

Friday 19 May 2023 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes



You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 Calculate.

$$\sqrt{5.2^2 - 4.8 \times 6.3}$$

Give your answer correct to 3 significant figures.

$$7.56355171$$

$$\approx 7.57$$

7.57

..... [2]

- 2 The price of petrol decreases from £1.32 per litre to £1.02 per litre.

Calculate the percentage decrease in the price.

$$\begin{aligned} \% \text{ decrease} &= \frac{\text{change}}{\text{original}} \times 100 \\ &= \frac{1.32 - 1.02}{1.32} \times 100 \\ &= 22.72 \\ &\approx 22.7 \% \end{aligned}$$

22.7

..... % [3]

- 3 (a) Eve, Jack and Ling share some money in the ratio 2 : 3 : 4.
Jack gets £720.

Work out how much Ling gets.

$$\begin{array}{ccccc}
 E & : & J & : & L \\
 2 & : & 3 & : & 4 \\
 & & \downarrow \times 240 & & \downarrow \times 240 \\
 & & £720 & & £960
 \end{array}$$

(a) £ 960 [2]

- (b) Amir, Beth and Casey share some money in the ratio 3 : 5 : c.
Casey's share is $\frac{2}{3}$ of the total.

Find the value of c.

$$\begin{array}{ccccc}
 A & : & B & : & C \\
 3 & : & 5 & : & c
 \end{array}$$

$$\begin{aligned}
 \text{Casey} &= \frac{c}{3+5+c} \\
 &= \frac{c}{8+c}
 \end{aligned}$$

$$\frac{c}{8+c} = \frac{2}{3}$$

$$3c = 2(8+c)$$

$$3c = 16 + 2c$$

$$-2c \quad -2c$$

$$c = 16$$

(b) c = 16 [3]

4 Alex invests £4500 at a rate of 7.5% per year simple interest.

(a) Find the value of the investment at the end of 4 years.

$$7.5\% \times 4 = 30\%$$

$$100\% + 30\% = 130\%$$

$$1.3 \times 4500 = 5850$$

(a) £ **5850** [3]

(b) At the end of t years, the value of the investment is over £13500 for the first time.

Find the value of t .

$$13500 - 4500 = £9000$$

$$\begin{aligned} 7.5\% \text{ of } 4500 &= 0.075 \times 4500 \\ &= 337.50 \end{aligned}$$

$$\begin{aligned} 9000 \div 337.50 &= 26.\dot{6} \\ &\approx 27 \end{aligned}$$

(b) $t =$ **27** [3]

- 5 (a) Write 0.00386 in standard form.

$$3.86 \times 10^{-3}$$

(a) 3.86×10^{-3} [1]

- (b) The speed of sound is 3.43×10^{-1} km/s.
An object is travelling at the speed of sound.

Work out how far the object travels in one day.

$$1 \text{ second} = 3.43 \times 10^{-1} \text{ km} \quad \times 3600$$

$$1 \text{ hour} = 1234.8 \text{ km} \quad \times 24$$

$$24 \text{ hours} = 29635.2 \text{ km}$$

(b) 29635.2 km [2]

- (c) In a science fiction story, a spacecraft travelling faster than the speed of light is said to be travelling at 'warp n ' where n is an integer.

Warp n is defined as $n^3 \times$ the speed of light.

In the story, a spacecraft needs to travel from Earth to Neptune in less than 2 minutes.

- The speed of light is 3.00×10^5 km/s.
- The distance from Earth to Neptune is 4.41×10^9 km.

Find the smallest possible warp n at which the spacecraft can travel.
You must show your working.

$$\text{Warp } n = n^3 \times 3 \times 10^5$$

$$\begin{array}{l} D \\ S \quad T \end{array} \quad D = 4.41 \times 10^9 \text{ km}$$

$$T = 2 \text{ mins} = 120 \text{ seconds}$$

$$S = \frac{4.41 \times 10^9}{120}$$

$$= 36750000 \text{ km/s}$$

$$n^3 = \frac{36750000}{3 \times 10^5}$$

$$n^3 = 122.5$$

$$n = \sqrt[3]{122.5}$$

$$= 4.966...$$

(c) $n = 5$ [3]

- 6 A six-sided numbered spinner is thrown 50 times. The score for each throw is recorded. Some of the results are shown in the table.

An 8 was thrown f times.

An unknown number on the spinner is represented by n .

Score	Frequency	Total score
1	12	12
3	2	6
5	9	45
6	16	96
8	$f = 7$	56
n	4	$4n$
Total	50	$215 + 4n$

The mean score of the 50 throws is 5.5 .

Find the value of f and the value of n .

$$f = 50 - (12 + 2 + 9 + 16 + 4)$$

$$= 7$$

$$\frac{215 + 4n}{50} = 5.5$$

$$\times 50 \quad \times 50$$

$$215 + 4n = 275$$

$$-215 \quad -215$$

$$4n = 60$$

$$\div 4 \quad \div 4$$

$$n = 15$$

$$f = \underline{7}$$

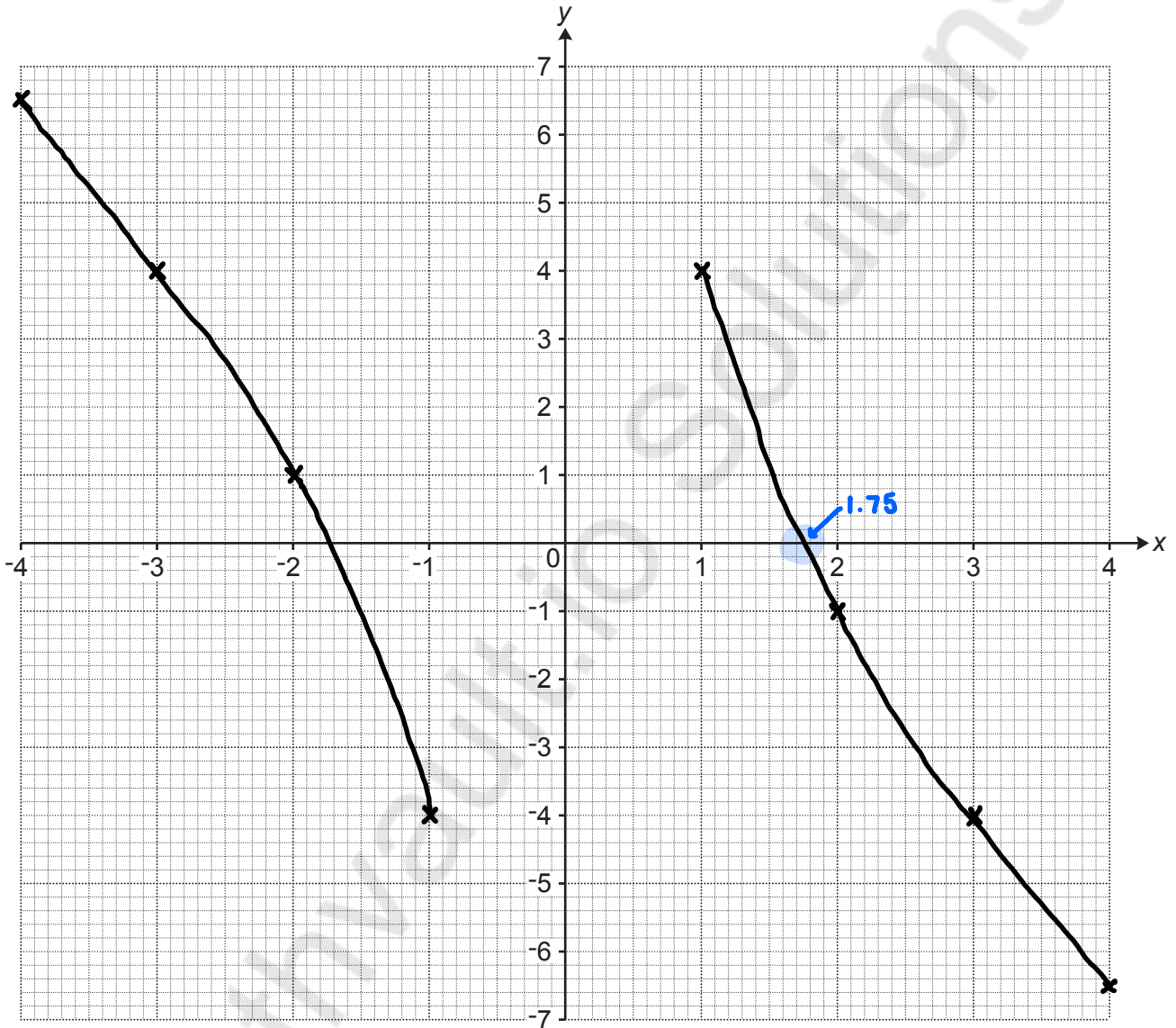
$$n = \underline{15}$$

[4]

7 Here is a table of values for $y = \frac{6}{x} - 2x$.

x	-4	-3	-2	-1	1	2	3	4
y	6.5	4	1	-4	4	-1	-4	-6.5

(a) Draw the graph of $y = \frac{6}{x} - 2x$ for $-4 \leq x \leq 4$, $x \neq 0$.



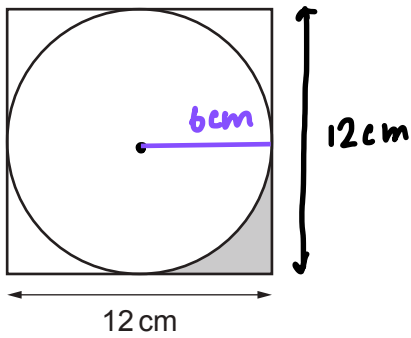
[3]

(b) Use your graph to find the positive solution of $\frac{6}{x} - 2x = 0$.

Give your answer to 1 decimal place.

(b) $x = \dots\dots\dots 1.8 \dots\dots\dots$ [1]

- 8 The diagram shows a circle inside a square of side 12 cm.



Work out the percentage of the square that is shaded.
You must show your working.

$$\begin{aligned} \text{Area of square} &= 12 \times 12 \\ &= 144 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \pi (6)^2 \\ &= 36\pi \text{ cm}^2 \end{aligned}$$

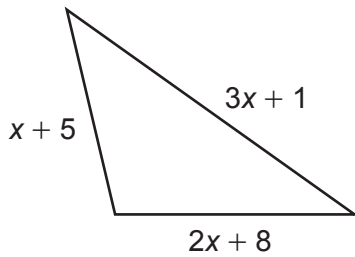
$$\text{Outside of circle area} = (144 - 36\pi) \text{ cm}^2$$

$$\begin{aligned} \text{Shaded area} &= \frac{144 - 36\pi}{4} \\ &= 7.725666118 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \frac{7.725\dots}{144} \times 100 &= 5.365045915 \\ &\approx 5.37\% \end{aligned}$$

..... 5.37 % [6]

- 9 The sides of this triangle are given in centimetres.
The perimeter of the triangle is 80 cm.



Not to scale

- (a) Find the length of each side of the triangle.
You must show your working.

$$x + 5 + 3x + 1 + 2x + 8 = 80$$

$$\begin{array}{r} 6x + 14 = 80 \\ - 14 \quad - 14 \\ \hline \end{array}$$

$$\begin{array}{r} 6x = 66 \\ \div 6 \quad \quad \div 6 \end{array}$$

$$x = 11$$

$$x + 5 = 11 + 5 = 16 \text{ cm}$$

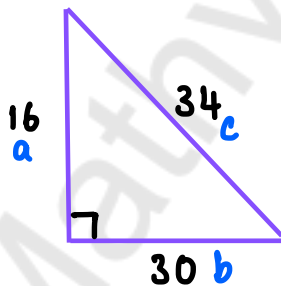
$$3x + 1 = 3(11) + 1 = 34 \text{ cm}$$

$$2x + 8 = 2(11) + 8 = 30 \text{ cm}$$

(a) 16 cm, 34 cm and 30 cm [5]

- (b) Is the triangle above a right-angled triangle?
Use calculations to show how you decide.

$$a^2 + b^2 = c^2$$



$$16^2 + 30^2 = 34^2$$

$$256 + 900 = 1156$$

$$1156 = 1156$$

..... Yes because $16^2 + 30^2 = 34^2$

.....
..... [3]

10 100 people were asked whether they had visited France (F) or Spain (S).

- 55 had visited France
- 60 had visited Spain
- 4 had not visited either country.

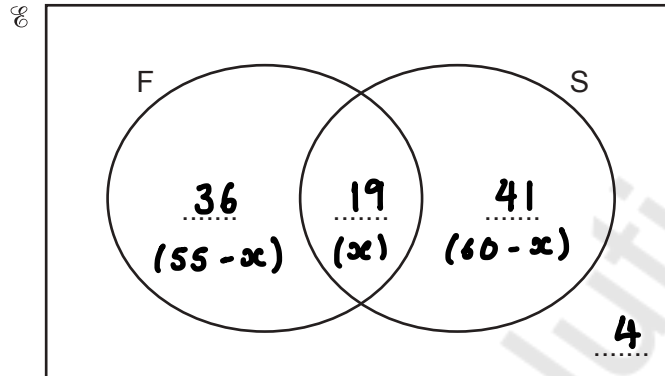
$$55 - x + x + 60 - x = 96$$

$$-x + 115 = 96$$

$$-x = -19$$

$$x = 19$$

(a) Complete the Venn diagram.



[3]

(b) One of these 100 people is chosen at random.

(i) Write down the probability that this person had visited exactly **one** of the countries.

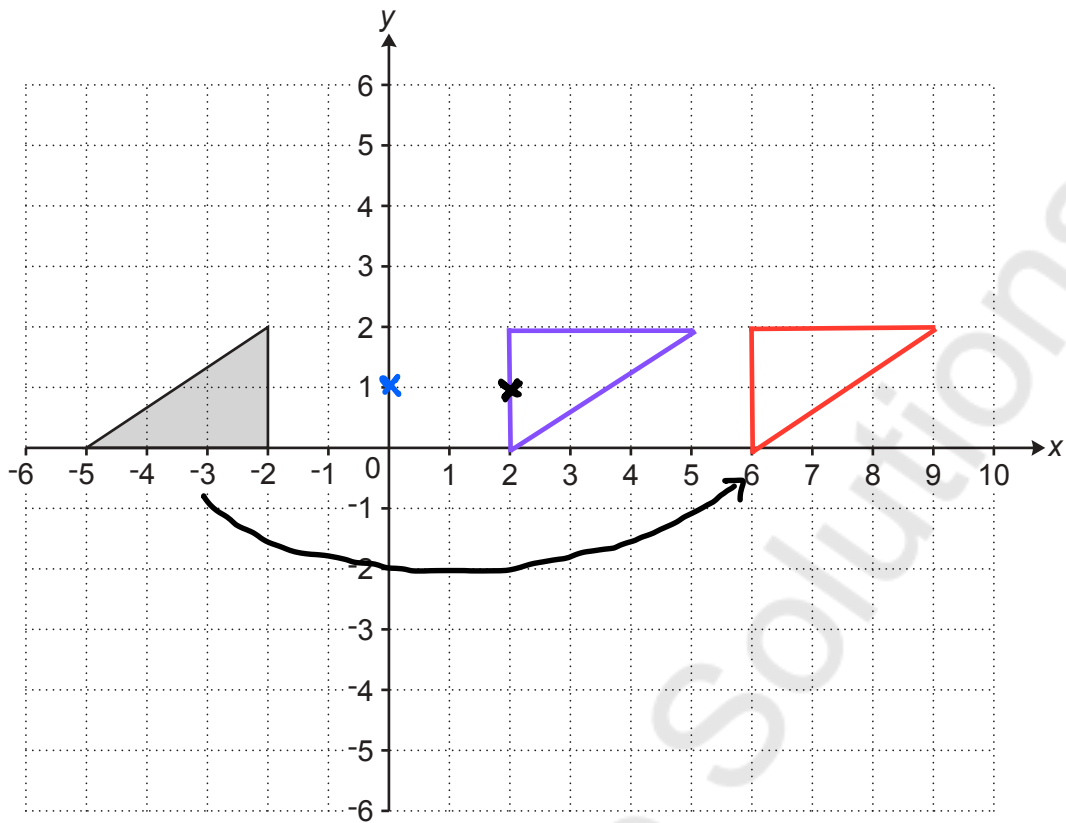
$$36 + 41 = 77$$

(b)(i) $\frac{77}{100}$ [2]

(ii) Write down the probability that this person had visited France given that they had also visited Spain.

(ii) $\frac{19}{60}$ [2]

11 You may use this coordinate grid to help you answer the following question.

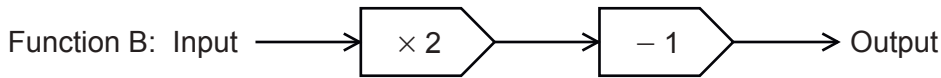
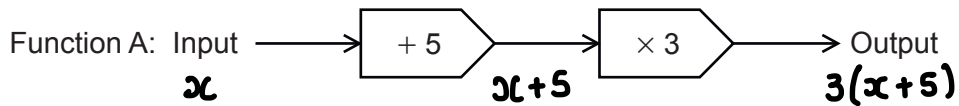


Describe fully the **single** transformation that is equivalent to:

- a rotation of 180° with centre $(0, 1)$, followed by
- a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

Rotation of 180° with centre $(2, 1)$
 OR Enlargement centre $(2, 1)$, scale factor -1 [3]

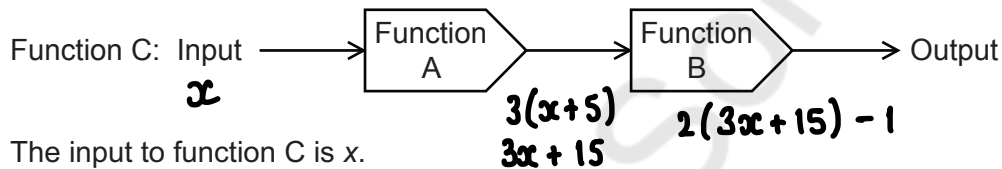
12 Here are two functions.



(a) Write an algebraic expression for the output of function A when the input is x .

(a) $3(x + 5)$ [1]

(b) Here is a composite function C.



Find the value of x .

You must show your working.

$$2(3x + 5) - 1$$

$$6x + 30 - 1$$

$$6x + 29$$

$$6x + 29 = 2x + 1$$

$$-2x \quad -2x$$

$$4x + 29 = 1$$

$$-29 \quad -29$$

$$4x = -28$$

$$\div 4 \quad \div 4$$

$$x = -7$$

(b) $x = -7$ [5]

13 Here are two pieces of work.

Each shows a question and the **first line** of an incorrect solution.

For each part, describe the error made in the first line of the solution.
You do **not** need to complete the solution.

(a)

Question:
Simplify. $\frac{2}{x-1} + \frac{3}{x+4}$

Solution:
 $\frac{2}{x-1} + \frac{3}{x+4} = \frac{2(x-1) + 3(x+4)}{(x-1)(x+4)}$

..... They did not cross-multiply the numerators
..... and denominators [1]

(b)

Question:
Solve. $x^2 + 7x + 5 = 0$

Solution:
 $x = -\frac{7 \pm \sqrt{7^2 - 4 \times 1 \times 5}}{2 \times 1}$

.....
..... [1]

- 14 A college offers 41 different subjects including 9 different languages. Students are asked to choose one subject from Option A, one subject from Option B and one subject from Option C.

Each of the 41 different subjects appears only once, either in Option A, or in Option B or in Option C.

Option A : 14 subjects including 2 languages
 Option B : 12 subjects including 3 languages
 Option C : 15 subjects including 4 languages

Work out the proportion of all the possible subject combinations that include **at least one** language.

You must show your working.

$$\begin{aligned} \text{Total including languages} &= 14 \times 12 \times 15 \\ &= 2520 \end{aligned}$$

$$\begin{aligned} \text{Total without languages} &= 12 \times 9 \times 11 \\ &= 1188 \end{aligned}$$

$$2520 - 1188 = 1332$$

$$\frac{1332}{2520}$$

$$\frac{1332}{2520}$$

..... [5]

- 15 y is inversely proportional to the square root of x .
 $y = 7$ when $x = 144$.

Find the value of y when $x = 16$.

$$\begin{aligned} y &\propto \frac{1}{\sqrt{x}} \\ y &= \frac{k}{\sqrt{x}} \\ 7 &= \frac{k}{\sqrt{144}} \\ 7 &= \frac{k}{12} \end{aligned}$$

$\times 12$

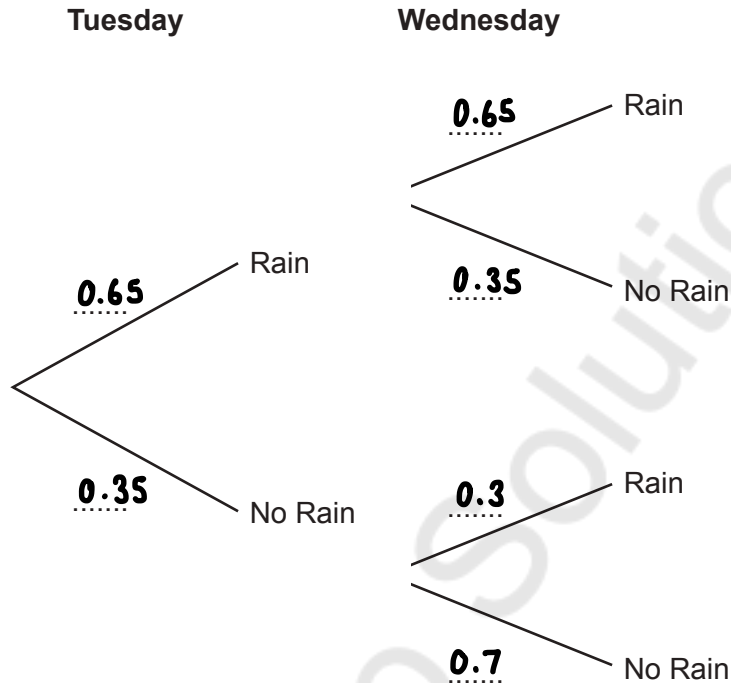
$\times 12$

$$\begin{aligned} k &= 84 \\ y &= \frac{84}{\sqrt{x}} \\ y &= \frac{84}{\sqrt{16}} \\ y &= \frac{84}{4} \\ &= 21 \end{aligned}$$

$y = 21$ [3]

- 16 If it rains on a given day the probability that it will rain the next day is 0.65.
If it does **not** rain on a given day the probability that it will rain the next day is 0.3.
It rained on Monday.

(a) Complete the tree diagram.



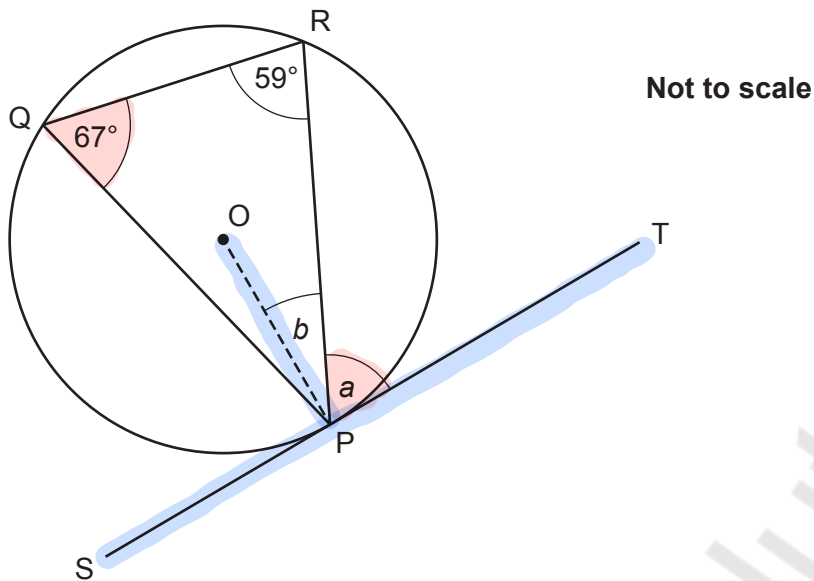
[2]

(b) Find the probability that it rains on Wednesday.

$$\begin{aligned}
 & p(\text{Rain - Tue Wed}) + p(\text{No Rain Tue - Rain Wed}) \\
 & 0.65 \times 0.65 + 0.35 \times 0.3 \\
 & 0.4225 + 0.105 = 0.5275
 \end{aligned}$$

(b) **0.5275** [3]

- 17 (a) P, Q and R are points on the circumference of a circle, centre O.



Angle $PRQ = 59^\circ$ and angle $PQR = 67^\circ$.
Line SPT is a tangent to the circle.

- (i) Work out angle a .
Give a reason for your answer.

Angle $a = 67^\circ$ because alternate segment theorem.

[2]

- (ii) Work out angle b .
Give a reason for your answer.

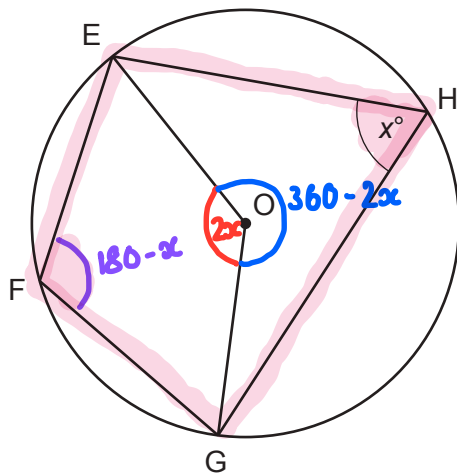
$$90 - 67 = 23$$

Angle $b = 23^\circ$ because tangent meets radius at

90°

[2]

- (b) E, F, G and H are points on the circumference of a circle, centre O.
Acute angle $\text{EHG} = x^\circ$.



Not to scale

- (i) Complete the following, giving the values of the angles in terms of x .

Obtuse angle EOG = $2x$ $^\circ$ because angle at the centre is twice the angle at the circumference.

Therefore, reflex angle EOG = $360 - 2x$ $^\circ$

Therefore, angle EFG = $180 - x$ $^\circ$ [3]

- (ii) Write down what your working in part (b)(i) has proved.

Opposite angles in a cyclic quadrilateral sum to 180° [1]

- 18 (a) Show that the equation $x^3 + x^2 - 5 = 0$ has a solution between $x = 1$ and $x = 2$. [3]

$$(1)^3 + (1)^2 - 5 = -3$$

$$(2)^3 + (2)^2 - 5 = 7$$

Change of sign \therefore solution between $x=1$ and $x=2$

- (b) Find this solution correct to 1 decimal place.
You must show calculations to support your answer.

$$(1.5)^3 + (1.5)^2 - 5 = 0.625$$

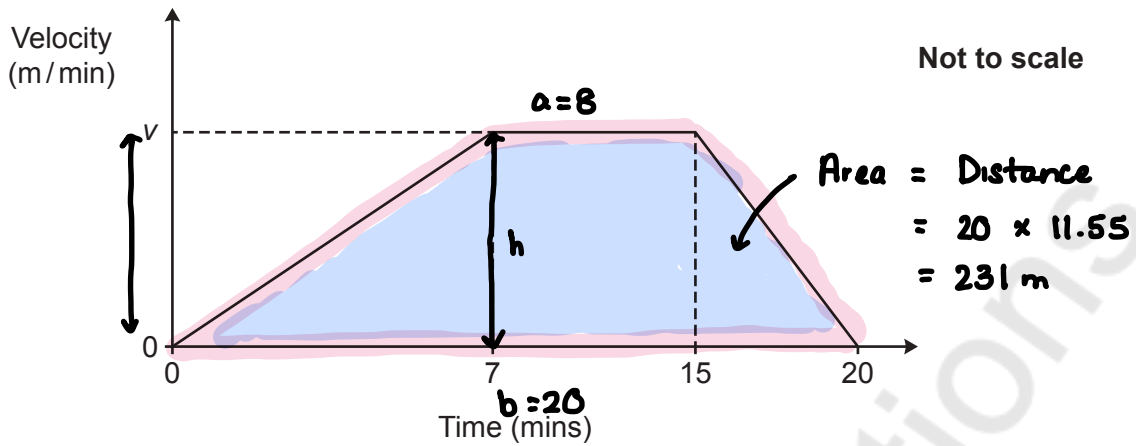
$$(1.4)^3 + (1.4)^2 - 5 = -0.296$$

$$(1.43)^3 + (1.43)^2 - 5 = -0.030893$$

$$(1.44)^3 + (1.44)^2 - 5 = 0.059584$$

(b) $x = \underline{1.4}$ [4]

- 19 The graph shows the velocity of a particle over the first 20 minutes of its motion.



Between 7 minutes and 15 minutes the velocity of the particle is v metres per minute. The average velocity of the particle over the 20 minutes is 11.55 metres per minute.

Find the value of v .

You must show your working.

$$\text{Area} = \frac{1}{2}(a+b) \times h$$

$$231 = \frac{1}{2}(8+20) \times h$$

$$231 = 14 \times h$$

$$\div 14 \quad \div 14$$

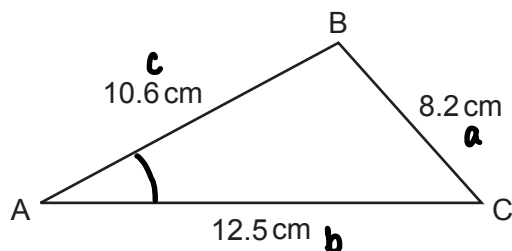
$$16.5 = h$$

$$= v$$

$$v = \underline{16.5} \dots \dots \dots [5]$$

TURN OVER FOR QUESTION 20

20 The diagram shows triangle ABC.



Not to scale

AB = 10.6 cm, BC = 8.2 cm and AC = 12.5 cm.

(a) Show that angle BAC = 40.5°, correct to 1 decimal place.

[3]

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{12.5^2 + 10.6^2 - 8.2^2}{2 \times 12.5 \times 10.6} \\ &= 0.7598867925\end{aligned}$$

$$\begin{aligned}A &= \cos^{-1}(\uparrow) \\ &= 40.54578122 \approx 40.5^\circ \text{ (1dp)}\end{aligned}$$

(b) Work out the area of triangle ABC.

$$\begin{aligned}\text{Area} &= \frac{1}{2} ab \sin C \quad a = 10.6 \quad b = 12.5 \quad C = 40.5 \\ &= \frac{1}{2} \times 10.6 \times 12.5 \times \sin(40.5) \\ &= 43.0259832 \\ &\approx 43.0 \text{ cm}^2\end{aligned}$$

(b) 43.0 cm² [2]

END OF QUESTION PAPER

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.