OCR
Oxford Cambridge and RSA

H

Tuesday 5 November 2019 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You may use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Carol makes birthday cards.
Each card takes the same amount of time to make.

She makes 3 cards in 48 minutes.
She has an order for 80 cards.

Can she complete this order in 3 days if she works 8 hours each day?
Show how you decide.

3 cards = 48 minutes
÷3 ÷3
1 card = 16 minutes
×80 ×80
80 cards = 1280 minutes
÷60
21.3 hours

3 × 8h = 24 hours

Yes because 21.3 hours < 24 hours.

[5]

2 Use the formula $F = \dfrac{s}{\sqrt{tm}}$ to find the value of F when

$s = 5.8 \times 10^{6}$
$t = 4.1 \times 10^{8}$
$m = 3.7 \times 10^{-2}$.

Give your answer in standard form, correct to 2 significant figures.

$$F = \frac{5.8 \times 10^{6}}{\sqrt{(4.1 \times 10^{8} \times 3.7 \times 10^{-2})}}$$

$= 1489.138881$

≈ 1500

$= 1.5 \times 10^{3}$

1.5×10^{3} ..... [4]

3 At a railway station, trains are either eastbound or westbound.
An eastbound train leaves the station every 25 minutes.
A westbound train leaves the station every 45 minutes.

An eastbound train and a westbound train both leave the station at 8 am.

When is the next time that two trains leave the station together?

Eastbound : 25 50 75 100 125 150 175 200 225

Westbound : 45 90 135 180 225

LCM = 225 minutes

$60 \overline{)225}$ = 3 r 45

3h 45 mins

8 am + 3h 45 = 11 : 45 am

11 : 45 am ..... [4]

4 Bob makes dry concrete by mixing cement, sand and stone in the ratio 1 : 2 : 3 by weight.
He buys the cement, sand and stone in bags as shown in this table.

	Weight of bag (kg)	Cost per bag (£)
Cement	25	5.50
Sand	20	2.00
Stone	15	3.90

He packs the dry concrete into 30 kg bags.

Bob buys just enough cement, sand and stone to make 50 bags of dry concrete.

(a) Show that Bob buys 500 kg of sand. **[3]**

C : Sand : Stone
1 : 2 : 3 = 6 parts

x 250

500 kg

30 kg x 50 bags = 1500 kg
1500 ÷ 6 = 250 kg

(b) Bob sells the 50 bags of dry concrete for a total of £396.

Calculate Bob's percentage profit.

Cement = 1 x 250 = 250 kg
250 kg ÷ 25 kg = 10 bags
10 x £5.50 = £55

Sand = 500 kg ÷ 20 kg = 25 bags
25 x £2 = £50

Stone = 3 x 250 = 750 kg
750 ÷ 15 = 50 bags
50 x 3.90 = £195

Total = 55 + 50 + 195
= £300 spent

% profit

$$\frac{396 - 300}{300} \times 100$$

= 32 %.

(b) 32 % **[5]**

5 Multiply out and simplify.

$(4x + y)(x - 3y)$

$4x^2 - 12xy + xy - 3y^2$

$4x^2 - 11xy - 3y^2$

$4x^2 - 11xy - 3y^2$ ..... [3]

6 A bag of sweets contains only mints, sherberts and toffees.

The ratio of the number of mints to sherberts is 2 : 3.
The ratio of the number of sherberts to toffees is 7 : 5.

What fraction of the sweets are sherberts?

M : S S : T

2 : 3 7 : 5

×7 ×7 ×3 ×3

14 : 21 21 : 15

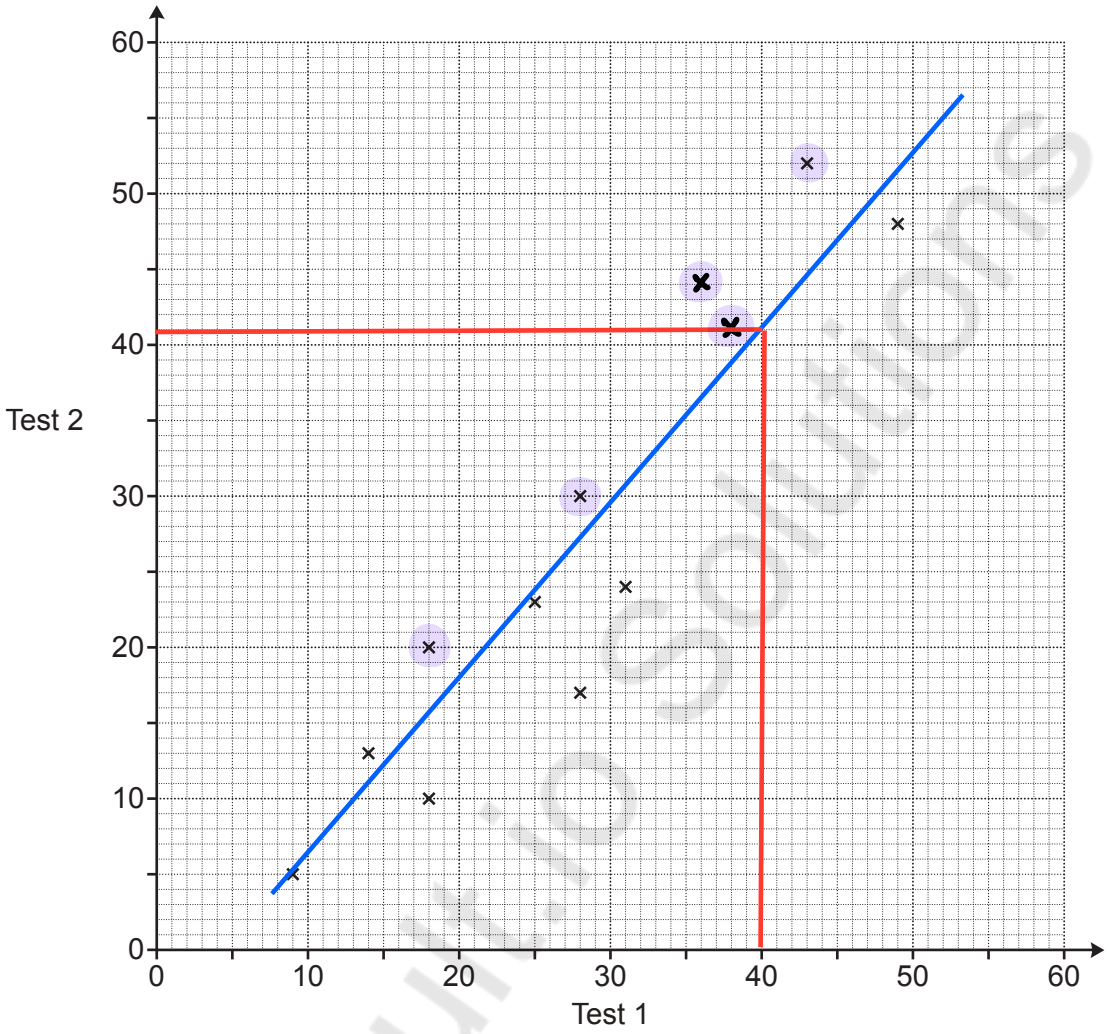
M : S : T

14 : 21 : 15 Total parts = 14 + 21 + 15 = 50

Sherberts = $\frac{21}{50}$

$\frac{21}{50}$ ..... [3]

7 12 students take two tests.
Each test is out of 60.
The scatter diagram shows the results for 10 of the students.



(a) The table shows the results for the other 2 students.

Test 1	36	38
Test 2	44	41

Plot these results on the scatter diagram. **[1]**

(b) Describe the type of correlation shown in the scatter diagram.

(b) ..... Positive ..... **[1]**

(c) **(i)** Draw a line of best fit on the scatter diagram. **[1]**

(ii) Another student was absent for Test 2.
The student scored 40 marks on Test 1.

Use your line of best fit to estimate a result for this student on Test 2.

(c)(ii) 41 **[1]**

(d) Work out the percentage of **the 12 students** whose result on Test 1 is **lower** than their result on Test 2.

$\frac{5}{12} \times 100 = 41.\dot{6}$

$\approx 41.7\%$

(d) 41.7 % **[4]**

8 The diagrams show the price paid by two groups of people visiting a funfair.

5 adults	£	
4 children	£	
Total	£	78

3 adults	£	
6 children	£	
Total	£	63

Assume each adult pays the same price and each child pays the same price.

Find the price for an adult and the price for a child.

$5a + 4c = 78$ ×3

$3a + 6c = 63$ ×5

$15a + 12c = 234$

$15a + 30c = 315$

$-18c = -81$

$\div -18 \quad \div -18$

$c = 4.50$

Child = £4.50

$5a + 4c = 78$

$5a + 4(4.5) = 78$

$5a + 18 = 78$

$-18 \quad -18$

$5a = 60$

$\div 5 \quad \div 5$

$a = 12$

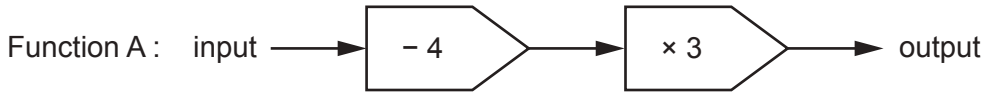
Adult = £12

Adult price = £ ....12....

Child price = £ ....4.50....

[5]

9 Here is function A.



(a) A number, k, is input into function A.
The output is also k.

Find the value of k.

$k - 4 \times 3 = k$

$3(k-4) = k$

$3k - 12 = k$

$-k \quad -k$

$2k - 12 = 0$

$+12 \quad +12$

$2k = 12$

$\div 2 \quad \div 2$

$k = 6$

(a) $k =$ 6 **[3]**

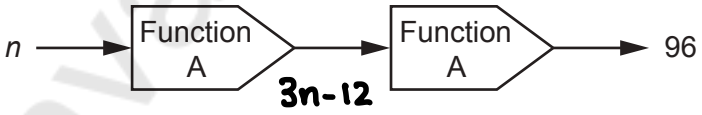
(b) The output of function A is y.

Write an algebraic expression, in terms of y, for the input of function A.

$\frac{y}{3} + 4$

(b) $\frac{y}{3} + 4$ **[2]**

(c) The diagram shows a composite function with an input, n, and an output of 96.



Find the value of n.

$3(n-4) = 3n-12$

$3((3n-12)-4)$

$3(3n-16)$

$9n - 48 = 96$

$+48 \quad +48$

$9n = 144$

$\div 9 \quad \div 9$

$n = 16$

(c) $n =$ 16 **[2]**

10 The value of a house, £H, is given by the formula

$$H = 165\,000 \times 1.03^t$$

where t is the number of years after 1st January 2010.

(a) Write down the value of the house on 1st January 2010.

165000×1.03^0

165000×1

(a) £ 165000 **[1]**

(b) Write down the annual percentage increase in the value of the house.

$1.03 \times 100 = 103\%$

$-\ 100\%$

3%

(b) 3 % **[1]**

(c) Show that the value of the house is over £200 000 on 1st January 2017. **[2]**

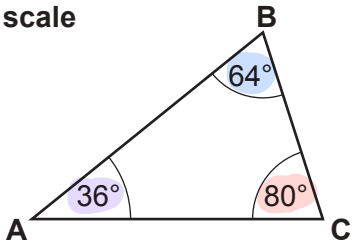
$t = 7$

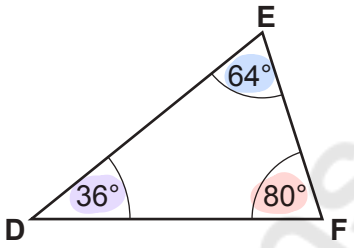
$165\,000 \times 1.03^7 = 202929.1878$

$\approx 202,929.19$

11 **(a)** Are these two triangles definitely congruent?
Give a reason.

Not to scale



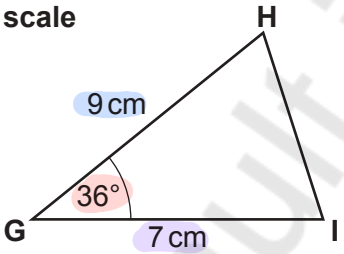


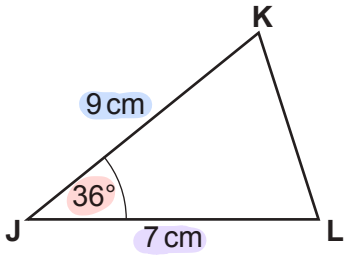
No because lengths are unknown.

[1]

(b) Prove that these two triangles are congruent.

Not to scale





Angle IGH = Angle LJK

GH = JK

GI = JL

SAS

[3]

12 The cumulative frequency graph summarises the annual salary, p (£ thousands), of the 60 workers in a factory.

(a) Use the graph to estimate the median annual salary.

60 ÷ 2 = 30

(a) £ ..... 24 ..... thousands **[1]**

(b) Complete this cumulative frequency table.

Annual salary, p (£ thousands)	**Cumulative frequency**
$p \leqslant 10$	14
$p \leqslant 20$	26
$p \leqslant 30$	36
$p \leqslant 50$	50
$p \leqslant 80$	60

[2]

(c) Use the information in the cumulative frequency table to calculate an estimate of the mean annual salary.

Annual salary	Frequency	Midpoint	Midpoint × frequency
$0 < p \leqslant 10$	14	5	$5 \times 14 = 70$
$10 < p \leqslant 20$	12	15	$15 \times 12 = 180$
$20 < p \leqslant 30$	10	25	$25 \times 10 = 250$
$30 < p \leqslant 50$	14	40	$40 \times 14 = 560$
$50 < p \leqslant 80$	10	65	$65 \times 10 = 650$
	60		1710

Mean = $1710 \div 60 = 28.5$

(c) £ ..... 28.5 ..... thousands **[5]**

(d) Explain why your estimate of the median is more reliable than your estimate of the mean.

..... For the mean, figures used are approximate / .....

..... midpoints are used. ..... **[1]**

13 (a) A transport lorry consists of a cab and a trailer.
The trailer has a volume of $90\,m^3$.
Alfie makes a model of this lorry using a scale of 1 : 72.

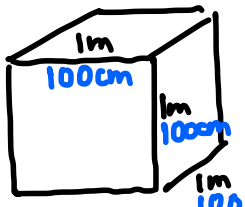
Work out the volume of the trailer in Alfie's model, giving your answer in cm^3.

Model : Real

□³ { Lengths 1 cm : 72 cm
Volume $1^3 : 72^3$

$1\,cm^3 : 372248\,cm^3$ ÷ 373248

Vol trailer = $90\,m^3$
↓ x 1,000,000
$= 90,000,000\,cm^3$



$V = 100^3 = 1,000,000$

$90,000,000 \div 373248 = 241.1265...$

(a) 241.1 ..... cm^3 **[3]**

(b) Alfie paints his model lorry.
He has eight colours available.

He decides to paint the cab in one colour and the trailer in a different colour.

He then wants to paint his name on the trailer.
The name must be in a different colour to the trailer.

In how many different ways can Alfie paint his model lorry?

Cab = 8

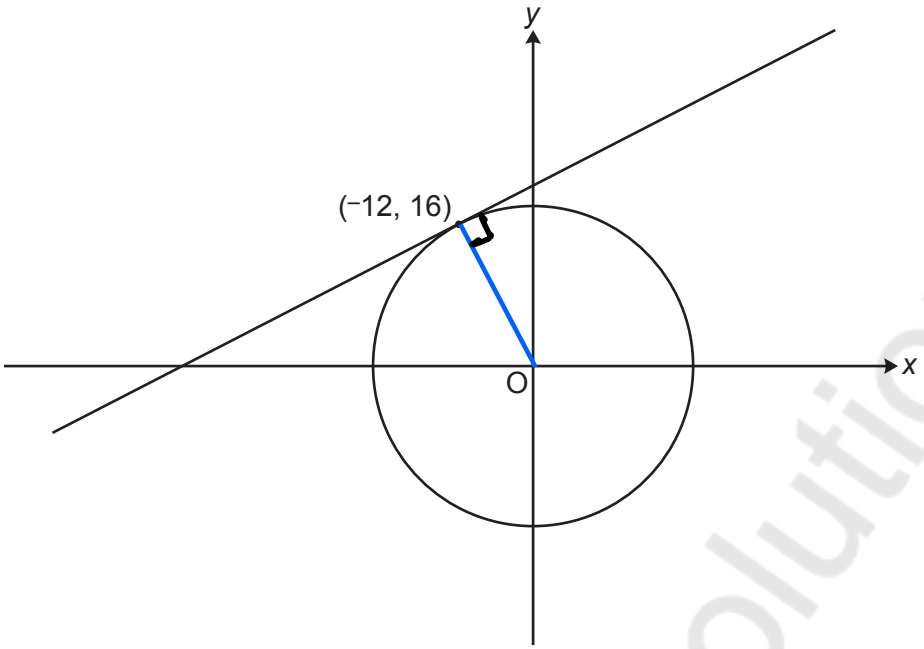
Trailer = 7

Name = 7

8 x 7 x 7 = 392

(b) 392 ..... **[3]**

14 The diagram shows a circle with centre (0, 0) and a tangent at the point (−12, 16).



Not to scale

The tangent crosses the y-axis at the point $(0, p)$.

Find the value of p.

Gradient of radius (-12, 16) (0, 0)

$$\frac{0-16}{0--12} = \frac{-16}{12} = \frac{-4}{3}$$

Gradient of tangent $= \frac{3}{4}$ ← m

(-12, 16)
↑ x ↑ y

$y = mx + c$

$16 = \frac{3}{4}(-12) + c$

$16 = -9 + c$

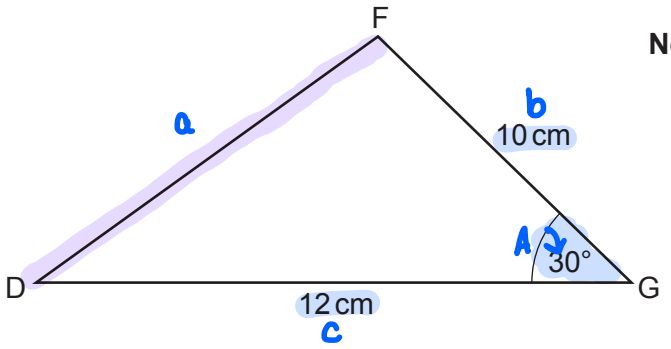
$+9 \quad +9$

$25 = c$ (y-intercept)

$25 = p$

$p =$ 25 **[5]**

15 **(a)** Calculate length DF in this triangle.



Not to scale

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$DF^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos(30)$

$DF^2 = 36.15...$

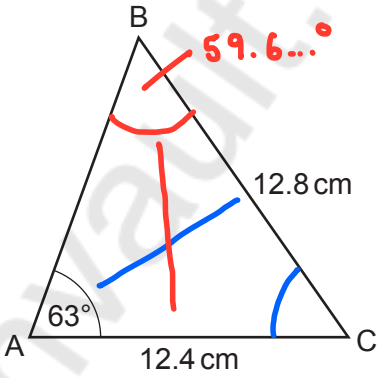
$\sqrt{\quad} \qquad \sqrt{\quad}$

$DF = 6.012811158$

≈ 6.01

(a) ..... 6.01 ..... cm **[3]**

(b) Calculate angle ACB in this triangle.



Not to scale

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin ABC}{12.4} = \frac{\sin 63}{12.8}$

$\sin ABC = \frac{\sin 63}{12.8} \times 12.4$

$= 0.8631...$

$ABC = \sin^{-1}(0.8631...)$

$= 59.67355245°$

$ACB = 180 - 63 - 59.6...$

$= 57.32644755$

$\approx 57.3°$

(b) ..... 57.3 ..... ° **[4]**

16 Show that $\dfrac{x+9}{x^2-1} + \dfrac{4}{x+1}$ can be written in the form $\dfrac{a}{x-1}$, where a is an integer. **[4]**

$x^2-1 = (x+1)(x-1)$

$$\frac{(x+9)(x+1) + 4(x+1)(x-1)}{(x+1)(x-1)(x+1)}$$

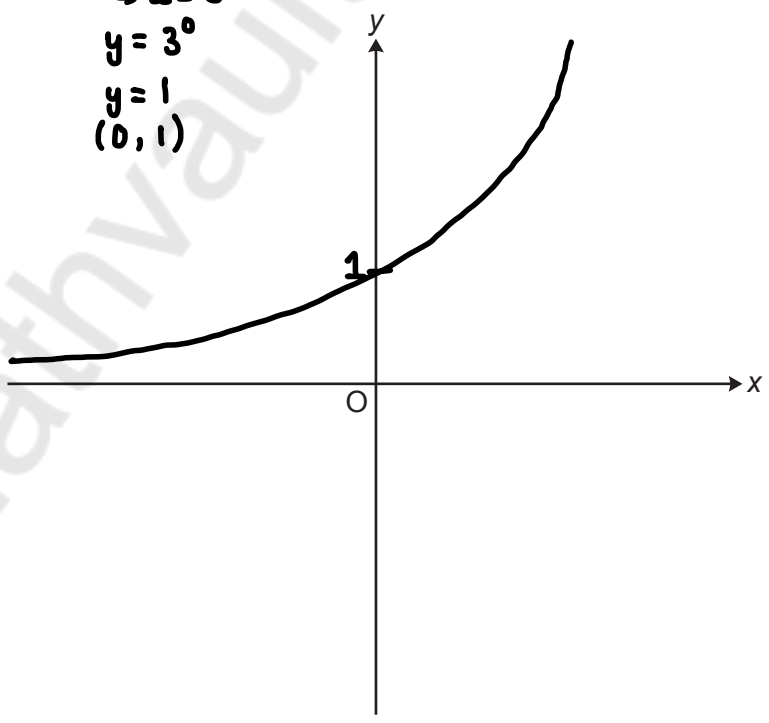
$$\frac{x+9 + 4(x-1)}{(x-1)(x+1)} \longrightarrow x+9+4x-4 \quad 5x+5$$

$$\frac{5x+5}{(x-1)(x+1)}$$

$$\frac{5(x+1)}{(x-1)(x+1)} = \frac{5}{x-1}$$

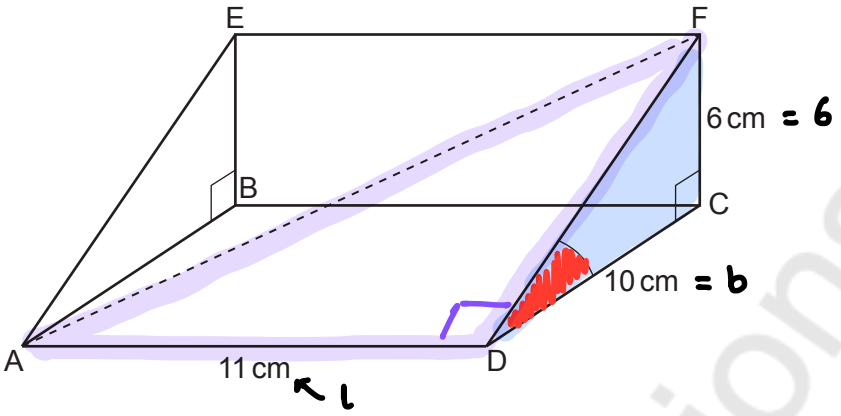
17 Sketch the graph of $y = 3^x$. Exponential asymptote = x-axis
Give the value of the y-intercept.

↳ $x=0$
$y = 3^0$
$y = 1$
$(0, 1)$



[2]

18 The diagram shows a right-angled triangular prism ABCDEF.



Length AD = 11 cm, length CD = 10 cm and length CF = 6 cm.

(a) Calculate the volume of the prism.

Vol = area cross-section × length

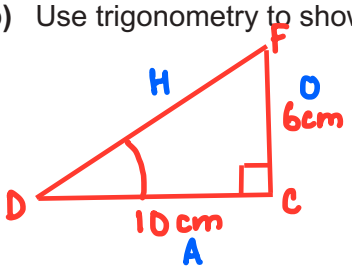
$= \frac{1}{2}bh \times l$

$= \frac{1}{2} \times 10 \times 6 \times 11$

$= 330$

(a) ..... 330 ..... cm^3 **[2]**

(b) Use trigonometry to show that angle FDC = 31°, correct to the nearest degree. **[2]**



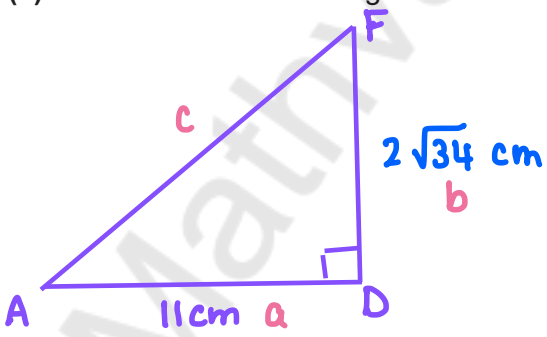
$\tan\theta = \frac{O}{A}$

$\tan FDC = \frac{6}{10}$

$FDC = \tan^{-1}\left(\frac{6}{10}\right)$

$= 30.963... \approx 31°$

(c) Calculate the exact length of AF.



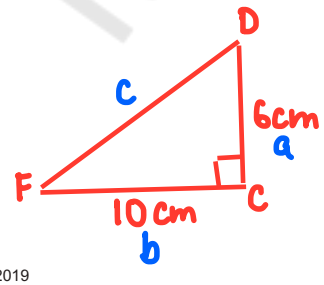
$a^2 + b^2 = c^2$

$11^2 + (2\sqrt{34})^2 = c^2$

$\sqrt{\quad}\qquad\sqrt{\quad}$

$\sqrt{11^2 + (2\sqrt{34})^2} = c$

$c = \sqrt{257}$



$a^2 + b^2 = c^2$

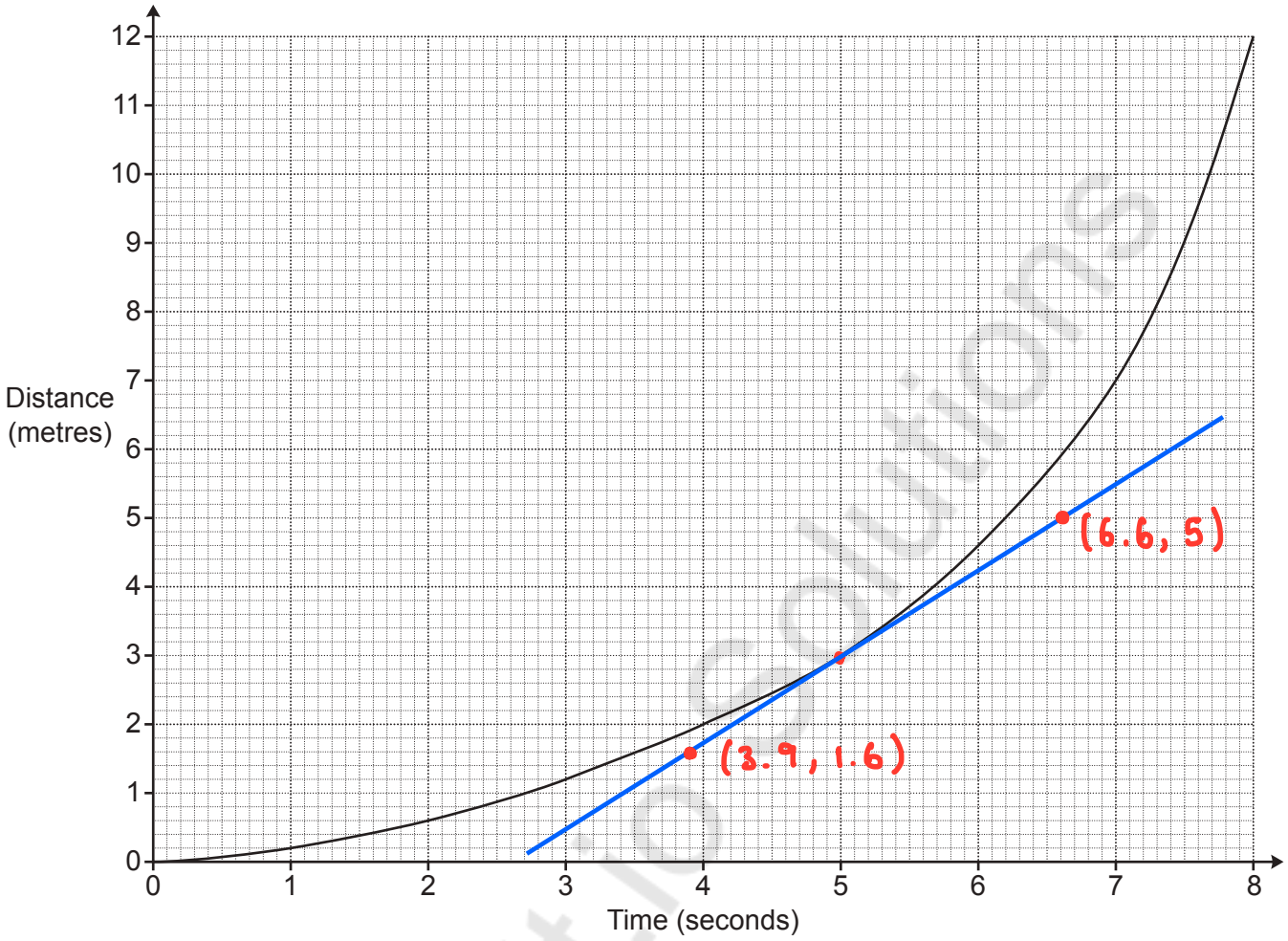
$6^2 + 10^2 = c^2$

$\sqrt{6^2 + 10^2} = c$

$2\sqrt{34} = c$

(c) ..... $\sqrt{257}$ ..... cm **[4]**

19 The graph shows the distance travelled by a particle over 8 seconds.



Estimate the speed of the particle at 5 seconds.

(3.9, 1.6) and (6.6, 5)

$$\frac{5 - 1.6}{6.6 - 3.9} = 1.\dot{2}5\dot{9}$$

$$\approx 1.26$$

..... 1.26 ..... m/s **[4]**

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).