OCR
Oxford Cambridge and RSA

Wednesday 8 November 2023 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The price of a phone increases from £240 to £262.80.

Calculate the percentage increase in the price of the phone.

$\frac{\text{change}}{\text{original}} \times 100$

change = £262.80 − £240
= £22.80

$\frac{22.8}{240} \times 100 = 9.5$

9.5 % **[3]**

2 A prime number is a whole number that has exactly two factors.

(a) Explain why 1 is not a prime number.

1 only has one factor.

[1]

(b) *a* and *b* are prime numbers.

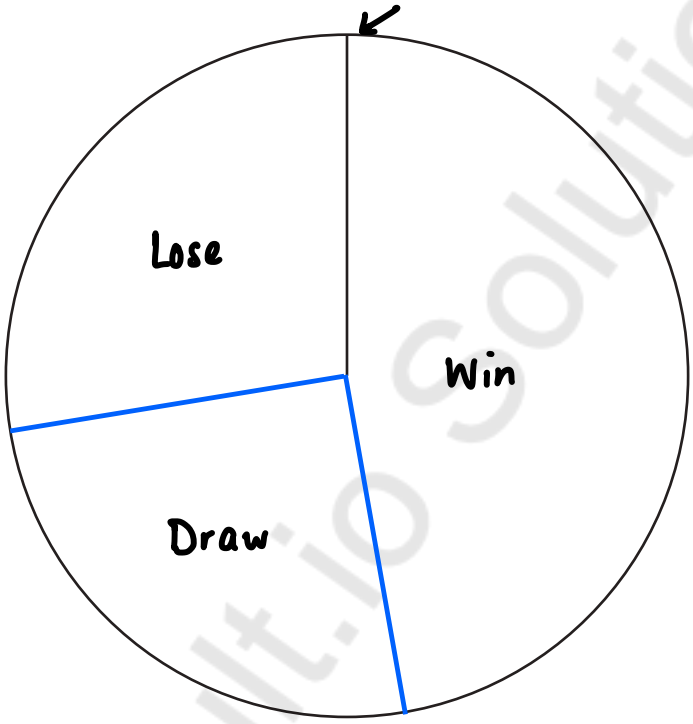
Write down the **6** factors of a^2b.

(b) 1 a b a^2 ab a^2b **[2]**

3 **(a)** The table shows the results for a sports club's 'A' team.

Result	Frequency		Angle
Win	18	×9	162°
Draw	10	×9	90°
Lose	12	×9	108°
Total	40	×9	360°

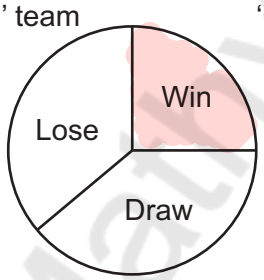
Complete a labelled pie chart to show these results.



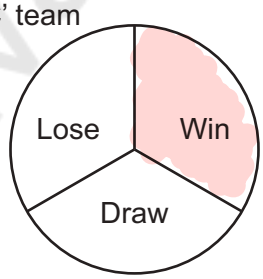
[4]

(b) Here are the results for the sports club's 'B' team and 'C' team.

'B' team



'C' team



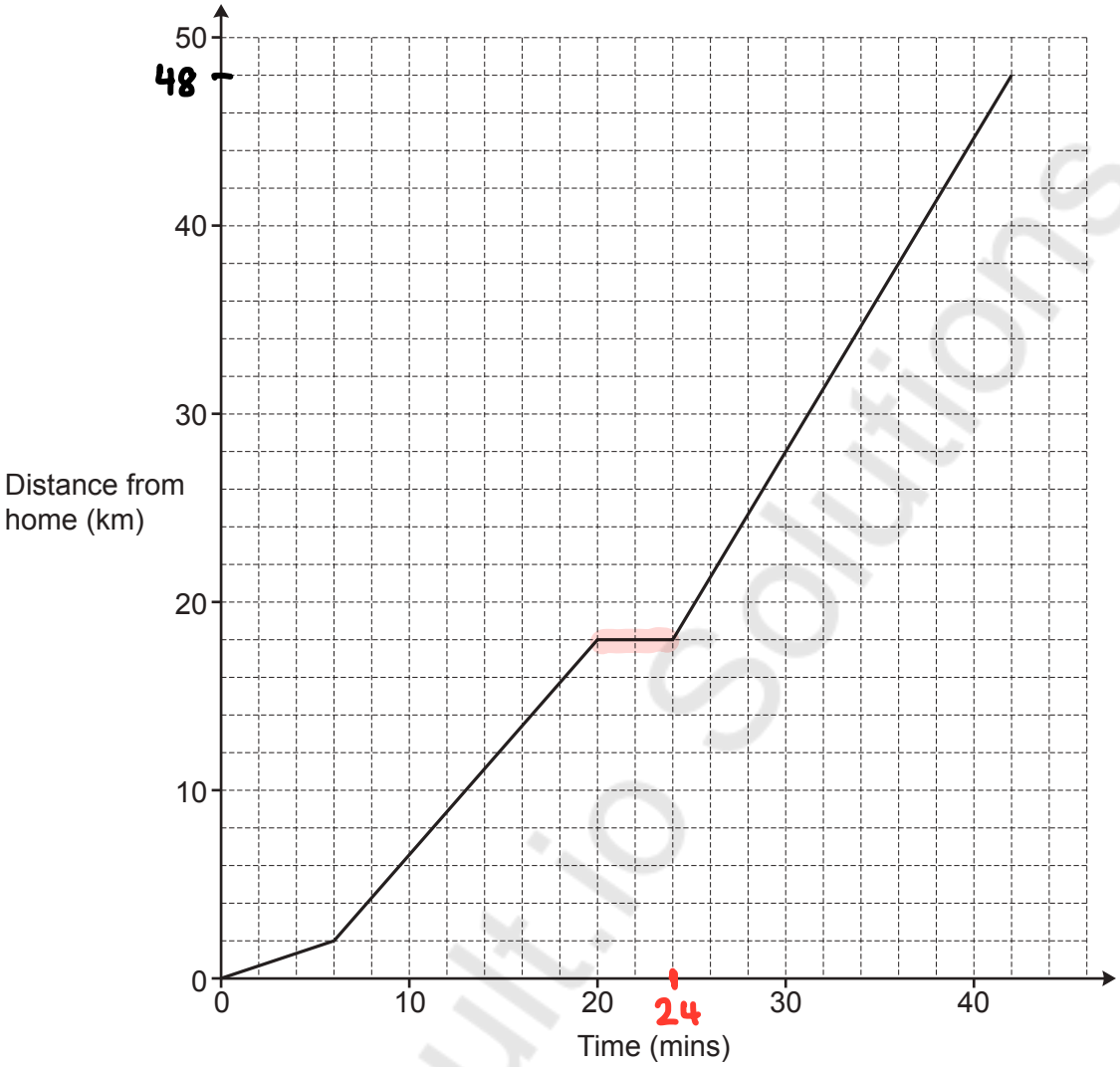
The 'C' team manager says

The pie charts show that the 'C' team won more games than the 'B' team.

Referring to the pie charts, explain why the 'C' team manager may not be correct.

The pie charts do not show how many matches were played by each team. **[1]**

4 The graph shows Taylor's journey from home to an airport.
During the journey Taylor stops for petrol.



(a) For how long did Taylor stop for petrol?

(a) ..... 4 ..... mins **[1]**

(b) Taylor drives the same route back home from the airport at an average speed of 45 km/h.
Taylor leaves the airport at 22:00.

Work out the time when Taylor arrives home.

D
S T $T = \frac{D}{S}$ D = 48 km S = 45 km/h $T = \frac{48}{45} = 1.06\dot{6}\text{h}$

$0.06 \times 60 = 4$

1h 4 mins

(b) ..... 23:04 ..... **[4]**

5 **(a)** Write an expression for the weight, in grams, of an object weighing x kilograms.

1kg = 1000g
×1000
$x$kg × 1000 = 1000x g

(a) ..... $1000x$ ..... g **[1]**

(b) Write an expression for the area, in m^2, of a garden of area y cm^2.

cm^2 ÷10,000 → m^2

$y\ cm^2$ ÷10,000 = $\frac{y}{10,000}\ m^2$

(b) ..... $\frac{y}{10,000}$ ..... m^2 **[1]**

6 2 kg of carrots and 5 kg of potatoes cost £6.36. → ×100 → 636p
3 kg of carrots and 2 kg of potatoes cost £5.25. → 525p

Find the cost of 1 kg of carrots and the cost of 1 kg of potatoes.
You must show your working.

c p

$2c + 5p = 636$ ×3
$3c + 2p = 525$ ×2

$6c + 15p = 1908$
$6c + 4p = 1050$
$11p = 858$
÷11 ÷11
$p = 78$ pence
÷100
£0.78

$2c + 5p = 636$
$2c + 5(78) = 636$
$2c + 390 = 636$
$-390 \quad -390$
$2c = 246$
÷2 ÷2
$c = 123$ pence
÷100
£1.23

1 kg of carrots cost £ ...1.23.....

1 kg of potatoes cost £ ...0.78.....

[5]

7 Find all the possible integer values that satisfy the inequality $-10 < 3x + 2 \leqslant 8$.

$-10 < 3x + 2 \leq 8$

$-2 \quad -2 \quad -2$

$-12 < 3x \leq 6$

$\div 3 \quad \div 3 \quad \div 3$

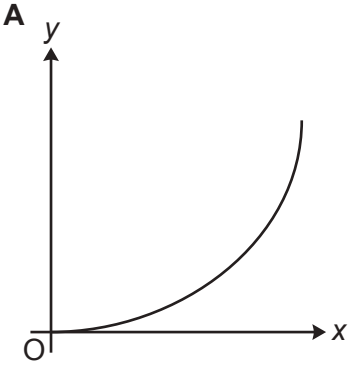
$-4 < x \leq 2$

$-3, -2, -1, 0, 1, 2$

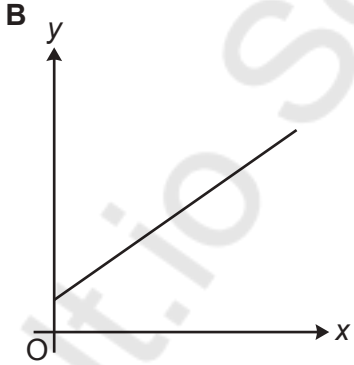
$x =$ $-3, -2, -1, 0, 1, 2$ **[3]**

8 Here are sketches of six graphs, labelled **A** to **F**.

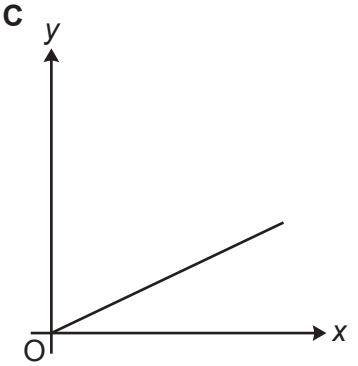
A



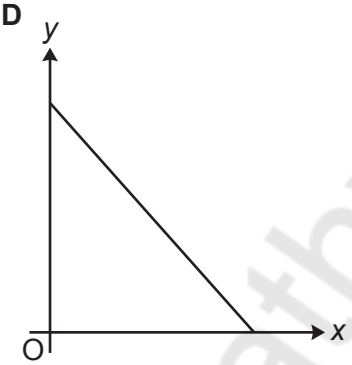
B



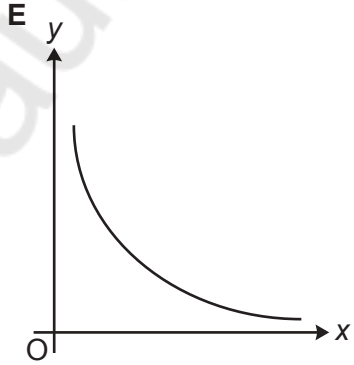
C



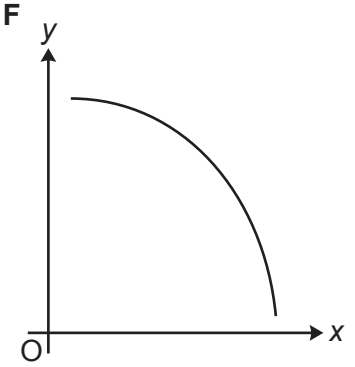
D



E



F



Write the letter of the graph that represents the following relationships.

(a) *y* is directly proportional to *x*.

(0,0) straight line

(a) C **[1]**

(b) *y* is inversely proportional to *x*.

reciprocal

(b) E **[1]**

9 Here are two pieces of work.
Each shows a question and an incorrect solution.

For each part, describe the error made and write out a correct solution.

(a)

Question:
Factorise. $x^2 + x - 20$

Solution:
$(x + 4)(x - 5)$

The error is expanding gives $x^2 - 5x + 4x - 20$
$= x^2 - x - 20$

A correct solution is $(x - 4)(x + 5)$ **[2]**

(b)

Question:
Solve. $4x + 5 = x + 2$

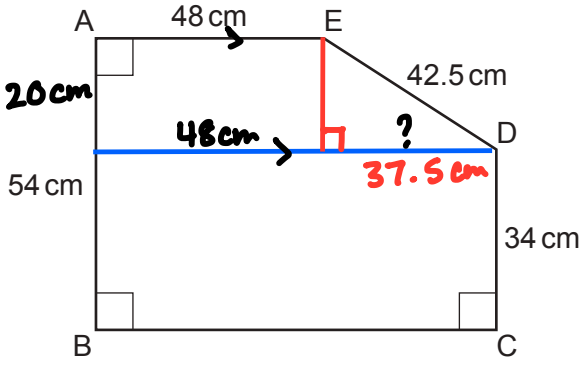
Solution:
$4x + 5 = x + 2$
$3x + 5 = 2$
$3x = 5 - 2$
$3x = 3$
$x = 1$

The error is $3x + 5 = 2$ so $3x = 2 - 5$ $3x = -3$

A correct solution is $3x = -3$
$x = -1$

[2]

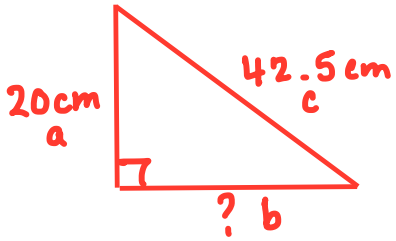
10 The diagram shows a pentagon ABCDE.



Not to scale

Find the area of the pentagon.
You must show your working.

Area trapezium = $\frac{1}{2}(a+b)h$



$a^2 + b^2 = c^2$

$20^2 + b^2 = 42.5^2$

$-20^2 \qquad -20^2$

$b^2 = 42.5^2 - 20^2$

$b = \sqrt{42.5^2 - 20^2}$

$= 37.5\text{cm}$

Trapezium $= \frac{1}{2}(48 + 85.5) \times 20$

$= 1335\text{ cm}^2$

Rectangle $= 85.5 \times 34$

$= 2907\text{ cm}^2$

Total area $= 1335 + 2907$

$= 4242\text{ cm}^2$

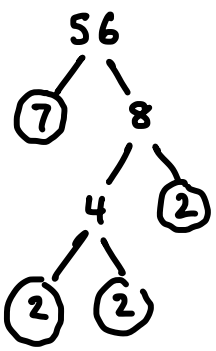
4242 cm^2 **[6]**

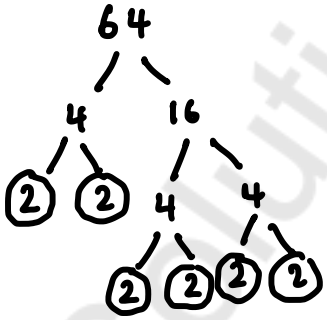
11 Riley and Sam are conducting surveys.
They are both given the same list of 12463 people from which to select their sample.

Riley selects every 56th person.
Sam selects every 64th person.
They both start counting from the first name in the list.

Work out how many people will be selected to be in both surveys.
You must show your working.

LCM of 56 and 64





$56 = 2 \times 2 \times 2 \times 7$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

HCF = $2 \times 2 \times 2$
= 8

LCM = $8 \times 2 \times 2 \times 2 \times 7$
= 448

$12463 \div 448 = 27.8$
≈ 27

27 **[5]**

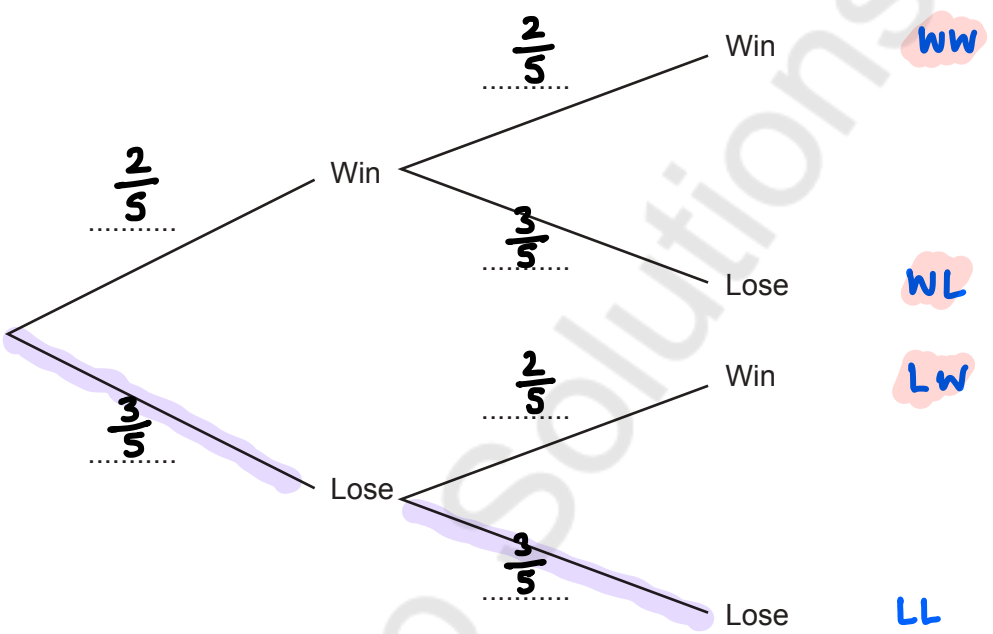
12 In a computer game the player can either win or lose.
A student thinks the ratio of the probability of winning to the probability of losing is 2 : 3.

The student plays two games.

p(win) = $\frac{2}{5}$

p(lose) = $\frac{3}{5}$

(a) Use the information to complete the tree diagram.



[3]

(b) Find the probability that the student wins at least one of the two games.

$1 - p(LL)$

$1 - \left(\frac{3}{5} \times \frac{3}{5}\right)$

$1 - \frac{9}{25}$

$= \frac{16}{25}$

(b) $\frac{16}{25}$ **[3]**

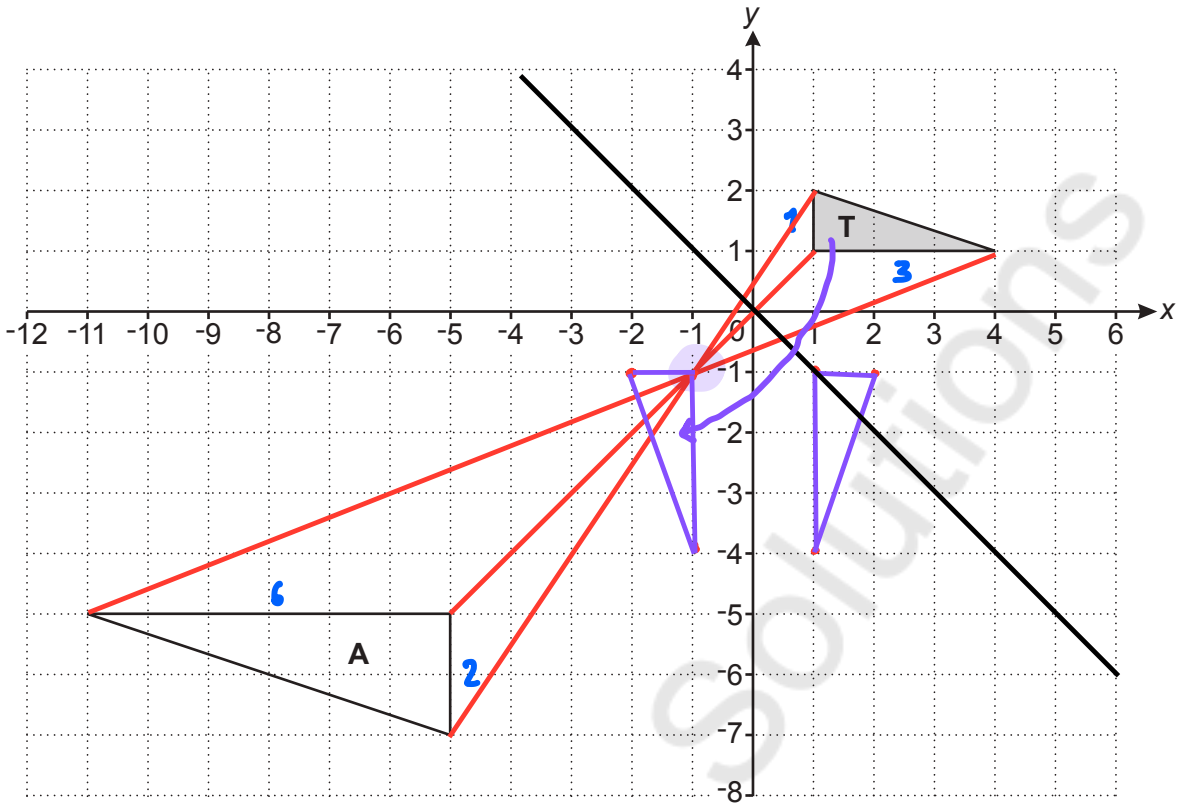
(c) The student now thinks the ratio of the probability of winning to the probability of losing has changed to 2 : 5.

Explain the effect this change will have on your answer to part **(b)**.

The answer will be smaller.

[1]

13 Triangle **T** and triangle **A** are drawn on the coordinate grid.



(a) Describe fully the **single** transformation that maps triangle **T** onto triangle **A**.

Enlargement, scale factor −2, centre (−1, −1)

[3]

(b) Describe fully the **single** transformation that is equivalent to:

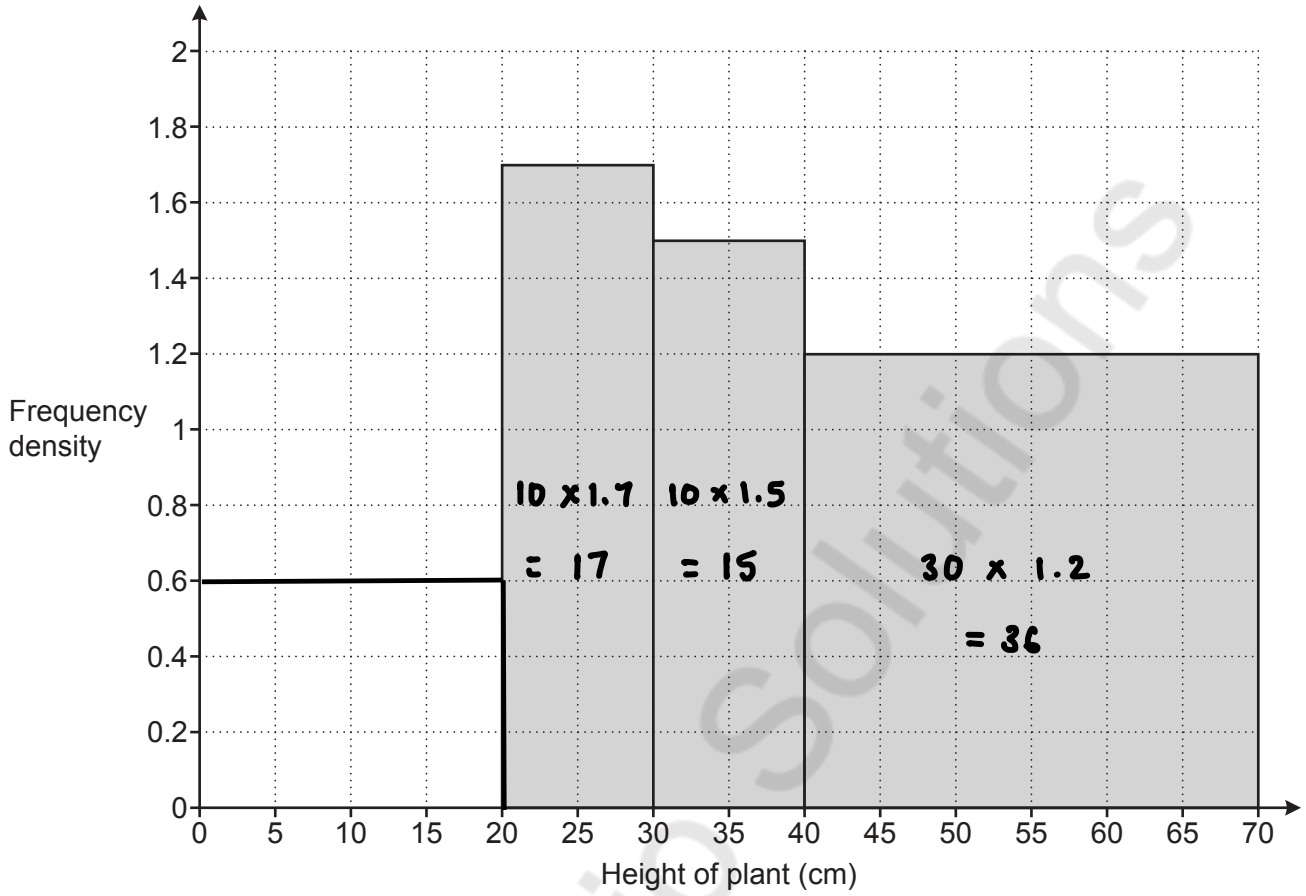
- a rotation of 90° clockwise about centre (0,0), followed by
- a reflection in the *y*-axis.

You may use the grid above to help you.

Reflection in the line $y = -x$

[3]

14 The histogram shows the heights of some plants out of a total of 80 plants.



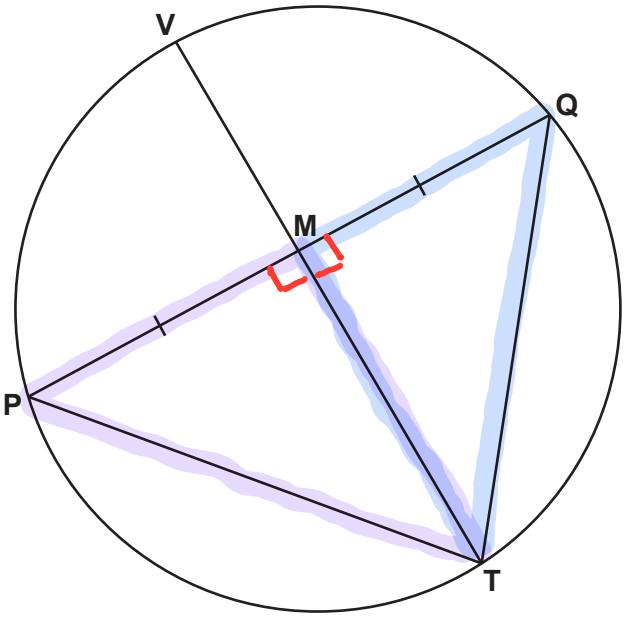
Complete the histogram to show the plants with heights between 0 cm and 20 cm. **[5]**

17 + 15 + 36 = 68

80 - 68 = 12

CW F FD 12 ÷ 20 = 0.6

15 P, Q, T and V are points on the circumference of a circle.
TV is a diameter of the circle.
M is a point on PQ such that PM = MQ.



Not to scale

Complete these sentences to show that triangle TMP is congruent to triangle TMQ.

S Side PM = side MQ because it is given to you.

A Angle PMT = angle TMQ because diameter bisects the chord PQ so VT is perpendicular to PQ

S Side MT is common.

Triangle TMP is congruent to triangle TMQ because SAS

[3]

16 A biologist assumes the population, P, of birds on an island can be predicted using the formula

$P = 3800 \times 1.042^n$

where n is the number of years after the start of 2020.

(a) Write down the percentage increase per year that is used in the formula.

$1.042 \times 100 = 104.2\%$

$104.2 - 100 = 4.2\%$

(a) 4.2 % **[1]**

(b) Calculate the predicted population at the start of 2024.

$P = 3800 \times 1.042^4$

$= 4479.757162$

≈ 4479

(b) 4479 **[2]**

(c) **(i)** Show that the number of birds is predicted to exceed 7000 during 2034. **[3]**

$P_{2034} = 3800 \times 1.042^{14}$

$= 6759.765995$

$P_{2035} = 3800 \times 1.042^{15}$

$= 7043.676167$

(ii) A researcher says that between 2022 and 2030 the percentage increase per year in the population will be 2.8%.

If the researcher is correct, explain how this new information will affect the answer in part **(c)(i)**.

It will take longer than 2034 for the population to reach 7000. **[1]**

17 **(a)** A sequence is defined by

$u_{n+1} = 3u_n + 7$ and $u_1 = -2$.

Work out the value of u_2 and the value of u_3.

$u_2 = 3(-2) + 7$
$= 1$

$u_3 = 3(1) + 7$
$= 10$

(a) $u_2 =$ 1

$u_3 =$ 10

[2]

(b) Here are the first four terms of a quadratic sequence.

-2 (+9) 7 (+15) 22 (+21) 43

(+6) (+6)

The sequence has the formula $x_n = an^2 + b$.

Find the value of a and the value of b.

$a = 6 \div 2$
$= 3$

n^2	1	4	9	16	) x 3
an^2	3	12	27	48	
Seq.	-2	7	22	43	
Seq. - an^2	0 -5	0 -5	0 -5	0 -5	) -5

(+0) (+0) (+0)

$0n - 5$

(b) $a =$ 3

$b =$ -5

[3]

18 Solve this quadratic equation by factorisation.

$2x^2 - 6x - 24 = 5x - 3$

$-5x + 3 \quad -5x + 3$

$2x^2 - 11x - 21 = 0$

$2 \times 21 = 42$

$+3 \quad -14$

$2x^2 + 3x - 14x - 21$

$x(2x + 3) - 7(2x + 3)$

$(2x + 3)(x - 7) = 0$

$2x + 3 = 0 \quad$ or $\quad x - 7 = 0$

$-3 \quad -3 \qquad +7 \quad +7$

$2x = -3 \qquad x = 7$

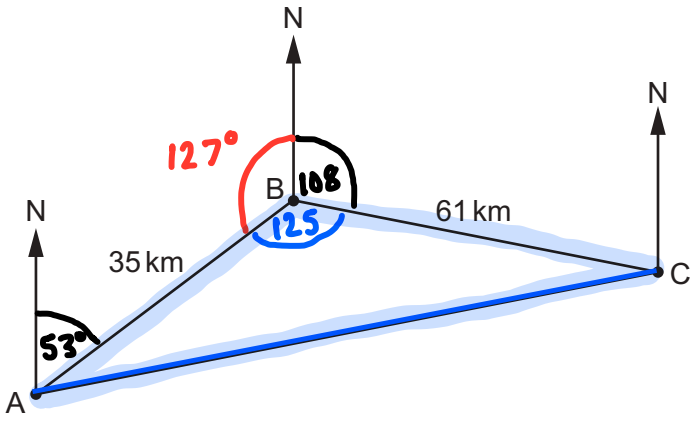
$\div 2 \qquad \div 2$

$x = \frac{-3}{2}$

$= -1.5$

$x =$ -1.5 ..... or $x =$ 7 ..... **[4]**

19 The diagram shows the positions of three towns A, B and C.



Not to scale

The bearing of town B from town A is 053°.
The bearing of town C from town B is 108°.
AB = 35 km and BC = 61 km.

Calculate AC.
You must show your working.

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$AC^2 = 35^2 + 61^2 - 2(35)(61)\cos(125)$

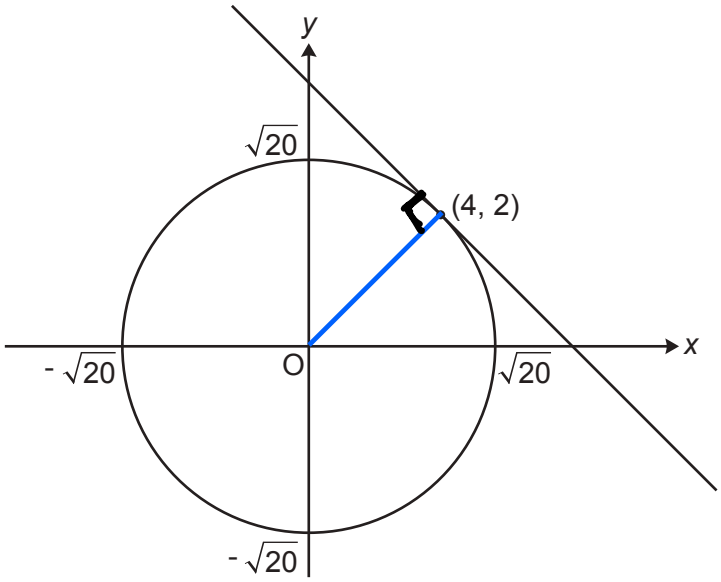
$= 7395.171381$

$AC = 85.99518233$

≈ 86 km

AC = ... 86 ..... km **[5]**

20 The diagram shows a circle, centre the origin, with the tangent to the circle at the point (4, 2).



(a) Write down the equation of the circle.

$x^2 + y^2 = r^2$

$r = \sqrt{20}$

$r^2 = 20$

(a) $x^2 + y^2 = 20$ **[2]**

(b) (i) Show that the tangent to the circle at the point (4, 2) has gradient -2. **[2]**

Gradient of radius $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$ (0, 0) $x_1\ y_1$ (4, 2) $x_2\ y_2$

Gradient of tangent $= -2$

(ii) Find the equation of the tangent to the circle at the point (4, 2).

$y = mx + c$ $\quad m = -2$ $\quad (4, 2)$ $x\ y$

$2 = -2(4) + c$

$2 = -8 + c$

$+8 \quad +8$

$10 = c$

$y = -2x + 10$

(b)(ii) $y = -2x + 10$ **[2]**

21 Solve.

$x^{-\frac{1}{6}} = \dfrac{5x^{\frac{1}{3}}}{x^{\frac{3}{4}}}$, where $x \neq 0$

$x^{-a} = \dfrac{1}{x^a}$

$\dfrac{1}{x^{\frac{1}{6}}} = \dfrac{5x^{\frac{1}{3}}}{x^{\frac{3}{4}}}$

$x^{\frac{3}{4}} = x^{\frac{1}{6}} \times 5x^{\frac{1}{3}}$

$x^{a} \times x^{b} = x^{a+b}$

$x^{\frac{3}{4}} = 5x^{\frac{1}{6}+\frac{1}{3}}$

$\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

$x^{\frac{3}{4}} = 5x^{\frac{1}{2}}$

$\div x^{\frac{1}{2}}$ $\quad \div x^{\frac{1}{2}}$

$x^{a} \div x^{b} = x^{a-b}$

$x^{\frac{3}{4}} \div x^{\frac{1}{2}} = 5$

$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

$x^{\frac{1}{4}} = 5$

$\sqrt[4]{x} = 5$

$x = 5^4$

$= 625$

$x =$ 625 ..... **[3]**

Turn over for question 22

22 You are given this identity.

$$\frac{2-3\sqrt{18}}{\sqrt{18}+4} = a\sqrt{2}+b$$

Find the value of a and the value of b.
You must show each step in your working.

$\sqrt{18} = \sqrt{9}\sqrt{2}$
$= 3\sqrt{2}$

$$\frac{2-3(3\sqrt{2})}{3\sqrt{2}+4}$$

$$\frac{2-9\sqrt{2}}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4}$$

$$\frac{(2-9\sqrt{2})(3\sqrt{2}-4)}{(3\sqrt{2}+4)(3\sqrt{2}-4)}$$

$$\frac{6\sqrt{2}-8-54+36\sqrt{2}}{18-12\sqrt{2}+12\sqrt{2}-16}$$

$$\frac{42\sqrt{2}-62}{2} = 21\sqrt{2}-31$$
$$a\sqrt{2}+b$$

$a =$ 21

$b =$ −31

[6]

END OF QUESTION PAPER