

Friday 10 November 2023 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- geometrical instruments
- tracing paper

Do not use:

- a calculator



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION

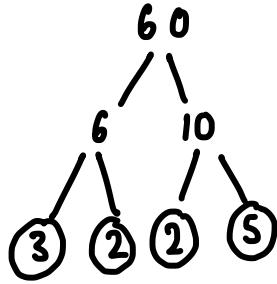
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.



- 1 Write 60 as a product of its prime factors.



$$2 \times 2 \times 3 \times 5$$

$$2^2 \times 3 \times 5$$

$$\dots\dots\dots 2^2 \times 3 \times 5 \dots\dots\dots [2]$$

- 2 By writing each number correct to 1 significant figure, find an estimate for this calculation.

$$\frac{486}{\sqrt{101.2}}$$

$$486 \approx 500$$

$$101.2 \approx 100$$

$$\frac{500}{\sqrt{100}}$$

$$= \frac{500}{10}$$

$$= 50$$

$$\dots\dots\dots 50 \dots\dots\dots [2]$$

- 3 (a) Simplify.

$$3a^2 \times 4a^5$$

$$3 \times 4 = 12$$

$$a^2 \times a^5 = a^7$$

$$12 \times a^7 = 12a^7$$

$$(a) \dots\dots\dots 12a^7 \dots\dots\dots [2]$$

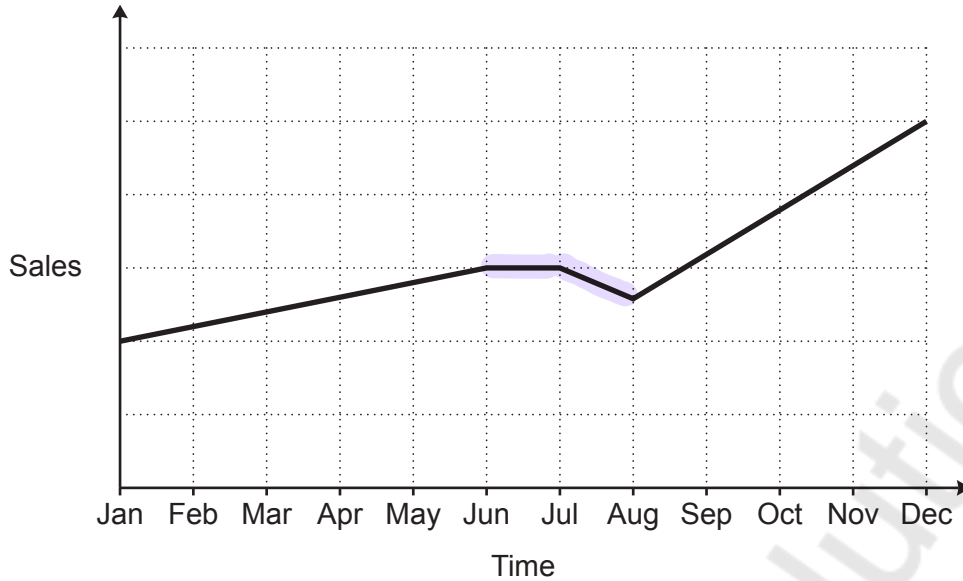
- (b) Factorise fully.

$$4x^2 - 12x$$

$$4x(x - 3)$$

$$(b) \dots\dots\dots 4x(x - 3) \dots\dots\dots [2]$$

- 4 A sales representative and a manager discuss this graph of sales over the last year.



- (a) The sales representative says

I can tell from the graph that, over the last year, sales have risen every month.

Is the sales representative correct?

Give a reason for your answer.

..... **No** because **there is no increase in sales between June and July.** [1]

- (b) The manager says

I can tell from the graph that sales are now more than double what they were at the start of the year.

Is the manager correct?

Give a reason for your answer.

..... **No** because **there is no scale on the vertical axis.** [1]

5 Work out.

(a) $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$$3 + -2 = 1$$

$$-1 + 4 = 3$$

(a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

[1]

(b) $\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

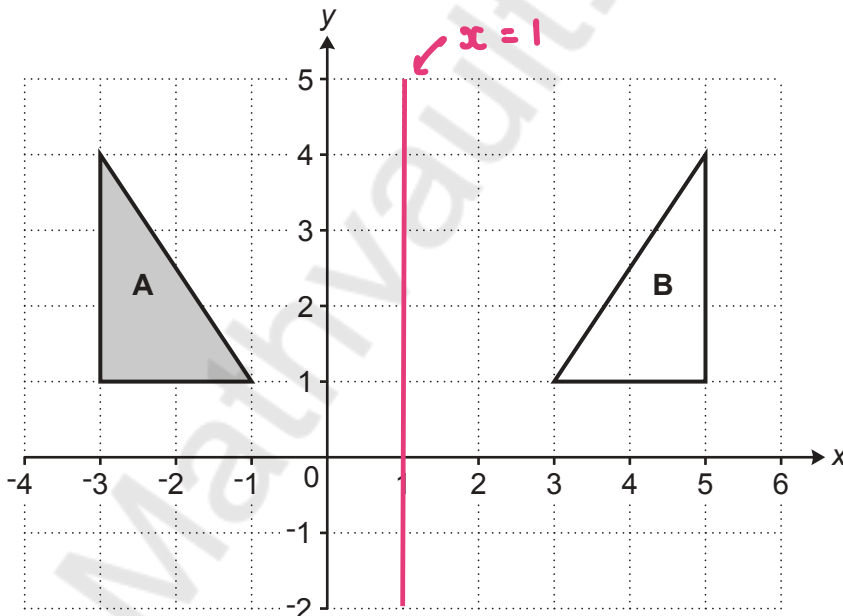
$$5 - 2 = 3$$

$$3 - -8 = 11$$

(b) $\begin{pmatrix} 3 \\ 11 \end{pmatrix}$

[2]

6 Triangle A and triangle B are drawn on the coordinate grid.



Describe fully the **single** transformation that maps triangle A onto triangle B.

..... Reflection in the line $x = 1$


..... [2]

- 7 A motorist wants to buy a new car but does not have enough money.

The price of the car is £16 000.

The motorist sees this notice for a deal on the car they want to buy.

The number of equal monthly payments is hidden.



Pay 15% of the price of the car now
and then
equal monthly payments of £300

Work out the number of monthly payments if the total cost of the car to the motorist is £17 400.

You must show your working.

$$\begin{array}{r}
 15\% \text{ of } \pounds 16000 \\
 10\% \text{ of } \pounds 16000 = \pounds 1600 \\
 + \quad 5\% \text{ of } \pounds 16000 = \pounds 800 \\
 \hline
 15\% \text{ of } \pounds 16000 = \pounds 2400
 \end{array}$$

$$\begin{array}{r}
 \pounds 17\ 400 \\
 - \pounds 2\ 400 \\
 \hline
 \pounds 15\ 000
 \end{array}$$

$$\frac{\pounds 15\ 000}{300} = \frac{150}{3} = 50$$

..... **50** [4]

- 8 The density of some concrete is 2.4 g/cm^3 .
A lump of this concrete has a volume of 400 cm^3 .

Work out the mass of the lump of concrete.

$$\begin{array}{l} m \\ D \quad V \end{array} \quad \begin{array}{l} D = 2.4 \\ V = 400 \\ \\ m = 2.4 \times 400 \\ \quad \downarrow \times 10 \\ \quad 24 \times 400 \end{array}$$

$$\begin{array}{r} 400 \\ \times 24 \\ \hline 1600 \\ 8000 \\ \hline 9600 \end{array} \xrightarrow{\div 10} 960$$

..... 960 g [2]

- 9 A box contains only red, green and black pens.
The ratio of red pens to green pens to black pens is $1 : 4 : 11$.

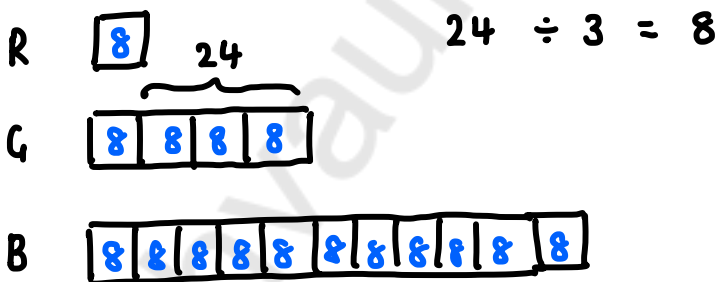
(a) Work out the percentage of the pens that are green.

$$\frac{4}{1+4+11} = \frac{4}{16} \div 4 = \frac{1}{4} \begin{array}{l} \times 25 \\ \times 25 \end{array} = \frac{25}{100} = 25\%$$

(a) 25 % [2]

(b) There are 24 more green pens than red pens.

Work out the total number of pens in the box.



$$\text{Red} = 8$$

$$\text{Green} = 32$$

$$\text{Black} = 88$$

$$\begin{aligned} \text{Total} &= 8 + 32 + 88 \\ &= 128 \end{aligned}$$

(b) 128 [4]

10 (a) Find the next term of this sequence.

$$\begin{array}{r} 81 \\ \times 3 \\ \hline 243 \end{array}$$

$$1 \quad \underbrace{\quad \times 3 \quad}_3 \quad 3 \quad \underbrace{\quad \times 3 \quad}_9 \quad 9 \quad 27 \quad 81 \quad \underbrace{\quad \times 3 \quad}_{\dots} \quad 243 \dots \quad [1]$$

(b) In the Fibonacci sequence below, the next term is found by adding the two previous terms. The third term is 0.83 and the fourth term is 1.29.

Complete the first, second and fifth terms of the sequence.

$$\begin{array}{r} 0.37 \\ \hline 0.73 \\ - 0.46 \\ \hline 0.37 \end{array} \quad \begin{array}{r} 0.46 \\ \hline 0.92 \\ - 0.83 \\ \hline 0.46 \end{array} \quad 0.83 \quad 1.29 \quad \begin{array}{r} 2.12 \\ \hline 0.83 \\ + 1.29 \\ \hline 2.12 \end{array} \quad [3]$$

11 (a) Work out.

$$\begin{array}{r}
 0.8 \div 0.004 \\
 \times 1000 \quad \downarrow \quad \downarrow \quad \times 1000 \\
 800 \div 4 = 200
 \end{array}$$

(a) **200** [1]

(b) A carpenter has a plank of wood of length w metres.

The carpenter cuts $\frac{3}{5}$ of the plank of wood into 20 equal pieces.

Each piece has length 0.06 metres.

Work out the value of w , the original length of the plank of wood.

You must show your working.

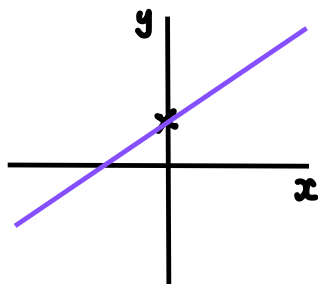
$$\begin{array}{r}
 20 \times 0.06 \\
 \quad \quad \quad \downarrow \times 100 \\
 20 \times 6 = 120 \\
 \quad \quad \quad \downarrow \div 100 \\
 \quad \quad \quad 1.2
 \end{array}$$

$$\begin{array}{l}
 \div 3 \left(\frac{3}{5} = 1.2 \text{ m} \right) \div 3 \\
 \downarrow \\
 \frac{1}{5} = 0.4 \text{ m} \\
 \uparrow \\
 \times 5 \left(\frac{5}{5} = 2 \text{ m} \right) \times 5
 \end{array}$$

$$\begin{array}{r}
 0.4 \\
 3 \overline{) 1.12} \\
 \underline{0.4} \\
 0.4 \times 5 \\
 4 \times 5 = 20 \xrightarrow{\div 10} 2
 \end{array}$$

(b) $w =$ **2** [4]

12 Write down an equation for the line that is parallel to $y = 3x - 5$ and passes through the point $(0, 1)$.



..... **$y = 3x + 1$** [2]

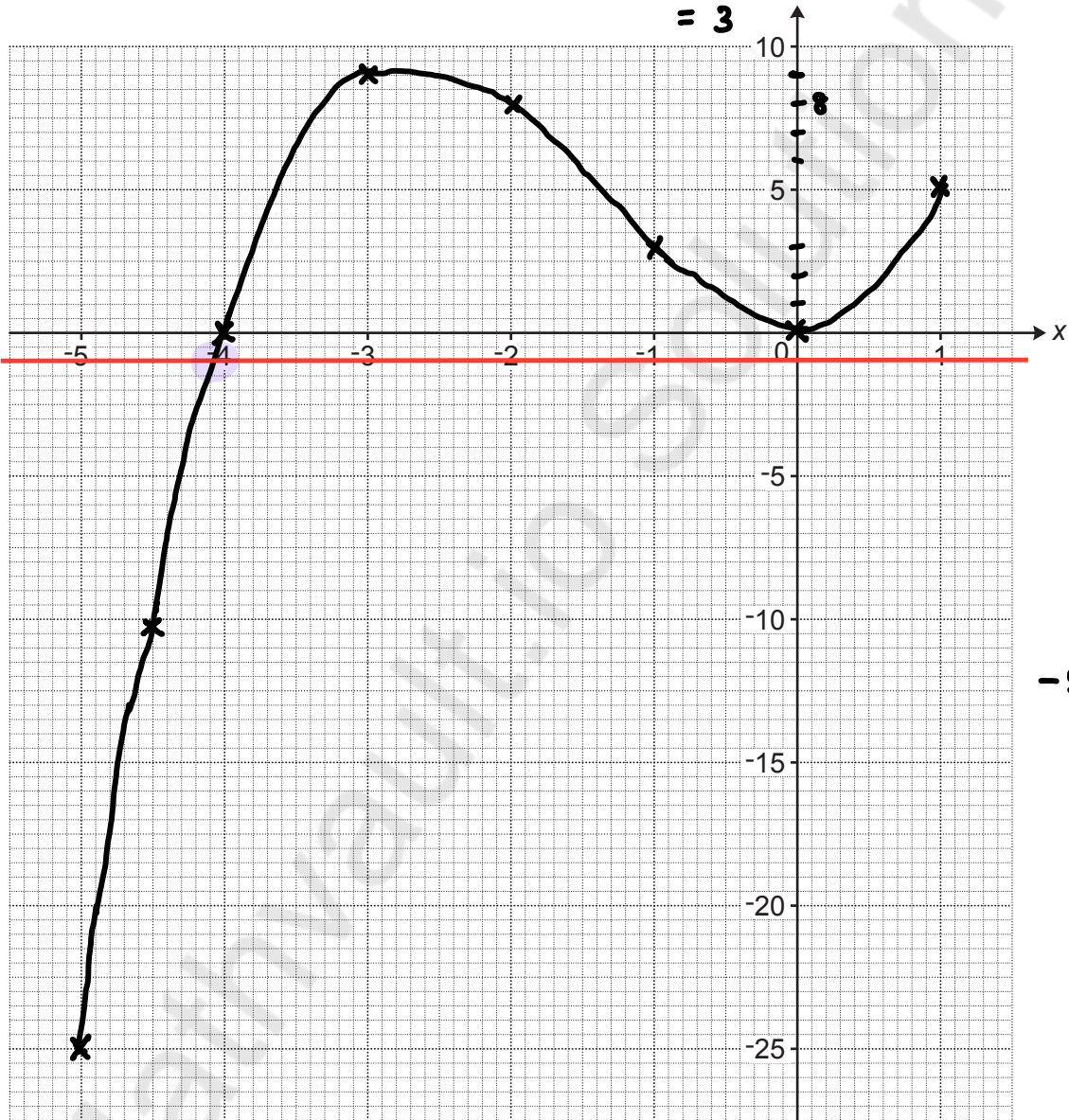
13 (a) Complete the table for $y = x^3 + 4x^2$.

x	-5	-4.5	-4	-3	-2	-1	0	1
y	-25	-10.1	0	9	8	3	0	5

[1]

(b) Draw the graph of $y = x^3 + 4x^2$ for $-5 \leq x \leq 1$.

$$\begin{aligned}
 y &= (-1)^3 + 4(-1)^2 \\
 &= -1 + 4(1) \\
 &= -1 + 4 \\
 &= 3
 \end{aligned}$$



$$-5 \div 10 = -0.5$$

[3]

(c) The equation $x^3 + 4x^2 = k$, where k is an integer, has exactly one solution for $-5 \leq x \leq 1$.

Find the greatest possible value of k .

(c) $k = \dots -1 \dots$ [1]

Turn over

- 14 A teacher is planning a theme day for the 500 pupils at their school. The teacher asks a sample of 20 pupils from year 8 which theme they would prefer.

The results are shown in the table.

Theme	Number of pupils
Sport	7
Art and design	3
Recipes	2
Music and movies	8

- (a) Describe **two** disadvantages of the teacher's sampling method.

1. **Small sample**.....
.....
2. **Sample limited to one year group**.....
..... [2]

- (b) Using the results from the table the teacher estimates that 175 pupils in the school would prefer a sport theme.

Here is the teacher's method.

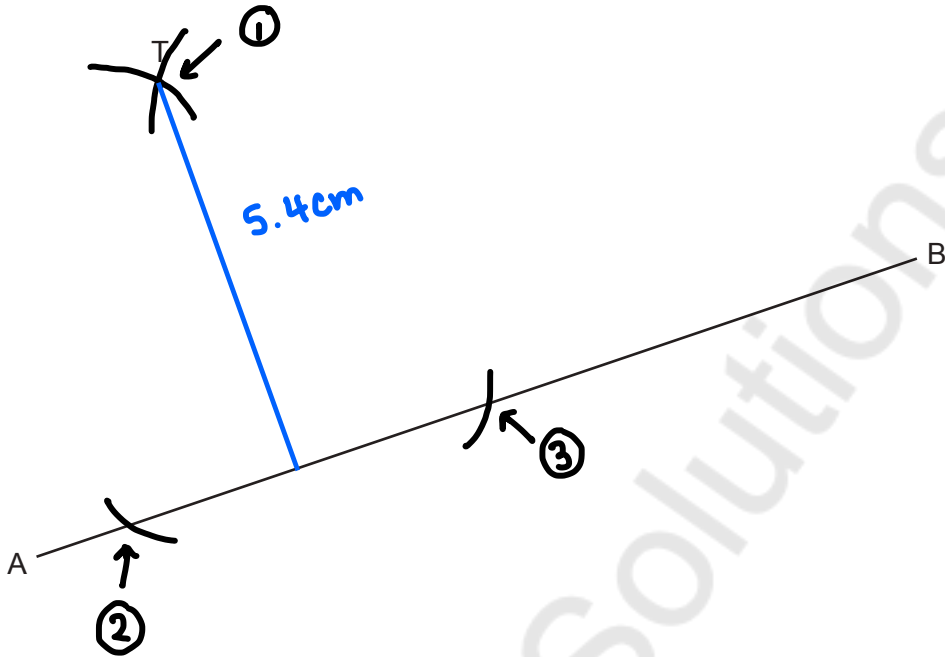
$$\frac{7}{20} \times 500 = 175$$

Write down **one** assumption the teacher has made when making their estimate.

- The sample is representative of the whole school**..... [1]

15 The diagram shows a town T and a straight road AB.

Scale: 2 cm represents 1 km



A new straight road is built from town T to the road AB.
The road is the shortest possible distance from town T to the road AB.

(a) Using ruler and compasses only, construct the road from town T to the road AB. [2]

(b) The new road costs £200 000 per kilometre to build.
The road constructor says

The new road will cost over £600 000 to build.

Show that the road constructor is incorrect.

$$2 \text{ cm} = 1 \text{ km}$$

$$5.4 \text{ cm} \div 2 = 2.7 \text{ km}$$

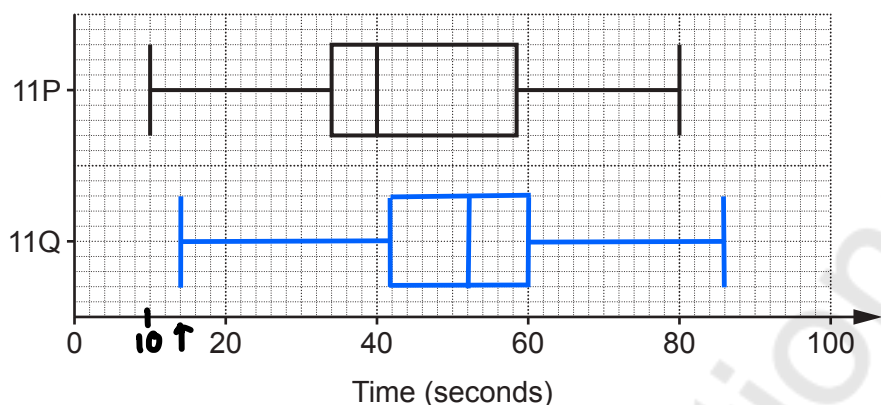
$$2.7 \times \text{£}200\,000 = \text{£}540\,000$$

$$\begin{array}{r} 2.7 \\ 2 \overline{) 5.4} \end{array}$$

$$\text{£}540,000 < \text{£}600,000$$

..... [3]

- 16 The box plot shows the distribution of the times, in seconds, taken by class 11P to complete a problem.



- (a) The times, in seconds, taken by class 11Q to complete the same problem are summarised below.

- median = 52
- lower quartile = 42
- interquartile range = 18
- range = 70
- highest score = 86

$$\text{lowest score} = 86 - 70 = 14$$

$$\text{upper quartile} = 42 + 18 = 60$$

Show the distribution of 11Q's times as a box plot on the diagram above. [3]

- (b) Which class has the more **consistent** times?
Give a reason for your answer.

..... 11Q because the interquartile range is smaller.

..... [1]

- 17 y is directly proportional to the cube of x .

Complete the table.

x	1	2	5
y	7	56	875

$$y \propto x^3$$

$$y = kx^3$$

$$7 = k(1)^3$$

$$7 = k$$

$$y = 7x^3$$

$$\begin{aligned} y &= 7(2)^3 \\ &= 7(8) \\ &= 56 \end{aligned}$$

$$875 = 7x^3$$

$$7 \overline{) 875}$$

$$125 = x^3$$

$$\sqrt[3]{125} = x$$

$$5 = x$$

[4]

18 Work out.

$$0.\dot{2}\dot{8} + \frac{4}{9}$$

Give your answer as a fraction in its simplest form.

$$\begin{aligned} x &= 0.\dot{2}\dot{8} \\ 10x &= 2.\dot{8}\dot{2} \\ 100x &= 28.\dot{2}\dot{8} \\ \hline 100x &= 28.\dot{2}\dot{8} \\ - x &= 0.\dot{2}\dot{8} \\ \hline 99x &= 28 \\ x &= \frac{28}{99} \end{aligned}$$

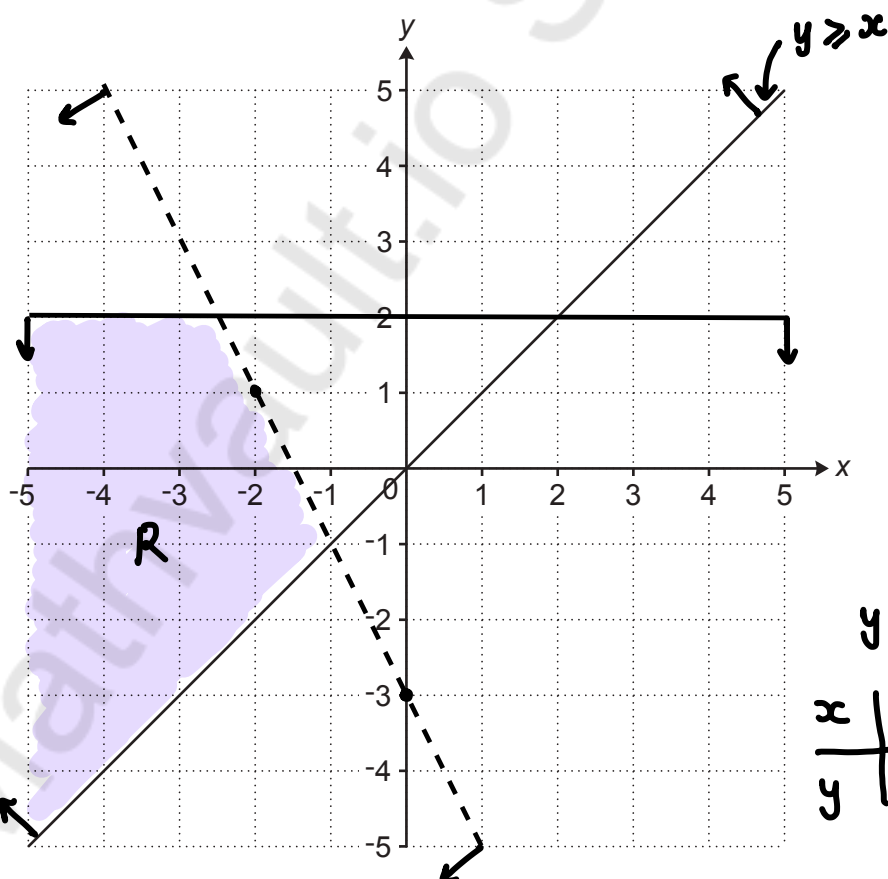
$$\frac{28}{99} + \frac{4}{9} \begin{matrix} \times 11 \\ \times 11 \end{matrix}$$

$$\frac{28}{99} + \frac{44}{99} = \frac{72}{99} \begin{matrix} \div 9 \\ \div 9 \end{matrix} = \frac{8}{11}$$

$$\frac{8}{11}$$

[4]

19 The graph of $y = x$ is drawn on the grid.



$$y = -2x - 3$$

x	-2	0	2
y	1	-3	-7

The region R satisfies the following inequalities.

$$y \geq x \quad y \leq 2 \quad y < -2x - 3$$

$$y = 2$$

By drawing two more straight lines on the grid, find and label the region R.

[6]

20 Work out.

$36^{-\frac{1}{2}}$

$$x^{-a} = \frac{1}{x^a}$$

$$x^{\frac{1}{2}} = \sqrt{x}$$

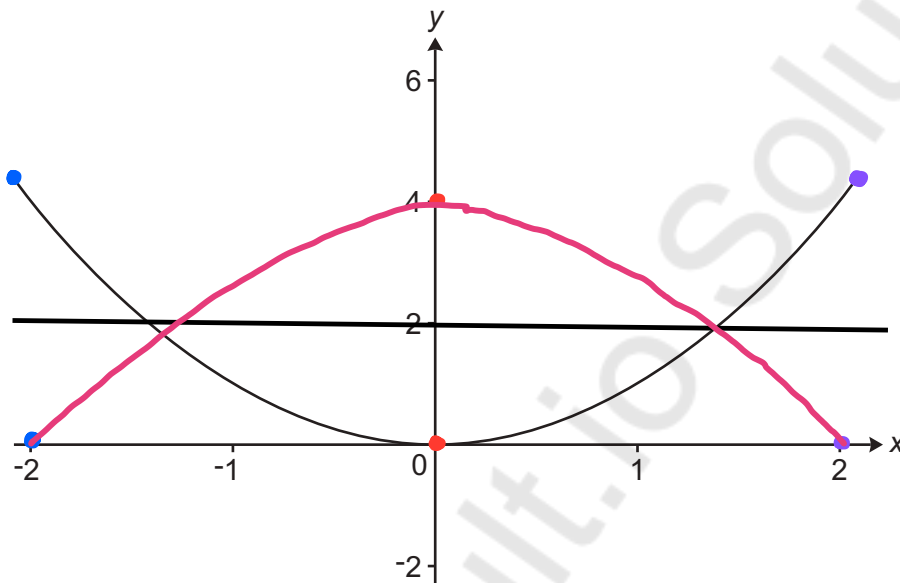
$$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{36}}$$

$$= \frac{1}{6}$$

$$\frac{1}{6}$$

[2]

21 The graph of $y = x^2$ is shown below for $-2 \leq x \leq 2$.(a) Find the equation of the image when $y = x^2$ is reflected in the line $y = 2$.

$$y = -x^2 + 4$$

(a)
$$y = 4 - x^2$$
 [2]

(b) Describe the **single** transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 6x - 5$.

$$\begin{aligned} \hookrightarrow y &= (x+3)^2 - 3^2 - 5 \\ &= (x+3)^2 - 9 - 5 \\ &= (x+3)^2 - 14 \end{aligned}$$

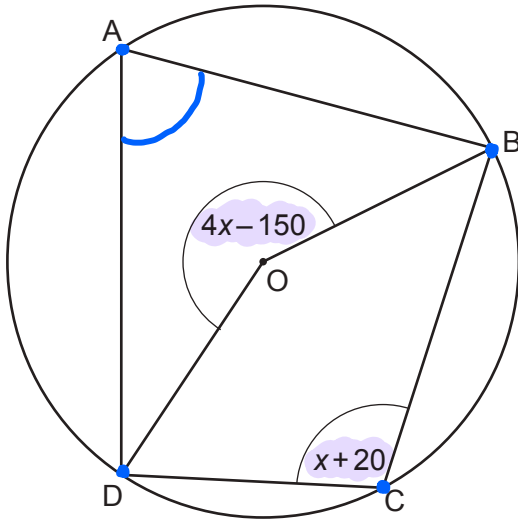
$$\hookrightarrow \text{Turning point} = (-3, -14)$$

Translation by the vector $\begin{pmatrix} -3 \\ -14 \end{pmatrix}$

[4]

22 In this question all angles are given in degrees.

A, B, C and D are points on the circumference of a circle, centre O.



Not to scale

(a) Find the size of angle BCD.

$$\begin{aligned}
 4x - 150 &= 2(x + 20) & \text{BCD} &= x + 20 \\
 4x - 150 &= 2x + 40 & &= 95 + 20 \\
 -2x & \quad -2x & &= 115 \\
 2x - 150 &= 40 \\
 +150 & \quad +150 \\
 2x &= 190 \\
 \div 2 & \quad \div 2 \\
 x &= 95
 \end{aligned}$$

(a) **115** ° [4]

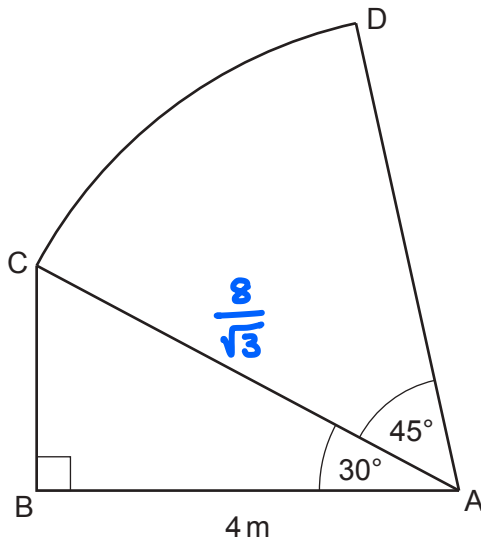
(b) Find the size of angle DAB.
Give a reason for your answer.

$$180 - 115 = 65^\circ$$

..... **65** ° because **opposite angles in a cyclic quadrilateral sum to 180°** [2]

23 In the diagram,

- ABC is a right-angled triangle
- ACD is the sector of a circle with centre A.

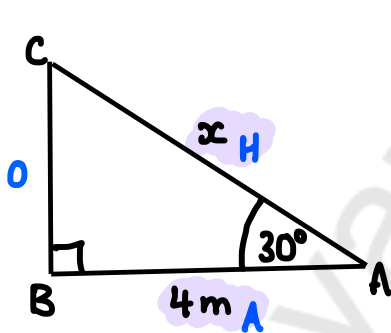


Not to scale

(a) Show that the area of the sector ACD is $\frac{8}{3}\pi \text{ m}^2$.

[6]

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$



$$\cos \theta = \frac{A}{H}$$

$$\cos(30) = \frac{4}{x}$$

$$x = 4 \div \cos(30)$$

$$= 4 \div \frac{\sqrt{3}}{2}$$

$$= \frac{4}{1} \times \frac{2}{\sqrt{3}}$$

$$= \frac{8}{\sqrt{3}}$$

	0	30	60	90	180
sin	0	1	2	3	4
cos	4	3	2	1	0
	2				

$$\text{radius} = \frac{8}{\sqrt{3}}$$

$$\text{Area of sector} = \frac{45}{360} \times \pi \left(\frac{8}{\sqrt{3}}\right)^2$$

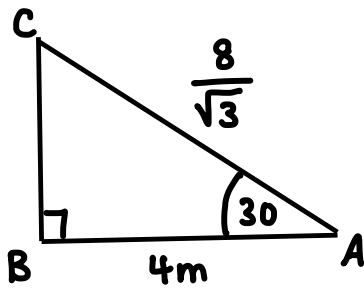
$$= \frac{1}{8} \times \pi \left(\frac{64}{3}\right)$$

$$= \frac{64}{24} \pi$$

$$= \frac{8}{3} \pi \text{ m}^2$$

(b) Work out the total area of the shape ABCD.

Give your answer in the form $\left(\frac{a\sqrt{k}}{b} + \frac{8}{3}\pi\right)\text{m}^2$.



$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 4 \times \frac{8}{\sqrt{3}} \times \sin(30) \\ &= \frac{16}{\sqrt{3}} \times \frac{1}{2} \\ &= \frac{16}{2\sqrt{3}} \quad \begin{array}{l} \div 2 \\ \div 2 \end{array} \\ &= \frac{8}{\sqrt{3}} \end{aligned}$$

$$\frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3} \text{ m}^2 \quad [\text{Triangle area}]$$

$$\text{Area ABCD} = \left(\frac{8\sqrt{3}}{3} + \frac{8}{3}\pi\right) \text{m}^2$$

(b) $\left(\frac{8\sqrt{3}}{3} + \frac{8}{3}\pi\right)$ m^2 [3]

- 24 The x-coordinates of the intersections of the graphs of $y = x^2 + ax + 7$ and $y = 3x + b$ are the solutions to the equation $x^2 + 8x + 13 = 0$.

Find the value of a and the value of b .

$$\begin{array}{r} x^2 + ax + 7 = 3x + b \\ -3x \quad -b \quad -3x \quad -b \end{array}$$

$$\begin{array}{r} x^2 + ax - 3x + 7 - b = 0 \\ x^2 \quad \underbrace{\quad + 8x \quad} \quad \underbrace{\quad + 13 \quad} \end{array}$$

$$\begin{array}{r} ax - 3x = 8x \quad \quad 7 - b = 13 \\ a - 3 = 8 \quad \quad -7 \quad -7 \\ a = 11 \quad \quad -b = 6 \\ \quad \quad \quad \quad b = -6 \end{array}$$

$$a = \underline{11} \dots\dots\dots$$

$$b = \underline{-6} \dots\dots\dots [3]$$

- 25 Riley has a set of cards.
Each card has a triangle or circle drawn on it and is coloured red or blue.

The table gives some information about the number of each type of card.

	Number of cards with a triangle	Number of cards with a circle
Number of blue cards	3	8
Number of red cards	9	2

$$= 10$$

Riley chooses three of these cards at random without replacement.
All three of these cards have a circle drawn on them.

Riley says

The probability that two of these three cards are blue and one is red is less than $\frac{1}{2}$.

Is Riley correct?

You must show your working.

$$p(\text{1st card blue}) = \frac{8}{10}$$

$$p(\text{2nd card blue}) = \frac{7}{9}$$

$$p(\text{3rd card red}) = \frac{2}{8}$$

$$\frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} = \frac{112}{720}$$

BBR
BRB
RBB

$$3 \times \frac{112}{720} = \frac{336}{720}$$

Yes because $\frac{336}{720}$ is less than $\frac{1}{2}$.

[5]

END OF QUESTION PAPER

