OCR
Oxford Cambridge and RSA

Tuesday 1 November 2022 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **(a)** Write 65400 in standard form.

6.54×10^4

(a) 6.54×10^4 **[1]**

(b) Write 8.2×10^{-4} as an ordinary number.

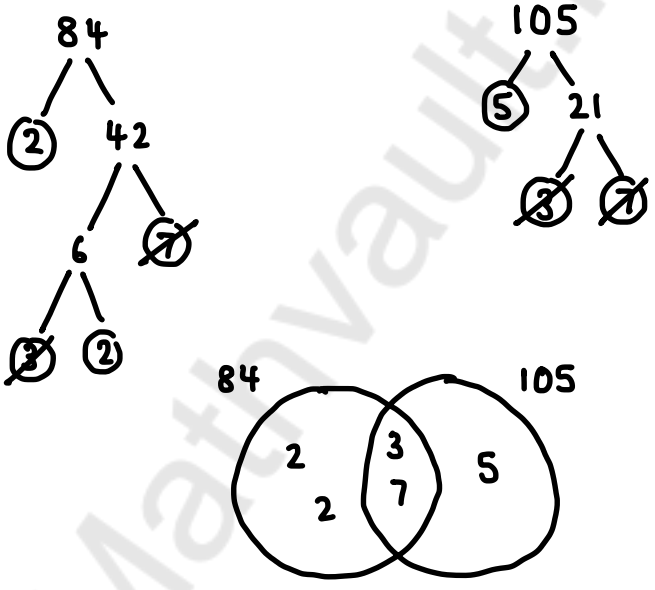
0.00082

(b) 0.00082 **[1]**

2 In 2019, comet A and comet B were both seen from Earth.
Comet A is seen from Earth every 84 years.
Comet B is seen from Earth every 105 years.

Find the next year when both comets will be seen from Earth.

LCM of 84 and 105



LCM = $2 \times 2 \times 3 \times 7 \times 5$

$= 420$

$2019 + 420 = 2439$

2439 **[4]**

3 An examination has three papers.
Paper 1 is marked out of 60.
Paper 2 is marked out of 40.
Paper 3 is marked out of 100.
The three marks are added together to form the total mark out of 200.

A student scored 65% on Paper 1 and 70% on Paper 2.

Find the mark they need to get on Paper 3 to achieve 64% of the total marks.
You must show your working.

$0.64 \times 200 = 128$ marks needed

Paper 1

$0.65 \times 60 = 39$ marks

Paper 2

$0.7 \times 40 = 28$ marks

Paper 3

$128 - 39 - 28 = 61$

61 **[5]**

4 A phone manufacturer records the faults that are reported.
Last week, in a batch of 96 phones, 6 were reported as faulty.

(a) Write down the relative frequency of faulty phones in this batch.

(a) $\frac{6}{96}$ **[1]**

(b) In 2020, the manufacturer sold a total of 12321 phones.

Work out how many of these phones the manufacturer should expect to be reported as faulty.

$\frac{6}{96} \times 12321 = 770.0625$

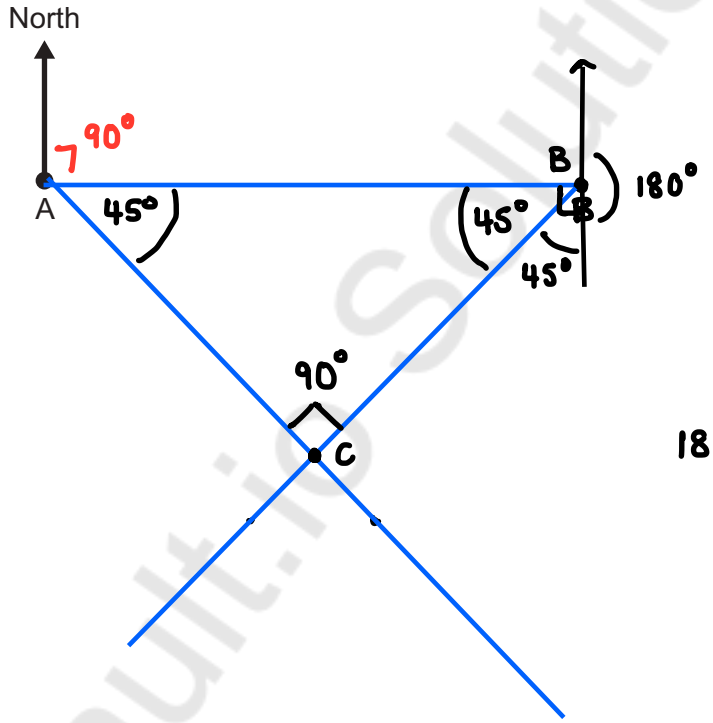
≈ 770

(b) 770 **[2]**

5 B is 12 km due east of A. 12km ÷ 2 = 6cm East = 90° clockwise
C is south-east of A and on a bearing of 225° from B.
135° clockwise
Complete the diagram to show the positions of A, B and C.
Show clearly the values of all three angles in triangle ABC.

Scale: 1 cm represents 2 km

÷2



225 - 180 = 45°

180 - 45 - 45 = 90°

[4]

6 **(a)** A solid block of wood is a cuboid which measures 3 cm by 4 cm by 5 cm.
Its density is 0.65 g/cm^3.

Work out the mass of the block of wood.

m
D V

$m = D \times V$

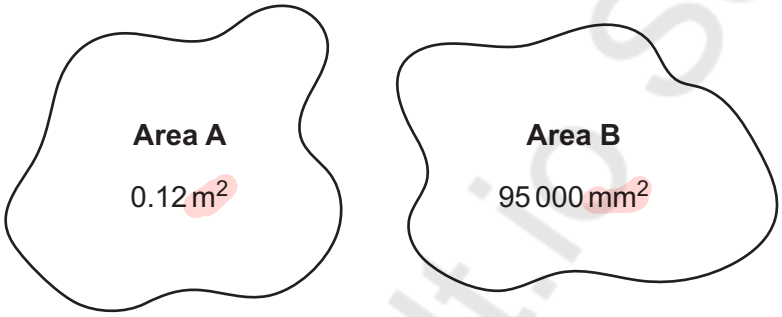
$D = 0.65\text{g}/\text{cm}^3$

$V = 3 \times 4 \times 5$
$= 60\ \text{cm}^3$

$m = 0.65 \times 60$
$= 39\ \text{g}$

(a) ..... 39 ..... g **[2]**

(b) Here are two areas.



Not to scale

10 mm = 1 cm
÷10
mm² → cm²
÷100

95,000 ÷ 100 = 950 cm²

100 cm = 1 m
÷100
cm² → m²
÷10,000

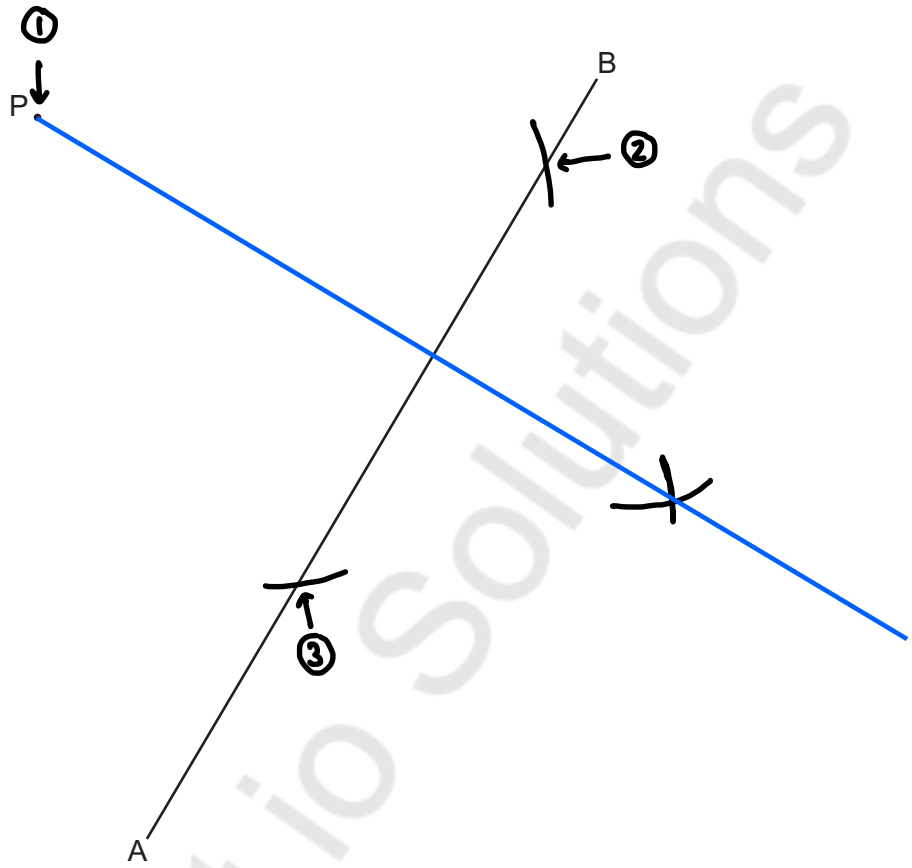
950 ÷ 10,000 = 0.095 m²

State which area is greater.
Show how you decide.

Area ..... A ..... is greater because ..... 0.12 > 0.095 .....

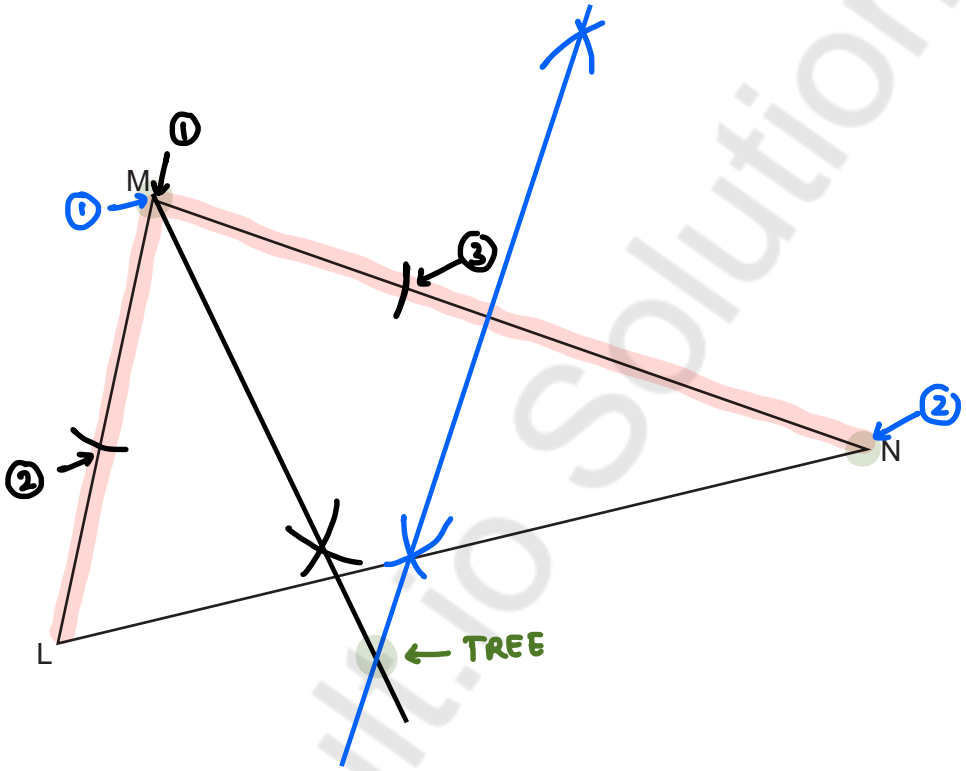
..... **[2]**

7 **(a)** Construct the perpendicular from the point P to the line AB.



[2]

(b) The diagram shows a field LMN.



A tree is to be planted in the field so that it is

- the same distance from the fences MN and ML angle bisector

and

- the same distance from corner M as from corner N. perpendicular bisector

Show, by construction, whether this can be done or cannot be done.

This .....cannot..... be done. **[5]**

8 A bag contains 35 balls.
Each ball is either red or green.
The ratio of red balls to green balls is 3 : 2.

Work out the smallest number of balls of each colour that have to be added to the bag so that the ratio of red balls to green balls becomes 7 : 3.
You must show your working.

R : G
3 : 2 5 parts
x7 ↓ ↓ x7 35 ÷ 5 = 7
21 14

R : G
7 : 3 = 10 parts
14 : 6 = 20 parts
21 : 9 = 30 parts
28 : 12 = 40 parts
35 : 15 = 50 parts

Red 35 - 21 = 14

Green 15 - 14 = 1

Number of red balls added to the bag = ...14....

Number of green balls added to the bag = ...1.... **[5]**

9 Here are two pieces of work.

For each one, describe the error in the method and give the correct answer.

(a)

Question:

Rearrange $y = 3x + 17$ to make x the subject.

Solution:

$y = 3x + 17$

$y + 17 = 3x$

$x = \frac{y + 17}{3}$

Error is it should be -17

Correct answer $x = \frac{y - 17}{3}$ **[2]**

(b)

Question:

Rearrange $A = 4x^2$ to make x the subject, where $x > 0$.

Solution:

$A = 4x^2$

$\sqrt{A} = \sqrt{4x^2}$

$\sqrt{A} = 4x$ $\quad \sqrt{A} = 2x$

$x = \frac{\sqrt{A}}{4}$ $\quad x = \frac{\sqrt{A}}{2}$

Error is it should be $\sqrt{A} = 2x$

Correct answer $x = \frac{\sqrt{A}}{2}$ **[2]**

10 You may use these kinematics formulae to answer this question.

$v = u + at$

$s = ut + \frac{1}{2}at^2$

A particle has an initial velocity of 3 m/s.
After 20 seconds the particle has a velocity of 11 m/s.
Work out the distance the particle has travelled after 20 seconds.

$v = u + at$

$v - u = at$

$\div t \qquad \div t$

$\frac{v-u}{t} = a$

$\frac{11-3}{20} = a$

$0.4 \text{ m/s}^2 = a$

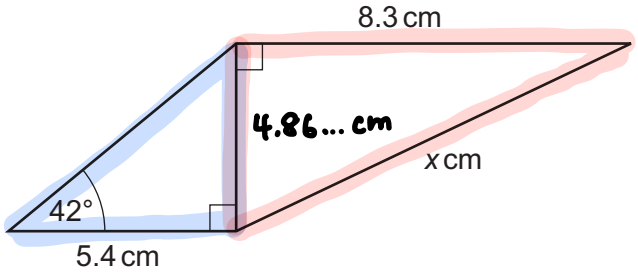
$s = 3 \times 20 + \frac{1}{2} \times 0.4 \times 20^2$

$= 140 \text{ m}$

..... 140 ..... m **[4]**

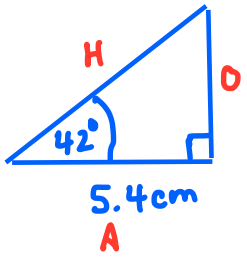
11 The diagram shows two right-angled triangles that are joined together.

All measurements are given accurate to 2 significant figures.



Not to scale

Work out the value of x.
Give your answer correct to an appropriate degree of accuracy.
You must show your working.



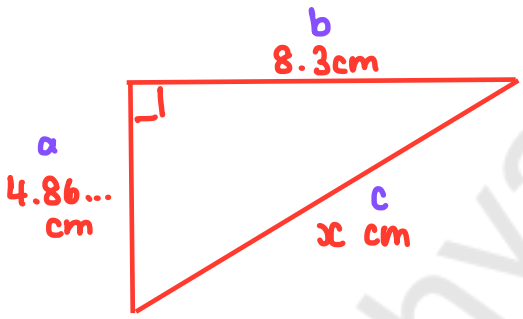
S O H C A H T O A

$\tan\theta = \frac{O}{A}$

$\tan(42) = \frac{O}{5.4}$

$O = 5.4 \times \tan(42)$

$= 4.862181839$ cm



$a^2 + b^2 = c^2$

$(4.86...)^2 + 8.3^2 = x^2$

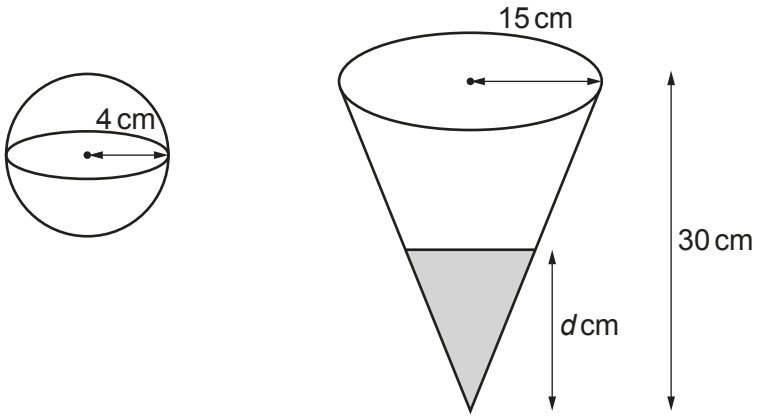
$\sqrt{(4.86...)^2 + 8.3^2} = x$

$x = 9.619293749$

≈ 9.6 cm

$x =$ 9.6 **[6]**

12 The diagram shows a sphere and a cone.



Not to scale

The sphere has radius 4 cm.
The cone has radius 15 cm and height 30 cm.

The sphere is completely filled with water.
The same amount of water is poured into the cone.

Work out the depth, d cm, of the water in the cone.
You must show your working.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. r = 4cm

The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] r = 15 h = 30

Vol. sphere = $\frac{4}{3}\pi(4)^3$

$= \frac{256}{3}\pi \text{ cm}^3$

Vol. cone = $\frac{1}{3}\pi(15)^2(30)$

$= 2250\pi \text{ cm}^3$

Volume SF = $\frac{\frac{256}{3}\pi}{2250\pi}$ (cm^3)

Length (cm) SF = $\sqrt[3]{\frac{\frac{256}{3}\pi}{2250\pi}}$

d = length SF × 30

= 10.1

$d =$ ....10.1.... **[6]**

13 y is directly proportional to $\sqrt{x}$.
$y = 1$ when $x = 16$.

Find a formula for y in terms of x.

$y \propto \sqrt{x}$

$y = k\sqrt{x}$ ⟶ $y = 0.25\sqrt{x}$

$1 = k\sqrt{16}$

$1 = k(4)$

$1 = 4k$

$k = \frac{1}{4} = 0.25$

.....$y = 0.25\sqrt{x}$..... **[3]**

14 An estimate for the number of seals on an island is given by the formula

$P = 5200 \times 1.02^t$

where P is the number of seals t years after the start of year 2015.

(a) Write down the annual percentage increase in the number of seals on the island.

$1.02 \times 100 = 102\%$.

(a) .....2%..... **[1]**

(b) Use the formula to show that there may have been about 4700 seals on the island at the start of year 2010. **[2]**

$t = -5$

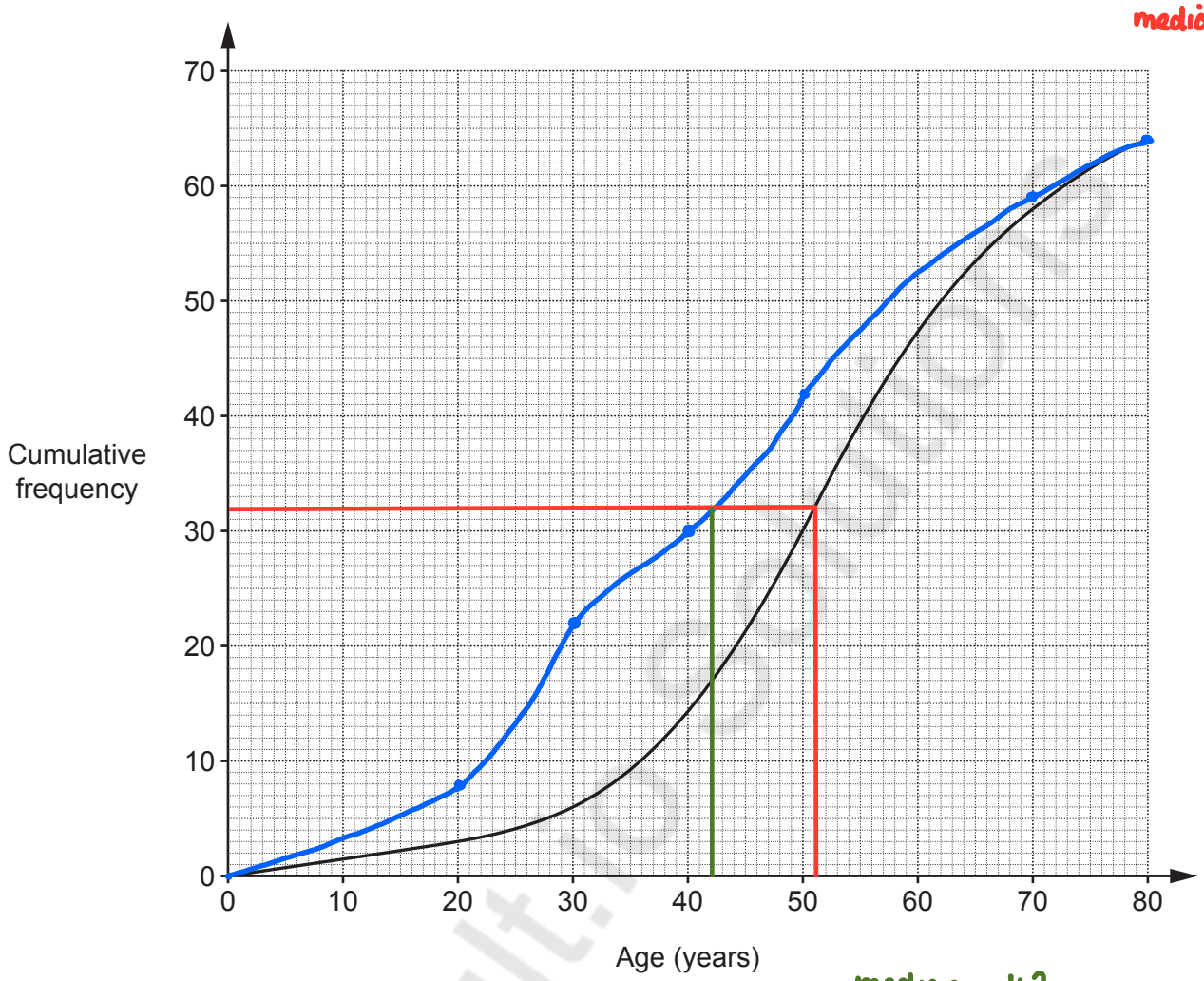
5200×1.02^{-5}

$= 4709.8...$

≈ 4710

15 The cumulative frequency graph shows the distribution of the ages of the members of a **tennis** club.

median = 51



median = 42

(a) The table summarises the ages of the members of a **cycling** club.

Age (*a* years)	$0 < a \leqslant 20$	$20 < a \leqslant 30$	$30 < a \leqslant 40$	$40 < a \leqslant 50$	$50 < a \leqslant 70$	$70 < a \leqslant 80$
Frequency	8	14	8	12	17	5
C.F.	8	22	30	42	59	64

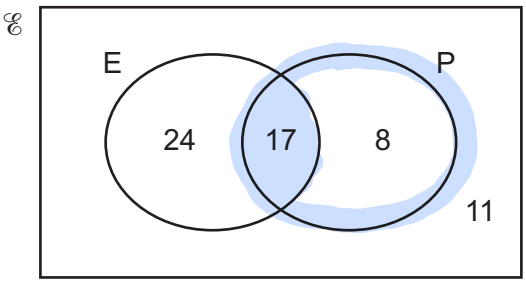
On the graph above, draw the cumulative frequency graph of the ages of the members of the **cycling** club. **[5]**

(b) Find out which club has younger members on average.
Give evidence to support your decision.

Cycling because median is 42 whereas median for tennis club is 51. **[2]**

16 A salesroom sells various types of car.
Some cars are electric (E), some are petrol (P), some are both and some are neither.

The Venn diagram below shows the salesroom's stock of cars.



A petrol car is picked at random.

Find the probability that the car is also electric.

$\frac{17}{25}$ ..... **[2]**

17 Find the equation of the line through (4, 5) that is perpendicular to $y = 2x - 3$.

$y = 2x - 3$

$y = mx + c$

m ↑ gradient

gradient $= 2$

Perpendicular gradient $= -\frac{1}{2}$

$y = -\frac{1}{2}x + c \quad (4, 5)$ (x, y)

$5 = -\frac{1}{2}(4) + c$

$5 = -2 + c$

$+2 \quad +2$

$7 = c$

$y = -\frac{1}{2}x + 7$ ..... **[3]**

18 (a) The next term in a Fibonacci sequence is found by adding together the two previous terms.

(i) The first and second terms of a particular Fibonacci sequence are x and y.

Show that the fourth term of the sequence can be written as $x + 2y$. **[2]**

1st	2nd	3rd	4th
x	y	$x + y$	$x + y + y$
			$= x + 2y$

(ii) The fourth term of the same Fibonacci sequence is 7.
The seventh term of the sequence is 31.

Work out the value of x and the value of y.
You must show your working.

4th $x + 2y = 7$

5th $x + 2y + x + y = 2x + 3y$

6th $2x + 3y + x + 2y = 3x + 5y$

7th $3x + 5y + 2x + 3y = 5x + 8y = 31$

$x + 2y = 7$ $\times 5$
$5x + 8y = 31$

$5x + 10y = 35$
$5x + 8y = 31$

$2y = 4$
$\div 2 \quad \div 2$
$y = 2$

$x + 2y = 7$
$x + 2(2) = 7$
$x + 4 = 7$
$x = 3$

(a)(ii) $x =$ 3

$y =$ 2 **[6]**

(b) Here are the first four terms of a sequence.

1 $\xrightarrow{\times\sqrt{3}}$ $\sqrt{3}$ $\xrightarrow{\times\sqrt{3}}$ 3 $\xrightarrow{\times\sqrt{3}}$ $3\sqrt{3}$

Write an expression for the nth term.

nth term ar^{n-1} a = 1st term r = common ratio

$1 \times (\sqrt{3})^{n-1}$

(b) $(\sqrt{3})^{n-1}$ **[2]**

(c) Here are the first four terms of a quadratic sequence.

−1 5 13 23

The nth term is $n^2 + bn + c$.

Find the value of b and the value of c. a = 1

n^2	1	4	9	16	) x 1
an^2	1	4	9	16	
Seq.	−1	5	13	23	
Seq. $- an^2$	−2	1	4	7	

−5 (−3) +3 +3 +3

$3n - 5$

b = 3

c = −5

(c) $b =$ 3

$c =$ −5 **[3]**

19 Describe the **single** transformation that maps the graph of $y = x^2$ onto the graph of $y = (x+3)^2 + 5$.

Translation by the vector $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$

[3]

20 Mrs Sweet has 8 different milk chocolates and 9 different plain chocolates.

Her daughter chooses one of the milk chocolates.
Her son then chooses one of the plain chocolates.
Mrs Sweet then chooses one of the remaining chocolates.

Work out how many different combinations of three chocolates they can choose.

Daughter Son Mrs. Sweet

$8 \times 9 \times 7_{\text{milk}} = 504$

$8 \times 9 \times 8_{\text{plain}} = 576$

$504 + 576 = 1080$

1080 **[3]**

21 60 people each try to solve a puzzle.
The table summarises their recorded times.

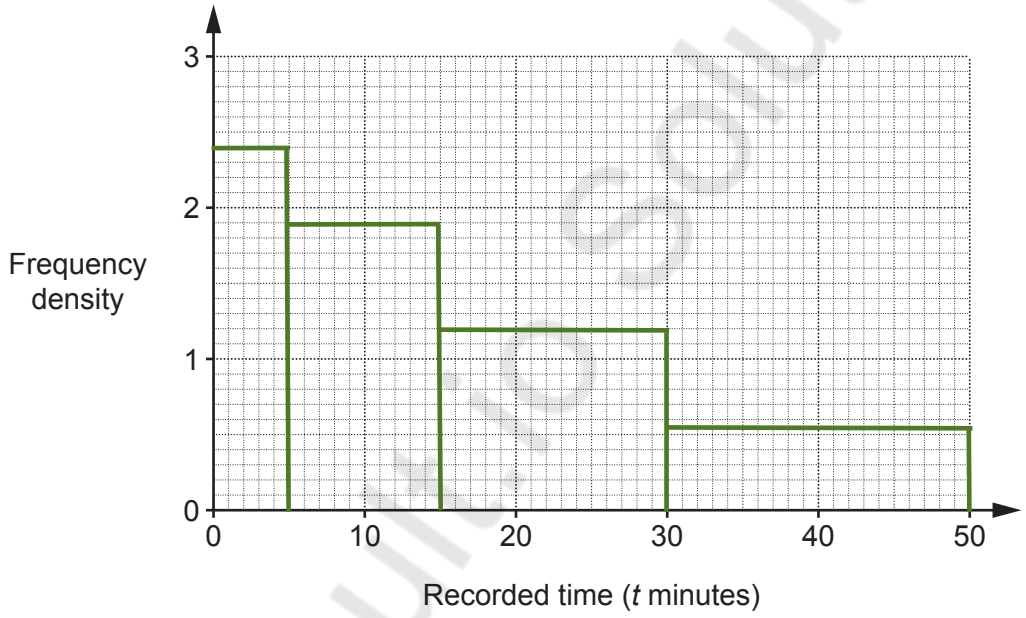
CW F FD

Recorded time (t minutes)	Frequency
$0 < t \leqslant 5$	12
$5 < t \leqslant 15$	19
$15 < t \leqslant 30$	18
$30 < t \leqslant 50$	11

F. D.

12 ÷ 5 = 2.4
19 ÷ 10 = 1.9
18 ÷ 15 = 1.2
11 ÷ 20 = 0.55

(a) Draw a histogram to show this information.



[3]

(b) Those people who failed to solve the puzzle within 50 minutes were given a recorded time of 50 minutes.

Nina uses mid-interval values to estimate the mean recorded time of the 60 people.

Explain why Nina's answer is likely to be an under-estimate for the mean of the actual time taken by the 60 people.

Those who failed took longer than 50 minutes, so the mid-interval value for the last interval would be greater. **[1]**

Turn over for Question 22

22 Solve algebraically.

$x^2 + y^2 = 18$

$y = x - 6$

$x^2 + (x-6)^2 = 18$

$x^2 + (x-6)(x-6) = 18$

$x^2 + x^2 - 6x - 6x + 36 = 18$

$2x^2 - 12x + 36 = 18$

$-18 \quad -18$

$2x^2 - 12x + 18 = 0$

$\div 2 \qquad \div 2$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x - 3 = 0$

$x = 3$

$y = x - 6$

$y = 3 - 6$

$= -3$

$x =$ 3

$y =$ −3 **[5]**

END OF QUESTION PAPER