OCR
Oxford Cambridge and RSA

H

Tuesday 21 May 2019 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You may use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

7736213689

J560/04

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **16** pages.

Answer **all** the questions.

1 Calculate.

$$\sqrt[3]{\frac{210}{10^2+5^2}}$$

Give your answer correct to 3 significant figures.

1.18784391

1.19

..... **[3]**

2 The ratio 50 grams to 1 kilogram can be written in the form 1 : *n*.

Find the value of *n*.

1 kg = 1000 g (×1000)

50 g : 1 kg

50 g : 1000 g

÷ 50 ÷ 50

1 : 20

n = 20 ..... **[2]**

3 **(a)** Anne, Barry and Colin share a prize in the ratio 3 : 4 : 5.
Colin gives $\frac{1}{3}$ of his share to a charity.

What fraction of the whole prize does Colin give to the charity?

A : B : C

3 : 4 : 5

$\text{Colin} = \frac{5}{3+4+5}$

$= \frac{5}{12}$

$\frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$

(a) $\frac{5}{36}$ ..... **[3]**

(b) Delia, Edwin and Freya share some money in the ratio 5 : 7 : 8.
Freya's share is £1600.

How much money did they share?

D : E : F

5 : 7 : 8 (× 200)

£1600

Total parts = 5 + 7 + 8 = 20

20 × 200 = 4000

(b) £ 4000 ..... **[2]**

4 A bus timetable shows the following information.

- A bus following route T leaves for the train station every 20 minutes.
- A bus following route A leaves for the airport every 18 minutes.
- A bus following route T and a bus following route A both leave at 8.37 am.

(a) When is the next time one of each bus is timetabled to leave at the same time?

T 20 40 60 80 100 120 140 160 180

A 18 36 54 72 90 108 126 144 162 180

LCM = 180 minutes $\xrightarrow{\div 60}$ 3 hours

8.37 am + 3 hours = 11:37 am (a) ..... 11:37 am ..... **[4]**

(b) Write down one assumption that was necessary to solve this problem.

..... There are no delays or cancellations .....

..... **[1]**

5 Bennie is 7 years older than Ayesha.
Chloe is twice as old as Bennie.
The sum of their three ages is 57.

Work out the ages of Ayesha, Bennie and Chloe.

Ayesha = x 9

Bennie = $x + 7$ 9 + 7 = 16

Chloe = $2(x + 7)$
= $2x + 14$ 2(9) + 14
18 + 14 = 32

$x + x + 7 + 2x + 14 = 57$

$4x + 21 = 57$
$-21 \quad -21$

$4x = 36$
$\div 4 \quad \div 4$

$x = 9$

Ayesha's age is ..... 9 .....

Bennie's age is ..... 16 .....

Chloe's age is ..... 32 ..... **[6]**

6 120 students in Year 10 and Year 11 sit a test.

- 61 of the students are in Year 10.
- 83 of the students are right-handed.
- 20 of the students in Year 11 are left-handed.

One of the students in Year 10 and one of the students in Year 11 are chosen at random.

Which one is more likely to be left-handed?
Show your working. You may use the table if you wish.

	Y10	Y11	Total
RH	44	39	83
LH	17	20	37
Total	61	59	120

Y10 - LH

$= \frac{17}{61}$

$17 \div 61 = 0.2786...$

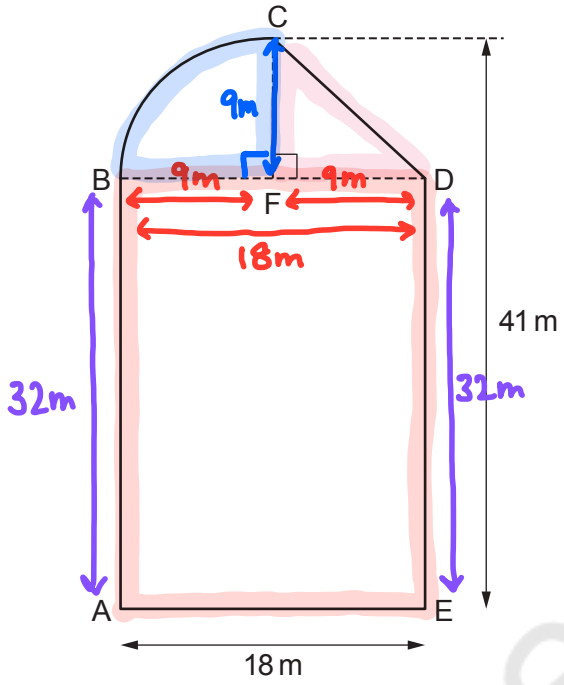
Y11 - LH

$= \frac{20}{59}$

$20 \div 59 = 0.33898...$

Year 11 as $0.34 > 0.28$ ..... **[6]**

7 The diagram shows a shape ABCDE.
The shape is made from a rectangle, a right-angled triangle and a quarter of a circle.



Not to scale

41m − 9m = 32m

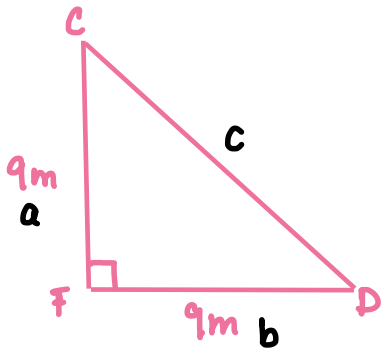
F is the mid-point of BD.
AE = 18 m and the perpendicular distance from C to AE is 41 m.

Work out the **perimeter** of the shape ABCDE.

Arc length $= \frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \pi \times 9$

$BC = \frac{9}{2}\pi$ m



$a^2 + b^2 = c^2$

$9^2 + 9^2 = CD^2$

$\sqrt{\quad} \qquad \sqrt{\quad}$

$\sqrt{9^2+9^2} = CD$

$9\sqrt{2} = CD$

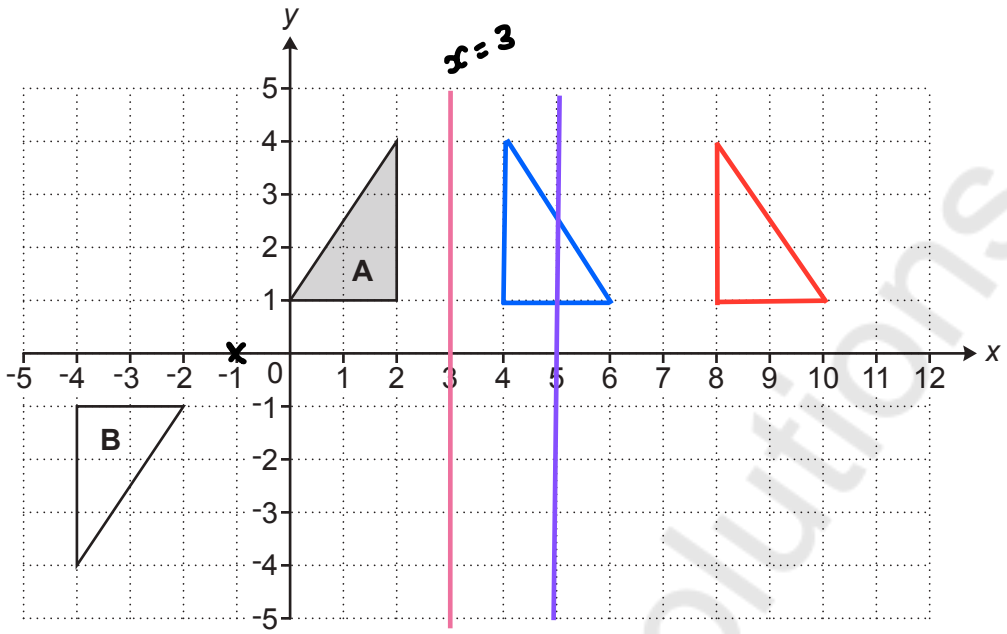
Perimeter ABCDE

$18 + 32 + 32 + \frac{9}{2}\pi + 9\sqrt{2}$

$= 108.865089 \approx 109$

109 m **[6]**

8 Triangle **A** and triangle **B** are drawn on the coordinate grid.



(a) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Rotation 180° centre (-1, 0)

.....

.....

[3]

(b) Describe fully the **single** transformation that is equivalent to:

- a reflection in the line $x = 3$, followed by
- a translation by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$. 4 right

You may use the grid above to help you.

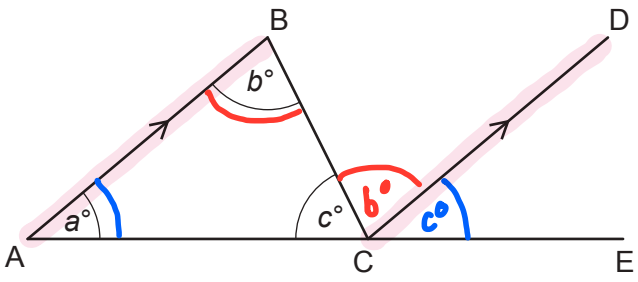
Reflection in the line $x = 5$

.....

.....

[3]

9 The diagram shows triangle ABC.
CD is parallel to AB.
A, C and E lie in a straight line.
Angles of size $a°$, $b°$ and $c°$ are shown.



Not to scale

(a) Insert $a°$, $b°$ or $c°$ to make this statement true.
Give a reason for your answer.

Angle DCE = ...a°... because ...corresponding angles are equal.....

..... **[2]**

(b) Use the diagram and the answer to part **(a)** to show that the angles of a triangle add up to 180°.
Give a reason for each statement you make. **[3]**

Angle BCD = b° because alternate angles are equal.

Angles on a straight line add to 180°

10 Claudia invests £25 000 at a rate of 2% per year compound interest.

Calculate the total amount of **interest** she will have earned after 5 years.
Give your answer correct to the nearest penny.

Final amount = investment × multiplier^n

Multiplier = 100% + 2%.
= 102%.
$\div 100$
1.02

Final amount = 25000×1.02^5
= 27602.02

Interest = 27602.02 − 25000
= 2602.02

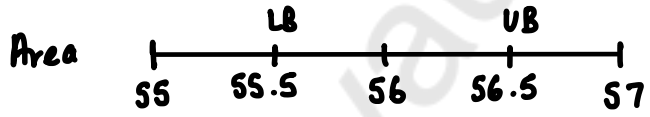
£ 2602.02 **[4]**

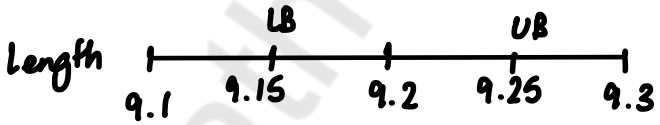
11 The area of a rectangle is 56 m^2, correct to the nearest m^2.
The length of the rectangle is 9.2 m, correct to the nearest 0.1 m.

Calculate the smallest possible width of the rectangle.

$A = l \times w$

$w = \frac{A}{l}$





$w_{LB} = \frac{A_{LB}}{L_{UB}}$

$= \frac{55.5}{9.25} = 6$

6 m **[4]**

12 **(a)** Here are the first four terms of a sequence.

-6 -1 4 9 14

-5 +5 +5 +5

Write an expression for the *n*th term of this sequence.

$5n - 6$

(a) ..... $5n-6$ ..... **[2]**

(b) The *n*th term of another sequence is given by

$$an^2 + bn$$

The third term is 9 and the sixth term is 126.

Find the value of *a* and the value of *b*.

Third term

$a(3)^2 + b(3) = 9$

$9a + 3b = 9$

Sixth term

$a(6)^2 + b(6) = 126$

$36a + 6b = 126$

$36a + 6b = 126$

$9a + 3b = 9$ x 2

$36a + 6b = 126$

$18a + 6b = 18$

$18a = 108$

$\div 18 \quad \div 18$

$a = 6$

$9a + 3b = 9$

$9(6) + 3b = 9$

$54 + 3b = 9$

$-54 \quad -54$

$3b = -45$

$\div 3 \quad \div 3$

$b = -15$

(b) $a =$ ..... 6 .....

$b =$ ..... -15 ..... **[5]**

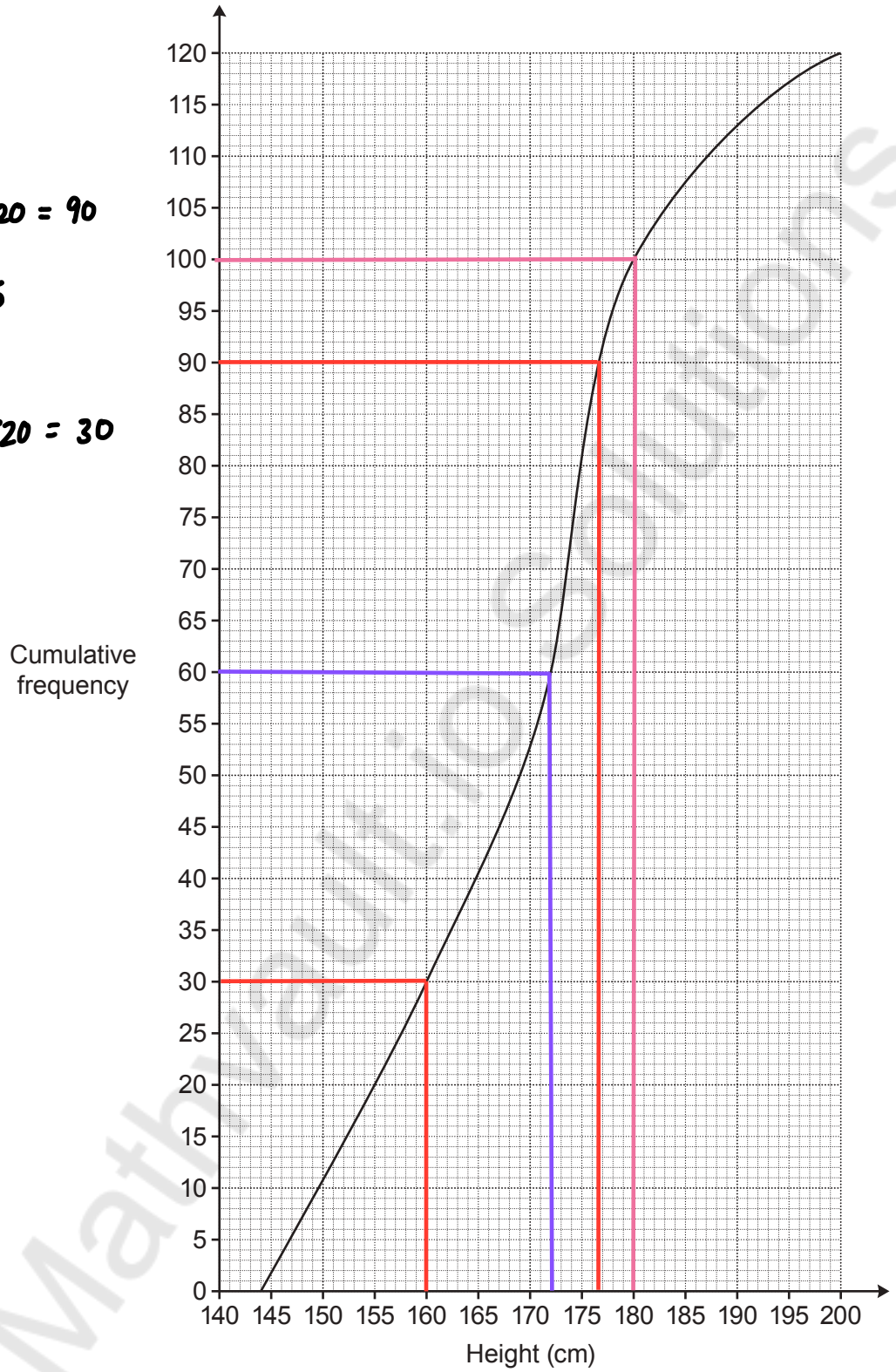
13 **(a)** The cumulative frequency graph shows the distribution of the heights of members of a rowing club.

UQ = $\frac{3}{4}$ of 120 = 90

UQ = 176.5

LQ = $\frac{1}{4}$ of 120 = 30

LQ = 160



(i) Find the median.

120 ÷ 2 = 60

(a)(i) ..... 172 ..... cm **[1]**

(ii) Find the interquartile range.

UQ − LQ
$\frac{3}{4}$ $\frac{1}{4}$

176.5 − 160 = 16.5

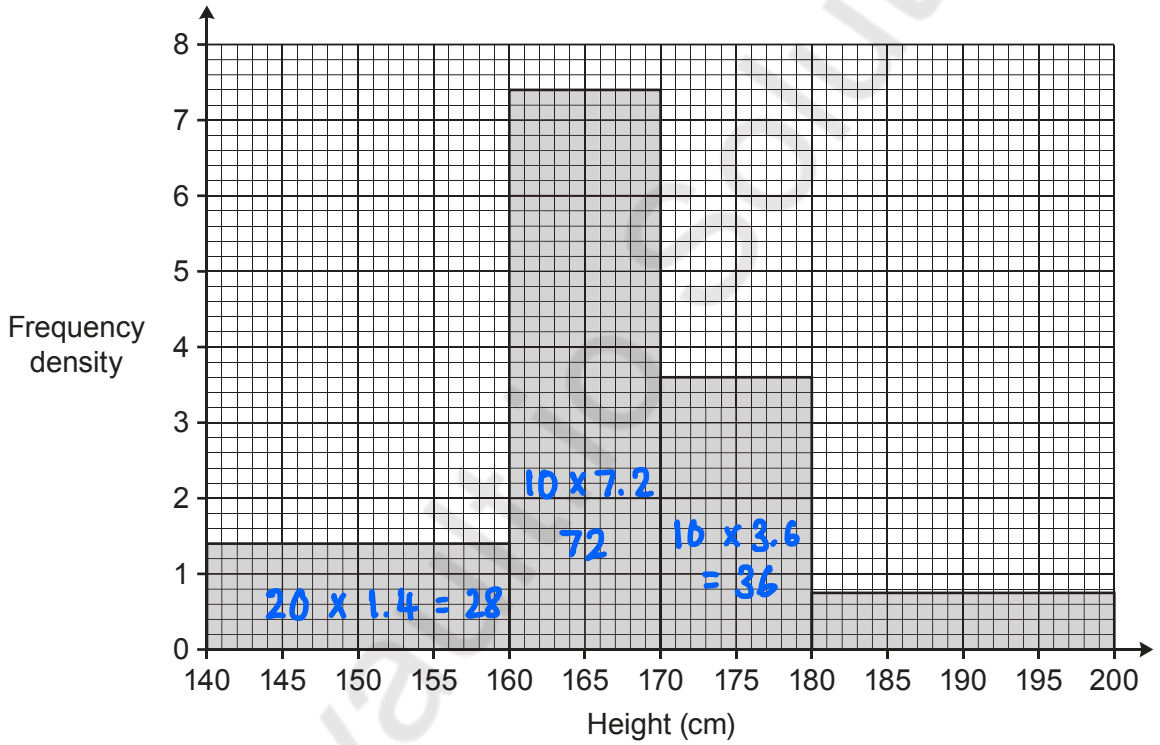
(ii) ..... 16.5 ..... cm **[2]**

(iii) Calculate the percentage of the members who are at least 180 cm tall.

$\frac{20}{120} \times 100 = 16.\dot{6}\ \%$

(iii) ..... $16.\dot{6}$ ..... % **[3]**

(b) The histogram summarises the heights of the 153 members of a swimming club.



Which club has the greater median height?
You must show all your working.

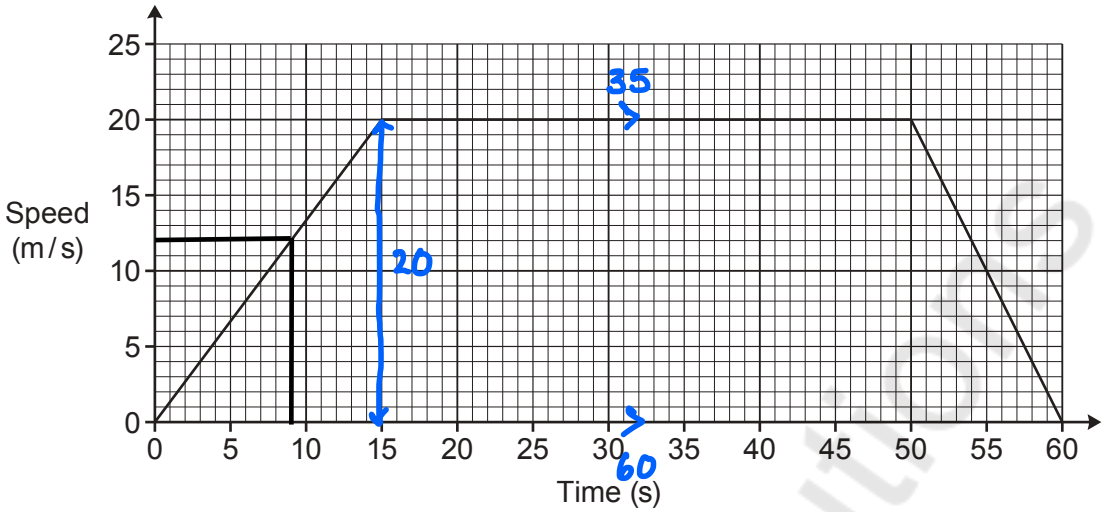
F
CW FD

153 ÷ 2 = 76.5

Median = 160 to 170cm

Rowing club as 172 > 160 to 170 ..... **[5]**

14 The graph shows the speed of a train during the first 60 seconds of motion.



(a) What is the speed of the train after 9 seconds?

(a) 12 m/s **[1]**

(b) What does the straight line suggest about the speed of the train over the first 15 seconds?

Acceleration is constant.

[1]

(c) Work out the average speed of the train, in m/s, during the 60 seconds.

$\text{Area} = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(35+60) \times 20$

$= 950\ \text{m}$

$\text{Speed} = \frac{D}{T}$

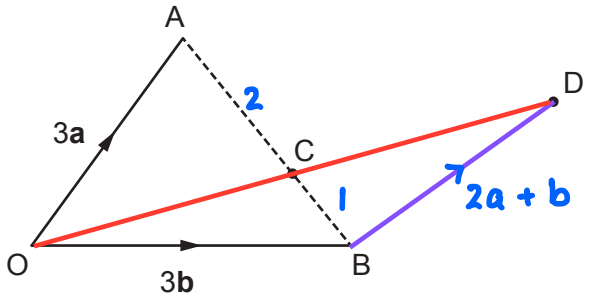
$= \frac{950}{60}$

$= 15.8\dot{3}\ \text{m/s}$

≈ 15.8

(c) 15.8 m/s **[5]**

15 The diagram shows triangle OAB and points C and D.



Not to scale

$\overrightarrow{OA} = 3\mathbf{a}$ and $\overrightarrow{OB} = 3\mathbf{b}$.
C lies on AB such that AC = 2CB.
D is such that $\overrightarrow{BD} = 2\mathbf{a} + \mathbf{b}$.

AC = 2CB
AC : CB
2 : 1

Show, using vectors, that OCD is a straight line. **[5]**

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= 3a + \frac{2}{3}\overrightarrow{AB}$

$= 3a + \frac{2}{3}(-3a + 3b)$

$= 3a + -2a + 2b$

$\overrightarrow{OC} = a + 2b$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3a + 3b$

$\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BD}$

$= \frac{1}{3}\overrightarrow{AB} + 2a + b$

$= \frac{1}{3}(-3a + 3b) + 2a + b$

$= -a + b + 2a + b$

$\overrightarrow{CD} = a + 2b$

$\overrightarrow{OC} = \overrightarrow{CD}$ ∴ OCD is a straight line.

16 **(a)** The table shows values of x and y.

x	4	16	36
y	6	3	2

Show that these values fit the relationship that y is inversely proportional to $\sqrt{x}$. **[2]**

$y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$6 = \frac{k}{\sqrt{4}}$

$6 = \frac{k}{2}$ $\times 2$

$\times 2$

$12 = k$

$y = \frac{12}{\sqrt{x}}$

$3 = \frac{12}{\sqrt{16}}$

$3 = \frac{12}{4}$ $\times 4$

$\times 4$

$12 = 12$

(b) a is inversely proportional to b^2 and $a = 3.75$ when $b = 4$.

Find a formula linking a and b.

$a \propto \frac{1}{b^2}$

$a = \frac{k}{b^2}$

$3.75 = \frac{k}{4^2}$

$3.75 = \frac{k}{16}$ $\times 16$

$\times 16$

$60 = k$

$a = \frac{60}{b^2}$

(b) $a = \frac{60}{b^2}$ ..... **[3]**

17 Show that $(a^3)^{-\frac{1}{3}} \times (a^2)^{\frac{1}{2}} = 1$. **[3]**

$(x^a)^b = x^{ab}$ $\quad x^a \times x^b = x^{a+b}$ $\quad x^0 = 1$

$a^{3 \times -\frac{1}{3}} \quad a^{2 \times \frac{1}{2}}$

$a^{-1} \times a^1 = a^{-1+1}$

$= a^0$

$= 1$

18 Region **R** satisfies these inequalities.

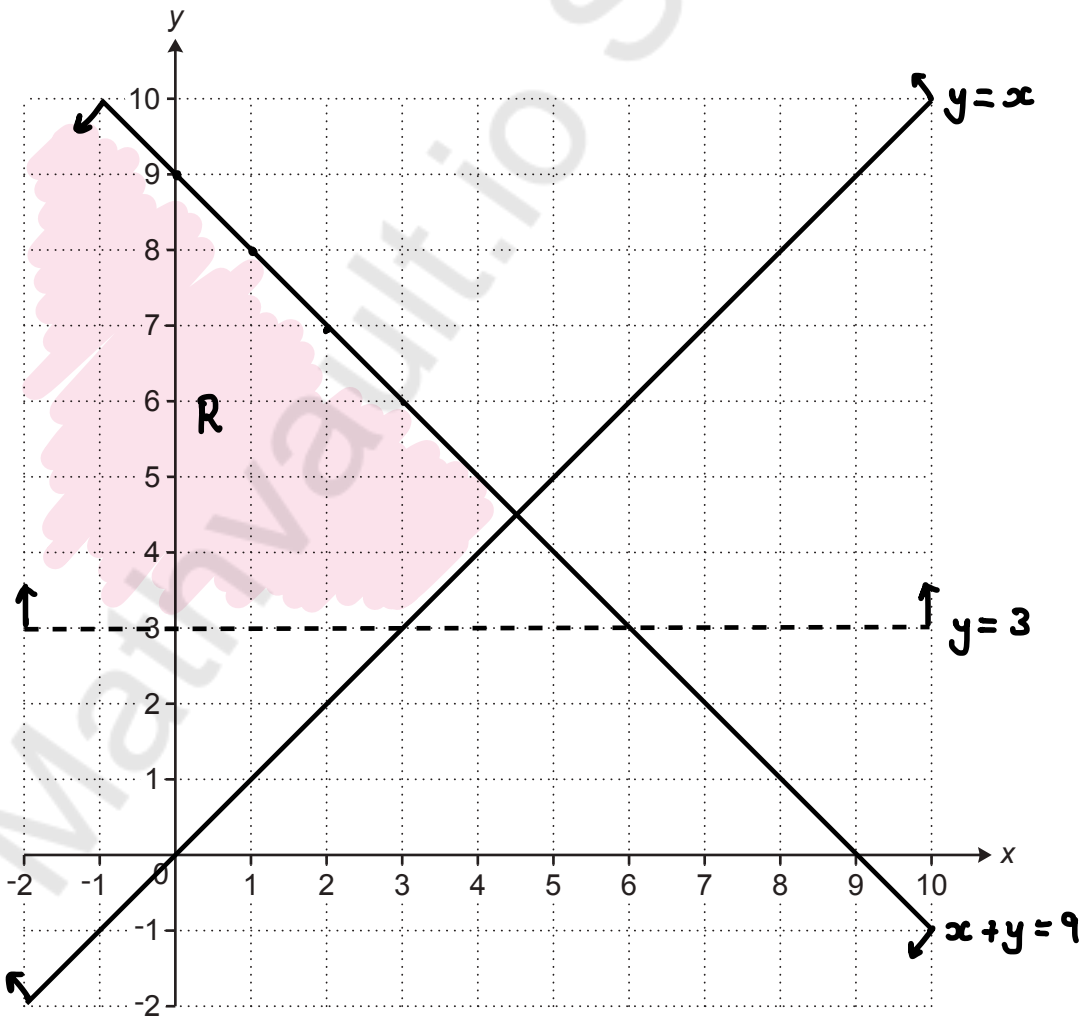
$y > 3$
$y \geqslant x$
$x + y \leqslant 9$

> < ≤ ≥

$x + y = 9$

x	0	1	2	3
y	9	8	7	6

By drawing three straight lines on the grid, find and label the region **R**.



[6]

Turn over for Question 19

19 Solve this equation algebraically.
Give your solutions correct to 2 decimal places.

$3x^2 + 8x - 5 = 0$

$ax^2 + bx + c = 0$

$a = 3 \quad b = 8 \quad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5)}}{2(3)}$$

$= 0.522588 1209$ or $= -3.189254788$

≈ 0.52 ≈ -3.19

$x =$ ....0.52.... or $x =$ ....-3.19.... **[4]**

END OF QUESTION PAPER