

Thursday 6 June 2019 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes



You may use:

- geometrical instruments
- tracing paper

Do not use:

- a calculator



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document consists of **20** pages.



No calculator can be used for this paper

Answer **all** the questions.

- 1 Work out $(2 \times 10^3) \times (4 \times 10^4)$, giving your answer in standard form.

$$2 \times 4 = 8$$

$$10^3 \times 10^4 = 10^7$$

$$8 \times 10^7$$

..... [2]

- 2 (a) Simplify fully.

$$\frac{3a^8 \times 2a^5}{a^2}$$

$$3a^8 \times 2a^5 = 6a^{13}$$

$$\frac{6a^{13}}{1a^2} = 6a^{11}$$

(a) $6a^{11}$ [3]

- (b) Solve.

$$\frac{6x - 10}{5} = 1$$

$$\times 5 \quad \times 5$$

$$6x - 10 = 5$$

$$+ 10 \quad + 10$$

$$6x = 15$$

$$\div 6 \quad \div 6$$

$$x = \frac{15}{6}$$

$$= \frac{5}{2}$$

(b) $x = \dots\dots\dots \frac{5}{2} \dots\dots\dots$ [3]

3 Ed has a card shop.

- (a) He buys a particular card for £1.20 and sells it for £1.68.

Calculate his percentage profit on this card.

$$\begin{aligned} \text{Profit} &= 168 - 120 \\ &= 48p \end{aligned}$$

$$\frac{48}{120} \div 12 = \frac{4}{10} = 0.4$$

$$0.4 \times 100 = 40$$

(a) 40 % [3]

- (b) Ed's profit on "Good Luck" cards in 2018 was £360. This was a decrease of 20% on his profit in 2017.

Work out Ed's profit on "Good Luck" cards in 2017.

$$100\% - 20\% = 80\%$$

$$\begin{array}{r} 80\% = £360 \\ \div 4 \qquad \qquad \qquad \div 4 \\ 20\% = £90 \\ \times 5 \qquad \qquad \qquad \times 5 \\ 100\% = £450 \end{array}$$

(b) £ 450 [3]

- 4 (a) A sunflower grows at a rate of 4 cm each day.

How many days does it take to grow from a height of 80 cm to more than 1.06 m?

$$1\text{m} = 100\text{cm}$$

$$1.06 \times 100 = 106\text{cm}$$

$$106 - 80 = 26$$

$$\begin{array}{r} 6.5 \\ 4 \overline{) 26.20} \end{array} \quad 6.5 \text{ days} \rightarrow 7$$

(a) 7 [3]

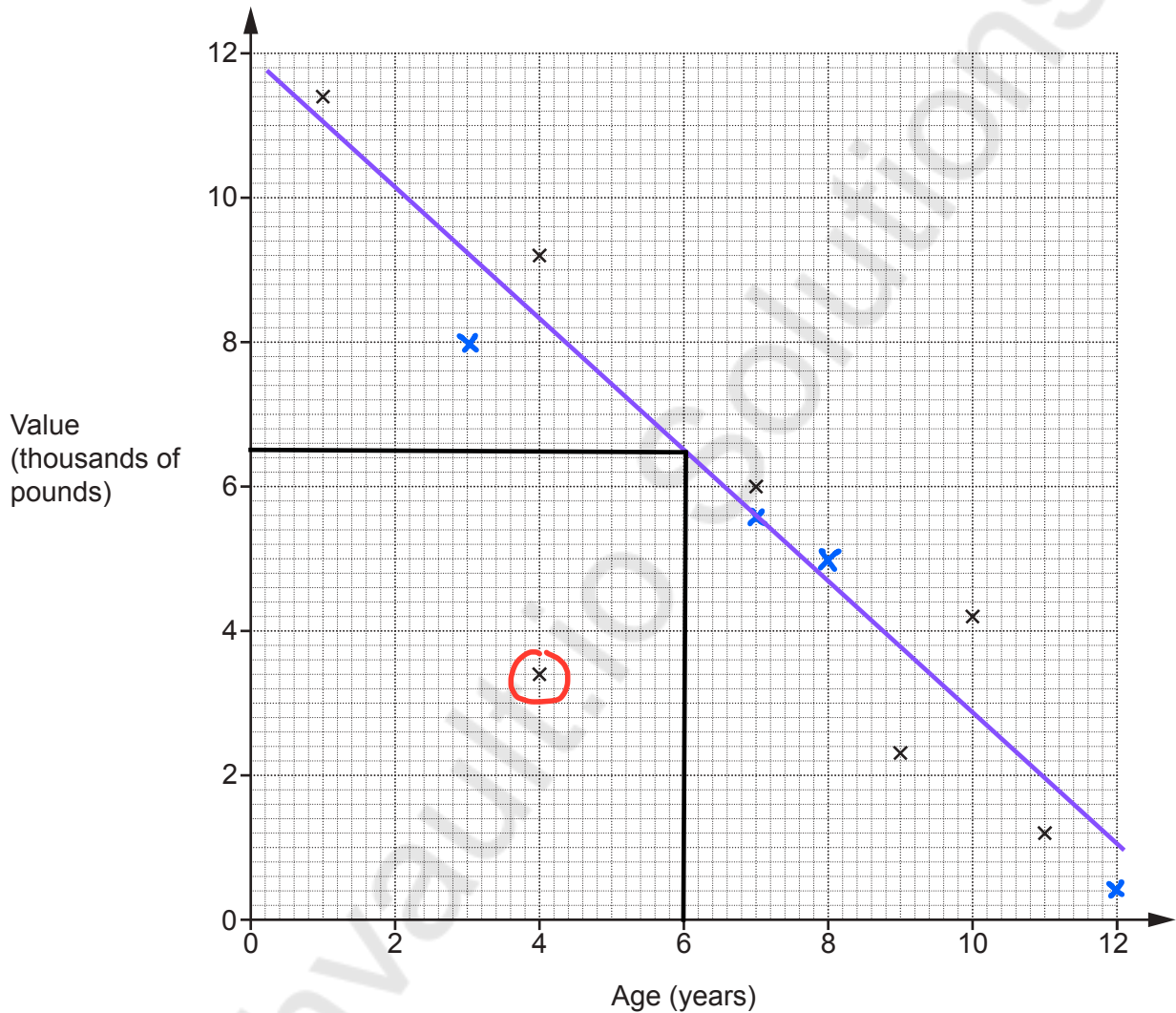
- (b) If the sunflower grows at a faster rate, how would this affect your answer to part (a)?

..... Fewer days. [1]

5 The table shows the ages and values of 11 cars of the same model.

Age (years)	4	7	11	1	9	10	4	3	7	8	12
Value (thousands of pounds)	9.2	6.0	1.2	11.4	2.3	4.2	3.4	8.0	5.6	5.0	0.4

The points for the first 7 cars are plotted on the scatter diagram.



(a) Plot the points for the remaining 4 cars. [2]

(b) Describe the type and strength of the correlation shown in the completed scatter diagram.

..... **Strong negative correlation** [2]

- (c) One car lost its value more quickly than the other cars.

On the scatter diagram, draw a circle around the point representing this car. [1]

- (d) By drawing a line of best fit, estimate the value of a car that is 6 years old.

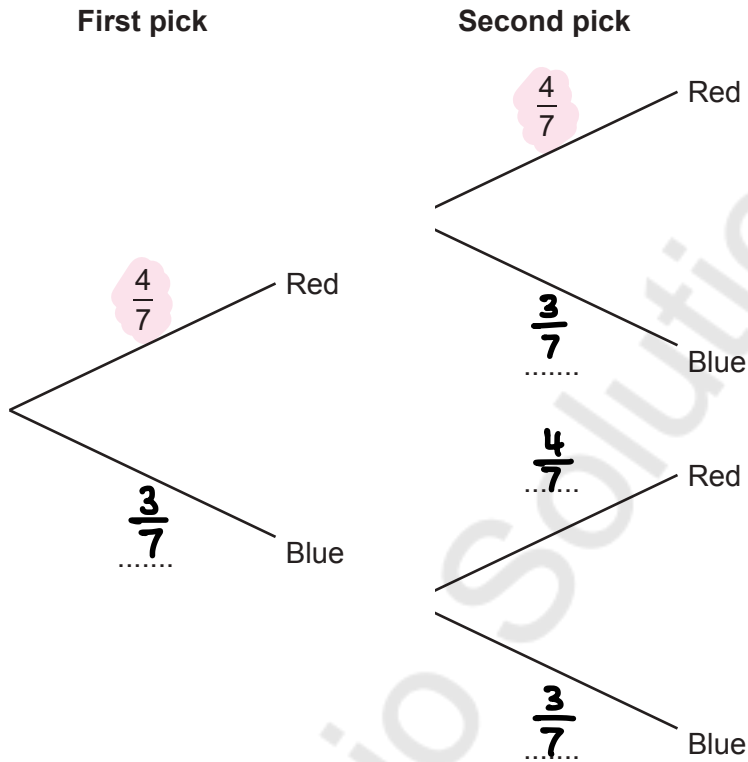
(d) £ 6500 [2]

- (e) Explain the limitations of using the equation of the line of best fit to estimate the value of a car that is 16 years old.

Only have data on cars up to 12 years old. [1]
.....

- 6 A bag contains 4 red counters and 3 blue counters only. Jack picks a counter at random and then replaces it. Jack then picks a second counter at random.

(a) Complete the tree diagram.



[2]

- (b) Work out the probability that Jack picks two red counters.

$$\begin{aligned}
 P(RR) &= \frac{4}{7} \times \frac{4}{7} \\
 &= \frac{16}{49}
 \end{aligned}$$

(b) $\frac{16}{49}$ [2]

7 Adam buys some theatre tickets in a sale.

The normal prices are:

£80 for each adult
£40 for each child.

In the sale, the prices are reduced by 15%.

Adam buys 2 adult tickets and 1 child ticket at the sale price.

A 2% booking fee is then added to the total cost of the tickets.

Calculate the total amount that Adam must pay.



Adult

$$100\% = \text{£}80$$

$$10\% = \text{£}8$$

$$5\% = \text{£}4$$

$$15\% = \text{£}12$$

$$\begin{aligned} \text{Sale price} &= \text{£}80 - \text{£}12 \\ &= \text{£}68 \end{aligned}$$

Child

$$100\% = \text{£}40$$

$$10\% = \text{£}4$$

$$5\% = \text{£}2$$

$$15\% = \text{£}6$$

$$\begin{aligned} \text{Sale price} &= \text{£}40 - \text{£}6 \\ &= \text{£}34 \end{aligned}$$

2 Adult tickets

$$= 2 \times 68$$

$$= \text{£}136$$

$$\begin{aligned} \text{Total cost} &= \text{£}136 + \text{£}34 \\ &= \text{£}170 \end{aligned}$$

Booking Fee

$$100\% = \text{£}170 \quad \div 100$$

$$1\% = \text{£}1.70$$

$$1\% = \text{£}1.70$$

$$2\% = \text{£}3.40$$

$$\text{£}170 + \text{£}3.40 = \text{£}173.40$$

$$\text{£} \dots \dots \dots 173.40 \dots \dots \dots [6]$$

8 Mrs Mills buys 4 packs of treats for her cats, Fluff and Tigger.

She gives Fluff $\frac{1}{6}$ of a pack each day.

She gives Tigger $\frac{1}{5}$ of a pack each day.

For how many complete days will the 4 packs of treats last?

$$\frac{1}{6} \times 5 + \frac{1}{5} \times 6$$

$$= \frac{5}{30} + \frac{6}{30}$$

$$= \frac{11}{30}$$

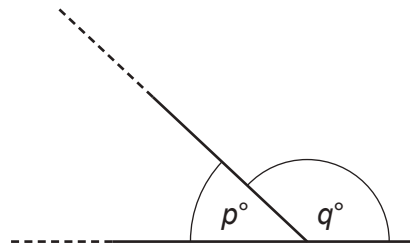
$$4 \div \frac{11}{30}$$

$$\frac{4}{1} \times \frac{30}{11} = \frac{120}{11}$$

$$11 \overline{) 120.000}$$

10.9

- 9 An interior angle of an isosceles triangle is p° and an exterior angle is q° .



Not to scale

It is given that $q = 5p$.

- (a) Write the ratio $p : q$ in its simplest form.

$$p : q$$

$$1 : 5$$

(a) **1** : **5** [2]

- (b) Work out the two different possible sets of angles for the isosceles triangle.

$$\text{Total parts} = 6$$

$$180 \div 6 = 30^\circ$$

$$p = 1 \times 30$$

$$= 30^\circ \text{ (if base angle)}$$

$$30^\circ, 30^\circ, 120^\circ$$

$$p = 30^\circ \text{ (if not base angle)}$$

$$30^\circ, 75^\circ, 75^\circ$$

$$180 - 30 = 150$$

$$150 \div 2 = 75^\circ$$

(b) Triangle 1: **30** $^\circ$, **30** $^\circ$, **120** $^\circ$

Triangle 2: **30** $^\circ$, **75** $^\circ$, **75** $^\circ$

[4]

- 10 (a) Write $\frac{1}{6}$ as a recurring decimal.

$$1 \div 6$$

$$\begin{array}{r} 0.166 \\ 6 \overline{) 1.104040} \end{array}$$

(a) $0.1\dot{6}$ [2]

- (b) Elsa divides a two-digit number by another two-digit number. She gets the answer 0.15.

She says that there is only one possible pair of numbers that will give this answer. Is she correct? Show how you decide.

$$x = 0.1\dot{5}$$

$$10x = 1.\dot{5}$$

$$100x = 15.\dot{5}$$

$$100x - 10x = 90x$$

$$15.\dot{5} - 1.\dot{5} = 14$$

$$90x = 14$$

$$x = \frac{14}{90} = \frac{7}{45}$$

..... She is correct. $14 \div 90 = 0.1\dot{5}$ [4]

- 11 (a) Simplify fully.

$$\sqrt{200}$$

$$= \sqrt{2} \times \sqrt{100}$$

$$= \sqrt{2} \times 10$$

(a) $10\sqrt{2}$ [2]

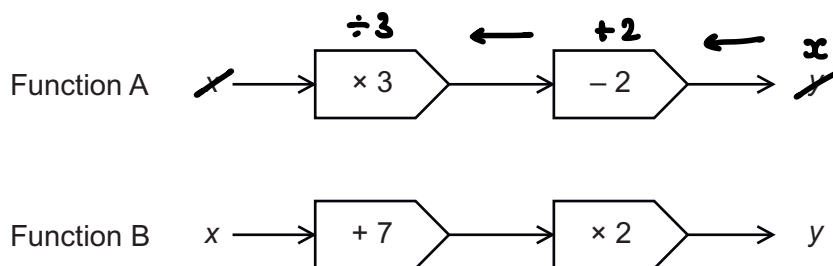
- (b) Evaluate.

$$8^{\frac{1}{3}}$$

$$\sqrt[3]{8}$$

(b) 2 [1]

12 Here are two functions.

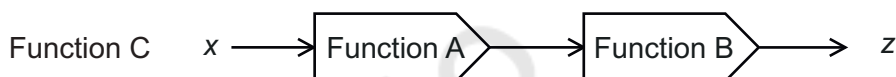


(a) Find an algebraic expression for the output of the **inverse** of function A when the input is x .

$$\frac{x+2}{3}$$

(a) $\frac{x+2}{3}$ [2]

(b) Here is a composite function C.



Find the value x when $z = 4x$.

$$\begin{aligned} \text{A : } \quad x \times 3 &= 3x \\ 3x - 2 & \end{aligned}$$

$$\begin{aligned} \text{B : } \quad (3x - 2) + 7 &= 3x + 5 \\ 2(3x + 5) &= 6x + 10 \end{aligned}$$

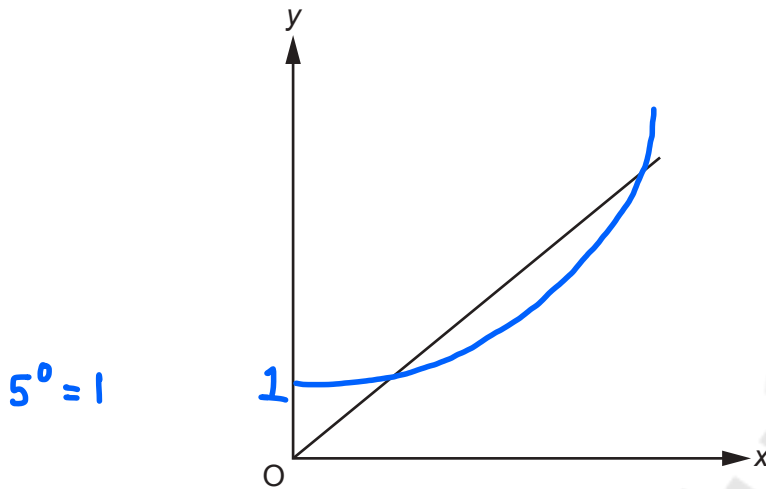
$$\begin{aligned} 6x + 10 &= 4x \\ -4x & \quad -4x \end{aligned}$$

$$\begin{aligned} 2x + 10 &= 0 \\ -10 & \quad -10 \end{aligned}$$

$$\begin{aligned} 2x &= -10 \\ \div 2 & \quad \div 2 \\ x &= -5 \end{aligned}$$

(b) $x = -5$ [5]

- 13 Shirley is asked to sketch a graph of $y = 5^x$ for $x \geq 0$. She produces the following.



The graph has two errors.

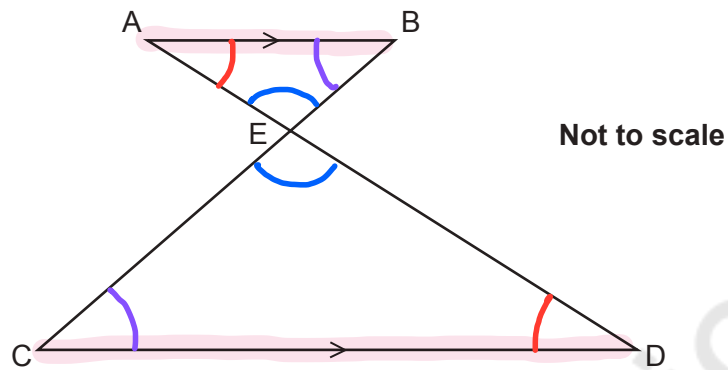
How should they be corrected?

1 It should be curved with an increasing gradient

2 It should go through (0, 1)

[2]

- 14 In the diagram AB is parallel to CD.
AED and BEC are straight lines.



Prove that triangle ABE is similar to triangle CDE.

Angle AEB = Angle CED - vertically opposite

Angle DAB = Angle ADC - alternate

Angle ABC = Angle DCB - alternate

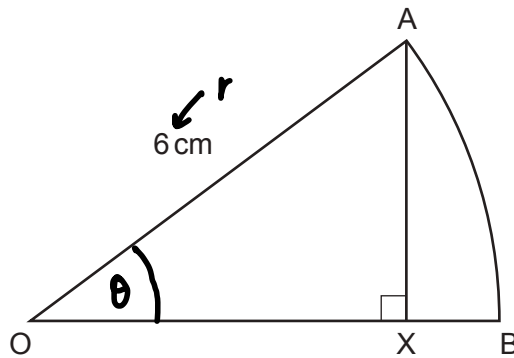
..... Angles are equal, therefore similar

.....

.....

..... [3]

- 15 OAB is a sector of a circle, centre O.
OA = 6 cm and AX is perpendicular to OB.



Not to scale

The area of sector OAB is $6\pi \text{ cm}^2$.

Show that $AX = 3\sqrt{3} \text{ cm}$.

[6]

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$6\pi = \frac{\theta}{360} \times \pi \times 6^2$$

$$\div \pi \qquad \qquad \qquad \div \pi$$

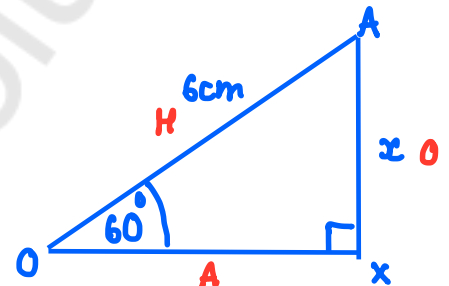
$$6 = \frac{\theta}{360} \times 36$$

$$6 = \frac{36\theta}{360}$$

$$\times 10 \qquad \qquad \qquad \times 10$$

$$60 = \theta$$

$$60 = \theta$$



S O^v H C A H T O A

$$\sin \theta = \frac{O}{H}$$

$$\sin 60 = \frac{AX}{6}$$

$$\frac{\sqrt{3}}{2} = \frac{AX}{6} \quad \times 6$$

$$\frac{6\sqrt{3}}{2} = AX$$

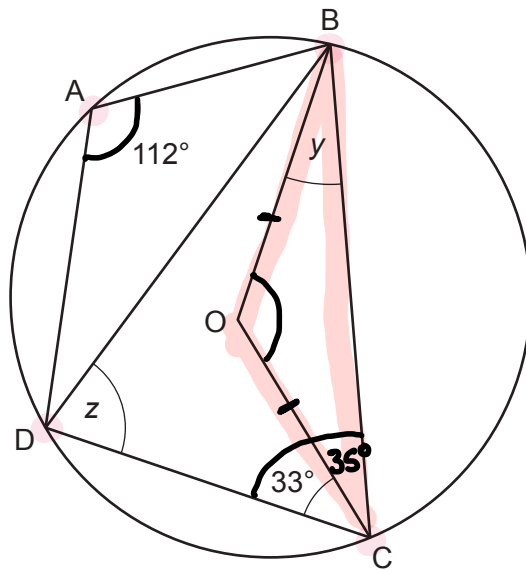
$$3\sqrt{3} = AX$$

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0

2

16 A, B, C and D are points on the circumference of a circle, centre O.

Angle BAD = 112° and angle DCO = 33° .



Not to scale

- (a) Show that angle $y = 35^\circ$.
Give reasons for each stage of your working.

[4]

$$\text{Angle } DCB = 180 - 112$$

$$= 68^\circ \quad \text{Opposite angles of a cyclic quadrilateral sum to } 180$$

$$68^\circ - 33^\circ = 35^\circ \quad \text{angle } OCB$$

$$y = 35^\circ \quad \text{base angles of an isosceles are equal}$$

- (b) Work out angle z .
Give reasons for your answer.

$$\begin{aligned} \text{Angle } BOC &= 180 - 35 - 35 \\ &= 180 - 70 \\ &= 110^\circ \end{aligned}$$

$$\begin{aligned} z &= 110^\circ \div 2 \\ &= 55^\circ \end{aligned}$$

Angle $z = 55^\circ$ because angle at the circumference is half the angle at the centre.

[3]

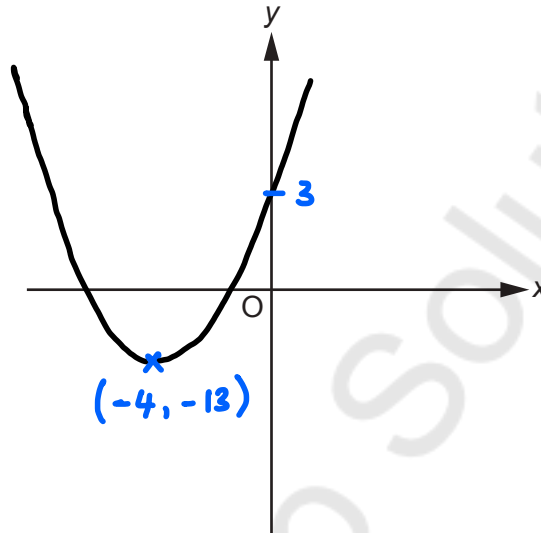
- 17 (a) Write $x^2 + 8x + 3$ in the form $(x + a)^2 - b$.

$$(x + 4)^2 - 4^2 + 3$$

$$(x + 4)^2 - 16 + 3$$

(a) $(x + 4)^2 - 13$ [3]

- (b) Sketch the graph of $y = x^2 + 8x + 3$.
Show clearly the coordinates of any turning points and the y-intercept.



Turning point
 $(-4, -13)$

y-intercept
 $0^2 + 8(0) + 3 = 3$
 $(0, 3)$

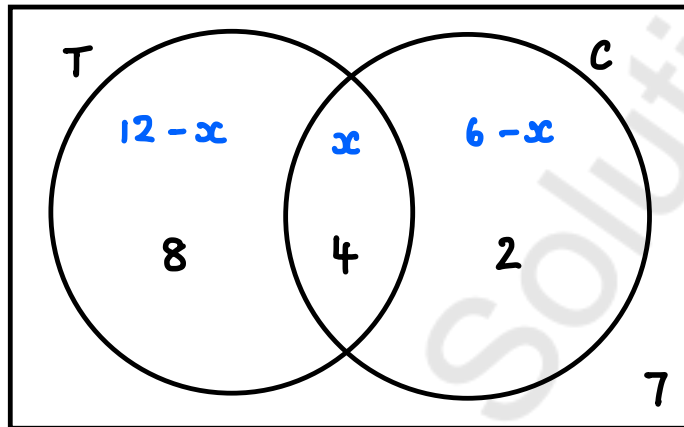
[4]

18 21 people travelled to a meeting.

- 12 used a train.
- 6 used a car.
- 7 did not use a train or a car. ✓
- Some used a train and a car.

Two people are chosen at random from those who used a train.

Find the probability that both these people also used a car.



$$12 - x + x + 6 - x = 14$$

$$18 - x = 14$$

$$\begin{array}{r} -18 \\ -18 \end{array}$$

$$-x = -4$$

$$x = 4$$

$$\text{Train} = 12$$

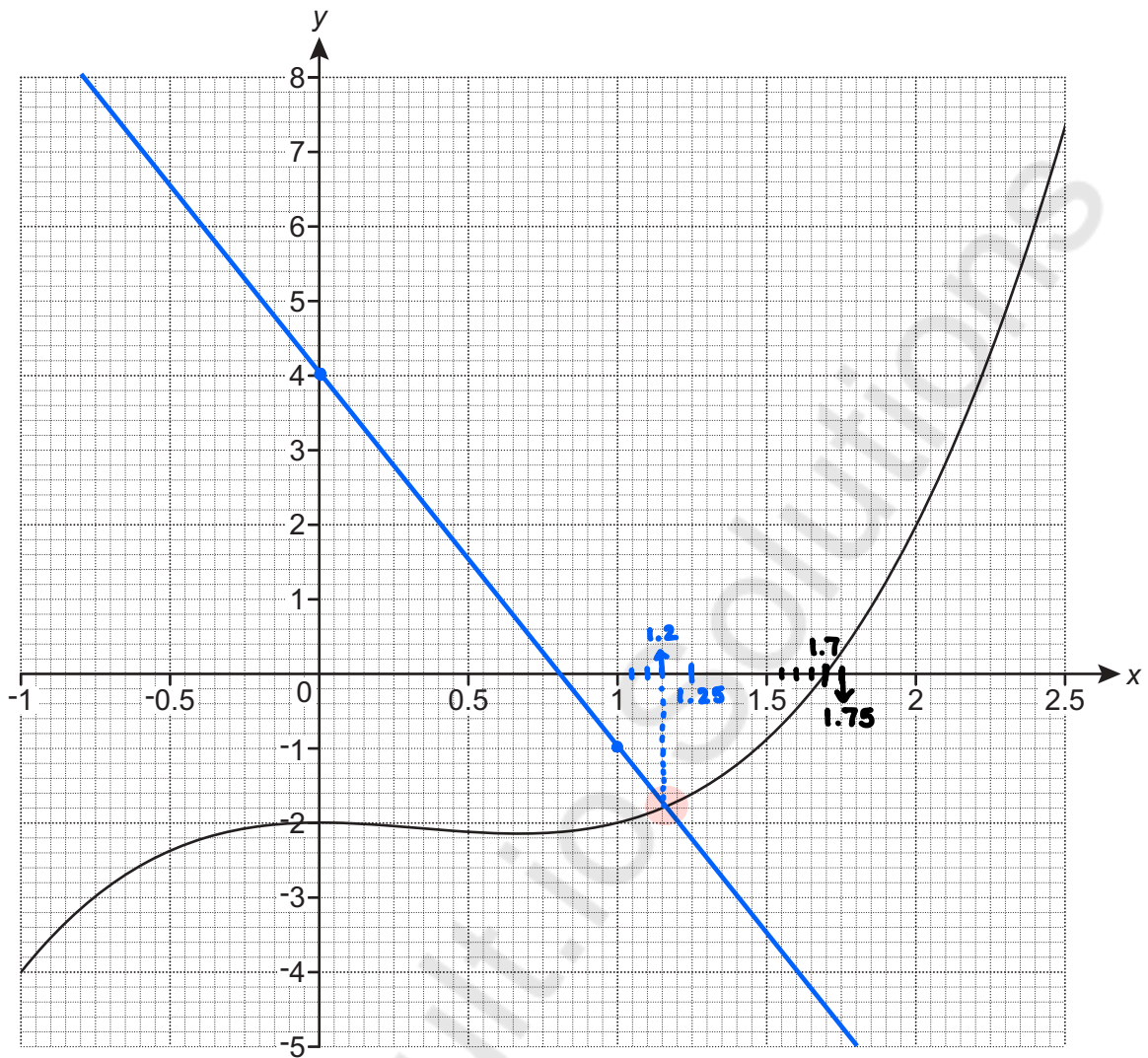
$$\text{Car} = 4$$

$$\frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$$

$$\frac{12}{132}$$

..... [6]

- 19 The graph of $y = x^3 - x^2 - 2$ is drawn on the grid.



- (a) Use the graph to solve $x^3 - x^2 - 2 = 0$.
Give your answer correct to 1 decimal place.

$x = \dots 1.7 \dots \dots \dots [1]$

- (b) The equation $x^3 - x^2 + 5x - 6 = 0$ can be solved by finding the intersection of the graph of $y = x^3 - x^2 - 2$ and the line $y = ax + b$.

- (i) Find the value of a and the value of b .

$$\begin{array}{r} x^3 - x^2 - 2 = ax + b \\ -ax - b \quad -ax - b \end{array}$$

$$x^3 - x^2 - ax - b - 2 = 0$$

$$\begin{array}{r} -ax = 5x \quad -b - 2 = -6 \\ a = -5 \quad \quad +2 \quad +2 \\ \quad \quad \quad -b = -4 \\ \quad \quad \quad b = 4 \end{array}$$

(b)(i) $a = -5$

$b = 4$ [2]

- (ii) Hence, use the graph to solve the equation $x^3 - x^2 + 5x - 6 = 0$.
Give your answer correct to 1 decimal place.

$$y = -5x + 4$$

x	0	1	2
y	4	-1	-6

(ii) $x = 1.2$ [3]

END OF QUESTION PAPER

