  $\text{cm}^2$  [5]

- 13 In the diagram, ABC is a right-angled triangle. P is a point on AB. BC = 40 m, AP = 20 m and angle ABC = 30°.



- (a) Show that AC = 20 m. [3]

S O H ✓  
C A H T O A  
↓  
 $\sin \theta = \frac{O}{H}$

$$\sin(30) = \frac{AC}{40}$$

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0
	2				

$$\times 40 \quad \frac{\sqrt{1}}{2} = \frac{AC}{40} \quad \times 40$$

$$\frac{1}{2} \times 40 = AC \quad \therefore AC = 20 \text{ m}$$

- (b) Find the length of PB. Give your answer in the form  $a(\sqrt{3} - b)$ , where  $a$  and  $b$  are integers.

C A H  
↓  
 $\cos \theta = \frac{A}{H}$

$$\cos(30) = \frac{AB}{40}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{40} \quad \times 40$$

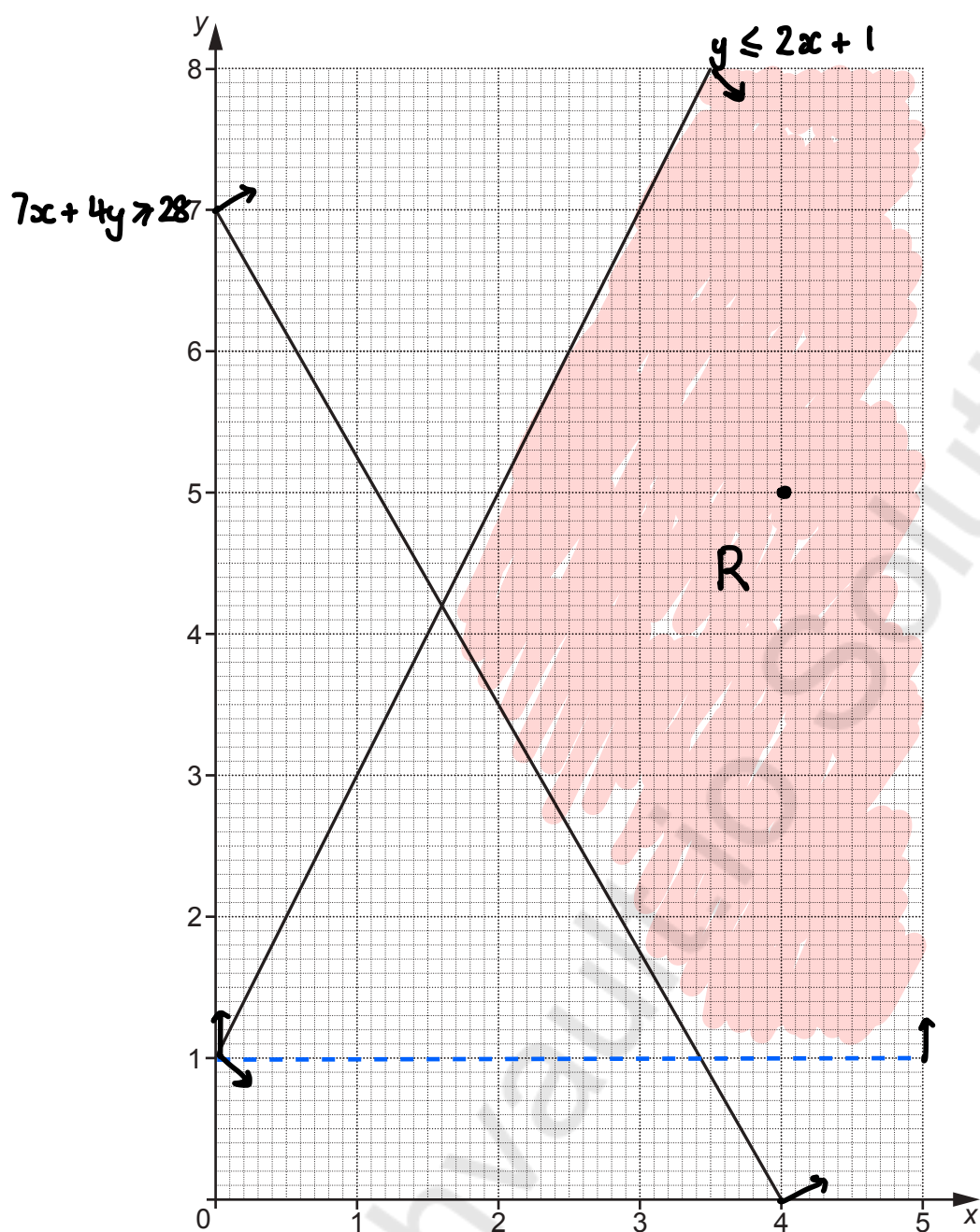
$$\begin{aligned} PB &= AB - AP \\ &= 20\sqrt{3} - 20 \\ &= 20(\sqrt{3} - 1) \end{aligned}$$

$$\frac{40\sqrt{3}}{2} = AB$$

$$20\sqrt{3} = AB$$

(b) .....  $20(\sqrt{3} - 1)$  ..... [5]

14 The diagram shows the lines  $y = 2x + 1$  and  $7x + 4y = 28$ .



$$5 \leq 2(4) + 1$$

$$5 \leq 9$$

$$7(4) + 4(5) \geq 28$$

$$28 + 20 \geq 28$$

$$48 \geq 28$$

The region R satisfies these inequalities.

$$y \leq 2x + 1$$

-ve gradient  
y-intercept

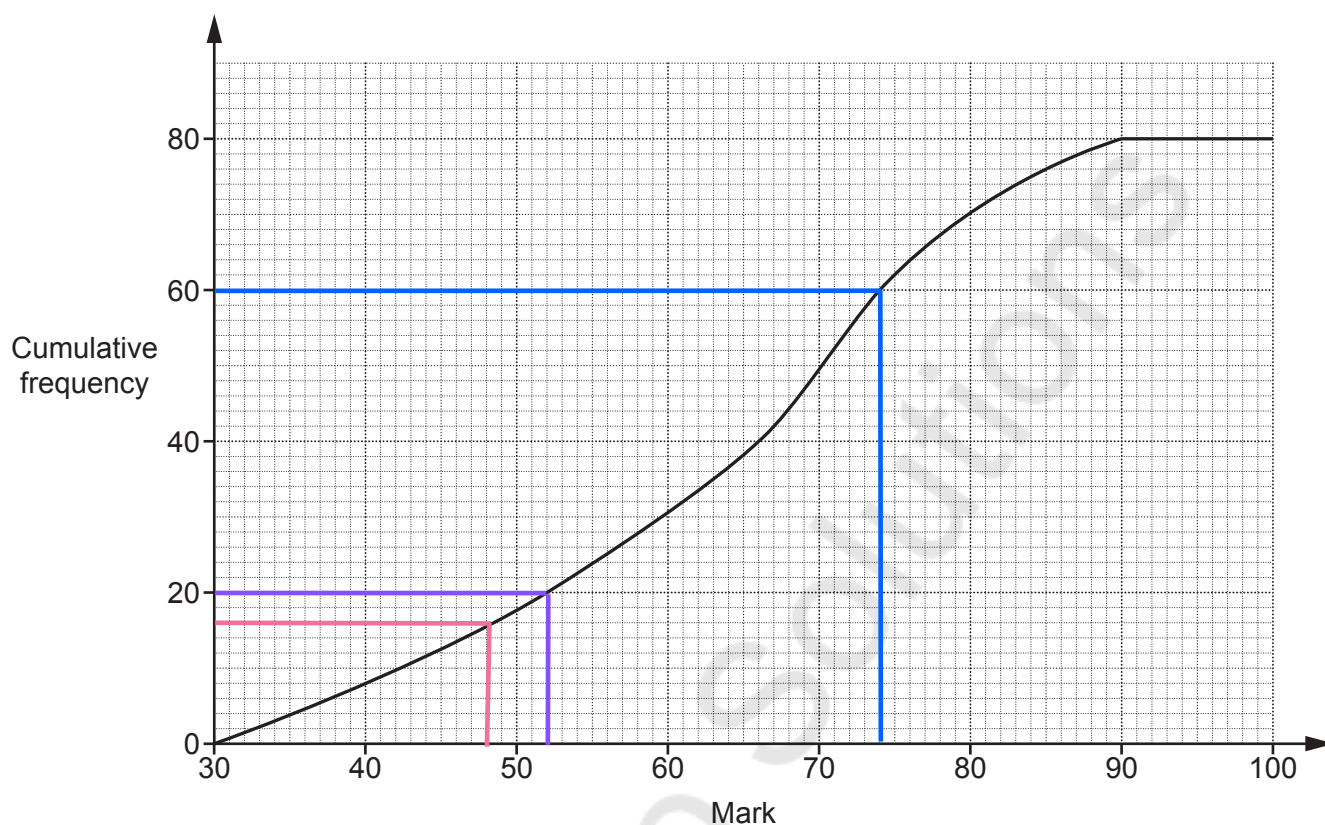
$$7x + 4y \geq 28$$

$$y > 1$$

y = 1  
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By drawing a third straight line, find and label the region R that satisfies these inequalities. [5]

- 15 The cumulative frequency graph shows information about the marks scored by a group of 80 students in a test.



- (a) Find the interquartile range.

$$IQR = UQ - LQ$$

$$\begin{aligned} UQ &= \frac{3}{4} \text{ of total} \\ &= \frac{3}{4} \times 80 \\ &= 60 \rightarrow 74 \text{ marks} \end{aligned}$$

$$\begin{aligned} LQ &= \frac{1}{4} \text{ of total} \\ &= \frac{1}{4} \times 80 = 20 \end{aligned}$$

$$\begin{aligned} 20 &\rightarrow 52 \text{ marks} \\ IQR &= 74 - 52 = 22 \end{aligned}$$

(a) ..... 22 [2]

- (b) The ratio of the number of students passing the test compared to failing the test is 4 : 1. Find the minimum mark needed to pass the test.

$$P : F$$

$$4 : 1 = 5 \text{ parts}$$

$$\begin{array}{ccc} \times 16 & \times 16 & 80 \div 5 = 16 \end{array}$$

$$64 : 16$$

(b) ..... 48 [3]

16 Here is a sequence.

$$5 \quad 5\sqrt{3} \quad 15 \quad 15\sqrt{3}$$

$\underbrace{\hspace{1.5cm}}_{\times \sqrt{3}} \quad \underbrace{\hspace{1.5cm}}_{\times \sqrt{3}} \quad \underbrace{\hspace{1.5cm}}_{\times \sqrt{3}}$

(a) Work out the next term.

$$15\sqrt{3} \times \sqrt{3}$$

$$15 \times 3 = 45$$

(a) ..... 45 ..... [1]

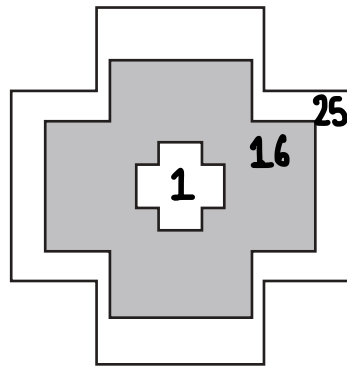
(b) Find the  $n$ th term.

$$a_n = ar^{(n-1)}$$

$$= 5\sqrt{3}^{(n-1)}$$

(b) .....  $5\sqrt{3}^{(n-1)}$  ..... [3]

- 17 The diagram consists of three mathematically similar shapes.  
The heights of the shapes are in the ratio 1 : 4 : 5.



Not to scale

Find the ratio

total shaded area : total unshaded area.

Give your answer in its simplest form.

$$\begin{array}{l} \text{Length} \quad 1 : 4 : 5 \\ \text{Area} \quad 1 : 16 : 25 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Length} \\ \text{Area} \end{array}} \right\} \square^2$$

$$\begin{aligned} \text{Shaded area} &= 16 - 1 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{Unshaded area} &= 25 - 15 \\ &= 10 \end{aligned}$$

Shaded: Unshaded

$$\begin{array}{ccc} 15 & : & 10 \\ \div 5 & & \div 5 \\ 3 & : & 2 \end{array}$$

total shaded area : total unshaded area ..... **3** ..... : ..... **2** ..... [4]

- 18 (a) (i) Write  $x^2 + 4x - 16$  in the form  $(x + a)^2 - b$ .

$$(x + 2)^2 - 20$$

$$\begin{array}{r} x^2 + 4x + 4 \\ - 20 \\ \hline x^2 + 4x - 16 \end{array}$$

(a)(i) .....  $(x + 2)^2 - 20$  ..... [3]

- (ii) Solve the equation  $x^2 + 4x - 16 = 0$ .  
Give your answers in surd form as simply as possible.

$$\begin{array}{r} (x + 2)^2 - 20 = 0 \\ + 20 \quad + 20 \\ \hline (x + 2)^2 = 20 \end{array}$$

$$\begin{array}{l} \sqrt{20} = \sqrt{4} \times \sqrt{5} \\ = 2\sqrt{5} \end{array}$$

$$\begin{array}{r} (x + 2)^2 = 20 \\ \sqrt{\quad} \quad \sqrt{\quad} \\ \hline x + 2 = \sqrt{20} \end{array}$$

$$\begin{array}{r} x + 2 = \sqrt{20} \\ - 2 \quad - 2 \\ \hline x = -2 \pm \sqrt{20} \end{array}$$

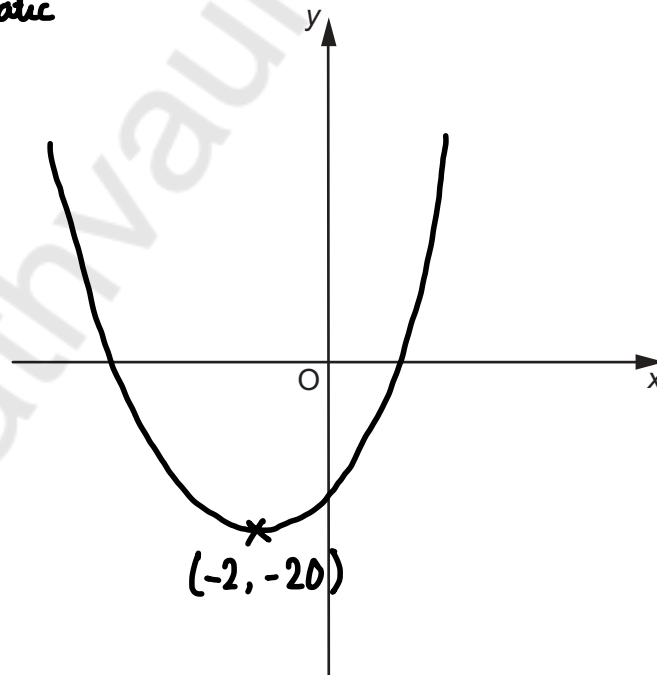
$$x = -2 \pm \sqrt{20}$$

$$= -2 \pm 2\sqrt{5}$$

(ii)  $x = -2 + 2\sqrt{5}$  or  $x = -2 - 2\sqrt{5}$  [4]

- (b) Sketch the graph of  $y = x^2 + 4x - 16$ , showing clearly the coordinates of any turning points.

Quadratic  $\rightarrow$



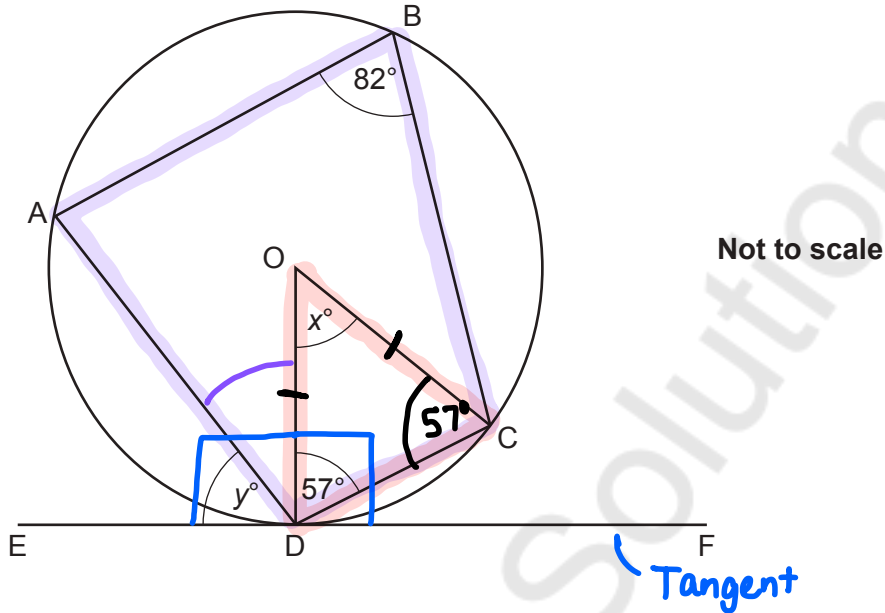
$$(x + 2)^2 - 20$$

$$\begin{array}{l} \text{Turning point} \\ = (-2, -20) \end{array}$$

[3]

- 19 The diagram shows a circle, centre O.

Points A, B, C and D lie on the circumference of the circle.  
EDF is a tangent to the circle.  
Angle ABC =  $82^\circ$  and angle ODC =  $57^\circ$ .



- (a) Work out the value of  $x$ .

$$\begin{aligned} x &= 180^\circ - 57 - 57 \\ &= 180^\circ - 114^\circ \\ &= 66^\circ \end{aligned}$$

(a)  $x = \dots 66 \dots$  [2]

- (b) Work out the value of  $y$ .

$$\begin{aligned} \angle ADO &= 180^\circ - 82^\circ - 57^\circ \\ &= 98^\circ - 57^\circ \\ &= 41^\circ \end{aligned}$$

$$\begin{aligned} y &= 90 - 41 \\ &= 49 \end{aligned}$$

(b)  $y = \dots 49 \dots$  [3]

- 20 (a) Prove that  $(2x+1)(3x+2) + x(3x+5) + 2$  is a perfect square.

$$(2x+1)(3x+2)$$

$$6x^2 + 4x + 3x + 2$$

$$6x^2 + 7x + 2$$

$$x(3x+5)$$

$$3x^2 + 5x$$

$$6x^2 + 7x + 2 + 3x^2 + 5x + 2$$

$$9x^2 + 12x + 4$$

$$9x^2 + 6x + 6x + 4$$

$$3x(3x+2) + 2(3x+2)$$

$$(3x+2)(3x+2)$$

$$= (3x+2)^2 \therefore \text{perfect square.}$$

$$9 \times 4 = 36$$

$$\frac{6}{6} \times \frac{6}{6} = 36$$

$$\frac{6}{6} + \frac{6}{6} = 12$$

$$\begin{array}{r} 36 \\ 1 \ 36 \\ 2 \ 18 \\ 3 \ 12 \\ 4 \ 9 \\ 6 \end{array}$$

.....  
 .....  
 .....  
 .....  
 ..... [6]

- (b) Gemma says

$$(3x+2)^2 = -12$$

The equation  $(2x+1)(3x+2) + x(3x+5) + 2 = -12$  has no solutions.

Explain Gemma's reasoning.

..... Cannot square root a negative value. ....

..... [1]

END OF QUESTION PAPER

**ADDITIONAL ANSWER SPACE**

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

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