

Friday 20 May 2022 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes



You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Write 6 050 000 in standard form.

$$6.05 \times 10^6$$

(a) 6.05×10^6 [1]

- (b) Write 4.58×10^{-3} as an ordinary number.

$$0.00458$$

(b) 0.00458 [1]

- 2 Calculate.

$$\frac{270}{2.5^2} - \frac{4.6 + 17.2}{8.4 - 6.8}$$

..... 29.575 [2]

- 3 In January 2018, an art collector bought an antique painting.
In January 2020, he sold it for £17 640.

Assume the value of the painting increased by 5% each year.

Compound interest

Calculate the art collector's profit.

You must show your working.

$$\text{Initial} \times \text{multiplier}^n = \text{final amount}$$

no. of years

$$x \times 1.05^2 = 17640$$

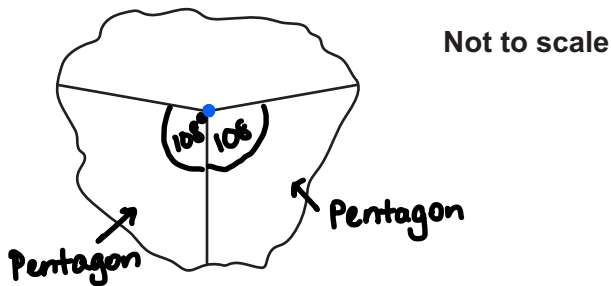
$$x = \frac{17640}{1.05^2}$$

$$= 16000$$

$$\begin{aligned} \text{Profit} &= 17640 - 16000 \\ &= 1640 \end{aligned}$$

£ 1640 [5]

- 4 Three **regular** polygons meet at a point.



Two of the polygons are pentagons.

Find the number of sides of the third polygon.
You must show your working.

$$\begin{aligned}
 \text{Sum of interior angles} &= (n-2) \times 180 && \swarrow \text{no. of sides} \\
 &= (5-2) \times 180 \\
 &= 3 \times 180 \\
 &= 540^\circ
 \end{aligned}$$

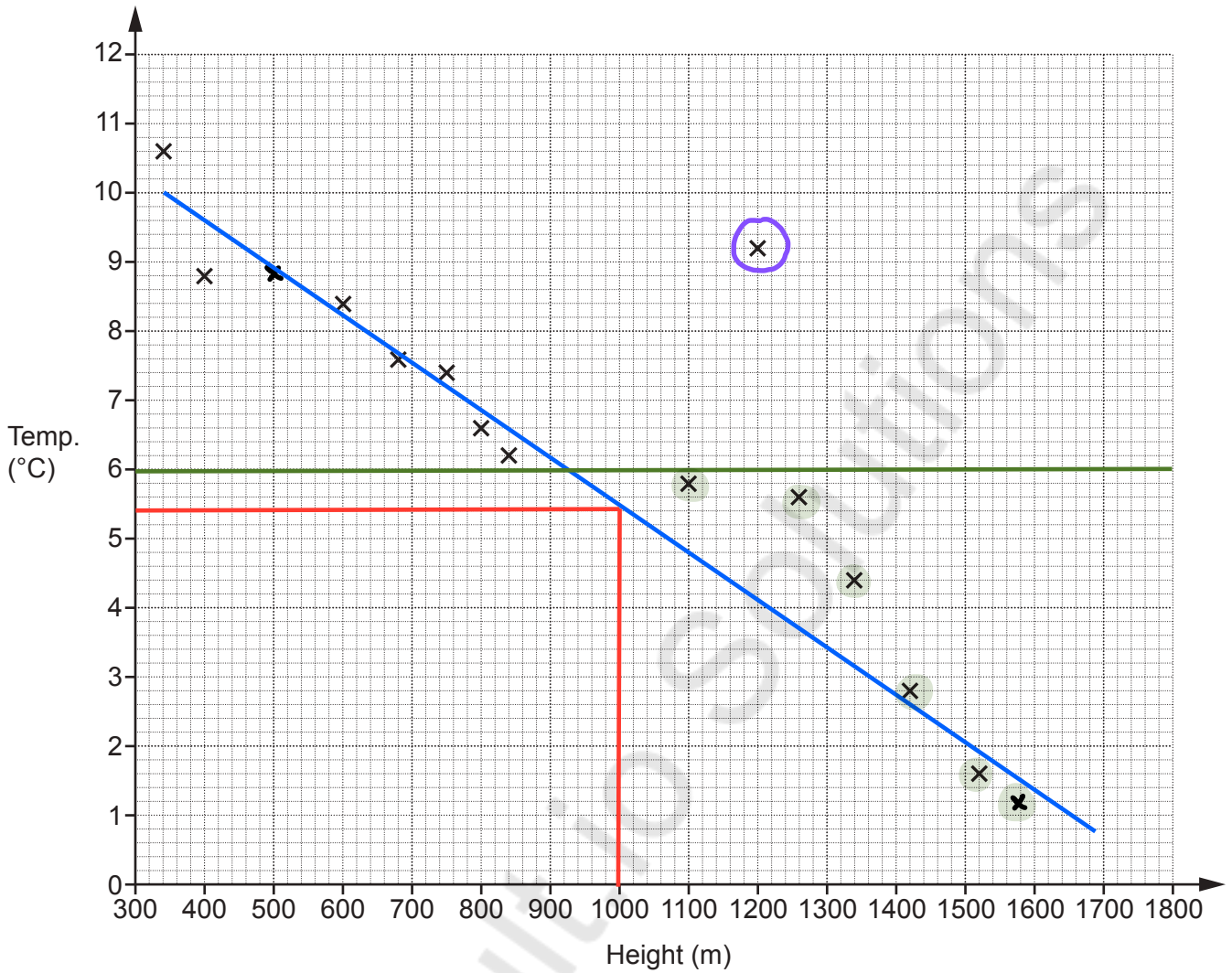
$$\begin{aligned}
 \text{Each interior angle} &= 540^\circ \div 5 \\
 &= 108^\circ
 \end{aligned}$$

$$360 - 108 - 108 = 144^\circ \text{ [Interior angle of other polygon]}$$

$$180^\circ - 144^\circ = 36^\circ \text{ [Exterior angle]}$$

$$\frac{360}{36} = 10 \text{ sides}$$

- 5 The scatter diagram shows the midday temperature at 13 different heights on a mountain.



- (a) The table has the information for 2 more heights.

Plot these on the scatter diagram.

Height (m)	500	1580
Temperature (°C)	8.8	1.2

[2]

- (b) Describe the type of correlation shown in the scatter diagram.

(b) **Negative** [1]

(c) By drawing a line of best fit, estimate the temperature at 1000 m.

(c) **5.4** °C [2]

(d) Circle the outlier on the scatter diagram.

[1]

(e) Explain why using the scatter diagram to estimate the temperature at 1800 m may be unreliable.

..... **1800m is outside the range of data.**

..... [1]

(f) Find the percentage of the 15 temperatures which are below 6 °C.

$$\frac{6}{15} \times 100 = 40$$

(f) **40** % [3]

- 6 A machine can dig, on average, 2 cm of tunnel each minute. It operates 24 hours each day.

(a) Work out how many days it should take to dig a tunnel of length 3.5 km. Give your answer to the nearest day.

$$1 \text{ minute} = 2 \text{ cm} \quad \times 60$$

$$1 \text{ hour} = 120 \text{ cm} \quad \times 24$$

$$24 \text{ hours} = 2880 \text{ cm}$$

$$2880 \text{ cm} \xrightarrow{\div 100} 28.8 \text{ m} \xrightarrow{\div 1000} 0.0288 \text{ km}$$

$$3.5 \text{ km} \div 0.0288 \text{ km} = 121.527 \text{ days}$$

$$\approx 122 \text{ days}$$

(a) **122** days [4]

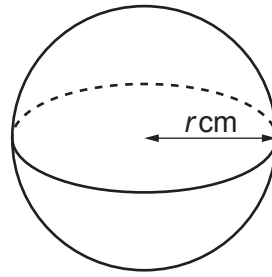
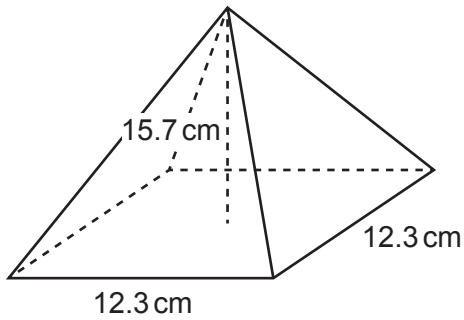
- (b) The machine actually digs an average of 2.5 cm of tunnel each minute for most of the time and an average of 1.5 cm each minute for the rest of the time.

How would this affect your answer to part (a)?

..... **It would be less.**

..... [1]

- 7 The diagram shows a square-based pyramid and a sphere.



The pyramid has base length 12.3 cm and perpendicular height 15.7 cm.
The sphere has radius r cm.

The pyramid and the sphere have the same volume.

Work out the radius of the sphere.
You must show your working.

[The volume of a pyramid is $\frac{1}{3} \times$ area of base \times perpendicular height.

The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$\begin{aligned} \text{Vol. pyramid} &= \frac{1}{3} \times 12.3 \times 12.3 \times 15.7 \\ &= 791.751 \text{ cm}^3 \end{aligned}$$

$$\text{Vol. sphere} = 791.751 \text{ cm}^3$$

$$791.751 = \frac{4}{3} \pi r^3$$

$$\times \frac{3}{4} \qquad \qquad \qquad \times \frac{3}{4}$$

$$593.81325 = \pi r^3$$

$$\div \pi \qquad \qquad \qquad \div \pi$$

$$189.016628 = r^3$$

$$\sqrt[3]{\quad} \qquad \qquad \sqrt[3]{\quad}$$

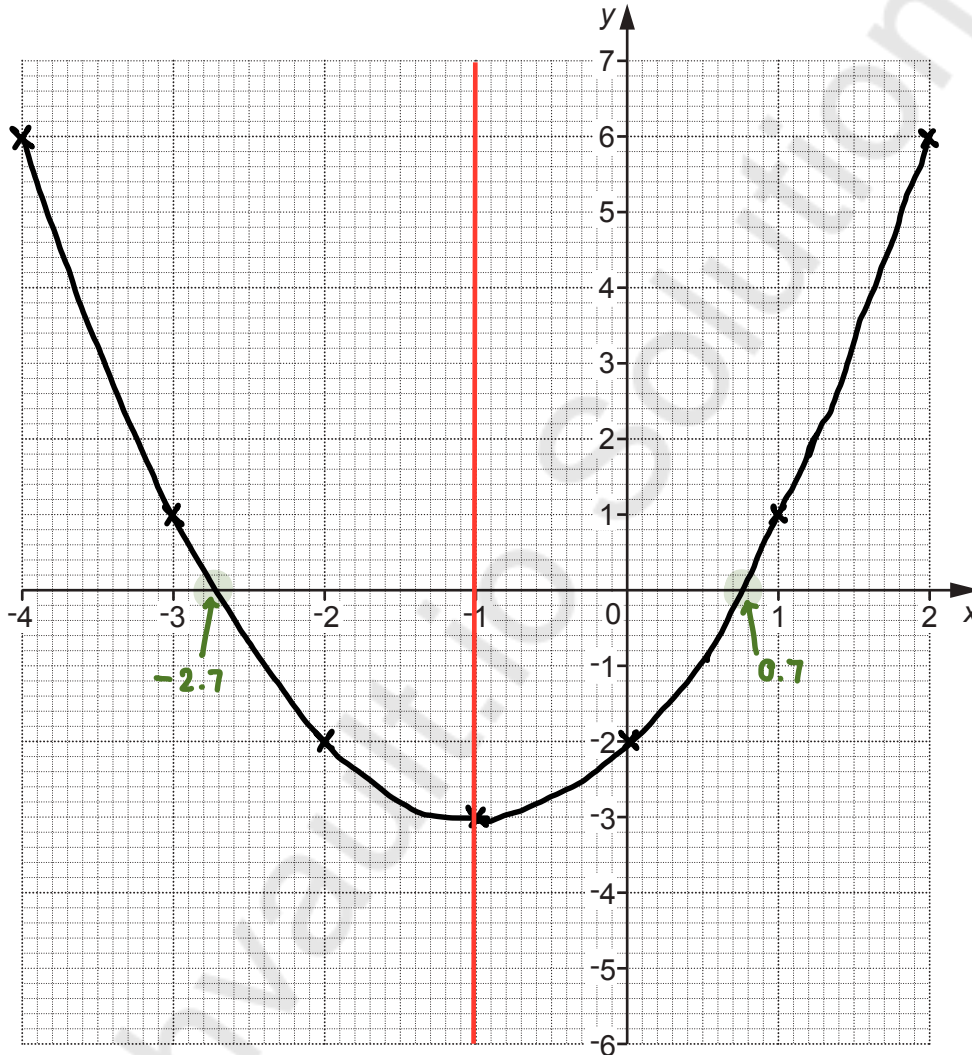
$$5.738961841 = r$$

..... **5.74** cm [5]

8 Here is a table of values for $y = x^2 + 2x - 2$.

x	-4	-3	-2	-1	0	1	2
y	6	1	-2	-3	-2	1	6

(a) Draw the graph of $y = x^2 + 2x - 2$ for $-4 \leq x \leq 2$.



[3]

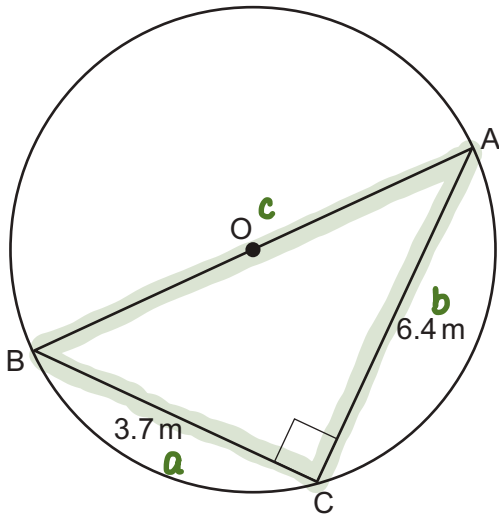
(b) Write down the equation of the line of symmetry of the graph.

(b) $x = -1$ [1]

(c) Use the graph to solve the equation $x^2 + 2x - 2 = 0$.
Give your answers to 1 decimal place.

(c) $x = -2.7$ or $x = 0.7$ [2]

- 9 Points A, B and C lie on the circumference of a circle, centre O.



Not to scale

Angle $ACB = 90^\circ$, $AC = 6.4 \text{ m}$ and $BC = 3.7 \text{ m}$.

Work out the circumference of the circle.
You must show your working.

$$a^2 + b^2 = c^2$$

$$3.7^2 + 6.4^2 = c^2$$

$$\sqrt{3.7^2 + 6.4^2} = c$$

$$7.392563831 = AB \text{ [diameter]}$$

$$C = \pi d$$

$$= \pi \times 7.39\dots$$

$$= 23.22442422$$

..... **23.2** m [5]

10 A student is researching the difference in how much exercise adults and children do. To collect their data, the student interviews the first 25 people found in the High Street at 11 am on one Monday morning.

(a) Make **three** different criticisms of the student's method of collecting data.

- 1 **Sample size is small.**
 - 2 **Not a random sample.**
 - 3 **May not be any children in the sample.**
- [3]

(b) Here is the data collection table that the student used.

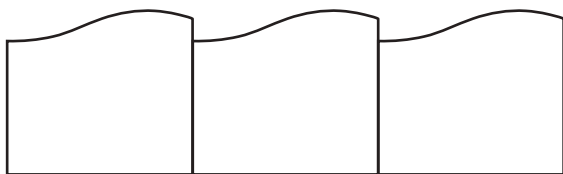
Hours exercised in a week (h)	Adult tally	Child tally
$0 \leq h \leq 2$		
$2 \leq h \leq 4$		
$4 \leq h \leq 8$		
$8 \leq h \leq 12$		
$12 \leq h \leq 20$		

Make **one** criticism of the student's table.

- The groups overlap.**
- [1]

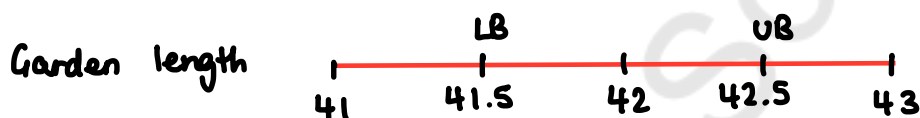
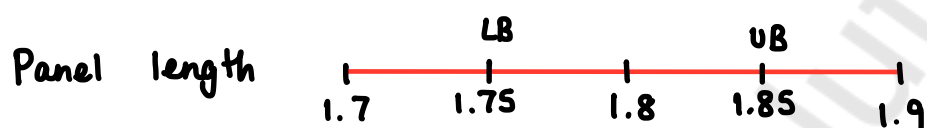
- 11 Jamie buys fence panels that fit tightly together.

Not to scale



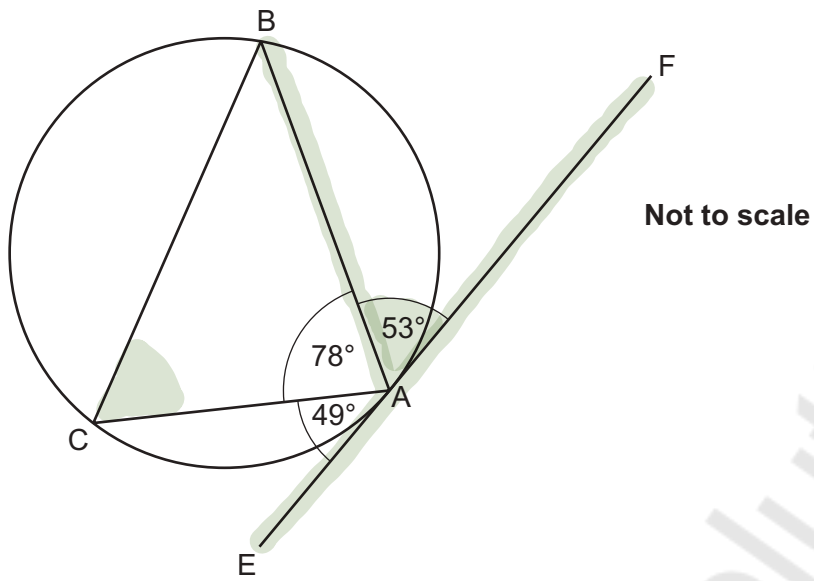
Each panel has a length of 1.8 m, correct to 1 decimal place.
 Jamie measures the length of a garden as 42 m, correct to the nearest metre.

Work out the minimum number of panels Jamie should buy in order to be certain that there are enough panels for the length of the garden.
 Show how you decide.



$$\begin{aligned}
 \text{No. of panels} &= \frac{\text{Garden UB}}{\text{Panel LB}} \\
 \text{UB} &= \frac{42.5}{1.75} \\
 &= 25
 \end{aligned}$$

- 12 (a) Points A, B and C lie on the circumference of a circle. EAF is a tangent to the circle.

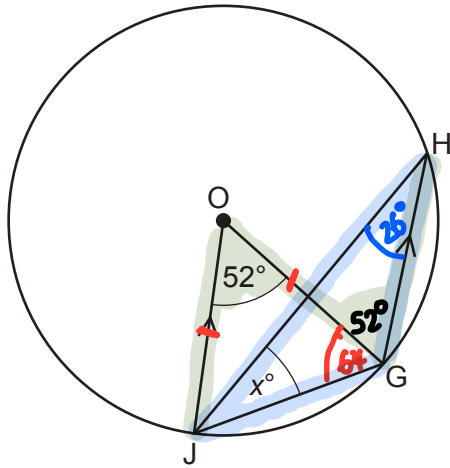


Write down the value of angle BCA giving a reason for your answer.

Angle BCA = **53** ° because **alternate segment theorem.**

.....
 [2]

(b) Points G, H and J lie on the circumference of a circle, centre O.



Not to scale

Angle $GOJ = 52^\circ$ and angle $GJH = x^\circ$.
Lines JO and GH are parallel.

Find the value of x .
You must show your working.

$$\angle OGH = 52^\circ \quad \text{Alternate angles}$$

$$\angle JHG = 26^\circ \quad \text{Angle at circumference is half angle at centre}$$

$$\angle OJG = \frac{180 - 52}{2} = 64^\circ \quad \text{Base angles of isosceles are equal.}$$

$$\begin{aligned} x &= 180 - 26 - 52 - 64 \\ &= 38 \end{aligned} \quad \text{Angles in a triangle sum to } 180^\circ$$

(b) $x = \underline{38}$ [5]

13 Here is a restaurant's menu.

Starter	Main	Dessert
Prawn Cocktail	Hunter's Chicken	Trifle
Duck Spring Rolls	Beef Curry	Ice Cream
Lamb Meatballs	Steak	Cheesecake
Leaf Salad (V)	Fish Pie	Chocolate Cake
Mushroom Soup (V)	Lasagne	Bakewell Tart
	Egg Salad (V)	Fruit Salad (V)
	Vegetable Hot Pot (V)	Cherry Pie (V)
	Macaroni Cheese (V)	

(V) denotes vegetarian

(a) A 3-course meal consists of one starter, one main and one dessert.

Work out how many different 3-course meals can be chosen from the menu.

$$5 \times 8 \times 7 = 280$$

(a) **280** [2]

(b) Find the fraction of the 3-course meals which are completely vegetarian (V).

$$2 \times 3 \times 2 = 12 \text{ veg meals}$$

(b) $\frac{12}{280}$ [2]

14 $(x+2)(3x+a)(bx+3) = 6x^3 + 11x^2 - 17x - 30$

Find the value of a and the value of b .

x	x	$+2$
$3x$	$3x^2$	$+6x$
$+a$	$+ax$	$+2a$

x	$3x^2$	$\frac{(6+a)}{x}$	$+2a$
bx	$6x^3$		
$+3$			-30

$$3x^2 \times bx = 3bx^3$$

$$3bx^3 = 6x^3$$

$$3b = 6$$

$$\div 3 \quad \div 3$$

$$b = 2$$

$$2a \times 3 = 6a$$

$$6a = -30$$

$$\div 6 \quad \div 6$$

$$a = -5$$

$$a = \dots -5 \dots$$

$$b = \dots 2 \dots [2]$$

- 15 Use algebra to prove that an odd number multiplied by a different odd number always gives an answer that is an odd number. [4]

$$n = \text{integer}$$

$$2n = \text{even integer}$$

$$2n+1 = \text{odd integer}$$

$$2m+1 = \text{other odd integer}$$

$$(2n+1)(2m+1)$$

$$4mn + 2n + 2m + 1$$

$$2(2mn + n + m) + 1 \quad \therefore \text{odd number}$$

- 16 Li bought a house at the start of 2016.

Li assumes the value of the house, £ V , can be predicted using the formula

$$V = 185000 \times 1.035^n$$

where n is the number of years after the start of 2016.

- (a) Explain how you know that the value of the house is predicted to increase each year.

1.035 is greater than 1
 [1]

- (b) Write down the percentage increase per year that is used in the formula.

$$1.035 \times 100 = 103.5\%$$

$$103.5\% - 100\% = 3.5\%$$

(b) 3.5 % [1]

- (c) Write down the value of the house at the start of 2016.

$$n = 0$$

$$185000 \times 1.035^0$$

$$185000 \times 1$$

(c) £ 185,000 [1]

- (d) Calculate the predicted value of the house at the start of 2020, giving your answer correct to 4 significant figures.

$$2020 - 2016 = 4$$

$$185,000 \times 1.035^4 = 212291.7551$$

$$\approx 212300$$

(d) £ 212,300 [2]

- (e) (i) Compared with its value at the start of 2016, show that the formula predicts the house will have doubled in value at some point during 2036. [3]

$$2036 - 2016 = 20$$

$$1.035^{20} = 1.9897... \quad [\text{start of } 2036]$$

$$1.035^{21} = 2.059... \quad [\text{start of } 2037]$$

- (ii) Give one reason why this may not happen.

This rate of increase might not continue.
 [1]

- 17 There are 15 sweets in a bag.
10 of the sweets are toffee and 5 are mint.
Reece takes two of the sweets at random.

Work out the probability that Reece takes one of each type of sweet.

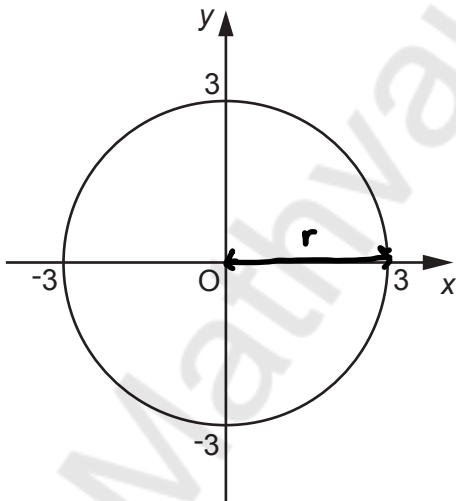
$$p(Tm) \quad \text{or} \quad p(mT)$$

$$\left(\frac{10}{15} \times \frac{5}{14}\right) + \left(\frac{5}{15} \times \frac{10}{14}\right) = \frac{10}{21}$$

$$\frac{10}{21}$$

..... [4]

- 18 The diagram shows a circle, centre the origin.



$$x^2 + y^2 = r^2$$

$$r = 3$$

$$r^2 = 3^2 = 9$$

Write down the equation of the circle.

$$x^2 + y^2 = 9$$

..... [2]

- 19 (a) Write as a single fraction in its simplest form.

$$\frac{4}{2n+3} - \frac{2n}{n^2+1}$$

$$\frac{4(n^2+1) - 2n(2n+3)}{(2n+3)(n^2+1)}$$

$$\frac{4n^2 + 4 - (4n^2 + 6n)}{(2n+3)(n^2+1)}$$

$$\frac{4 - 6n}{(2n+3)(n^2+1)}$$

(a) $\frac{4 - 6n}{(2n+3)(n^2+1)}$ [4]

(b) Simplify.

$$\frac{x^2 - x - 12}{2x^2 - 3x - 20}$$

$$x^2 - x - 12 = (x - 4)(x + 3)$$

$$\frac{-12}{-4 \quad 3}$$

$$2x^2 - 3x - 20$$

$$2x - 20 = -40$$

$$\frac{-40}{5 \quad -8}$$

$$2x^2 + 5x - 8x - 20$$

$$x(2x+5) - 4(2x+5)$$

$$(2x+5)(x-4)$$

$$\frac{(x-4)(x+3)}{(2x+5)(x-4)}$$

$$\frac{x+3}{2x+5}$$

$$(b) \quad \frac{x+3}{2x+5} \quad [5]$$

TURN OVER FOR QUESTION 20

20 Solve this inequality.

$$x^2 + 4x - 12 \leq 0$$

Give your answer using set notation.
You must show your working.

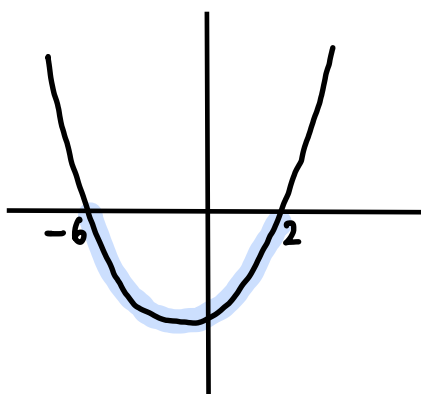
$$\frac{-12}{6 \quad -2}$$

$$(x + 6)(x - 2) = 0$$

$$x + 6 = 0 \quad x - 2 = 0$$

$$-6 \quad -6 \quad +2 \quad +2$$

$$x = -6 \quad x = 2$$



$$-6 \leq x \leq 2$$

$$\{x : -6 \leq x \leq 2\} \quad [5]$$

END OF QUESTION PAPER

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