OCR
Oxford Cambridge and RSA

H

Tuesday 2 November 2021 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

8208779593

J560/04

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start to write your answer.

Answer **all** the questions.

1 Calculate.

(a) $(6^2 + 5)^3$

(a) 68921 [1]

(b) $\sqrt{\dfrac{8.4^2 - 1.9^2}{2.5 + 5.7}}$

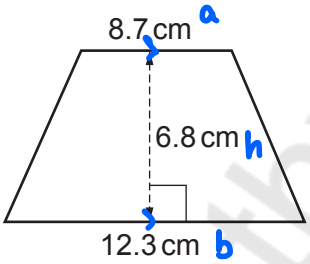
Write your answer correct to **3** significant figures.

2.857382394

≈ 2.86 (3sf)

(b) 2.86 [3]

2 Calculate the area of this trapezium.



Not to scale

$A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8.7 + 12.3) \times 6.8$

$= 71.4\ cm^2$

71.4 cm^2 [2]

3 Simplify.

$x^{12} \div x^4$

x^8 ..... **[1]**

4 Li throws two fair four-sided dice, each numbered 1, 2, 3 and 4.
Li multiplies together the two numbers that the dice land on to produce a score.

Find the probability that Li's score is a prime number.

Dice 1

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Dice 2

$= \frac{4}{16}$

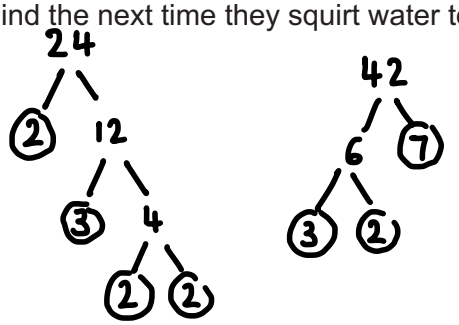
$\frac{4}{16}$ ..... **[4]**

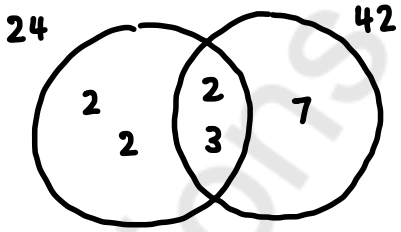
5 **(a)** Fountain A squirts water every 24 minutes.
Fountain B squirts water every 42 minutes.
They squirt water together at 15:19.

Find the next time they squirt water together.

24 = 2 × 2 × 2 × 3

42 = 2 × 3 × 7





LCM = 2 × 2 × 2 × 3 × 7
= 168 mins

60 | 168 = 2 r 48 = 2h 48 mins

15:19 + 2h = 17:19 + 41 mins = 18:00 + 7 mins = 18:07

(a) 18:07 **[4]**

(b) A school sends 60 students from Year 8 and 105 students from Year 9 to a museum.

The school divides these students into groups using the following rules.

- The groups must all be the same size.
- All students in any group must be from the same year.
- There should be as few groups as possible.

Find the size of each group and the total number of groups.

60 = 2 × 2 × 3 × 5

105 = 3 × 5 × 7

HCF = 3 × 5
= 15

Year 8 = 60 ÷ 15 = 4 groups

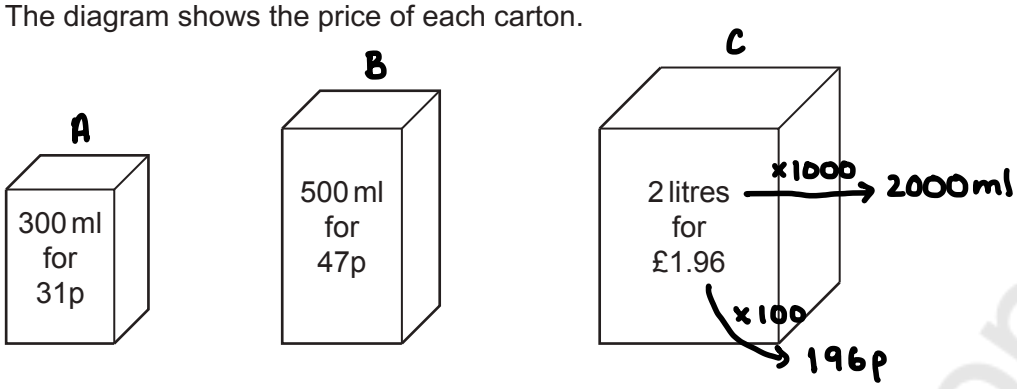
Year 9 = 105 ÷ 15 = 7 groups

Total = 11 groups

Size of each group = 15

Total number of groups = 11 **[4]**

6 A shop sells the same milk in three different sized cartons.
The diagram shows the price of each carton.



(a) Which carton is the best value for money?
Show how you decide.

A 300ml = 31p
÷3 ÷3
100ml = 10.3̇p

B 500ml = 47p
÷5 ÷5
100ml = 9.4p

C 2000ml = 196p
÷20 ÷20
100ml = 9.8p

.....B.....

.....

..... **[3]**

(b) A student only buys milk on a Saturday morning.
They use 120 ml of milk each day.
Any unused milk has to be thrown away at the end of the following Friday.

Show that it is cheaper for the student to buy the milk they need in 300 ml cartons than in 500 ml cartons. **[3]**

Sat - Fri = 7 days / mornings

120ml × 7 = 840ml

300ml cartons

840ml ÷ 300ml = 2.8
≈ 3

31p × 3 = 93p

500ml cartons

840ml ÷ 500ml = 1.68
≈ 2

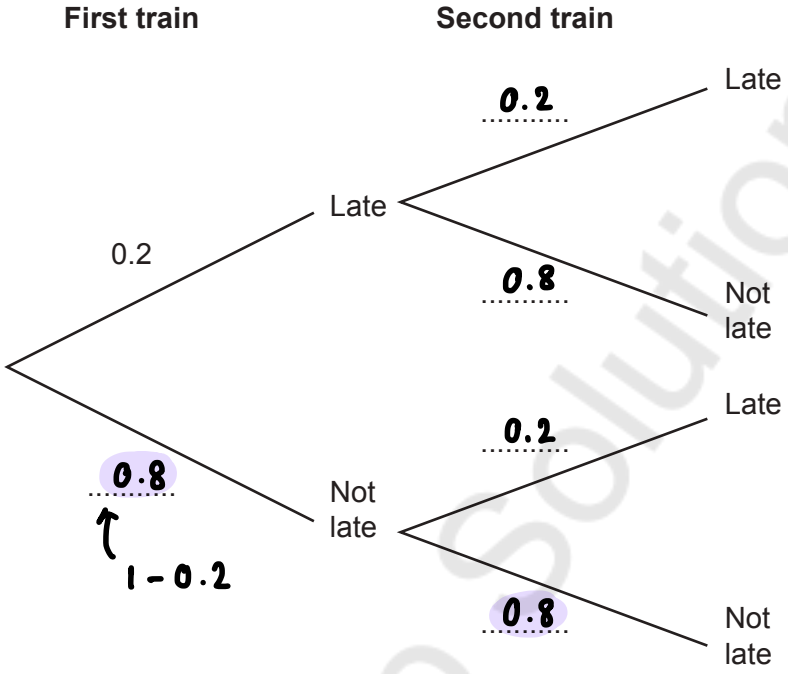
47p × 2 = 94p

93p < 94p

7 **(a)** Over a long period of time, it is found that the probability of a train from Bewford to London being late is 0.2.

(i) One morning there are two trains from Bewford to London.

Use the information to complete the tree diagram.



[2]

(ii) Work out the probability that both trains are **not late**.

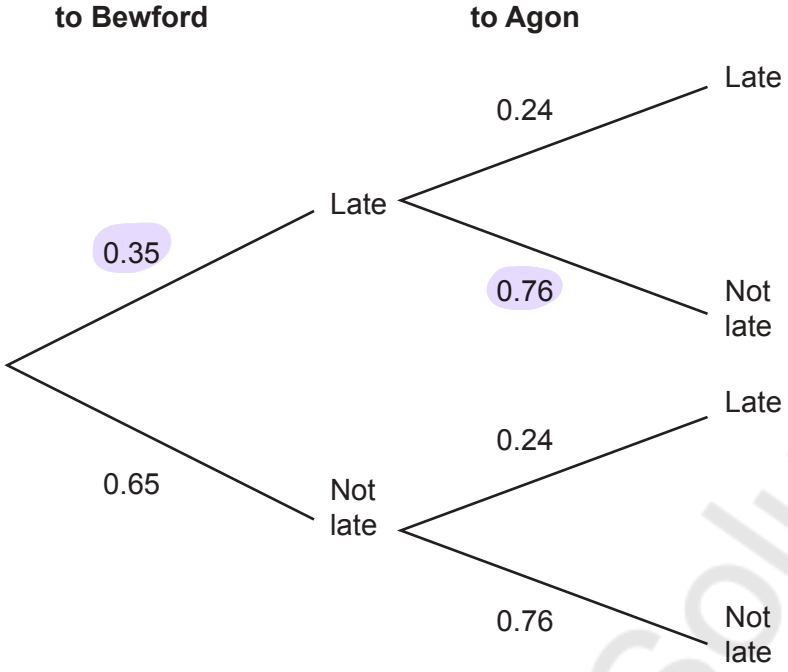
$0.8 \times 0.8 = 0.64$

(a)(ii) 0.64 **[2]**

(iii) Give a reason why the probabilities used in the tree diagram for the second train may **not** be reliable.

The first train being late might affect the second train. **[1]**

(b) Morgan takes a train from London to Bewford and then another train to Agon.
The tree diagram shows the probabilities of Morgan's trains being late or not late.



Morgan will **not catch** the train to Agon if the train to Bewford is late and the train to Agon is not late.

Work out the probability that Morgan will **catch** the train to Agon.

p (not catch) = 0.35 × 0.76
= 0.266

p (catch) = 1 − 0.266
= 0.734

(b) ..... 0.734 ..... **[3]**

8 Jamie invests £6000 at a simple interest rate of r% each year.
After 6 years the value of their investment is £7170.

Find the value of r.

7170 - 6000 = £1170 interest

£1170 ÷ 6 = £195 interest per year

$\frac{195}{6000} \times 100 = 3.25$

$r =$ 3.25 **[4]**

9 The price of a plane ticket is increased by 15% to £1426.

Find the original price of the plane ticket.

100% + 15% = 115%

115% = £1426 ÷1.15

÷1.15 100% = £1240

£ 1240 **[3]**

10 Alex, Blake and Charlie play a computer game.

Alex goes first and scores n points.

- Blake scores 8 points less than 3 times the number of points scored by Alex.
- Charlie scores 25 more points than Blake.
- The three people score a total of 618 points.

Work out how many points they each score.
You must show your working.

Alex = n

Blake = $n \times 3 - 8$
= $3n - 8$

Charlie = $3n - 8 + 25$
= $3n + 17$

$n + 3n - 8 + 3n + 17 = 618$

$7n + 9 = 618$
$-9 \quad -9$

$7n = 609$
$\div 7 \quad \div 7$

$n = 87$

Alex = 87

Blake = $3(87) - 8$
= 253

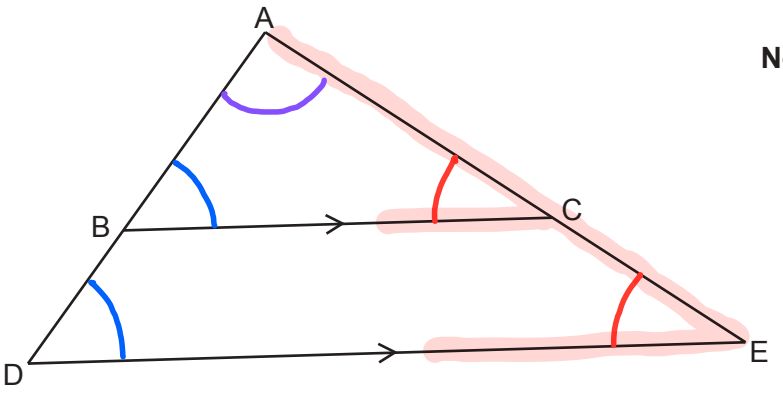
Charlie = $3(87) + 17$
= 278

Alex = 87

Blake = 253

Charlie = 278 **[7]**

11 The diagram shows triangles ABC and ADE.



Not to scale

B lies on AD and C lies on AE.
BC is parallel to DE.

Complete these statements to show that triangles ABC and ADE are similar.

Angle ABC = angle ADE because they are corresponding angles.

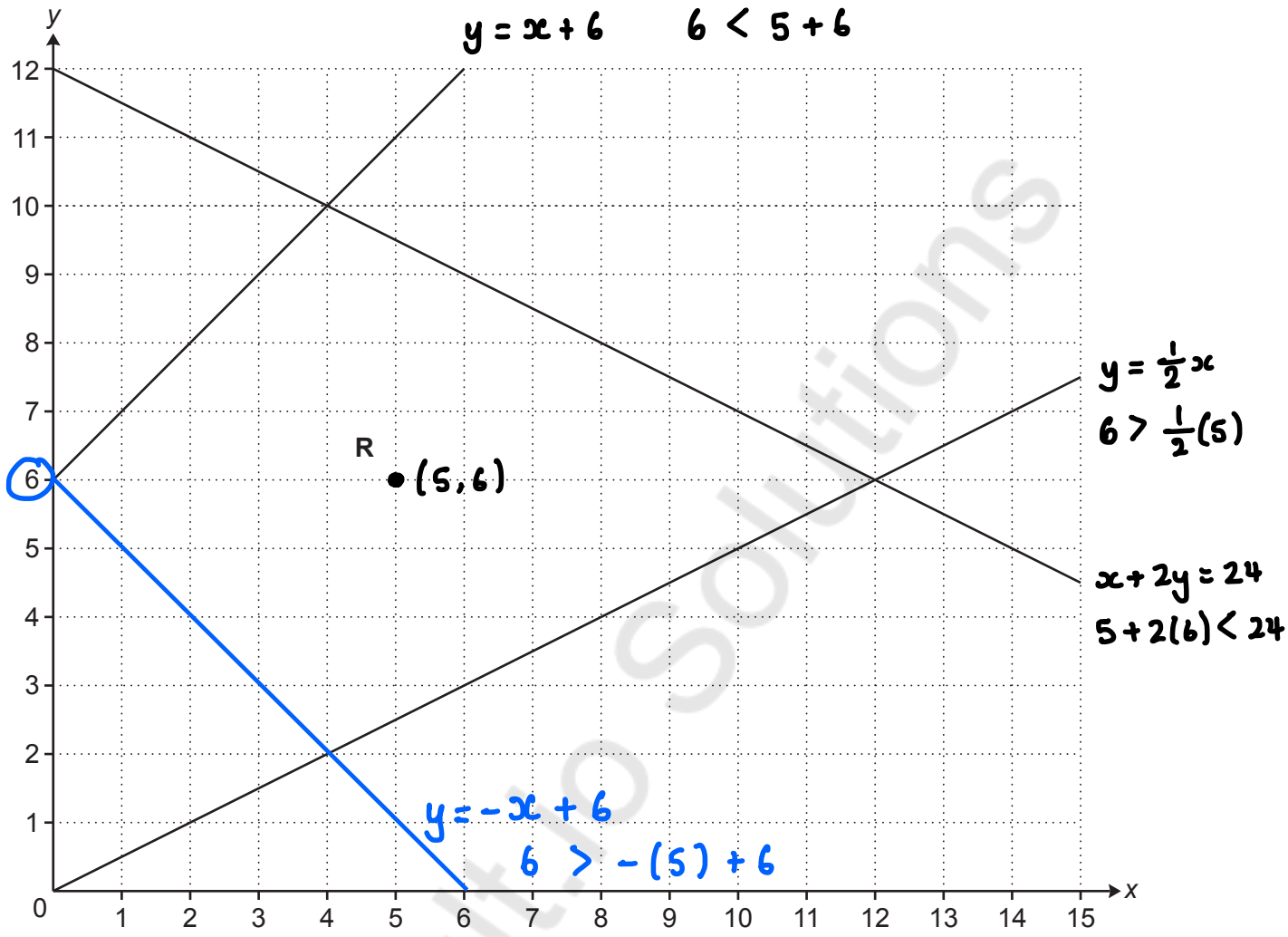
Angle ACB = angle AED because they are corresponding angles

Angle BAC is shared

Triangles ABC and ADE are similar because they have the same angles.

[3]

12 **(a)** The region **R** is shown on this grid.



Region **R** is defined by four inequalities.
One of the inequalities is $x \geqslant 0$.

Use the symbols $\leqslant$ and $\geqslant$ to complete the other three inequalities.

$x \geqslant 0$

$y \geqslant \frac{1}{2}x$

$x + 2y \leqslant 24$

$y \leqslant x + 6$

$x + 2y = 24$

$2y = -x + 24$

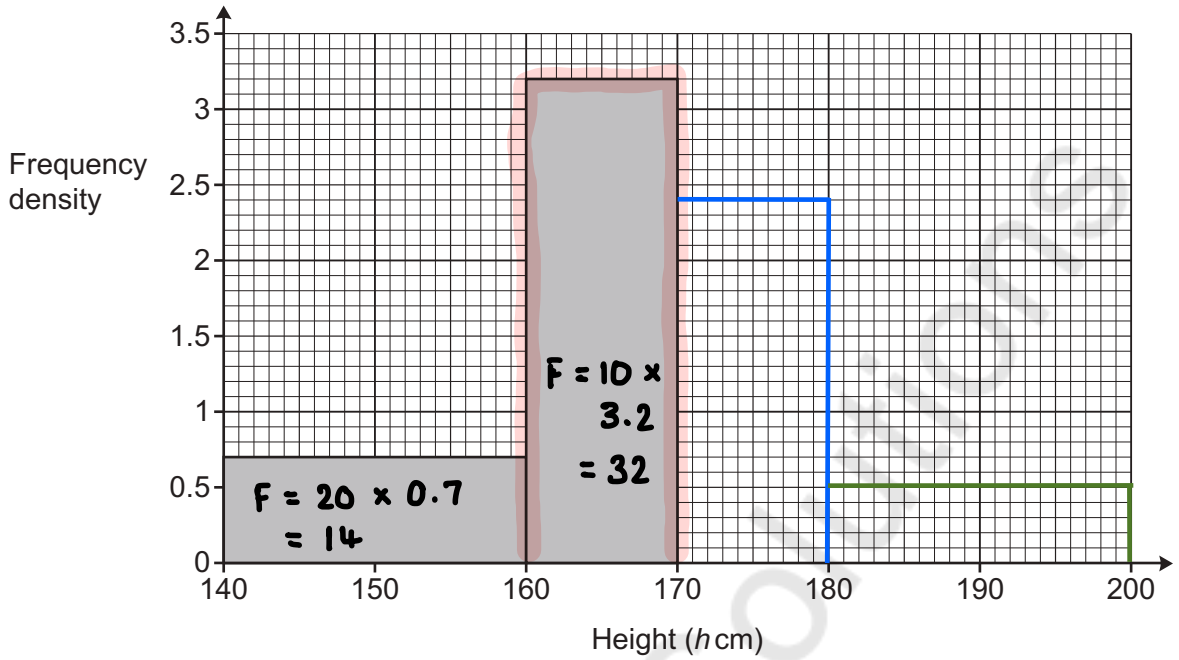
[2]

(b) The inequality $x \geqslant 0$ is replaced by a new inequality.
Region **R** is then a kite.

Write down the new inequality.

(b) $y \geqslant -x + 6$ **[3]**

13 The height, h cm, of each member of a tennis club is recorded.
The histogram shows some of the results.



40% of the members have a height in the interval $160 \leqslant h < 170$.
30% of the members have a height in the interval $170 \leqslant h < 180$.
100% of the members have a height in the interval $140 \leqslant h < 200$.

F
CW FD

Complete the histogram for the intervals $170 \leqslant h < 180$ and $180 \leqslant h < 200$. **[6]**

40% = 32
÷4 ÷4
10% = 8
×3 ×3
30% = 24

10% = 8
100% = 80
80 − 32 − 24 − 14 = 10

$FD = \frac{F}{CW}$
$= \frac{24}{10}$
$= 2.4$

$FD = \frac{10}{20}$
$= 0.5$

14 Find the coordinates of the turning point of the graph of $y = x^2 + 6x + 17$.

$(x + 3)^2 - 3^2 + 17$

$(x + 3)^2 - 9 + 17$

$(x + 3)^2 + 8$

$(x + 3)^2 = 0$

$\sqrt{} \quad \sqrt{}$

$x + 3 = 0$

$-3 \quad -3$

$x = -3$

(......-3...... , ...8......) **[4]**

15 Here are the first four terms of a quadratic sequence.

-1 $\quad$ 3 $\quad$ 13 $\quad$ 29

+4 $\quad$ +10 $\quad$ +16

+6 $\quad$ +6

The *n*th term is $an^2 + bn + c$.

Find the values of *a*, *b* and *c*.

$a = 6 \div 2 = 3$

n^2	1	4	9	16
an^2	3	12	27	48
Seq	-1	3	13	29
Seq - an^2	-4	-9	-14	-19

× 3

-5 +1 $\quad$ -10 +1 $\quad$ -15 +1 $\quad$ -20 +1

-5 $\quad$ -5 $\quad$ -5

$-5n + 1$

↑ b $\quad$ ↑ c

a = ...3......

b = ...-5......

c = ...1...... **[4]**

16 The formula

$P = 6800 \times 1.045^n$

is used to predict the population, P, of an island n years after 2018.

(a) Write down the population of the island in 2018.

$n = 0$

6800×1.045^0

6800×1

(a) ..... 6800 ..... **[1]**

(b) Write down the percentage growth rate used in the formula.

$1.045 \times 100 = 104.5\%$

(b) ..... 4.5% ..... % **[1]**

(c) **(i)** Work out the population predicted by the formula for the year 2030.

$n = 2030 - 2018$

$= 12$

$P = 6800 \times 1.045^{12}$

$= 11531.99374$

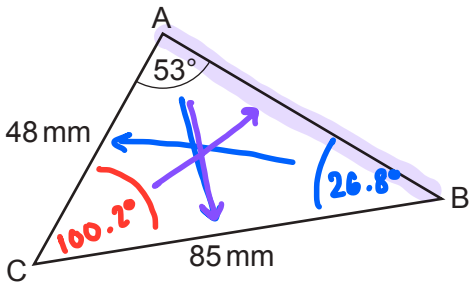
≈ 11532

(c)(i) ..... 11532 ..... **[2]**

(ii) Give **one** reason why the answer to **(c)(i)** may **not** be reliable.

The population may not continue to increase at this same rate. ..... **[1]**

17 The diagram shows triangle ABC.



Not to scale

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

AC = 48 mm, BC = 85 mm and angle BAC = 53°.

Calculate length AB.
You must show your working.

∠ ABC

$$\frac{\sin ABC}{48} = \frac{\sin 53}{85}$$

× 48 × 48

$$\sin ABC = \frac{\sin 53}{85} \times 48$$

$$\sin ABC = 0.450994170\ldots$$

$$ABC = \sin^{-1}(\ )$$

$$= 26.8074868^\circ$$

∠ ACB = 180 − 53 − 26.8…

= 100.1925132°

Length AB

$$\frac{AB}{\sin(100.19\ldots)} = \frac{85}{\sin 53}$$

$$AB \sin 53 = 85 \sin(100.19\ldots)$$

÷ sin 53 ÷ sin 53

$$AB = \frac{85 \sin(100.19\ldots)}{\sin 53}$$

= 104.7519072

≈ 105 mm

105 ..... mm **[6]**

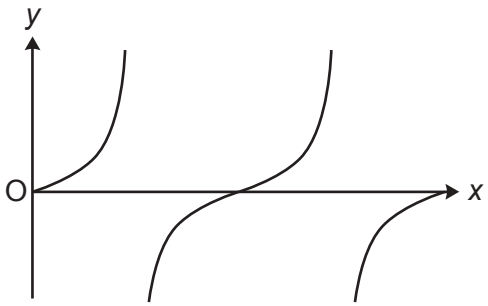
18 (a) For each graph below, select its possible equation from this list.

$y = x$ $\quad$ $y = x^2$ $\quad$ $y = \frac{1}{x}$

$y = \sin x$ $\quad$ $y = \cos x$ $\quad$ $y = \tan x$

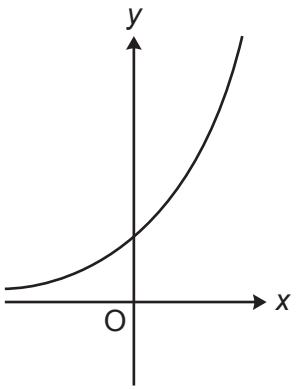
$y = 3^x$ $\quad$ $y = \left(\frac{1}{3}\right)^x$

(i)



(a)(i) $y =$ tan x ..... **[1]**

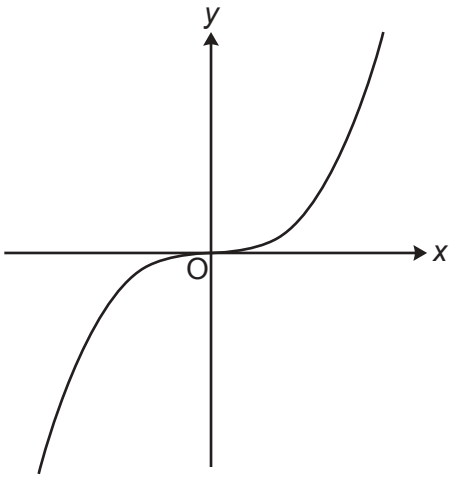
(ii)



$y = a^x$

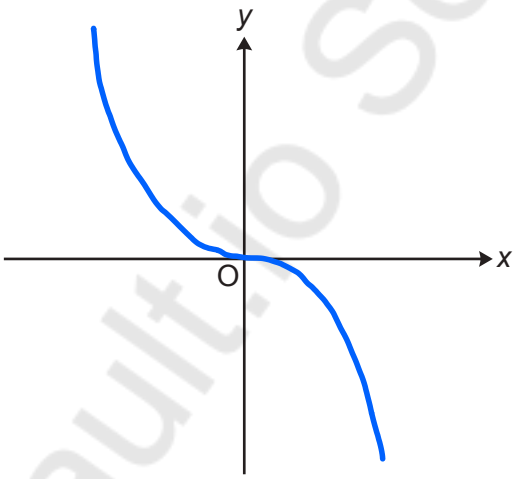
(ii) $y =$ 3^x ..... **[1]**

(b) Here is a sketch of $y = x^3$.



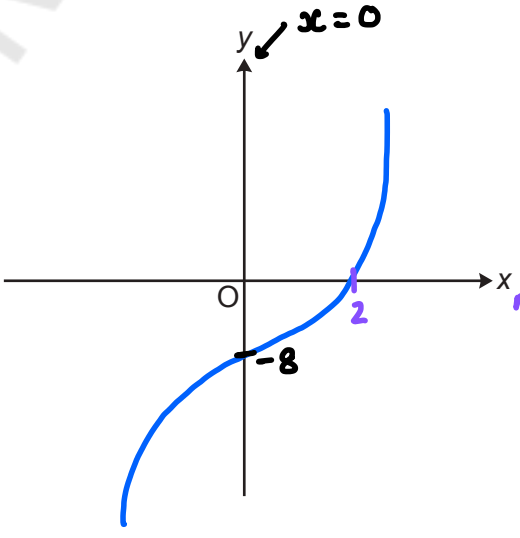
On the axes below, sketch the graphs of

(i) $y = -x^3$



[1]

(ii) $y = x^3 - 8$, showing the values of any intercepts with the axes.



$x = 0$

$y = (0)^3 - 8$

$y = -8$

$y = 0$

$x^3 - 8 = 0$

$+8 \quad +8$

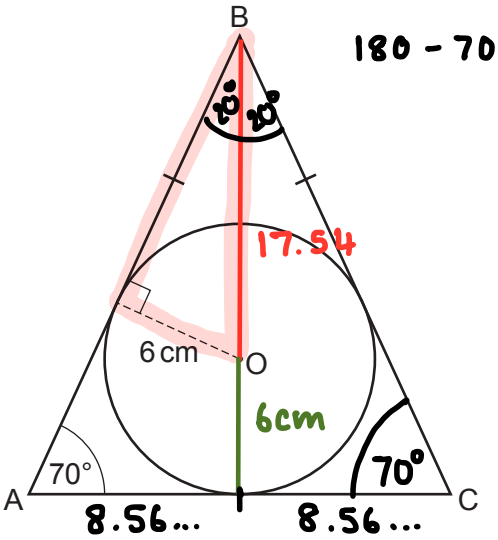
$x^3 = 8$

$\sqrt[3]{\quad} \quad \sqrt[3]{\quad}$

$x = 2$

[3]

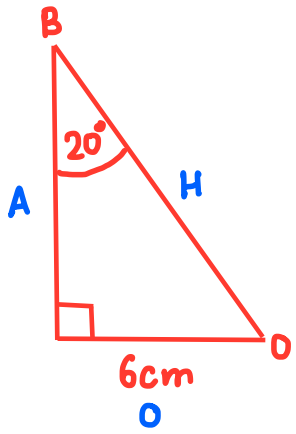
19 ABC is an isosceles triangle.
The sides of the triangle ABC are all tangents to a circle of radius 6 cm, centre O.



Not to scale

Angle BAC = 70° and BA = BC.

(a) Show that length BO is 17.54 cm, correct to **2** decimal places. **[4]**



SOH CAH TOA

$\sin\theta = \frac{O}{H}$

$\sin 20 = \frac{6}{BO}$

$\times BO \quad \times BO$

$BO \sin 20 = 6$

$\div \sin 20 \quad \div \sin 20$

$BO = 6 \div \sin 20$

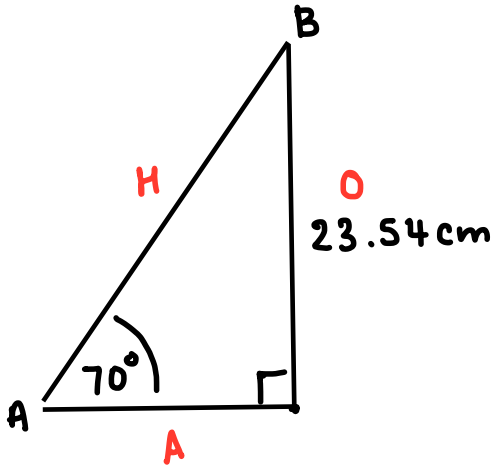
$= 17.5428264$

≈ 17.54 (2dp)

(b) Find the area of triangle ABC.
You must show your working.

Area = $\frac{1}{2}bh$

$h = 17.54 + 6$
$= 23.54$ cm



SOH CAH TOA ✓

$\tan\theta = \frac{O}{A}$

$\tan 70 = \frac{23.54}{A}$ ×A

×A

$A\tan 70 = 23.54$

$\div \tan 70 \quad \div \tan 70$

$A = 23.54 \div \tan 70$
$= 8.56888804$ cm

Base = A × 2
= 8.56... × 2
= 17.13777608 cm

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 17.1... \times 23.54$
$= 201.7116245$
≈ 201.7 cm² (1dp)

201.7 cm² **[5]**

20 Solve algebraically.

$$y = x + 3$$
$$(x-3)^2 + y^2 = 50$$

You must show your working.

$(x-3)^2 + (x+3)^2 = 50$

$(x-3)(x-3) + (x+3)(x+3) = 50$

$x^2 - 3x - 3x + 9 + x^2 + 3x + 3x + 9 = 50$

$2x^2 + 18 = 50$

$-18 \quad -18$

$2x^2 = 32$

$\div 2 \qquad \div 2$

$x^2 = 16$

$\sqrt{} \quad \sqrt{}$

$x = \pm 4$

$y = 4 + 3$

$= 7$

$y = -4 + 3$

$= -1$

$x =$ 4 $y =$ 7

$x =$ −4 $y =$ −1 **[5]**

END OF QUESTION PAPER