OCR
Oxford Cambridge and RSA

Thursday 3 November 2022 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Jamie was paid £14.00 per hour.
Jamie receives a pay increase of 20%.

Work out how much Jamie is now paid per hour.

100% = £14.00
÷10 ↓ 10% = £1.40 ÷10
×2 ↓ 20% = £2.80 ×2

£14.00 + £2.80 = £16.80

£ 16.80 **[3]**

2 Find all the possible integer values that satisfy the inequality $-4 \leqslant x - 3 < 1$.

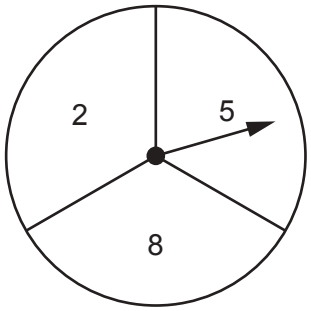
+3 +3 +3

$-1 \leqslant x < 4$

-1, 0, 1, 2, 3

-1, 0, 1, 2, 3 **[3]**

3 Azmi has a fair spinner numbered 2, 5 and 8.



Azmi spins the spinner twice and adds the two scores to get a total.

(a) Complete the table to show all of the possible totals.

		First spin		
	Total	2	5	8
Second spin	2	4	7	10
	5	7	10	13
	8	10	13	16

[1]

(b) Find the probability that the total is a square number.

1 4 9 16 ...

(b) $\frac{2}{9}$ ..... **[2]**

4 Layla and Jamal open a box of sweets.
Layla and Jamal share all of the sweets in the ratio 2 : 3.

L : J

= 5 parts

(a) Write down the fraction of the sweets that Layla receives.

(a) $\frac{2}{5}$ **[1]**

(b) Layla eats some of **her** sweets.
She is then left with 18% of the sweets that were in the box.

Work out the percentage of **her** sweets that Layla has eaten.

$\frac{2}{5} \overset{\times 20}{=} \frac{40}{100} = 40\%$

$40\% - 18\% = 22\%$

$\frac{22}{40} \overset{\div 2}{=} \frac{11}{20} \overset{\times 5}{=} \frac{55}{100} = 55\%$

(b) 55 % **[4]**

5 Ashley goes on a journey.

She travels by taxi for $\frac{1}{8}$ of the journey.

She travels by train for $\frac{4}{5}$ of the journey.

She walks for the remaining 900 m of the journey.

Find the length of this journey in kilometres.
You must show your working.

$\frac{1}{8} \overset{\times 5}{\underset{\times 5}{+}} \frac{4}{5} \overset{\times 8}{\underset{\times 8}{}}$

$\frac{5}{40} + \frac{32}{40} = \frac{37}{40}$

$\frac{40}{40} - \frac{37}{40} = \frac{3}{40}$

$\frac{3}{40} = 900\text{m}$

$\div 3 \qquad \div 3$

$\frac{1}{40} = 300\text{m}$

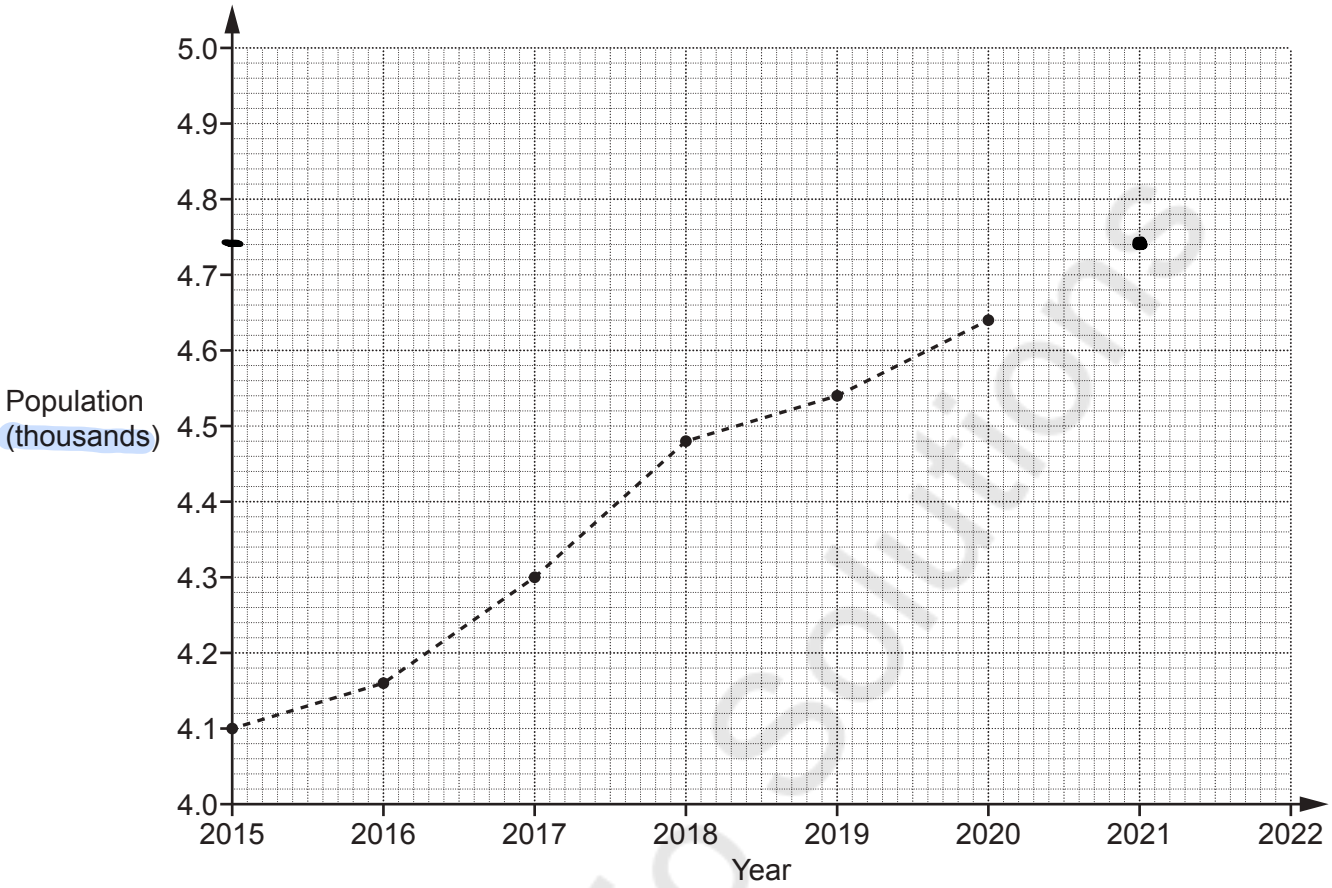
$\times 40 \qquad \times 40$

$\frac{40}{40} = 12{,}000\text{m}$

$12000\text{ m} \div 1000 = 12\text{ km}$

..... 12 ..... km **[6]**

6 The graph shows information about the population of a village.



(a) The population of the village in 2021 was 4740. Plot this point on the graph. **[1]**

4740 ÷ 1000 = 4.74

(b) Work out the increase in the population of the village between 2016 and 2018.

2016 = 4.16

2018 = 4.48

4.48 - 4.16 = 0.32

0.32 × 1000 = 320

(b) 320 **[2]**

(c) Rowan says that there was a huge increase in the population of the village between 2015 and 2020.

Describe how Rowan may have been misled by the graph.

Only part of the vertical scale is shown.

[1]

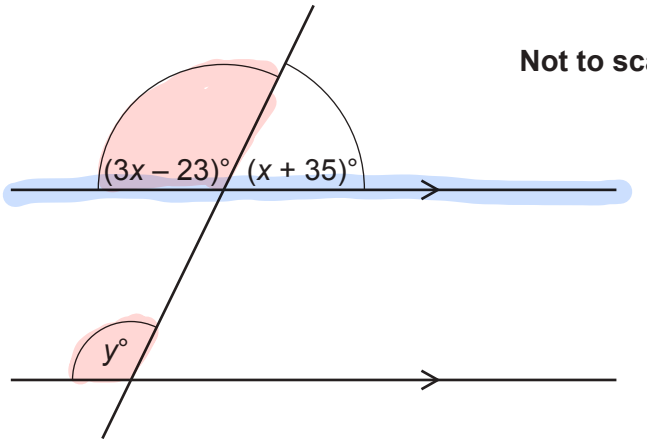
(d) Blake says that the population of the village will be greater than 4800 in 2022.

Write down an assumption Blake has made.

Increasing trend continues.

[1]

7 The diagram shows a straight line crossing a pair of parallel lines.



Not to scale

Find the value of y.
You must show your working.

$3x - 23 + x + 35 = 180$

$4x + 12 = 180$

$-12 \quad -12$

$4x = 168$

$x = 42$

$4 \overline{)1^{1}6\ 8} = 0\ 4\ 2$

$3x - 23$

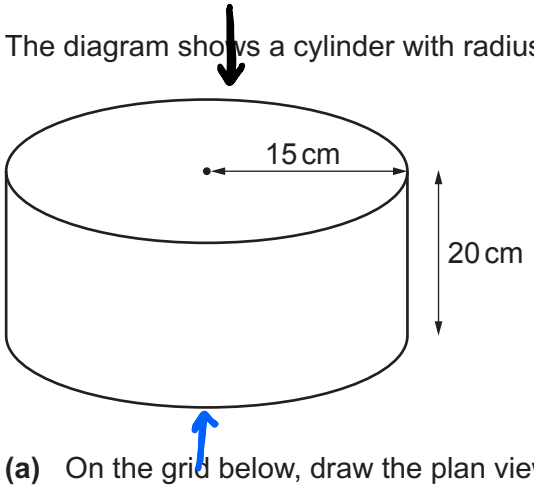
$3(42) - 23$

$126 - 23 = 103^{\circ}$

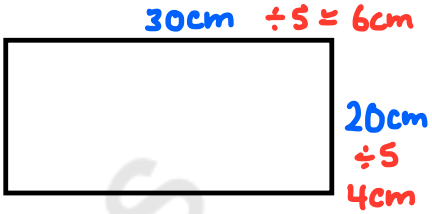
$y = 103^{\circ}$

$y =$ 103 **[5]**

8 The diagram shows a cylinder with radius 15 cm and height 20 cm.



Not to scale



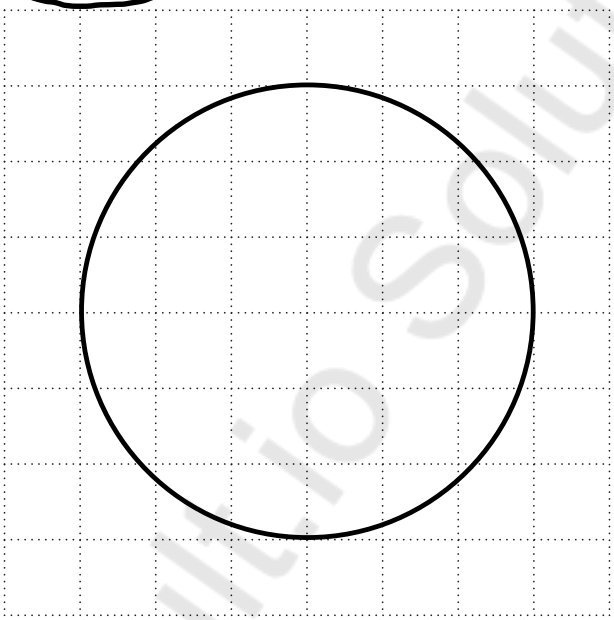
(a) On the grid below, draw the plan view of the cylinder.
Use the scale 1 cm represents 5 cm.

÷5

r = 15cm

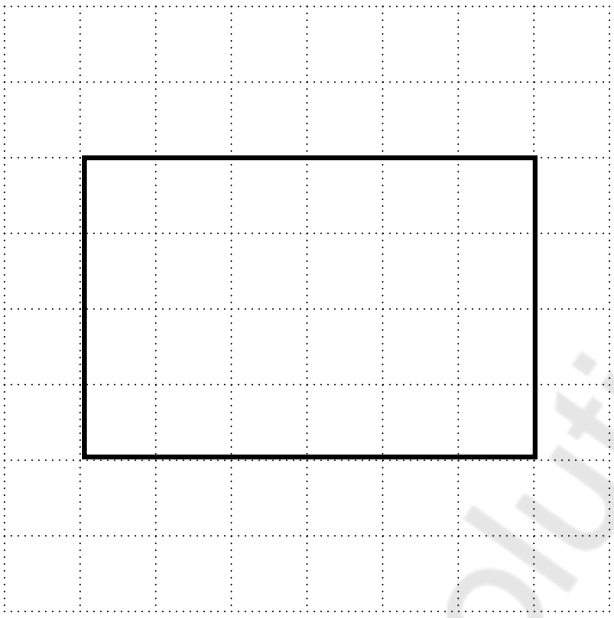
d = 30cm

30 ÷ 5 = 6cm



[2]

(b) On the grid below, draw the front elevation of the cylinder.
Use the scale 1 cm represents 5 cm.



[2]

9 A student says that they have placed the following values in order starting with the smallest.

$\left(\frac{1}{10}\right)^2$ ✓ $\sqrt{0.25}$ 4^{-1}

Has the student done this correctly?
Show how you decide.

$\left(\frac{1}{10}\right)^2 = \frac{1}{100} = 0.01$

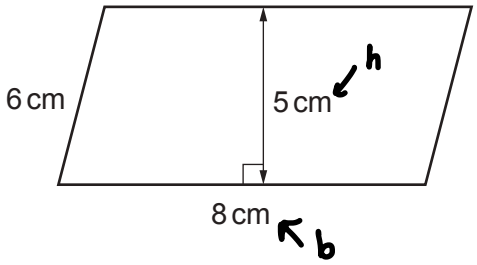
$\sqrt{0.25} = 0.5$

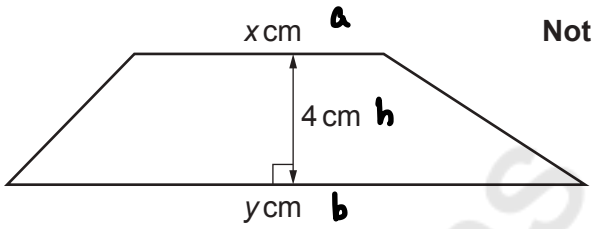
$4^{-1} = \frac{1}{4} = 0.25$

No ..... because $4^{-1} < \sqrt{0.25}$ .....

..... **[4]**

10 The parallelogram and the trapezium have the same area.





Not to scale

The ratio of x : y is 3 : 5.

Find the value of x and the value of y.
You must show your working.

Area parallelogram = b x h

= 8 x 5

= 40 cm²

Area trapezium = $\frac{1}{2}(a+b)h$

$40 = \frac{1}{2}(x+y) \times 4$

÷4 ÷4

$10 = \frac{1}{2}(x+y)$ x2

x 2

$20 = x + y$

x : y

3 : 5 = 8 parts

20 ÷ 8 = 2.5

x = 3 x 2.5 = 7.5

y = 5 x 2.5 = 12.5

$x =$ 7.5

$y =$ 12.5 **[6]**

11 Write $0.\dot{2}\dot{7}$ as a fraction in its simplest form.

$x = 0.\dot{2}\dot{7}$

$10x = 2.\dot{7}\dot{2}$

$100x = 27.\dot{2}\dot{7}$

$$\begin{array}{rl} 100x = & 27.\dot{2}\dot{7} \\ - \quad x = & 0.\dot{2}\dot{7} \\ \hline 99x = & 27 \end{array}$$

$\div 99 \qquad \div 99$

$$x = \frac{27}{99} \begin{matrix} \div 9 \\ \div 9 \end{matrix} = \frac{3}{11}$$

$\frac{3}{11}$ ..... **[3]**

12 The time, t seconds, taken by each of 60 students to complete a puzzle is recorded.

The table shows information about these times.

Time (t seconds)	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 50$	$50 < t \leqslant 70$	$70 < t \leqslant 90$
Frequency	8	0	12	30	10

20 students

(a) Two students are picked at random.
Reece works out the probability that they both took longer than 50 seconds to complete the puzzle.
Reece's working is shown below.

> The number of students who took longer than 50 seconds is 30 + 10 = 40
>
> The probability that one student took longer than 50 seconds is $\frac{40}{60} = \frac{2}{3}$
>
> The probability they both took longer than 50 seconds is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Explain the error in their method and write the correct calculation that Reece needs to do.
You do not need to work out the answer to the calculation.

The error is ..... the probability for second student is not $\frac{2}{3}$ .....

.....

The correct calculation is ..... $\frac{40}{60} \times \frac{39}{59}$ ..... **[2]**

(b) Two students are picked at random from those who took 50 seconds or less.

Find the probability that one of them took 30 seconds or less and the other took more than 40 seconds.
You must show your working.

$p(\leq 30 \text{ seconds}) = \frac{8}{20}$

$p(> 40 \text{ seconds}) = \frac{12}{20}$

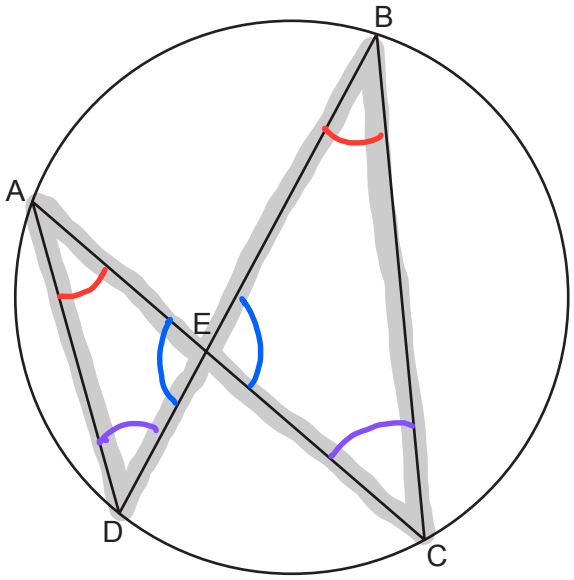
$\frac{8}{20} \times \frac{12}{19}$ $\qquad$ $\frac{12}{20} \times \frac{8}{19}$

$= \frac{96}{380}$ $\qquad$ $= \frac{96}{380}$

$\frac{96}{380} + \frac{96}{380} = \frac{192}{380}$

(b) $\frac{192}{380}$ ..... **[5]**

13 Points A, B, C and D lie on the circumference of a circle.
Line AC intersects line BD at point E.



Not to scale

Prove that triangle AED is similar to triangle BEC.

$\angle AED = \angle BEC$ Vertically opposite angles are equal

$\angle DAE = \angle EBC$ Angles in the same segment are equal

$\angle ADE = \angle ECB$ " "

AAA ∴ similar

.....

.....

.....

..... **[3]**

14 The number of bees, P, in a colony is given by the formula

$P = ab^x$

where x is the number of months after the start of July.

At the start of July, there were 25 000 bees in the colony.
After one month, there were 23 500 bees in the colony.

Find the value of a and the value of b.
Give the value of b as a decimal.

July

$25000 = ab^0$

$b^0 = 1$

$25000 = a$

After 1 month

$23500 = ab^1$

$23500 = 25000b$

$\frac{23500}{25000} = b$

$\frac{235}{250} \begin{matrix} \div 5 \\ \div 5 \end{matrix} = b$

$\frac{47}{50} = b$

$\frac{94}{100} = 0.94 = b$

$a =$ 25000

$b =$ 0.94 **[4]**

15 **(a)** Simplify.

$\sqrt{3} \times \sqrt{15}$

$\sqrt{15} = \sqrt{3} \times \sqrt{5}$

$\underbrace{\sqrt{3} \times \sqrt{3}}_{3} \times \sqrt{5}$

$3 \times \sqrt{5} = 3\sqrt{5}$

(a) $3\sqrt{5}$ **[2]**

(b) Rationalise the denominator and simplify.

$\frac{40}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$

$\frac{40\sqrt{15}}{15}$ $\div 5$ $\div 5$ $\frac{8\sqrt{15}}{3}$

(b) $\frac{8\sqrt{15}}{3}$ **[3]**

(c) Work out.

$27^{\frac{4}{3}}$

$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$

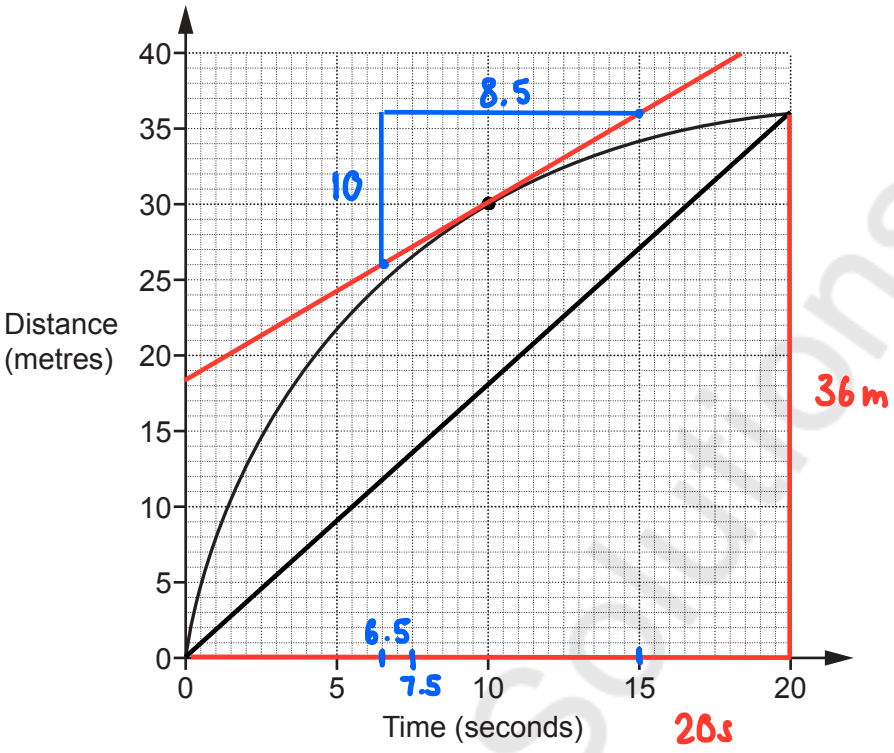
$27^{\frac{4}{3}} = (\sqrt[3]{27})^4$

$= 3^4$

$= 81$

(c) 81 **[2]**

16 The graph shows the distance travelled by a particle over the first 20 seconds of its motion.



(a) Show that the average speed of the particle over the first 20 seconds of its motion is 1.8 m/s. **[1]**

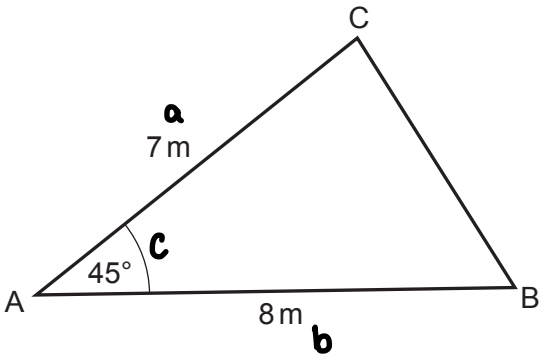
$\frac{36}{20} = 1.8$

(b) Estimate the speed of the particle at 10 seconds.
You must show working to support your estimate.

$\frac{10}{8.5}$ ×10 ×10 $\frac{100}{85}$

(b) $\frac{100}{85}$ ..... m/s **[3]**

17 The diagram shows triangle ABC.



Not to scale

Find the area of the triangle.
Give your answer in the form $a\sqrt{b}$ where a and b are integers.

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 7 \times 8 \times \sin 45$$

$$= 28 \times \sin 45$$

$$= 28 \times \frac{\sqrt{2}}{2}$$

$$= \frac{28\sqrt{2}}{2}$$

$$= 14\sqrt{2}$$

	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0

2

..... $14\sqrt{2}$ ..... m^2 **[3]**

18 **(a)** By factorising, find the roots of $y = x^2 + 18x + 77$.

77

7 11

$(x + 7)(x + 11) = 0$

$x + 7 = 0$ $\quad$ $x + 11 = 0$

$-7 \quad -7$ $\quad$ $-11 \quad -11$

$x = -7$ $\quad$ $x = -11$

(a) $x =$ ..... -7 ..... and $x =$ ..... -11 ..... **[3]**

(b) **(i)** Write $y = x^2 + 18x + 77$ in the form $y = (x + a)^2 - b$.

$(x + 9)^2 - 9^2 + 77$

$(x + 9)^2 - 81 + 77$

$(x + 9)^2 - 4$

(b)(i) $y =$ ..... $(x + 9)^2 - 4$ ..... **[3]**

(ii) Write down the coordinates of the turning point of the graph of $y = x^2 + 18x + 77$.

$(x + 9)^2 - 4$

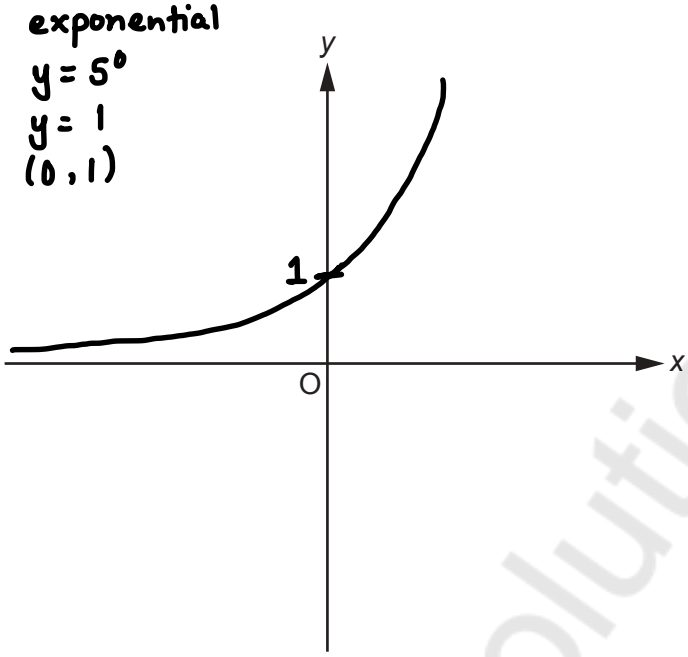
$x + 9 = 0$ $\quad$ $y = -4$

$-9 \quad -9$

$x = -9$

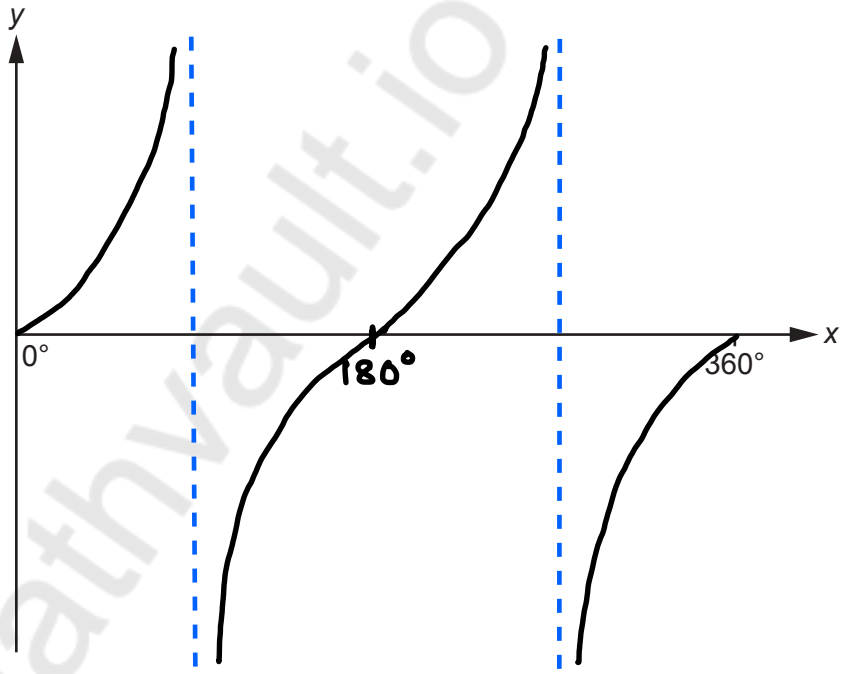
(ii) (..... -9 ..... , ... -4 .....) **[2]**

19 **(a)** Sketch the graph of $y = 5^x$ indicating any values where the graph crosses the axes.



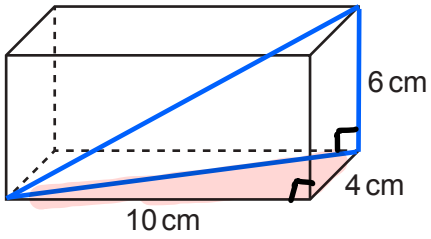
[2]

(b) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$ indicating any values where the graph crosses the axes.



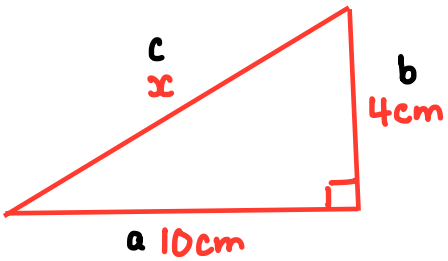
[2]

20 Kai has a box in the shape of a cuboid.
The internal dimensions of the box are 10 cm by 4 cm by 6 cm.



Kai is given a pencil of length 13 cm.

Show that the pencil does not fit completely inside the box. **[4]**



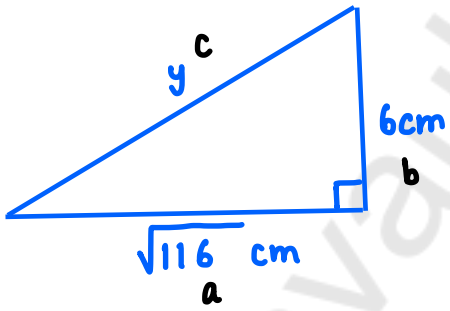
$a^2 + b^2 = c^2$

$10^2 + 4^2 = x^2$

$100 + 16 = x^2$

$116 = x^2$

$\sqrt{116} = x$



$a^2 + b^2 = c^2$

$(\sqrt{116})^2 + 6^2 = y^2$

$116 + 36 = y^2$

$152 = y^2$

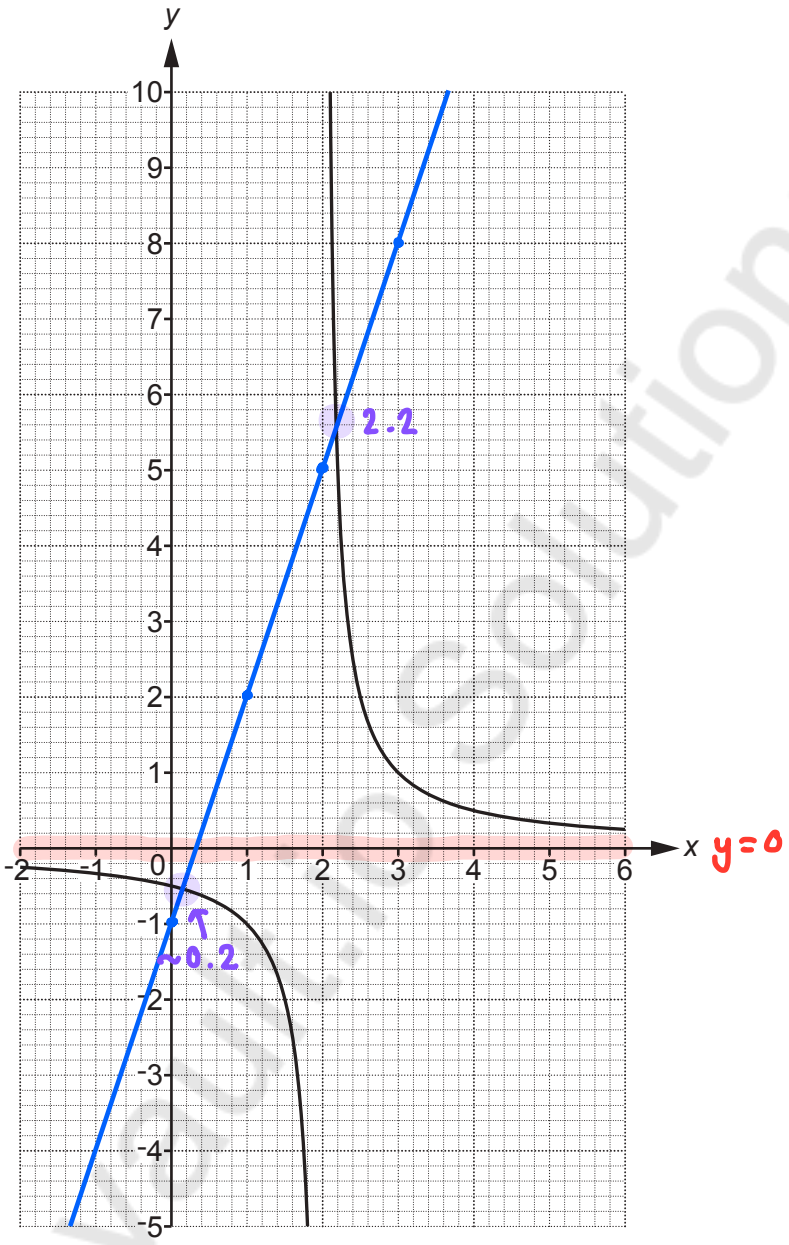
$\sqrt{152} = y$

$\sqrt{169} = 13$

$\sqrt{152} < 13$

$\therefore$ pencil may not fit.

21 The graph of $y = \dfrac{1}{x-2}$ is drawn on the grid for $-2 \leqslant x \leqslant 6$.



(a) There are no values of x for which $\dfrac{1}{x-2} = k$.

Find the value of k.

(a) $k =$ 0 **[1]**

(b) (i) **Use the graph** to find approximate solutions to the equation $\frac{1}{x-2} = 3x - 1$.
Give your answers to **1** decimal place.
Show your working on the graph.

$y = 3x - 1$

x	0	1	2	3
y	-1	2	5	8

(b)(i) $x =$ 0.2 or $x =$ 2.2 **[4]**

(ii) Show algebraically that $\frac{1}{x-2} = 3x - 1$ has the same solutions as $3x^2 - 7x + 1 = 0$. **[4]**

$$\frac{1}{x-2} = 3x - 1$$

$$x(x-2) \qquad x(x-2)$$

$$1 = (3x-1)(x-2)$$

$$1 = 3x^2 - 6x - 1x + 2$$

$$1 = 3x^2 - 7x + 2$$

$$-1 \qquad -1$$

$$0 = 3x^2 + 7x + 1$$

$$3x^2 + 7x + 1 = 0$$

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).