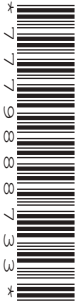


Monday 11 November 2019 – Afternoon

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes



You may use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

| | | | | |
|--|--|--|--|--|
| | | | | |
|--|--|--|--|--|

Candidate number

| | | | |
|--|--|--|--|
| | | | |
|--|--|--|--|

First name(s)

Last name

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer all the questions.

- 1 Solve $3x - 5 \geq 10$.
Show your solution on the number line.

$$\begin{array}{r} 3x - 5 \geq 10 \\ +5 \quad +5 \\ \hline 3x \geq 15 \\ \div 3 \quad \quad \div 3 \\ \hline x \geq 5 \end{array}$$

[4]

- 2 Amrit's income is 32% more than Bethan's income.
Amrit and Bethan's combined income is £54 868.

Calculate Amrit's income.

$$\text{Bethan} = 100\% \xrightarrow{\div 100} 1$$

$$\text{Amrits} = 100 + 32 = 132\% \xrightarrow{\div 100} 1.32$$

$$A : B$$

$$1.32 : 1$$

$$\text{Total} = 2.32$$

$$54868 \div 2.32 = 23650$$

$$\downarrow \times 23650$$

$$31218$$

£ 31218 [5]

3

1 2 3 4 5 6

- 3 Jacob, Amelie and Reuben each roll a fair six-sided dice.
What is the probability that all three roll a number less than 3?

Give your answer as a fraction in its simplest form.

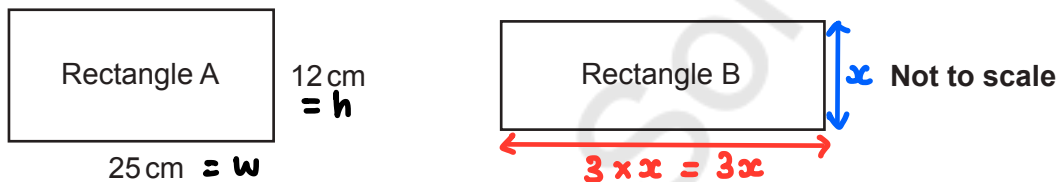
$$P(< 3) = \frac{2}{6} = \frac{1}{3}$$

$$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

$$\frac{1}{27}$$

[3]

- 4 The diagram shows two rectangles, A and B.



Rectangle A has a width of 25 cm and a height of 12 cm.
The width of rectangle B is three times the height of rectangle B.

The area of rectangle A is equal to the area of rectangle B.

Find the perimeter of rectangle B.

$$\begin{aligned} \text{Area of A} &= w \times h \\ &= 25 \times 12 \\ &= 300 \text{ cm}^2 \end{aligned}$$

$$\text{Area of B} = 300 \text{ cm}^2$$

$$300 = 3x \times x$$

$$300 = 3x^2$$

$$\div 3 \qquad \div 3$$

$$100 = x^2$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$10 = x$$

$$\text{Width} = 30$$

$$\text{Height} = 10$$

$$\begin{aligned} \text{Perimeter} &= 30 + 30 + 10 + 10 \\ &= 80 \end{aligned}$$

..... 80 cm [5]

Turn over

- 5 Kay invests £1500 in an account paying 3% **compound** interest per year.
Neil invests £1500 in an account paying $r\%$ **simple** interest per year.

At the end of the 5th year, Kay and Neil's accounts both contain the same amount of money.

Calculate r .

Give your answer correct to 1 decimal place.

Kay

$$\begin{aligned} \text{Final amount} &= \text{investment} \times \text{multiplier}^n \\ &= 1500 \times 1.03^5 \\ &= \pounds 1738.91 \end{aligned}$$

Multiplier
 $100\% + 3\% = 103\%$
 $1.03 \leftarrow \div 100$

Neil

$$\begin{aligned} \text{Interest} &= \pounds 1738.91 - \pounds 1500 \\ &= \pounds 238.91 \end{aligned}$$

$$\begin{aligned} \text{Interest per year} &= 238.91 \div 5 \\ &= \pounds 47.78 \end{aligned}$$

$$\begin{aligned} \% \text{ interest} &= \frac{47.78}{1500} \times 100 \\ &= 3.185\dot{3} \\ &\approx 3.2 \end{aligned}$$

$$r = \underline{3.2} \dots \dots \dots [6]$$

- 6 The table shows the children nominated to win the subject prize in Mathematics and the subject prize in English.

| Mathematics | English |
|-------------|---------|
| Alice | Alice |
| Ben | Claire |
| Emma | Gabi |
| Paddy | Simon |

The winner of each subject prize is picked at random.
It is possible for Alice to win both prizes.

In what percentage of the combinations of prize winners does Alice win **at least** one prize?

AA BA EA PA
 AC BC EC PC
 AG BG EG PG
 AS BS ES PS

= 16 combinations

$$\frac{7}{16} \times 100 = 43.75$$

..... 43.75 % [4]

7 (a) Complete the table for $y = x^2 - 4x + 1$.

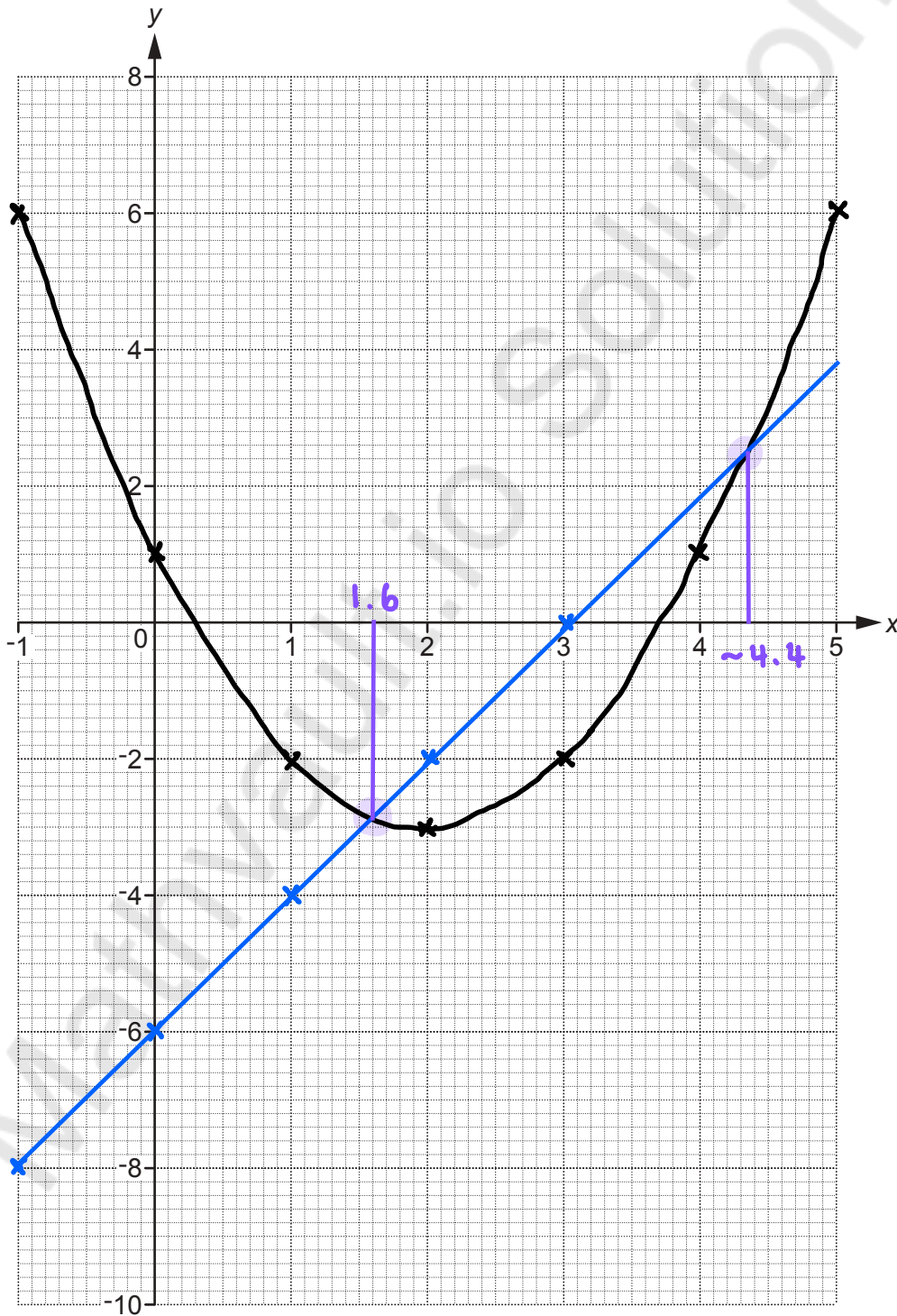
| | | | | | | | |
|---|----|---|----|----|----|---|---|
| x | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 6 | 1 | -2 | -3 | -2 | 1 | 6 |

$$y = (-1)^2 - 4(-1) + 1$$

$$y = (1)^2 - 4(1) + 1$$

$$y = (4)^2 - 4(4) + 1 \quad [2]$$

(b) Draw the graph of $y = x^2 - 4x + 1$ for $-1 \leq x \leq 5$.



[3]

(c) On the same grid, draw the graph of $y = 2x - 6$ for $-1 \leq x \leq 5$.

[3]

| | | | | | |
|-----|-----------------|----|----|----|---|
| x | -1 | 0 | 1 | 2 | 3 |
| y | -8 | -6 | -4 | -2 | 0 |
| | ↑ | | | | |
| | $y = 2(-1) - 6$ | | | | |

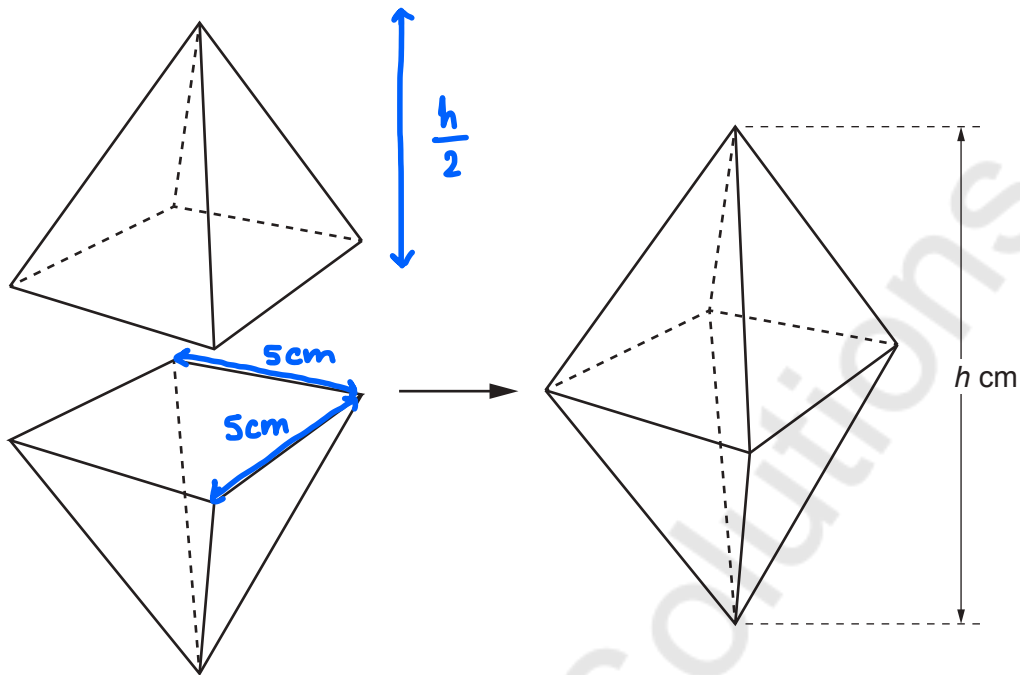
(d) Use your graphs to solve the equation $x^2 - 4x + 1 = 2x - 6$.

Give your answers to 1 decimal place.

Intersection points

(d) $x = \dots 1.6 \dots$ or $x = \dots 4.4 \dots$ [2]

- 8 An octahedron is formed from two identical square based pyramids. The square bases are stuck together as shown.



The volume of the octahedron is 60 cm^3 .
The length of the side of each pyramid's square base is 5 cm.

Work out the height h cm of the **octahedron**.

[The volume of a pyramid is $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$]

$$\text{Vol pyramid} = \frac{60}{2} = 30 \text{ cm}^3$$

$$30 = \frac{1}{3} \times (5 \times 5) \times \frac{h}{2}$$

$$30 = \frac{1}{3} \times \frac{25}{1} \times \frac{h}{2}$$

$$30 = \frac{25h}{6}$$

$$\times 6 \quad \times 6$$

$$180 = 25h \quad \div 25$$

$$\div 25$$

$$7.2 = h$$

$$h = \underline{7.2} \dots \dots \dots \text{ cm [4]}$$

9 Vector $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and vector $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(a) Find the values of k and n so that

$$k(\mathbf{a} + \mathbf{b}) = \begin{pmatrix} 10 \\ n \end{pmatrix}.$$

$$k \left[\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right] = \begin{pmatrix} 10 \\ n \end{pmatrix}$$

$$k \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ n \end{pmatrix}$$

$$\begin{array}{l} 4k = 10 \\ \div 4 \end{array}$$

$$k = 2.5$$

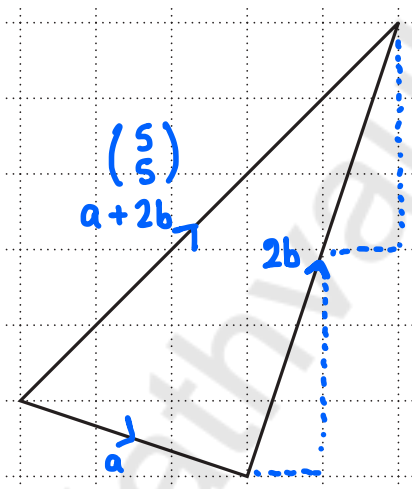
$$\begin{array}{l} 2k = n \\ 2(2.5) = n \end{array}$$

$$5 = n$$

(a) $k = 2.5$

$n = 5$ [3]

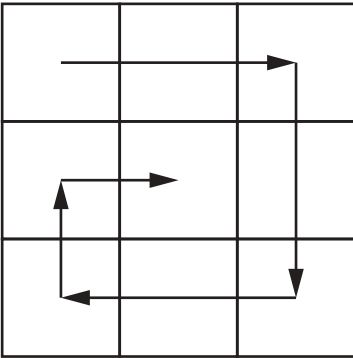
(b) Gavin starts to draw a diagram to show that $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$.



Complete Gavin's diagram.

[3]

- 10 Nine consecutive numbers are written on a 3-by-3 grid. They are arranged, in ascending order, in a spiral as shown.



- (a) Karen writes the numbers 3 to 11 on her grid.

| | | |
|----|----|---|
| 3 | 4 | 5 |
| 10 | 11 | 6 |
| 9 | 8 | 7 |

The total of the first column is $3 + 10 + 9 = 22$.

Karen says

The total of the first column is one less than the total of the second column.

Show that this is correct for Karen's grid.

[1]

$$\begin{aligned} \text{Total 2nd column} &= 4 + 11 + 8 \\ &= 23 \end{aligned}$$

$$22 = 23 - 1$$

(b) Victor says

If **any** nine consecutive numbers are arranged in ascending order in this spiral on a 3-by-3 grid, the total of the first column will **always** be one less than the total of the second column.

Prove that Victor is correct.

[5]

| | | |
|-------|-------|-------|
| x | $x+1$ | $x+2$ |
| $x+7$ | $x+8$ | $x+3$ |
| $x+6$ | $x+5$ | $x+4$ |

$$\begin{aligned} \text{Total 1st column} &= x + x+7 + x+6 \\ &= 3x+13 \end{aligned}$$

$$\begin{aligned} \text{Total 2nd column} &= x+1 + x+8 + x+5 \\ &= 3x+14 \end{aligned}$$

$$3x+14 - 3x+13 = 1$$

11 A sequence is defined by the rule $u_{n+1} = 5u_n - 15$.

(a) If $u_3 = 6$, calculate

(i) u_5

$$u_{3+1} = 5u_3 - 15$$

$$\begin{aligned} u_4 &= 5(6) - 15 \\ &= 30 - 15 \\ &= 15 \end{aligned}$$

$$u_{4+1} = 5u_4 - 15$$

$$\begin{aligned} u_5 &= 5(15) - 15 \\ &= 75 - 15 \\ &= 60 \end{aligned}$$

(a)(i) $u_5 = \underline{60}$ [3]

(ii) u_2

$$u_{2+1} = 5u_2 - 15$$

$$\begin{aligned} 6 &= 5u_2 - 15 \\ +15 &\quad +15 \\ 21 &= 5u_2 \\ \div 5 &\quad \div 5 \\ 4.2 &= u_2 \end{aligned}$$

(ii) $u_2 = \underline{4.2}$ [3]

(b) Trevor says

$$\text{If } u_1 = 3.75 \text{ then } u_{100} = 3.75$$

Show that Trevor is correct.

[2]

$$u_{n+1} = 5u_n - 15$$

$$\begin{aligned} u_2 &= 5(3.75) - 15 \\ &= 3.75 \end{aligned}$$

$$\begin{aligned} u_3 &= 5(3.75) - 15 \\ &= 3.75 \end{aligned}$$

⋮

$$u_{100} = 3.75$$

Since $u_1 = u_2$, all terms are equal.

- 12 (a) Arron ran a distance of 5 km at an average speed of 2.2 m/s.

How long did Arron run for?

Give your answer in minutes and seconds, to the nearest second.

$$T = \frac{D}{S}$$

$$1000\text{m} = 1\text{km}$$

↖
x1000

$$5\text{km} = 5000\text{m}$$

$$D = 5000\text{m}$$

$$S = 2.2\text{ m/s}$$

$$T = \frac{5000}{2.2}$$

$$= 2272.7\dot{2}\text{ seconds}$$

D
S T

$$60\text{ sec} = 1\text{m}$$

↖
÷60

$$2272.7\dot{2} \div 60$$

$$37.8\dot{7}\text{ mins}$$

$$\downarrow \times 60$$

$$52.7\dot{2}\text{s} \approx 53$$

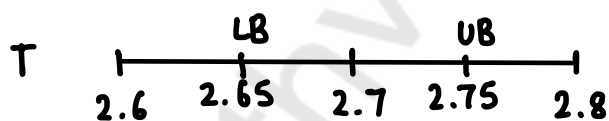
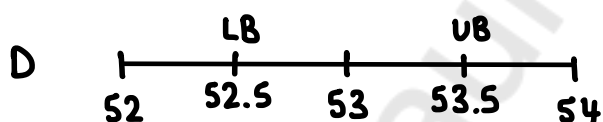
(a)37..... minutes53..... seconds [4]

- (b) Claudine cycled a distance of 53 km in 2.7 hours.
The distance is measured correct to the nearest km.
The time is given correct to 1 decimal place.

$$\frac{D}{S T} \quad S = \frac{D}{T}$$

Calculate the lower and upper bounds of her average speed.

Give your answers correct to 2 decimal places.



$$S_{UB} = \frac{D_{UB}}{T_{LB}}$$

$$= \frac{53.5}{2.65}$$

$$= 20.188\dots$$

$$\approx 20.19$$

$$S_{LB} = \frac{D_{LB}}{T_{UB}}$$

$$= \frac{52.5}{2.75}$$

$$= 19.0\dot{9}$$

$$\approx 19.09$$

(b) lower bound =19.09..... km/h

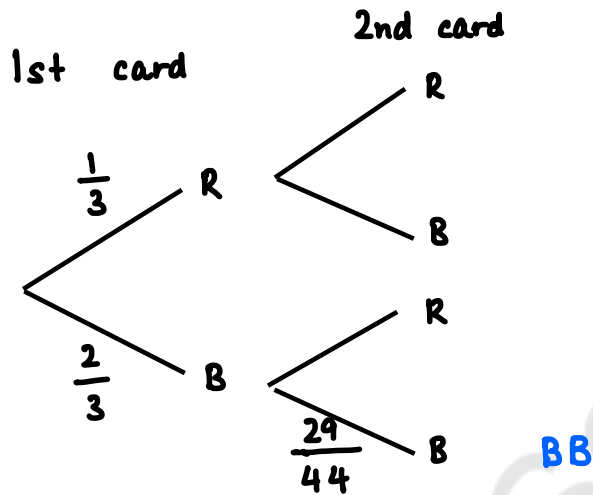
upper bound =20.19..... km/h [6]

- 13 Dani has a pack of 45 cards.
Each card is either red or black.

One-third of the cards in the pack are red. $p(\text{red}) = \frac{1}{3}$ $p(\text{black}) = \frac{2}{3}$

She picks two cards from the pack, **without replacement**.

Calculate the probability that Dani picks **two black cards**.



$$\text{Red} = \frac{1}{3} \times 45$$

$$= 15$$

$$\text{Black} = 45 - 15$$

$$= 30$$

$$p(\text{BB}) = \frac{2}{3} \times \frac{29}{44}$$

$$= \frac{29}{66}$$

$$\frac{29}{66}$$

..... [5]

- 14 Write $(\sqrt[4]{8})^5$ as a power of 2.

$$8 = 2 \times 2 \times 2$$

$$= 2^3$$

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

$$(\sqrt[4]{2^3})^5$$

$$\left((2^3)^{\frac{1}{4}} \right)^5$$

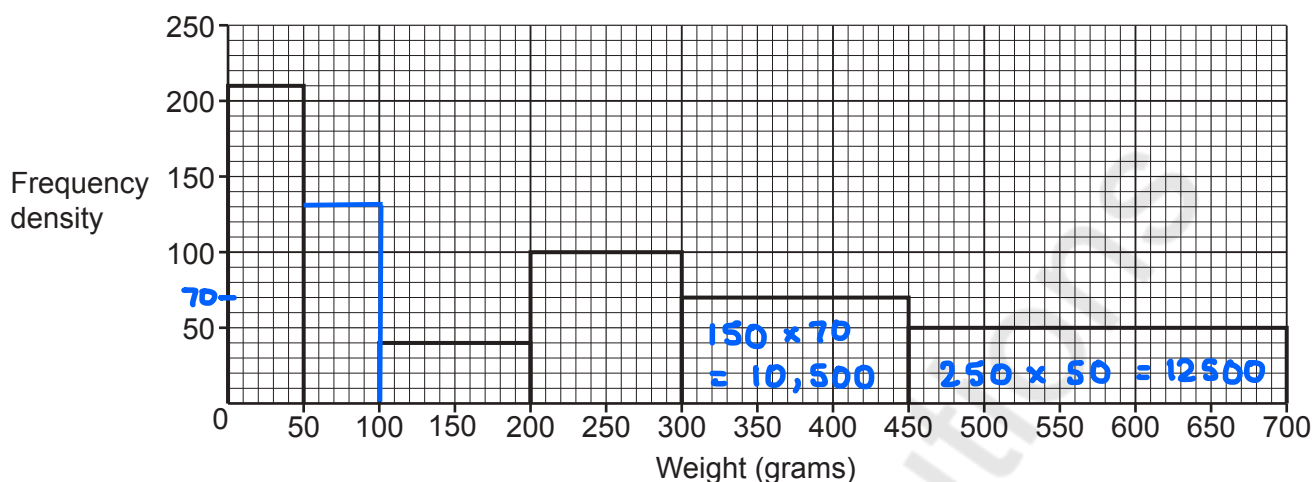
$$\left(2^{\frac{3}{4}} \right)^5$$

$$2^{\frac{15}{4}}$$

$$2^{\frac{15}{4}}$$

..... [3]

- 15 The histogram shows information about the weights of some of the parcels handled by a delivery company in one month.



- (a) Zoe says

There are fewer parcels weighing between 450g and 700g than parcels weighing between 300g and 450g.

Is Zoe correct?

Show how you decide.

$$\text{Freq.} = \text{cw} \times \text{fd}$$

No. There are 12,500 parcels between 450g and 700g, and 10,500 between 300g and 450g.

[4]

- (b) The delivery company delivered 6500 parcels weighing between 50g and 100g.

Complete the histogram to show this information.

$$\begin{aligned} \text{fd} &= \frac{f}{\text{cw}} \\ &= \frac{6500}{50} \\ &= 130 \end{aligned}$$

[2]

- (c) Zoe uses the histogram to calculate the number of parcels weighing between 200g and 250g.

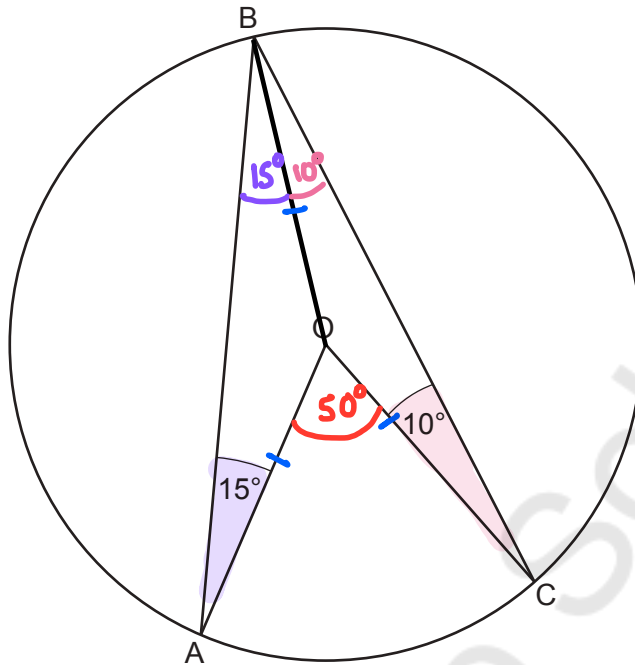
Explain why Zoe's answer is unlikely to be reliable.

The weights of parcels may not be evenly distributed between 200g and 250g.

[1]

16 (a) In the diagram,

- A, B and C are points on the circumference of a circle
- O is the centre of the circle
- angle OAB = 15°
- angle BCO = 10° .



Calculate the acute angle AOC.

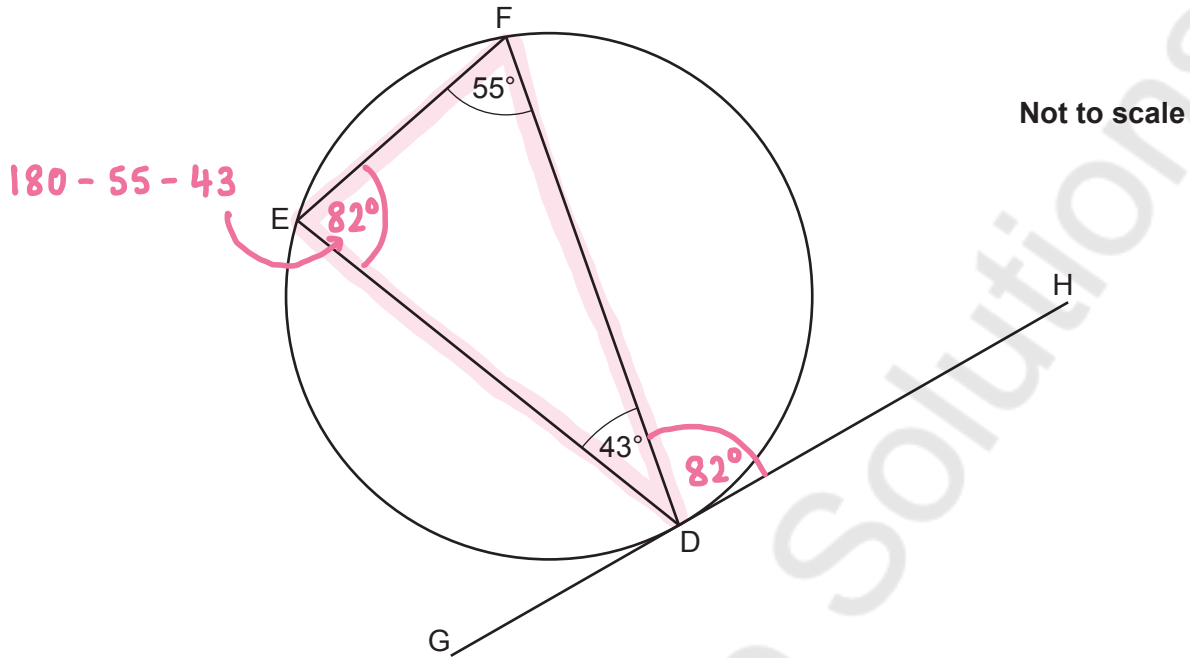
$$\begin{aligned} \angle ABC &= 10 + 15 \\ &= 25^\circ \end{aligned}$$

$$\begin{aligned} \angle AOC &= 2 \times 25^\circ \\ &= 50^\circ \end{aligned}$$

(a) 50 ° [4]

(b) In the diagram,

- E, F and D are points on the circumference of the circle
- G, D and H lie on a tangent to the circle
- angle EFD = 55°
- angle FDE = 43° .

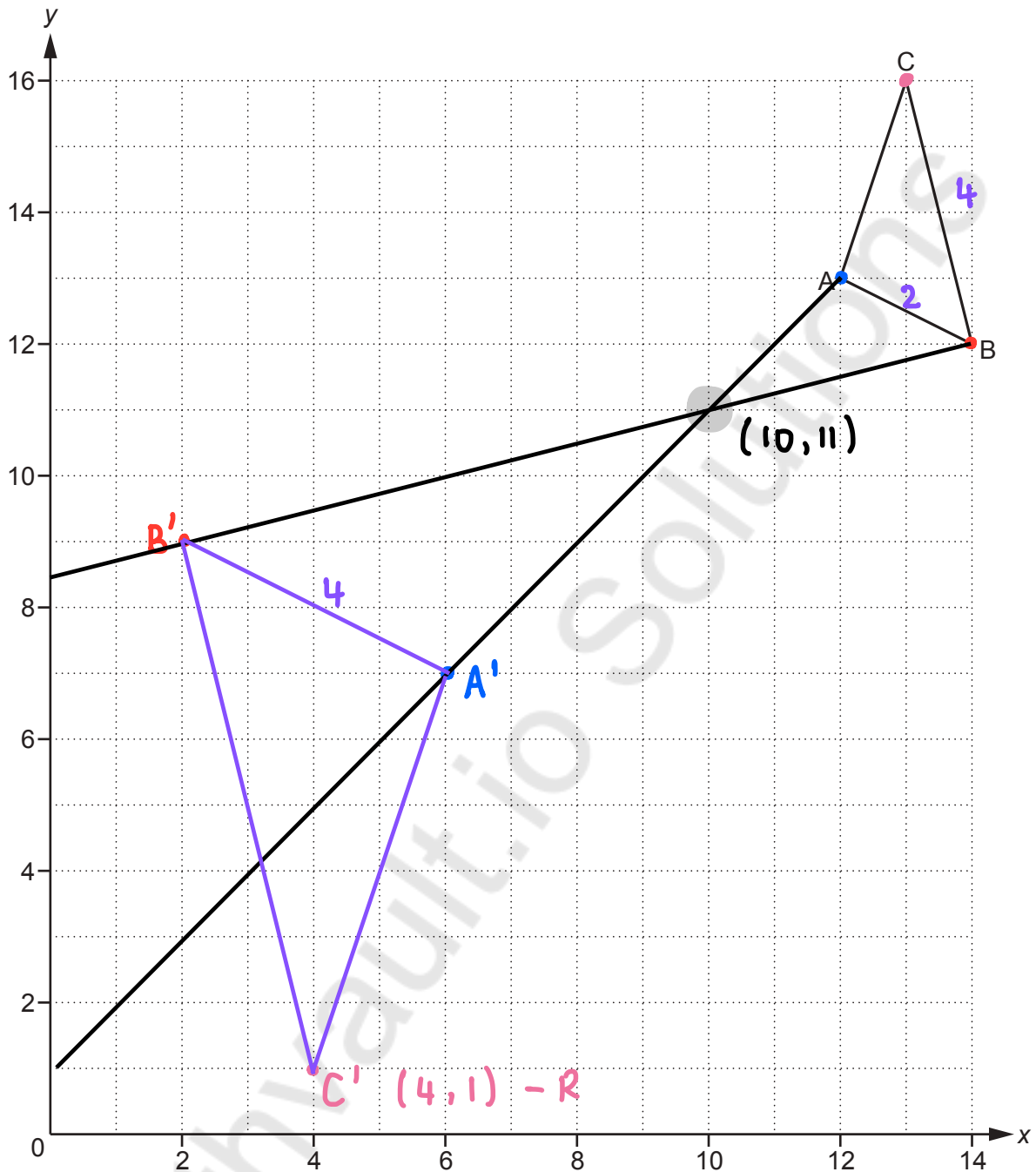


Explain why angle HDF is 82° .

Angle DEF = Angle HDF - Alternate segment theorem

[4]

17 A triangle has vertices A, B and C.



The triangle is enlarged with scale factor f and centre of enlargement E.

Vertex A maps to (6, 7).

Vertex B maps to (2, 9).

(a) Find the coordinates of the centre of enlargement, E.

(a) (.....,) [2]

(b) Find the scale factor, f .

(b) ⁻² [2]

(c) Vertex C maps to the point R.
Find the coordinates of R.

(c) (..... ⁴, ¹) [2]

END OF QUESTION PAPER

