OCR
Oxford Cambridge and RSA

Wednesday 7 June 2023 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Work out.

$$\frac{33}{35} \div 1\frac{4}{7}$$

Give your answer as a fraction in its simplest form.

$1\frac{4}{7} = \frac{11}{7}$

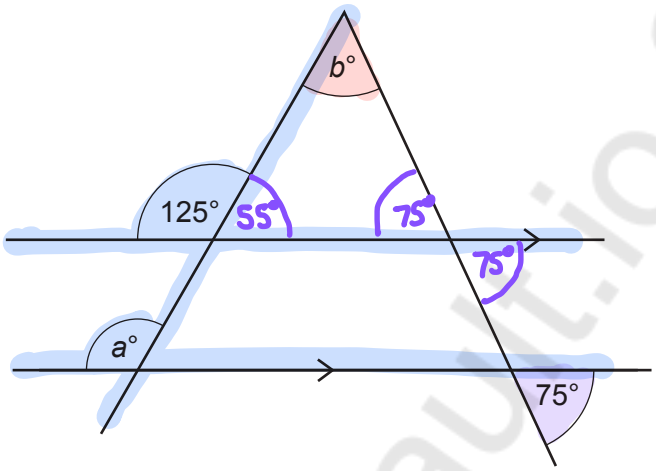
$\frac{33}{35} \div \frac{11}{7}$

$\frac{\cancel{33}^{3}}{\cancel{35}_{5}} \times \frac{\cancel{7}^{1}}{\cancel{11}_{1}}$ $\quad \frac{3}{5} \times \frac{1}{1} = \frac{3}{5}$

$\frac{3}{5}$

..... **[3]**

2 The diagram shows two straight lines crossing a pair of parallel lines.



Not to scale

(a) Write down the value of a.
Give a reason for your answer.

$a =$ 125° because corresponding angles are equal.

..... **[2]**

(b) Work out the value of b.

$55 + 75 = 130°$

$180 - 130° = 50°$

(b) $b =$ 50 **[3]**

3 Work out.

3.8 ÷ 0.02

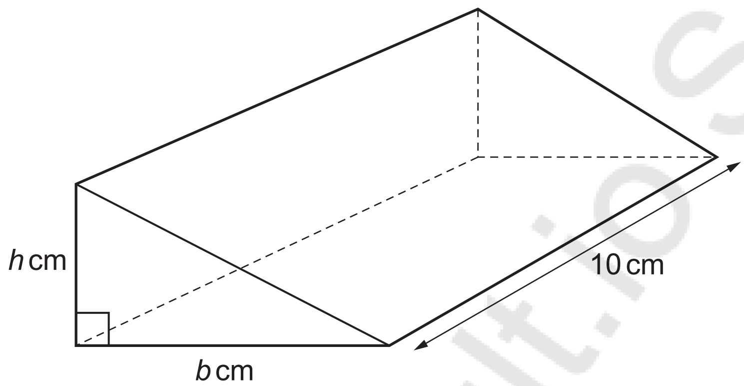
×100 ×100

380 ÷ 2

$$2 \overline{\smash{)}3^{1}80}$$ = 190

190 **[2]**

4 The diagram shows a prism of length 10 cm.



The cross-section of the prism is a right-angled triangle.
The base, b cm, is 2 cm longer than the height, h cm.
The volume of the prism is 240 cm^3.

A student is explaining how they worked out the value of b.

They say

b is 6 because that means h is 4 and 6 × 4 × 10 = 240.

Describe the student's error and find the correct value of b.

$\frac{1}{2}bh \times \text{length}$

$\frac{1}{2} \times 6 \times 4 \times 10$

The error is they did not multiply by $\frac{1}{2}$ for area of the cross-section.

$\frac{1}{2}b \times h \times 10 = 240$

$\frac{1}{2}b \times h = 24$

$b \times h = 48$

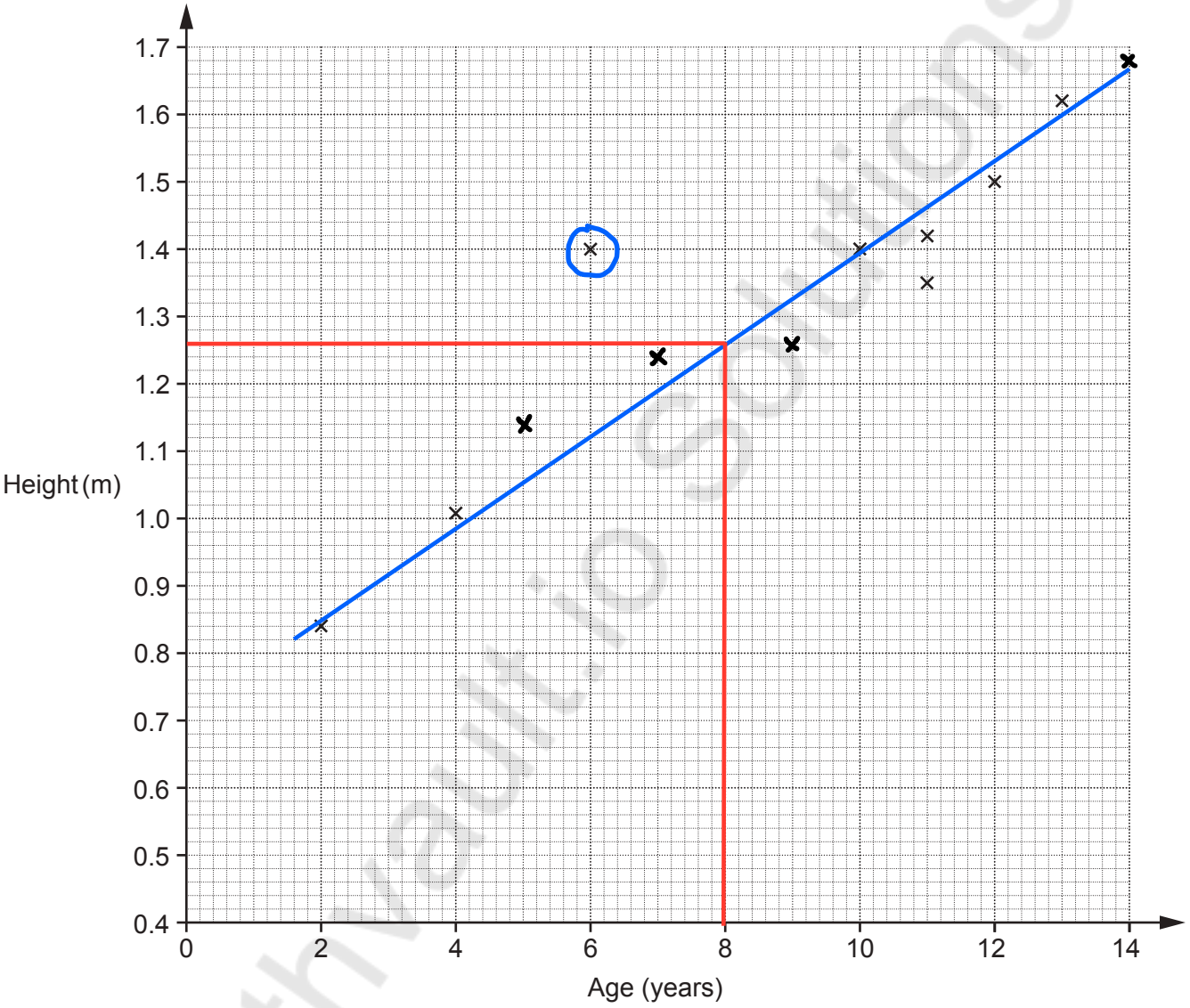
$8 \times 6 = 48$

b = 8 **[3]**

5 The table shows the ages and heights of 12 children.

Age (years)	2	4	12	6	10	11	13	11	5	7	9	14
Height (m)	0.84	1.01	1.5	1.4	1.4	1.35	1.62	1.42	1.14	1.24	1.26	1.68

The points for the first eight children (shaded in the table above) are plotted on the scatter diagram.



(a) Plot the points for the remaining four children. **[2]**

(b) Describe the type of correlation shown in the completed scatter diagram.

Positive ..... **[1]**

(c) One of these children is taller than expected for their age.

On the scatter diagram, circle the point representing this child. **[1]**

(d) (i) Kai is 8 years old.
By drawing a line of best fit, estimate Kai's height.

(d)(i) 1.26 m **[2]**

(ii) Describe an assumption you have made in giving your answer to part **(d)(i)**.

Kai is of average height and is not too tall or short for their age. **[1]**

(e) Explain why using this data to estimate the height of a child that is 17 years old may be unreliable.

Only have data up to age 14. **[1]**

6 Taylor has a full bottle of medicine.
The bottle holds 20 doses of medicine.

Each day Taylor takes one dose of medicine out of the bottle.
After 8 days, there are 180 millilitres of medicine left in the bottle.

Work out how many millilitres of medicine the bottle holds when full.

20 - 8 = 12 doses left

12 doses = 180 ml

÷6 ÷6

2 doses = 30 ml

x10 x10

20 doses = 300 ml

..... 300 ml **[4]**

7 A volunteer packs boxes for a charity.
They can pack 5 boxes in 45 seconds.

(a) Use this information to show that they can pack 55 boxes in less than 9 minutes. **[4]**

1 min = 60s

×9 ×9

9 mins = 540 s

5 boxes = 45s

×11 ×11

55 boxes = 495s

$$\begin{array}{r} 45 \\ \times\ 11 \\ \hline 45 \\ 450 \\ \hline 495 \end{array}$$

495s < 540s

(b) What assumption did you make in part **(a)**?

They continue to pack the boxes at the same rate. **[1]**

8 A block made of iron is in the shape of a cuboid.
The block is 3.1 cm by 4.9 cm by 2.2 cm.
The density of iron is 7.87 g / cm^3.
Sam thinks that the mass of the block is about 2.4 kg.

m
D V

Use estimation to decide if Sam's answer is reasonable.
Show how you decide.

$D = 7.87\ g/cm^3$

$\approx 8\ g/cm^3$

$V = l \times w \times h$

$\approx 3 \times 5 \times 2$

$= 30\ cm^3$

$m = 8 \times 30$

$= 240g$

$2.4kg \times 1000 = 2400g$

Sam's answer is not reasonable because 2400g is much greater than 240g.

[5]

9 A zoo counts its animals.
The ratio of antelope to zebra is 3 : 2.
The ratio of meerkats to zebra is 7 : 3.

(a) Write the number of antelope as a percentage of the number of zebra.

A : Z
3 : 2

$\frac{3}{2} = 1.5 \times 100 = 150$

(a) ..... 150 ..... % **[2]**

(b) There are 15 more meerkats than antelope.

Work out the number of zebra in the zoo.

A : Z	m : Z
3 : 2	7 : 3
x 3	x 2
9 : 6	14 : 6

A : m : Z
9 : 14 : 6

14 − 9 = 5 more parts

15 ÷ 5 = 3

6 × 3 = 18 zebras

(b) ..... 18 ..... **[4]**

10 A student draws two different regular polygons.
The exterior angle of one polygon is $p°$.
The exterior angle of the other polygon is $q°$.

The sum of p and q is 112°.
The difference between p and q is 32°.

Find the **number of sides** of each polygon.
You must show your working.

$p + q = 112$
$p - q = 32$

$2q = 80$

$\div 2 \quad \div 2$

$q = 40$

$\frac{360}{40} = 9$ sides

$p + q = 112$

$p + 40 = 112$

$-40 \quad -40$

$p = 72$

$\frac{360}{72} = 5$ sides

.....5..... sides and .....9..... sides **[6]**

11 y is directly proportional to the square of x.

Find the percentage decrease in y when x is decreased by 30%.

$y \propto x^2$

$y = kx^2$

$y = k(0.7)^2$

$y = k(0.49)$

$y = 0.49k$

51% decrease

↳ 100% − 30% = 70%

÷100

0.7

$0.7 \times 0.7 = 0.49$

$100 - 51 = 49\% \rightarrow 0.49$

51 ..... % **[4]**

12 Here are the first four terms of a sequence.

$\frac{2}{5}$ (+3) $\frac{5}{10}$ (+3) $\frac{8}{17}$ (+3) $\frac{11}{26}$ (+3) $\frac{14}{37}$

(+5) (+7) (+9) (+11)

(a) Find the next term.

$\frac{14}{37}$

(a) ..... **[1]**

(b) Find the nth term.

3n

2 (+3) 5 (+3) 8 (+3) 11

3n − 1

5 (+5) 10 (+7) 17 (+9) 26

(+2) (+2)

$an^2 + bn + c$

$a = 2 \div 2 = 1$

$n^2 + 2n + 2$

n^2	1	4	9	16	×1
an^2	1	4	9	16	
Seq	5	10	17	26	
Seq − an^2	4 (+2)	6 (+2)	8 (+2)	10 (+2)	= 2n + 2

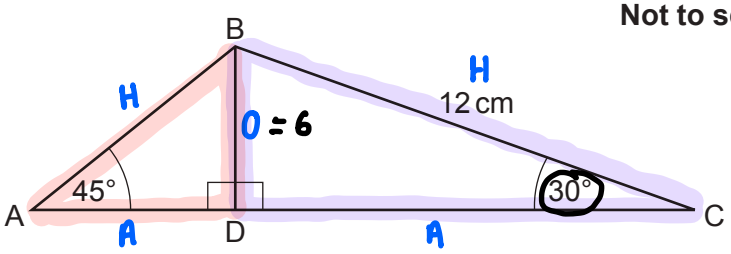
(+2) (+2) (+2)

$\frac{3n-1}{n^2+2n+2}$

(b) ..... **[3]**

13 The diagram shows a triangle, ABC, with perpendicular height BD.

Not to scale



S O/H V C A/H T O/A

BC = 12 cm, angle BCD = 30° and angle BAD = 45°.

(a) Work out the length of BD.

$\sin\theta = \frac{O}{H}$

$\sin 30 = \frac{BD}{12}$ ×12 ×12

$BD = 12 \times \sin 30$

$= 12 \times \frac{1}{2}$

$= 6$

	0	30	45	60	90
sin	0	1	2	3	4
cos	4	3	2	1	0

2

$\sin 30 = \frac{\sqrt{1}}{2} = \frac{1}{2}$

(a) ..... 6 ..... cm **[3]**

(b) Work out the exact length of AB.
Give your answer in its simplest form.

$\sin\theta = \frac{O}{H}$

$\sin 45 = \frac{6}{AB}$ ×AB ×AB

$AB \times \sin 45 = 6$

÷ sin 45 ÷ sin 45

$AB = 6 \div \sin 45$

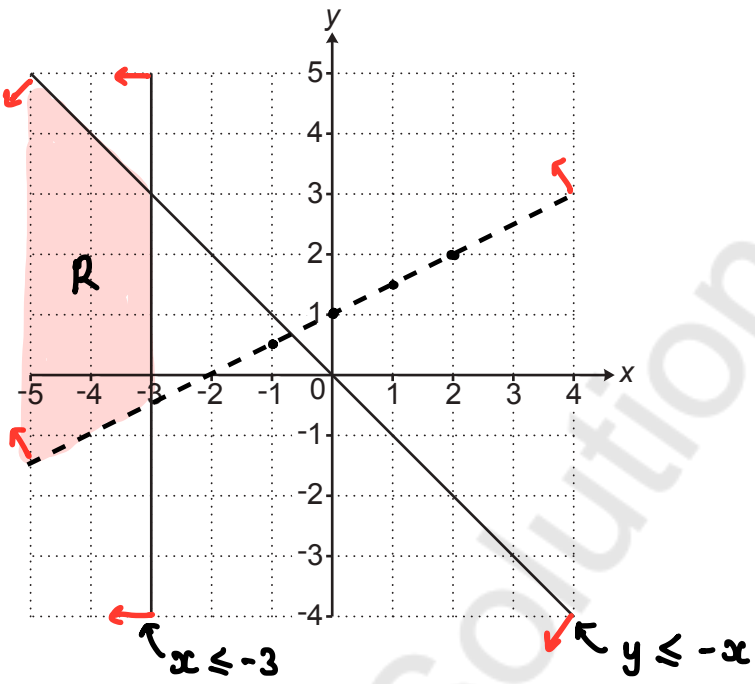
$= 6 \div \frac{\sqrt{2}}{2}$

$= \frac{6}{1} \times \frac{2}{\sqrt{2}}$

$= \frac{12}{\sqrt{2}}$ cm

(b) ..... $\frac{12}{\sqrt{2}}$ ..... cm **[3]**

14 The graphs of $x = -3$ and $y = -x$ are drawn on the grid.



The region **R** satisfies the following inequalities.

$x \leqslant -3$ $y \leqslant -x$ $y - 1 > \frac{1}{2}x$

By drawing one more line, find and label the region **R**. **[5]**

$y - 1 > \frac{1}{2}x$

$+1 \quad +1$

$y > \frac{1}{2}x + 1$

x	-1	0	1	2
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2

$\frac{1}{2}(-1) + 1$

$-\frac{1}{2} + 1 = \frac{1}{2}$

15 **(a)** Factorise.

$9x^2 - 4$

$\sqrt{\ }$ $\sqrt{\ }$

$(3x + 2)(3x - 2)$

(a) $(3x+2)(3x-2)$ **[2]**

(b) Solve by factorisation.

$3x^2 - 2x - 8 = 0$

$ax^2 + bx + c = 0$

$a = 3 \quad b = -2 \quad c = -8$

$a \times c = -24$

-24

$-6 \quad +4$

$3x^2 - 6x + 4x - 8$

$3x(x-2) \quad +4(x-2)$

$(3x+4)(x-2) = 0$

$3x + 4 = 0$ or $x - 2 = 0$

$-4 \quad -4 \qquad +2 \quad +2$

$3x = -4 \qquad x = 2$

$\div 3 \quad \div 3$

$x = -\frac{4}{3}$

(b) $x =$ $-\frac{4}{3}$ or $x =$ 2 **[3]**

(c) Solve.

$\frac{2(x-5)}{1-3x} = 2$

$x(1-3x) \qquad x(1-3x)$

$2(x-5) = 2(1-3x)$

$2x - 10 = 2 - 6x$

$+6x \qquad +6x$

$8x - 10 = 2$

$+10 \quad +10$

$8x = 12$

$\div 8 \qquad \div 8$

$x = \frac{12}{8} = \frac{3}{2}$

(c) $x =$ $\frac{3}{2}$ **[4]**

16 **(a)** Work out.

$64^{\frac{2}{3}}$

$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2$

$= 4^2$

$= 16$

(a) 16 **[2]**

(b) $\frac{p}{q} + 0.\dot{1}\dot{3} = \frac{5}{9}$

where $\frac{p}{q}$ is a fraction in its lowest terms.

Find the value of p and the value of q.

$x = 0.\dot{1}\dot{3}$

$10x = 1.\dot{3}\dot{1}$

$100x = 13.\dot{1}\dot{3}$

$100x = 13.\dot{1}\dot{3}$

$-\quad x = 0.\dot{1}\dot{3}$

$99x = 13$

$\div 99 \qquad \div 99$

$x = \frac{13}{99}$

$\frac{p}{q} + \frac{13}{99} = \frac{5}{9}$

$-\frac{13}{99} \qquad -\frac{13}{99}$

$\frac{p}{q} = \frac{5}{9}^{\times 11} - \frac{13}{99}$

$\times 11$

$\frac{p}{q} = \frac{55}{99} - \frac{13}{99}$

$\frac{p}{q} = \frac{42}{99}^{\div 3} = \frac{14}{33}$

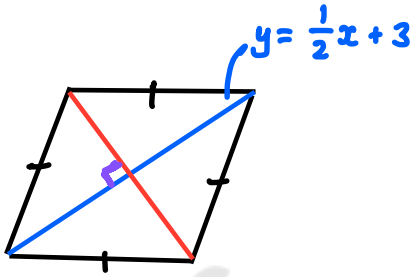
$p = 14$

$q = 33$

(b) $p =$ 14

$q =$ 33 **[4]**

17 A rhombus is drawn on a coordinate grid.



One diagonal of the rhombus has equation $y = \frac{1}{2}x + 3$.

The other diagonal passes through the point (1, 7).

Find the equation of the other diagonal of the rhombus.
Give your answer in the form $y = mx + c$.

$y = \frac{1}{2}x + 3 \qquad m = \frac{1}{2}$

Gradient of other diagonal = −2

(1, 7)
x y

$y = mx + c$

$y = -2x + c$

$7 = -2(1) + c$

$7 = -2 + c$

$+2 \quad +2$

$9 = c$

$y = -2x + 9$

$y =$ −2x + 9 **[4]**

18 $\sqrt[5]{p^2} = (\sqrt[3]{m})^2$ and $p = m^x$, where $p > 0$, $m > 0$ and $p \neq m$.

Show that the value of x is $\frac{5}{3}$. **[3]**

$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$

$(\sqrt[3]{m})^2 = m^{\frac{2}{3}}$

$\sqrt[5]{p^2} = m^{\frac{2}{3}}$

$\square^5 \qquad \square^5$

$p^2 = (m^{\frac{2}{3}})^5$

$p^2 = m^{\frac{10}{3}}$

$\sqrt{\ } \qquad \sqrt{\ }$

$p = \sqrt{m^{\frac{10}{3}}}$

$= (m^{\frac{10}{3}})^{\frac{1}{2}}$

$= m^{\frac{10}{6}}$

$p = m^{\frac{5}{3}}$

$x = \frac{5}{3}$

$\sqrt{x} = x^{\frac{1}{2}}$

19 A box contains 25 discs.
The discs are either blue or yellow in the ratio 4 : 1.
Two discs are chosen at random from the box without replacement.

Find the probability that the two discs are different colours.
You must show your working.

B : Y

4 : 1 = 5 parts $25 \div 5 = 5$

$\times 5$ ↓ ↓ $\times 5$

20 : 5

BY or YB

$$p(BY) = p(B) \times p(Y)$$

$$= \frac{20}{25} \times \frac{5}{24} = \frac{100}{600}$$

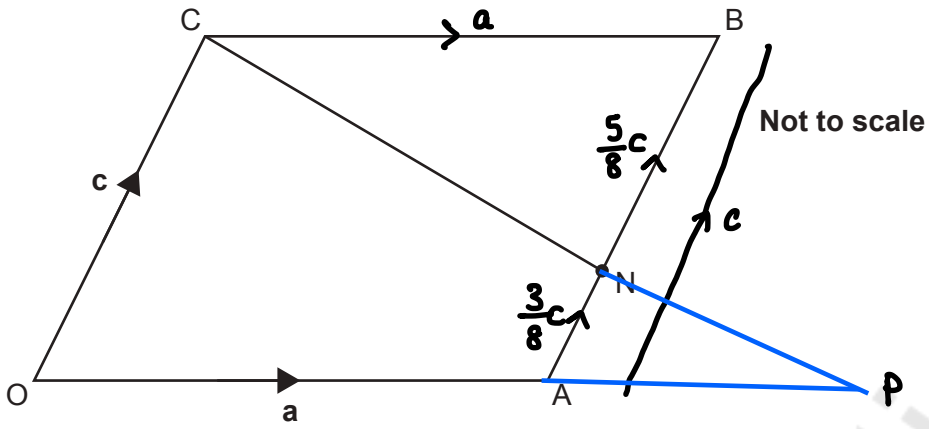
$$p(YB) = p(Y) \times p(B)$$

$$= \frac{5}{25} \times \frac{20}{24} = \frac{100}{600}$$

$$\frac{100}{600} + \frac{100}{600} = \frac{200}{600} = \frac{1}{3}$$

$\frac{1}{3}$ ..... **[5]**

20 OABC is a parallelogram.



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
The point N lies on line AB such that AN : NB = 3 : 5.

(a) Find the following vectors in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$= c + a$

(a)(i) $\overrightarrow{OB} =$ $c + a$ **[1]**

(ii) $\overrightarrow{ON}$

$\overrightarrow{OA} + \overrightarrow{AN}$

$a + \frac{3}{8}c$

(ii) $\overrightarrow{ON} =$ $a + \frac{3}{8}c$ **[2]**

(b) Line CN is extended to reach point P, such that $\overrightarrow{CP} = \frac{8}{5}\overrightarrow{CN}$.

Show, using vectors, that OAP is a straight line. **[4]**

$\overrightarrow{CN} = \overrightarrow{CB} + \overrightarrow{BN}$

$= a - \frac{5}{8}c$

$\overrightarrow{CP} = \frac{8}{5}\overrightarrow{CN}$

$= \frac{8}{5}(a - \frac{5}{8}c)$

$= \frac{8}{5}a - c$

$\overrightarrow{OP} = \overrightarrow{OC} + \overrightarrow{CP}$ $\qquad \overrightarrow{OA} = a$

$= c + \frac{8}{5}a - c$

$= \frac{8}{5}a$

$\overrightarrow{OP} = \frac{8}{5}\overrightarrow{OA}$

$\overrightarrow{OP}$ is a multiple of $\overrightarrow{OA}$ $\quad \therefore$ OAP is a straight line.

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).