OCR
Oxford Cambridge and RSA

Wednesday 14 June 2023 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

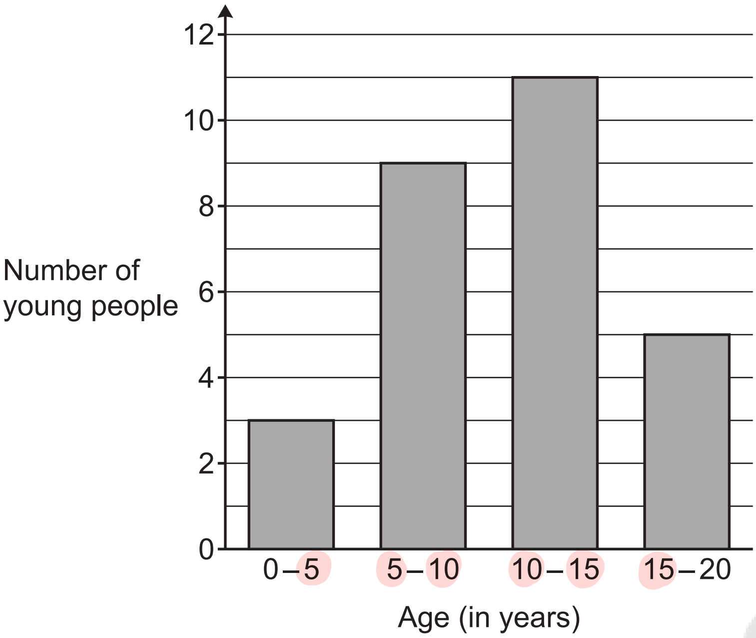
Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Alex draws a bar chart to show the age of the young people attending a youth club.



Make **one** criticism of Alex's bar chart.

The age groups overlap.

....

.... **[1]**

2 **(a)** Rearrange this formula to make u the subject.

$$v^2 = u^2 + 2as$$

$-2as \quad -2as$

$v^2 - 2as = u^2$

$\sqrt{\ } \quad \sqrt{\ }$

$\sqrt{v^2 - 2as} = u$

(a) $u = \sqrt{v^2 - 2as}$ .... **[2]**

(b) A rocket accelerates at 90 m/s² and travels 270 km.
The rocket's final velocity is 8000 m/s.

a s v

Using part **(a)**, or otherwise, calculate the rocket's initial velocity in m/s.

u

270 km × 1000 = 270,000 m

$u = \sqrt{v^2 - 2as}$

$= \sqrt{(8000)^2 - 2(90)(270{,}000)}$

$= 3924.283374$

≈ 3924.3 m/s

(b) ....3924.3...................... m/s **[3]**

3 A bag contains 150 counters.
The counters are either red or yellow.

(a) Riley picks a counter from the bag, records its colour, and replaces it.
They do this nine times.

Here are Riley's results.

Red	𝍸	5
Yellow	||||	4

Use Riley's results to work out how many red counters are likely to be in the bag.

p(red) × 150

$\frac{5}{9} \times 150 = 83.\dot{3}$

(a) ....83 or 84........ red counters **[3]**

(b) Ling uses the same bag of counters and picks the counters in the same way.

Here are Ling's results.

Red	𝍸 𝍸 ||	12
Yellow	𝍸 |||	8

Use Ling's results to estimate the probability of choosing a red counter from the bag.
Give your answer as a fraction in its simplest form.

$p(\text{red}) = \frac{12}{20} \begin{matrix} \div 4 \\ \div 4 \end{matrix} = \frac{3}{5}$

(b) ....$\frac{3}{5}$........ **[2]**

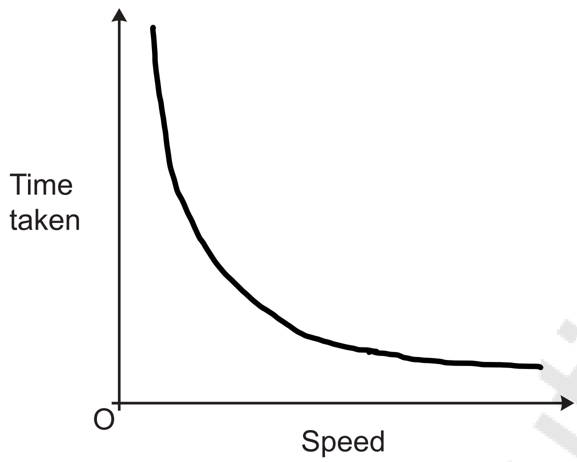
(c) Explain why Ling's results are likely to give a better estimate of the probability of choosing a red counter from the bag than Riley's results.

....Ling has more results than Riley.....

.... **[1]**

4 **(a)** The time taken to complete a journey halves as the speed doubles.

On the axes below, sketch a graph to show this relationship. Inverse proportion



[2]

(b) It takes 40 minutes to fill a garden pond using water from 5 identical hose pipes.

Assuming the rate of flow of water from each hose pipe is the same, work out how many minutes it would take to fill the same garden pond using 2 of these hose pipes.

÷5   5 pipes = 40 mins   ×5

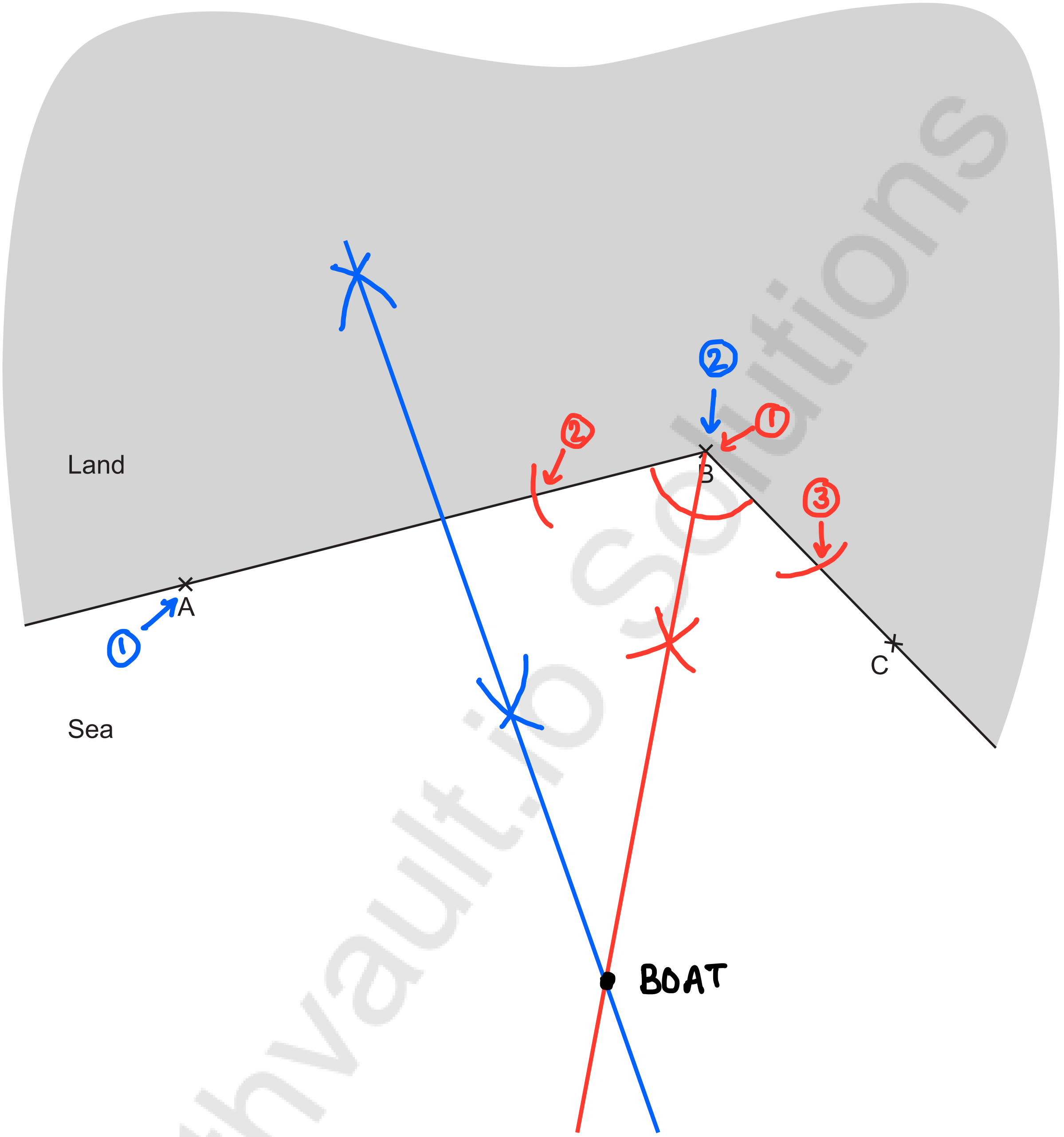
1 pipe = 200 mins

×2   2 pipes = 100 mins   ÷2

(b) ....100.......... minutes **[2]**

5 The diagram represents a coastline.

A, B and C are lighthouses.



A boat is perpendicular bisector of AB
- the same distance from A and B
- the same distance from AB and BC. angle bisector ABC

Using a ruler and compasses only, construct the position of the boat.
Label the position of the boat clearly. **[5]**

6 At the end of each year, a driver records how many kilometres they have driven.

In 2021, they drove 18% more kilometres than in **2020**.
In 2022, they drove 25% more kilometres than in **2020**.

In 2022, they drove 3500 km.

(a) Kai says

I can work out how many kilometres were driven in 2020 by reducing 3500 by 25%.
3500 × 0.75 = 2625 km.

Explain why 2625 is **not** the correct number of kilometres driven in 2020.

$2625 \times 1.25 = 3281.25$ not 3500 km

[1]

(b) Calculate the number of kilometres driven in **2021**.

2022 $100\% + 25\% = 125\%$

$125\% = 3500$ km

$\div 1.25$ $\div 1.25$

2020 $100\% = 2800$ km

2021 $100\% + 18\% = 118\%$

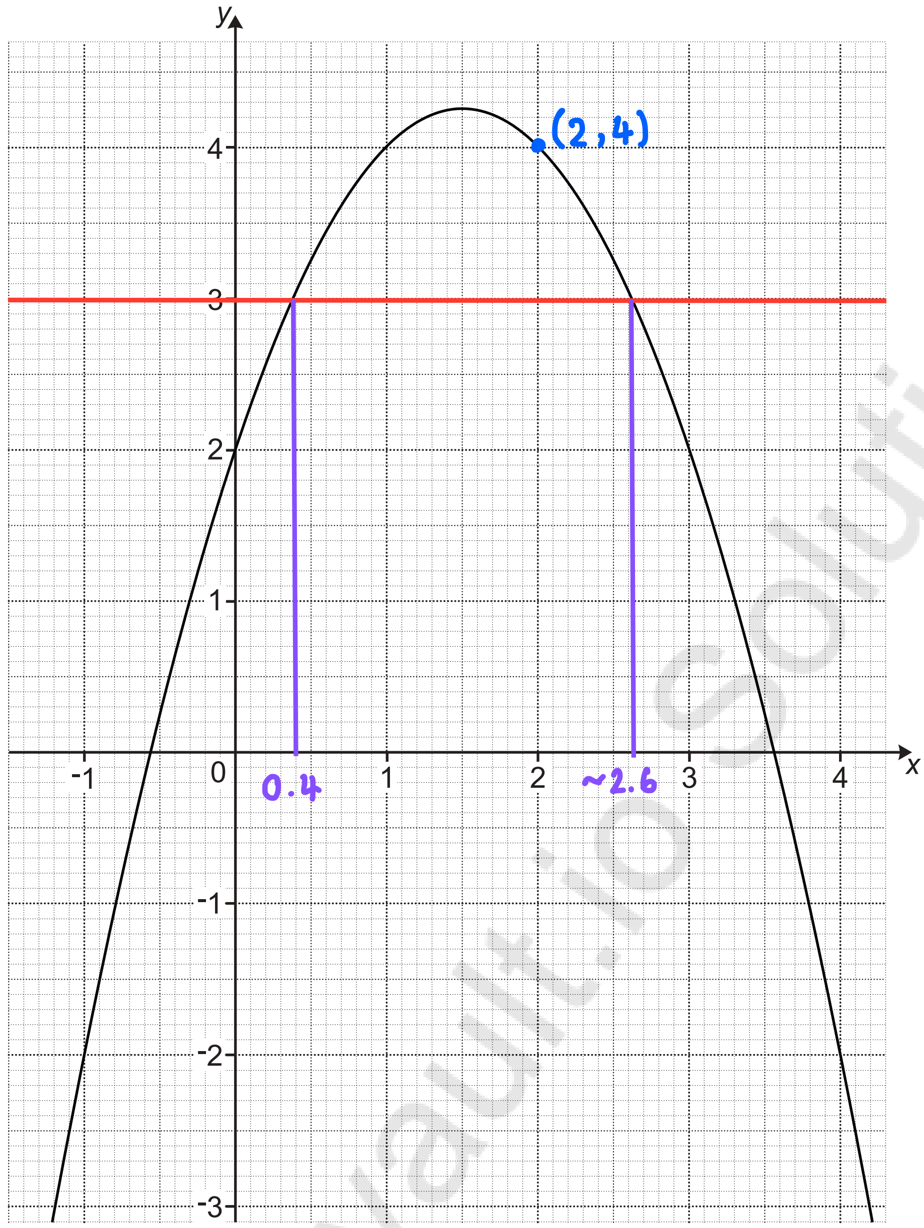
$100\% = 2800$ km

$\times 1.18$ $\times 1.18$

$118\% = 3304$ km

(b) 3304 km **[4]**

7 The diagram shows the graph of $y = kx - x^2 + 2$, where k is an integer.



(a) Show that $k = 3$. **[2]**

$(2, 4)$
$x \quad y$

$y = kx - x^2 + 2$

$4 = k(2) - (2)^2 + 2$

$4 = 2k - 4 + 2$

$4 = 2k - 2$

$+2 \qquad +2$

$\div 2 \quad 6 = 2k \quad \div 2$

$\underline{3 = k}$

(b) Use the graph to solve $3x - x^2 + 2 = 3$.
Give your answers to **1** decimal place.

$y = 3x - x^2 + 2$

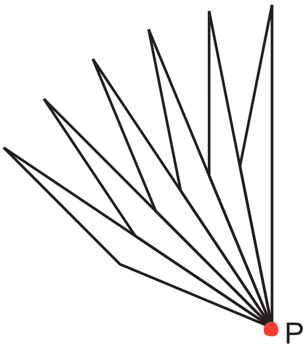
$3 = 3x - x^2 + 2$

draw $y = 3$

(b) $x =$ ......0.4...... or $x =$ ......2.6...... **[2]**

8 Taylor designs a logo using isosceles triangles joined at a central point, P.

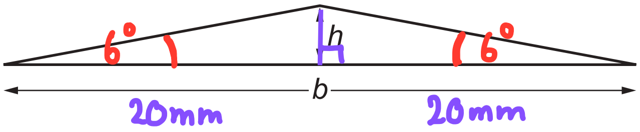
This is the start of Taylor's design.



Not to scale

The completed design will have rotational symmetry, order 60 about point P.

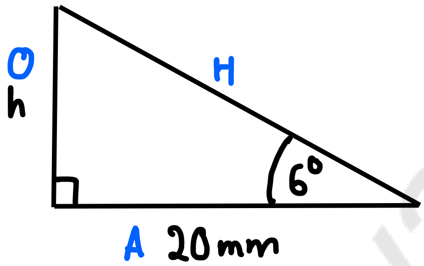
Each triangle has base, b, and height, h, measured in mm.



Not to scale

Calculate h when $b = 40$ mm.
Give your answer correct to **1** decimal place.

$360° \div 60 = 6°$



S O H   C A H   T O A

$\text{Tan}\,\theta = \frac{O}{A}$

$\text{Tan}(6) = \frac{h}{20}$ $\times 20$

$\times 20$

$h = 20 \times \tan(6)$

$= 2.102084705$

≈ 2.1 mm

.... 2.1 mm **[4]**

9 On Heidi's bookcase, the ratio of fiction to non-fiction books is 2 : 3.
Heidi removes 2 fiction books from the bookcase.
The ratio of fiction to non-fiction books is then 5 : 8.

How many books are left on the bookcase in total?

F : NF

$2x$: $3x$

-2

$2x - 2 : 3x = 5 : 8$

$$\frac{2x-2}{3x} = \frac{5}{8}$$

$8(2x-2) = 5(3x)$

$16x - 16 = 15x$

$-15x$ $-15x$

$x - 16 = 0$

$+16$ $+16$

$x = 16$

F : NF

$2x - 2$: $3x$

$2(16) - 2$ $3(16)$

$30 + 48 = 78$

......................78...... books **[4]**

10 **(a)** Show that 95 is **not** a prime number.

Factors of 95 are 1, 5, 19, 95

.... **[1]**

(b) **(i)** 2000 and 8750 are written below as the product of their prime factors.

$2000 = 2^4 \times 5^3$
$8750 = 2 \times 5^4 \times 7$

Find the highest common factor (HCF) of 2000 and 8750.

$2000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$8750 = 2 \times 5 \times 5 \times 5 \times 5 \times 7$

$HCF = 2 \times 5 \times 5 \times 5$

$= 250$

(b)(i) ....250.......... **[2]**

(ii) Write 2×10^{12} as a product of its prime factors.

10
/ \
2 5

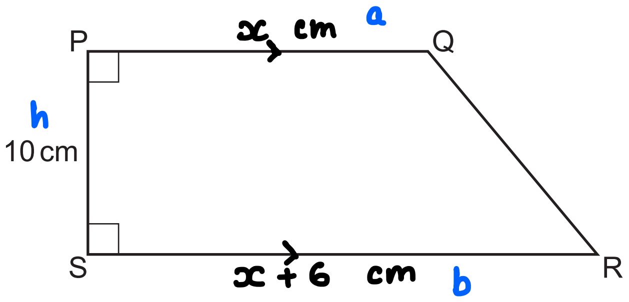
$2 \times (2 \times 5)^{12}$

$2^1 \times 2^{12} \times 5^{12}$

$2^{13} \times 5^{12}$

(ii) ....$2^{13} \times 5^{12}$.......... **[2]**

11 The diagram shows a quadrilateral, PQRS.



Not to scale

PS = 10 cm.
Angle QPS = Angle PSR = 90°.

SR is 6 cm longer than PQ.
The area of quadrilateral PQRS is A cm^2.

Write a simplified expression for the length PQ in terms of A.
You must show your working.

$$\text{Area} = \frac{1}{2}(a + b)h$$

$$A = \frac{1}{2}(x + x + 6) \times 10$$

$$A = \frac{1}{2}(2x + 6) \times 10$$

$$A = (x + 3) \times 10$$

$$A = 10x + 30$$

-30 $\quad$ -30

$$A - 30 = 10x$$

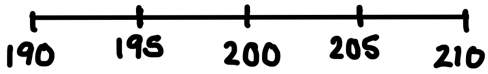
$\div 10$ $\quad$ $\div 10$

$$\frac{A - 30}{10} = x$$

$\frac{A - 30}{10}$ .... **[5]**

12 A box contains 200 matches, correct to the nearest ten matches.

(a) Complete the error interval for n, the number of matches in the box.



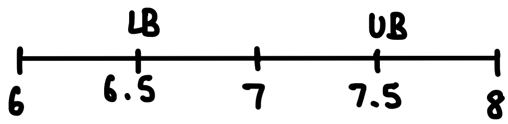
(a) ......195...... $\leqslant n \leqslant$ ......204...... **[2]**

(b) The box is a cuboid with

- length 7 cm, correct to the nearest cm
- width 5 cm, correct to the nearest cm
- volume 248 cm^3, correct to the nearest cm^3.

Show that the smallest possible height of the box is 6 cm. **[3]**

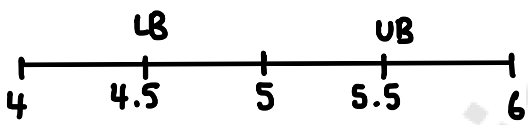
Length



$$V = l \times w \times h$$

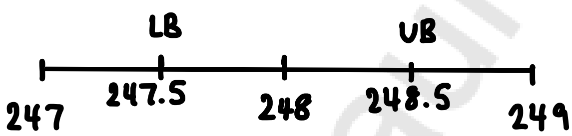
$$h = \frac{V}{l \times w}$$

Width



$$h_{LB} = \frac{V_{LB}}{l_{UB} \times w_{UB}}$$

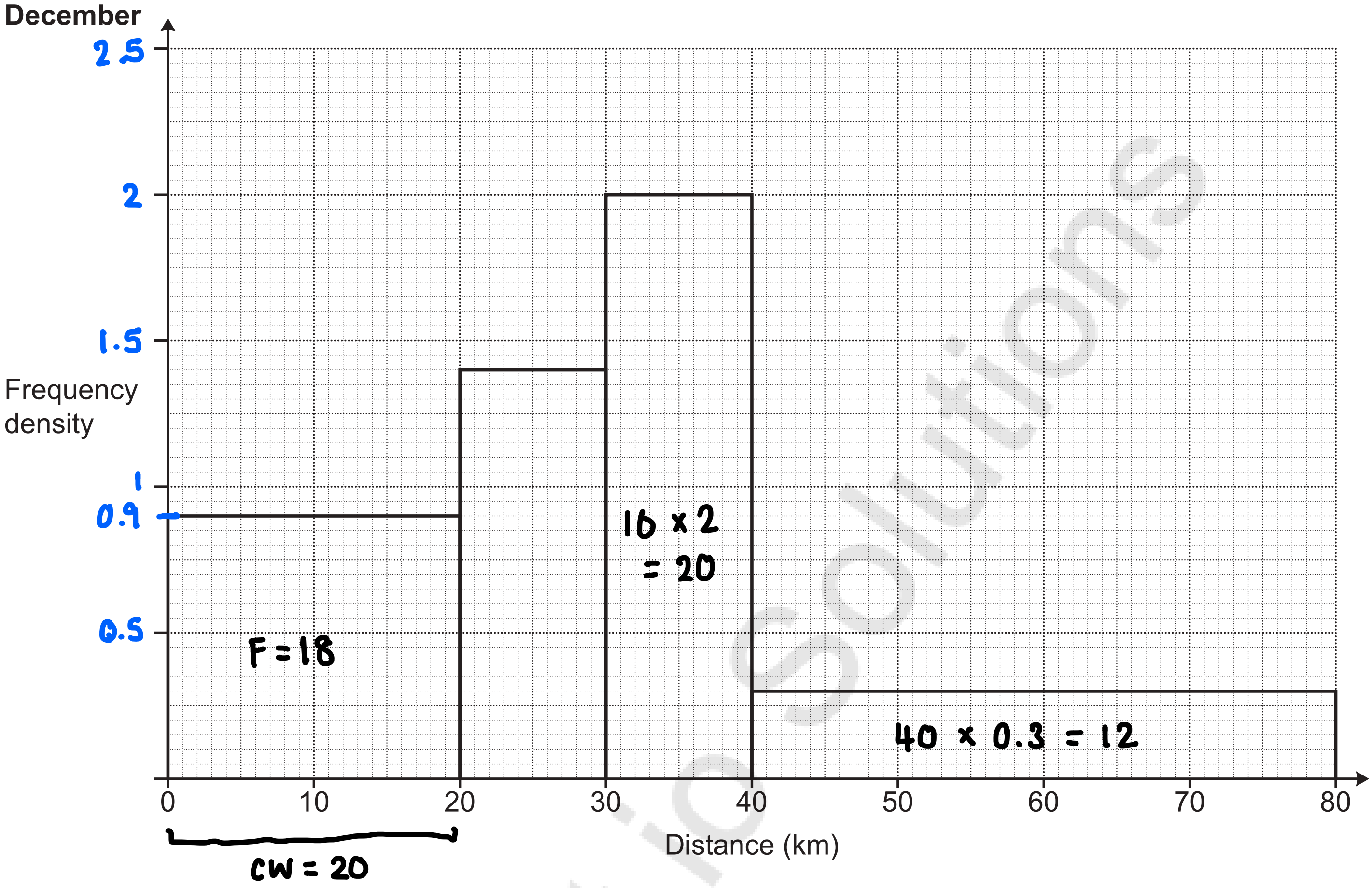
Volume



$$= \frac{247.5}{7.5 \times 5.5}$$

$$= 6$$

13 A running club records the distances run by each member during December.
The results are shown in this histogram.



(a) 18 members run less than 20 km.

(i) Work out the number of members who run more than 30 km.

F
CW   FD

$FD = \frac{18}{20} = 0.9$

$20 + 12 = 32$

(a)(i) ....32........ **[3]**

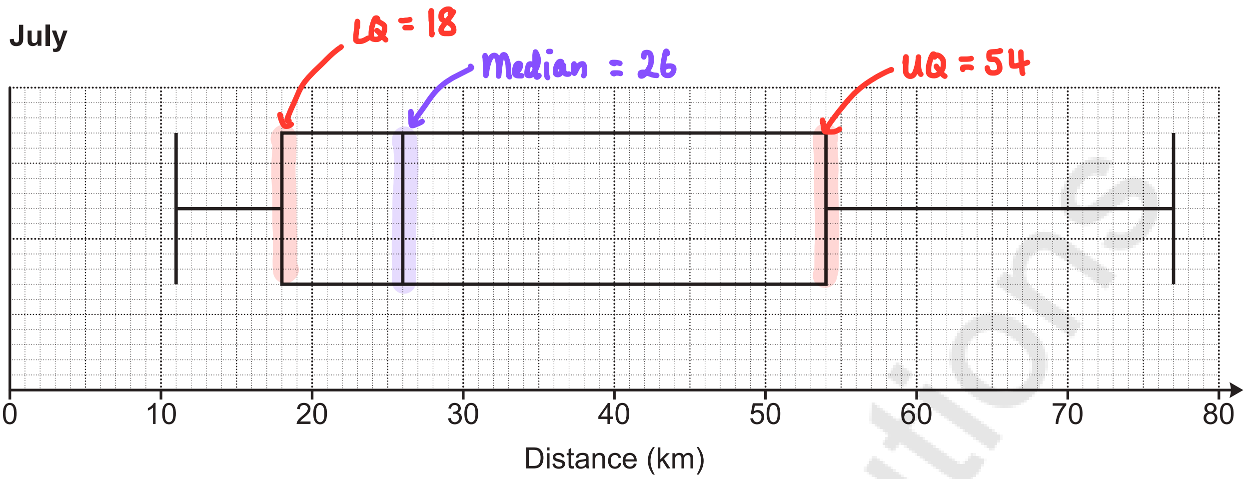
(ii) Finley says

To estimate the range, I subtracted the smallest possible value from the largest possible value. So, 80 – 0 = 80 km.

Explain why Finley's method is likely to overestimate the true value of the range.

The largest value could be anywhere between 40 and 80. **[1]**

(b) This box plot shows the distribution of the distance run by each member of the running club during July.



During **December**,
- the median distance run was 30 km
- the interquartile range of the distance run was 20 km.

Make **two** comparisons between the distances run during December and the distances run during July.
Include values to support your comparisons.

IQR = UQ − LQ
= 54 − 18
= 36

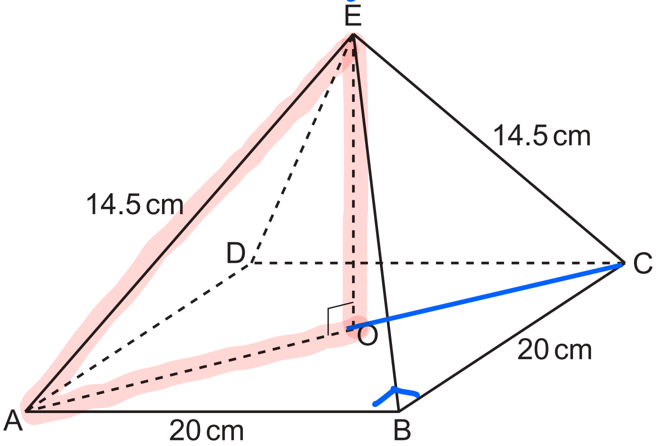
1. Median of July distances lower than December.

2. IQR for July distances is greater than December, they are more varied / spread out.

[4]

14 The diagram shows a square-based pyramid ABCDE.
O is the centre of the base.

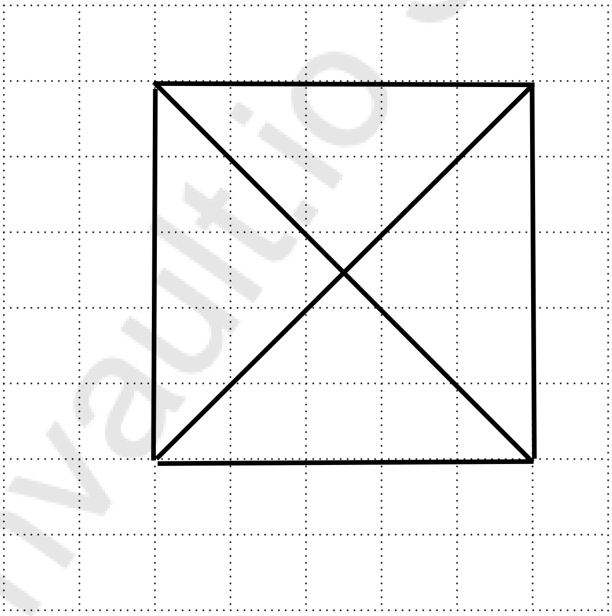
PLAN



The pyramid has base length 20 cm and each sloping edge has length 14.5 cm.

(a) Draw the plan view of the pyramid on the one-centimetre grid below.

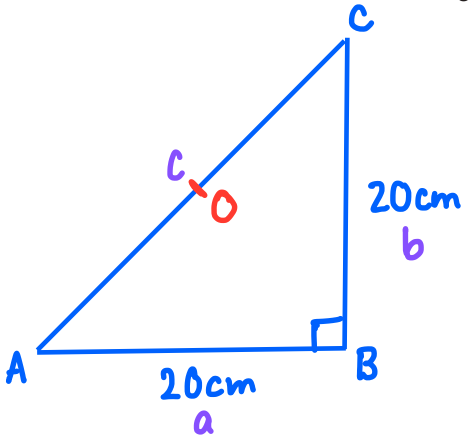
Scale: 1 cm represents 4 cm.



[2]

(b) Calculate the volume of the pyramid.
You must show your working.

[The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ perpendicular height]



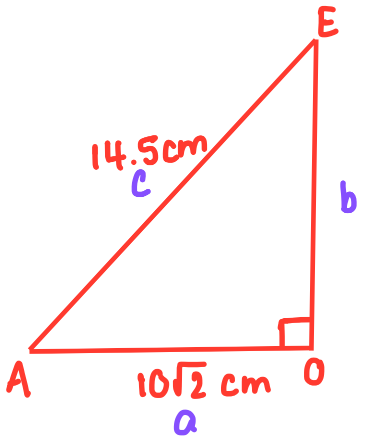
$a^2 + b^2 = c^2$

$20^2 + 20^2 = AC^2$

$\sqrt{20^2 + 20^2} = AC$

$20\sqrt{2} = AC$

$AO = 20\sqrt{2} \div 2 = 10\sqrt{2}$



$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$EO^2 = 14.5^2 - (10\sqrt{2})^2$

$EO = \sqrt{14.5^2 - (10\sqrt{2})^2}$

$= \frac{\sqrt{41}}{2}$

$= 3.201\ldots$ cm

[perpendicular height]

$\text{Volume} = \frac{1}{3} \times 20 \times 20 \times \frac{\sqrt{41}}{2}$

$= 426.8749492$

$\approx 426.9 \text{ cm}^3$

(b) ......426.9...... cm^3 **[5]**

15 Two bottles are mathematically similar.

$1L = 1000\,cm^3$ ($\times 1000$)

The small bottle holds 0.5 litres and has a height of 35 cm.
The large bottle holds 2 litres.

Calculate the height of the large bottle.

	Small	Large	SF
Height (cm)	35	55.559... $35 \times \sqrt[3]{4}$	$\sqrt[3]{4}$
Volume (cm^3)	500 ↑ 0.5×1000	2000 ↑ 2×1000	$\frac{2000}{500} = 4$

$4 \xrightarrow{\sqrt[3]{4}} \sqrt[3]{4}$

55.6 .... cm **[4]**

16 The price of a seat on a flight, £P, is given by

$P = 49 \times 1.009^n$

where n is the number of seats already sold on this flight.

(a) Write down the percentage increase in price of the second seat sold compared to the first seat sold.

$1.009 \times 100 = 100.9\%$

$100.9\% - 100\% = 0.9$

(a) ....0.9........................ % **[1]**

(b) Show that the price of the 40th seat sold is less than £70. **[2]**

$n = 39$

$49 \times 1.009^{39} = 69.49468061$

< 70

17 The kth term of a sequence is r^k, where $r \neq 0$.
The sixth term is equal to three times the second term.

Find the value of r, giving your answer correct to **3** decimal places.

6th term $= r^6$

2nd term $= r^2$

$r^6 = 3 \times r^2$

$\div r^2 \qquad \div r^2$

$\frac{r^6}{r^2} = 3$

$r^4 = 3$

$\sqrt[4]{\ } \qquad \sqrt[4]{\ }$

$r = \sqrt[4]{3}$

$= 1.316074013$

≈ 1.316

$r =$ ....1.316........................ **[4]**

18 **(a)** Describe fully the graph of $x^2 + y^2 = 20$.

$x^2 + y^2 = r^2$

$r^2 = 20$

$r = \sqrt{20}$

Circle , centre $(0, 0)$, radius is $\sqrt{20}$.

[3]

(b) The graph of $y = 3x + 10$ intersects the graph of $x^2 + y^2 = 20$ at two points.

Use an algebraic method to work out the coordinates of the two points.
You must show your working.

$x^2 + (3x + 10)^2 = 20$

$x^2 + (3x + 10)(3x + 10) = 20$

$x^2 + 9x^2 + 30x + 30x + 100 = 20$

$10x^2 + 60x + 100 = 20$

$-20 \quad -20$

$10x^2 + 60x + 80 = 0$

$\div 10 \quad \div 10$

$x^2 + 6x + 8 = 0$

$(x + 4)(x + 2) = 0$

$x + 4 = 0$ or $x + 2 = 0$

$-4 \quad -4 \qquad -2 \quad -2$

$x = -4 \qquad x = -2$

$y = 3x + 10$

$y = 3(-4) + 10$

$y = -2$

$y = 3x + 10$

$y = 3(-2) + 10$

$y = 4$

(b) (-4 , -2) and (-2 , 4) **[6]**

19 **(a)** Show that $\sqrt{11} \times \sqrt{22} = 11\sqrt{2}$. **[1]**

$$\sqrt{22} = \sqrt{2} \times \sqrt{11}$$

$$\sqrt{11} \times \sqrt{2} \times \sqrt{11} = 11\sqrt{2}$$

(b) Show that $\dfrac{\sqrt{11}}{13+\sqrt{22}}$ can be written in the form $\dfrac{a\sqrt{11}-11\sqrt{2}}{b}$ where a and b are integers. **[4]**

$$\frac{\sqrt{11}}{13+\sqrt{22}} \times \frac{13-\sqrt{22}}{13-\sqrt{22}}$$

$$\frac{\sqrt{11}(13-\sqrt{22})}{(13+\sqrt{22})(13-\sqrt{22})}$$

$$\frac{13\sqrt{11} - 11\sqrt{2}}{169 - \cancel{13\sqrt{22}} + \cancel{13\sqrt{22}} - 22}$$

$$\frac{13\sqrt{11} - 11\sqrt{2}}{147}$$

20 **(a)** Write $(2x-5)(x+4)$ in the form $2(x+a)^2-b$.

You must show your working.

$(2x-5)(x+4)$

$2x^2+8x-5x-20$

$2x^2+3x-20$

$2\left(x^2+\frac{3}{2}x-10\right)$

$2\left[\left(x+\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-10\right]$

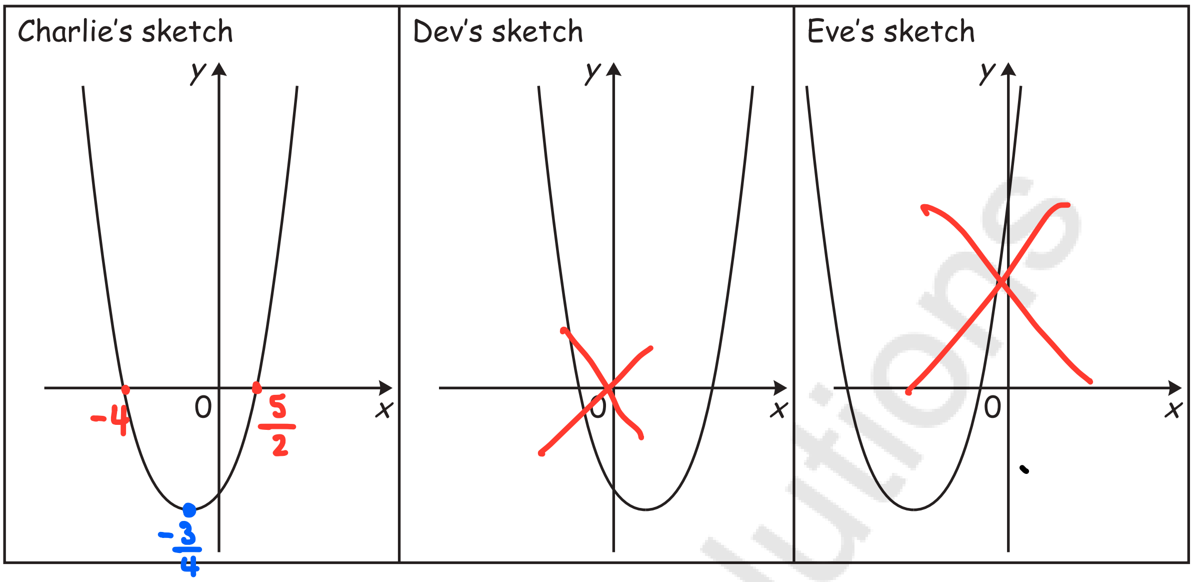
$2\left[\left(x+\frac{3}{4}\right)^2-\frac{9}{16}-10\right]$

$2\left(x+\frac{3}{4}\right)^2-\frac{18}{16}-20$

$2\left(x+\frac{3}{4}\right)^2-\frac{169}{8}$

(a) $2\left(x+\frac{3}{4}\right)^2-\frac{169}{8}$ .... **[5]**

(b) Charlie, Dev and Eve all attempt to sketch the graph of $y = (2x - 5)(x + 4)$.



Whose sketch is the most accurate?
Write down the properties of the graph that you used in making your decision.

$(2x - 5)(x + 4) = 0$

$2x - 5 = 0 \qquad x + 4 = 0$

$x = \frac{5}{2} \qquad x = -4$

Charlie's because the roots are -4 and $\frac{5}{2}$

[2]

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).