OCR
Oxford Cambridge and RSA

Monday 13 November 2023 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

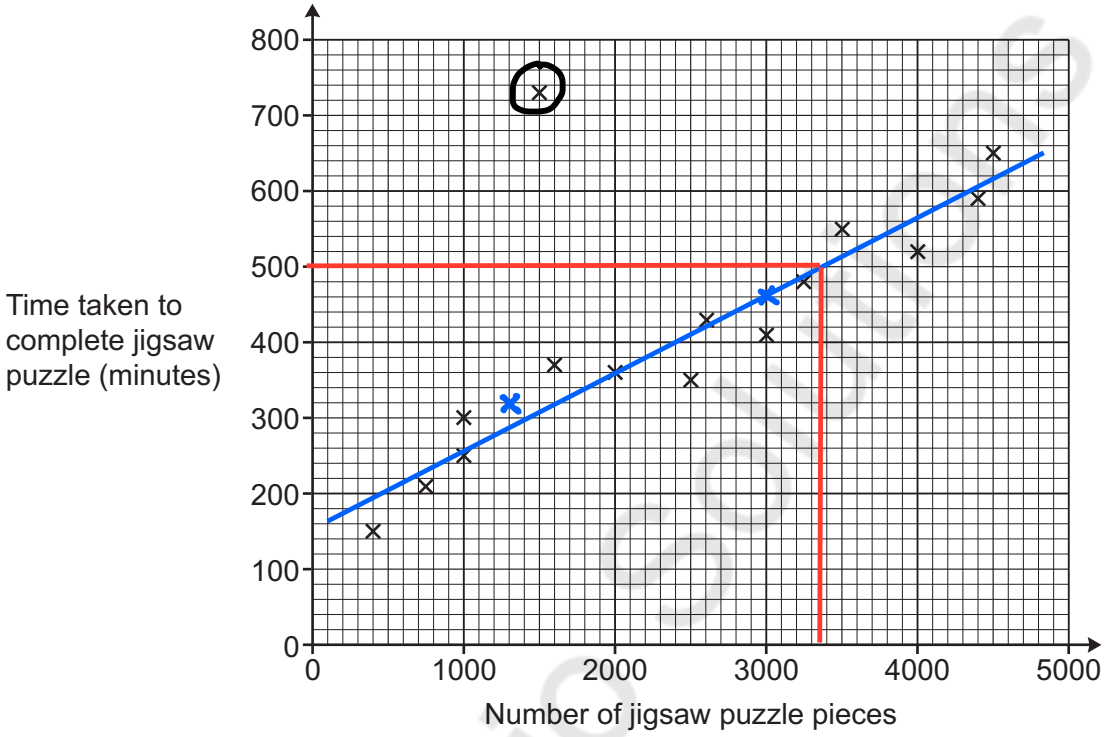
INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Beth completes some jigsaw puzzles and records the following information.

- The number of pieces in the jigsaw puzzle.
- The time taken to complete the jigsaw puzzle, in minutes.

Beth shows this information in a scatter diagram.



(a) **(i)** Beth completes two more jigsaw puzzles.

- A 3000 piece jigsaw puzzle taking 460 minutes.
- A 1300 piece jigsaw puzzle taking 320 minutes.

Show this information on the scatter diagram. **[1]**

(ii) Describe the type of correlation shown on the scatter diagram.

(a)(ii) Positive **[1]**

(b) One of Beth's jigsaw puzzles was described as "the most difficult jigsaw puzzle you will ever try".

(i) Circle the most likely jigsaw puzzle on the scatter diagram. **[1]**

(ii) Give a reason why you chose this jigsaw puzzle.

The jigsaw took a long time for a similar number of pieces.

[1]

(c) **(i)** Draw a line of best fit on the scatter diagram. **[1]**

(ii) Use your line of best fit to estimate how many pieces are in a jigsaw puzzle that takes Beth 500 minutes to complete.

(c)(ii) ..... 3350 ..... pieces **[1]**

(d) Explain why Beth should **not** use her scatter diagram to estimate how long it will take to complete a jigsaw puzzle containing 8000 pieces.

..... 8000 pieces is outside the range of data. .....

.....

..... **[1]**

2 A restaurant menu has 4 main courses and 3 side dishes.
For their meal, each customer chooses 1 main course and 1 side dish.

Main course	
Beef burger	£6
Lasagna	£7
Veggie burger	£5
Turkey stew	£6

Side dish	
Salad	£2
Chips	£3
Garlic bread	£1

Work out the percentage of possible meals that cost less than £8.

Total combinations = 4 x 3
= 12

$\frac{4}{12} \times 100 = 33.\dot{3}$

33.3 % **[4]**

3 Write these numbers in order of size, starting with the smallest.

0.36% $\quad \frac{1}{333}$ $\quad 0.03$ $\quad 3.1 \times 10^{-3}$

$\div 100$ $\quad 1 \div 333$ $\quad$ $\downarrow$

0.0036 $\quad 0.00\dot{3}$ $\quad 0.0031$

③ $\quad$ ① $\quad$ ②

$\frac{1}{333}$, 3.1×10^{-3} , 0.36% , 0.03 **[4]**

smallest

4 Casey's mobile phone gives a weekly report showing the amount of time they use their phone.

This week the report says that the phone was used

- for 217 minutes
- 24% more than last week.

Calculate Casey's phone usage last week.

$100\% + 24\% = 124\%$

$\div 1.24$ ($124\% = 217$ mins) $\div 1.24$

$100\% = 175$ mins

175 minutes **[3]**

5 A sheet of A4 card weighs 1.19×10^{-2} kg.

(a) Work out the weight of 500 sheets of the A4 card.

$500 \times (1.19 \times 10^{-2}) = 5.95$

(a) ..... 5.95 ..... kg **[2]**

(b) Card is classified using *W*, the weight in grams per square metre (gsm).

$$W = \frac{\text{weight in grams}}{\text{area in square metres}}$$

A sheet of A4 card is a rectangle that is 21 cm by 29.7 cm.

Calculate *W* for this A4 card.

1 kg = 1000 g
×1000

1.19×10^{-2} kg × 1000 = 11.9 g

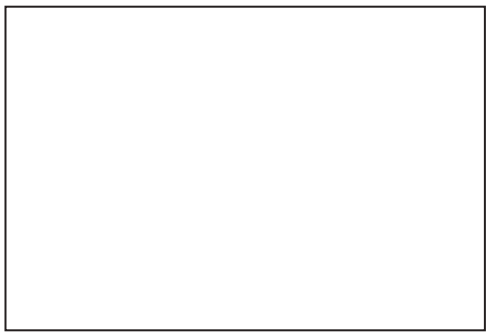
100 cm = 1m
÷100

Area = l × w
= 0.297 × 0.21
= 0.06237

$W = \frac{11.9}{0.06237} = 190.7968575$

(b) ..... 190.8 ..... gsm **[4]**

6 The area of this rectangle can be written as $ax^2 + bx - 10$.



Not to scale

Find the values of a, b and c.
You must show your working.

Area = l x w

$= (x+c)(3x+2)$

$= 3x^2 + 2x + 3xc + 2c$

$ax^2 \quad + bx \quad -10$

$a = 3$

$2x + 3xc = bx$

$\div x \qquad \div x$

$2 + 3c = b$

$2 + 3(-5) = b$

$2 - 15 = b$

$-13 = b$

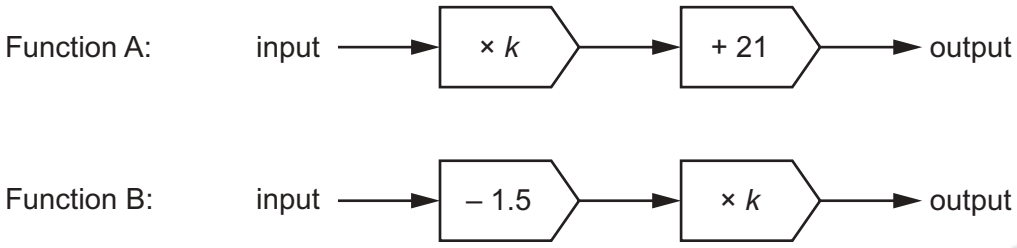
$2c = -10$

$\div 2 \qquad \div 2$

$c = -5$

$a =$ 3 , $b =$ -13 and $c =$ -5 **[5]**

7 Here are two functions.



5 is input into Function A.
5 is also input into Function B.
The output of Function A is 10 times the output of Function B.

Work out the value of k.
You must show your working.

Function A

$5 \times k = 5k$

$5k + 21$

Output : $5k + 21$

Function B

$5 - 1.5 = 3.5$

$3.5 \times k = 3.5k$

Output : $3.5K$

$5k + 21 = 10(3.5k)$

$5k + 21 = 35k$

$-5k \qquad -5k$

$21 = 30k$

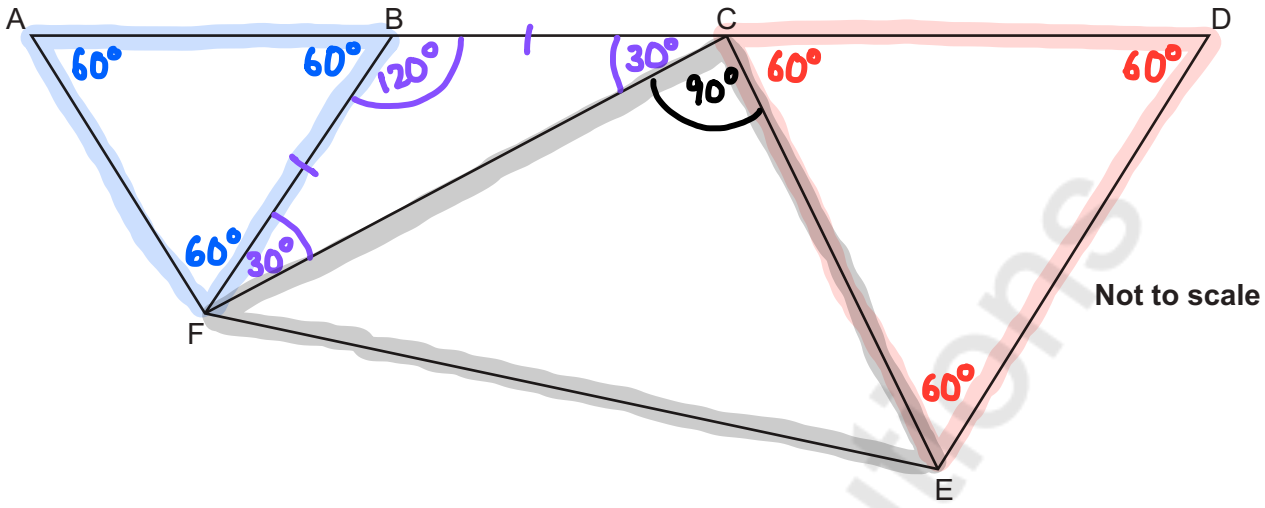
$\div 30 \qquad \div 30$

$\frac{21}{30} = k$

$0.7 = k$

$k =$ 0.7 **[5]**

8 The diagram shows four triangles that are joined together.



- Points A, B, C and D lie on a straight line.
- Triangles ABF and CDE are equilateral triangles.
- Triangle BCF is an isosceles triangle with BF = BC.

Show that triangle CEF is a right-angled triangle.
Give a reason for each stage of your working.
Use the template below to help present your work. You may not need all of the lines.

Angle ABF = 60° because all angles are equal.

Angle FBC = 120° because angles on a straight line sum to 180°

Angle BCF = 30° because base angles of an isosceles are equal

Angle ECD = 60° because all angles are equal

Angle FCE = 90° because angles on a straight line sum to 180°

Angle ..... = ...° because .....

Angle ..... = ...° because .....

[5]

9 A large box of chocolates contains dark, milk and white chocolates.
When Riley opens the box, the ratio of dark to milk to white chocolates is 3 : 2 : 4.
Riley's family eat 6 of the dark chocolates, none of the milk chocolates and all of the white chocolates.
The ratio of dark to milk chocolates is now 9 : 8.

How many **white** chocolates did Riley's family eat?

D : m : w

3x : 2x : 4x

−6 −4x

D : m

3x−6 : 2x

D : m

9 : 8

$$\frac{3x-6}{2x} = \frac{9}{8}$$

$8(3x-6) = 9(2x)$

$24x - 48 = 18x$

$-18x \quad -18x$

$6x - 48 = 0$

$+48 \quad +48$

$6x = 48$

$\div 6 \quad \div 6$

$x = 8$

$W = 4x$

$= 4(8)$

$= 32$

..... 32 ..... white chocolates **[4]**

10 **(a)** x and y are related by the equation $xy = 36$.

$\div x \quad \div x \qquad y = \frac{36}{x}$

Tick the correct statement.

☐	y is directly proportional to x
☑	y is inversely proportional to x
☐	y is not proportional to x

[1]

(b) y is inversely proportional to x^4.
$y = 2.5$ when $x = 2$.

Find a formula linking x and y.

$y \propto \frac{1}{x^4}$

$y = \frac{k}{x^4}$ ⟶ $y = \frac{40}{x^4}$

$2.5 = \frac{k}{2^4}$

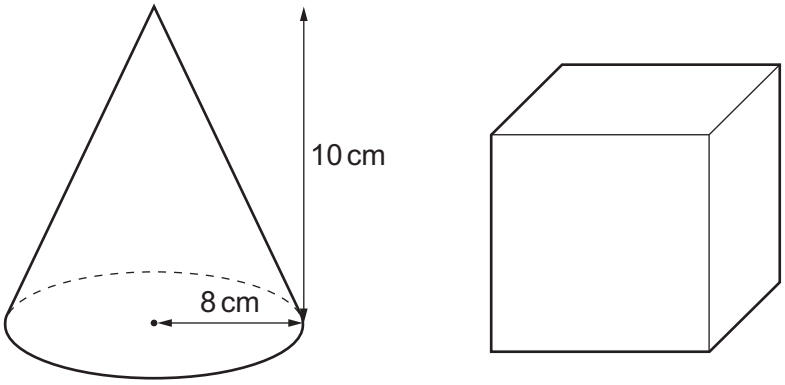
$2.5 = \frac{k}{16}$

$\times 16 \qquad \times 16$

$40 = k$

(b) $y = \frac{40}{x^4}$ **[3]**

11 The diagram shows a cone and a cube.
The cone has radius 8 cm and height 10 cm.



The volume of the cone is equal to the volume of the cube.

Work out the length of one side of the cube.

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$V = \frac{1}{3}\pi r^2 h \qquad r = 8 \quad h = 10$

$= \frac{1}{3}\pi (8)^2 (10)$

$= \frac{640}{3}\pi \text{ cm}^3$

$V = l \times w \times h \qquad l = w = h$

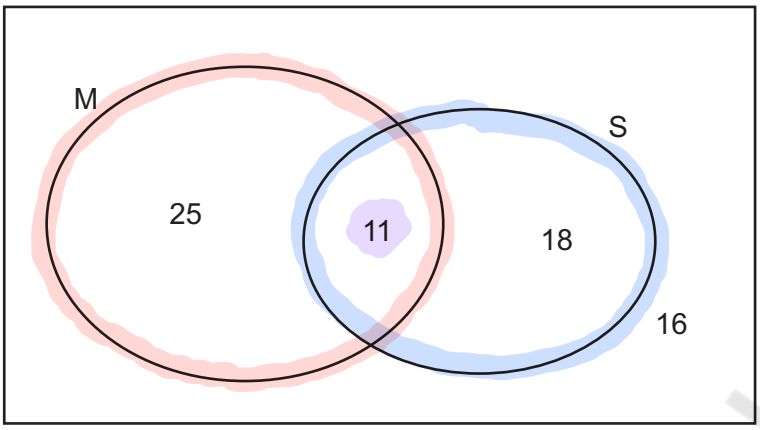
$\sqrt[3]{\frac{640}{3}\pi} = 8.751238716$

$= 8.75 \text{ cm}$

..... 8.75 ..... cm **[4]**

12 A cafe owner recorded information about customer orders for coffee.
They recorded whether the customer asked for milk (M) and whether the customer asked for sugar (S).

The results are shown in this Venn diagram.



(a) One of the customers is chosen at random.

Find the probability that the customer asked for sugar.

$$\frac{11+18}{25+11+18+16} = \frac{29}{70}$$

(a) $\frac{29}{70}$ **[2]**

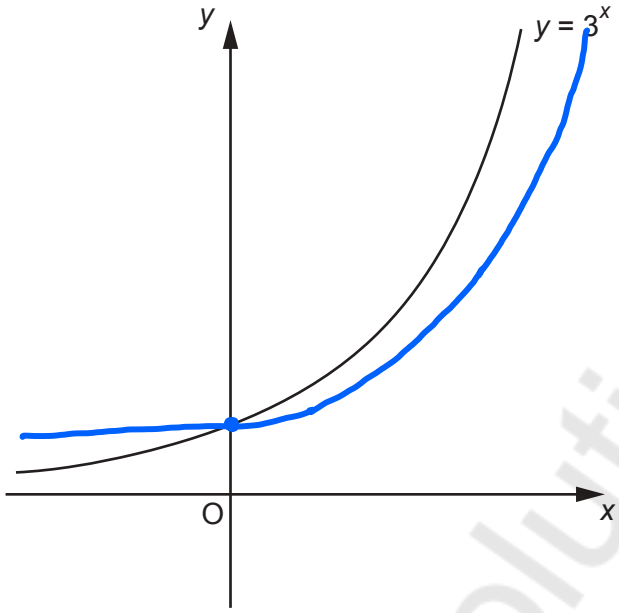
(b) One of the customers is chosen at random.

Find the probability that the customer asked for sugar given that they asked for milk.

$$\frac{11}{25+11} = \frac{11}{36}$$

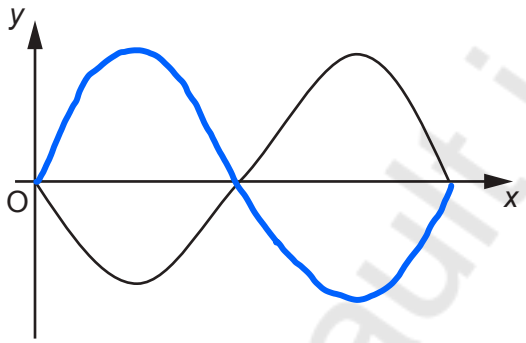
(b) $\frac{11}{36}$ **[2]**

13 **(a)** The graph of $y = 3^x$ is sketched below.



On the same axes, sketch the graph of $y = 2^x$. **[3]**

(b) Charlie sketches this graph.



Charlie says

The equation of my graph is y = sinx.

(i) Explain how you know that Charlie is **not** correct.

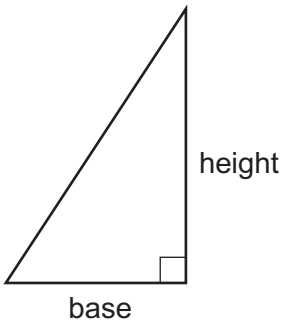
Sin x is positive for $0 \leqslant x \leqslant 90$

..... **[1]**

(ii) Write down a possible equation for Charlie's graph.

(b)(ii) $y = -\sin x$ **[1]**

14 Here is a right-angled triangle.

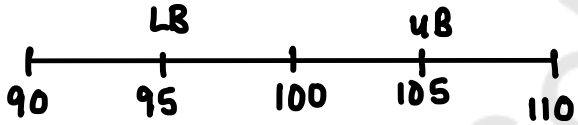


Not to scale

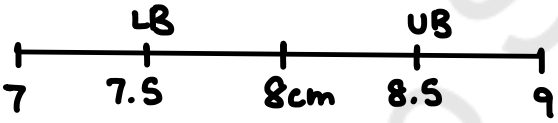
The area of the triangle is 100 cm^2, correct to the nearest 10 cm^2.
The length of the base of the triangle is 8 cm, correct to the nearest cm.

Calculate the largest possible height of the triangle.

Area



Base



$A = \frac{1}{2}bh$

$\div \frac{1}{2}b \qquad \div \frac{1}{2}b$

$h = \frac{A}{\frac{1}{2}b}$

$h_{UB} = \frac{A_{UB}}{\frac{1}{2}b_{LB}}$

$= \frac{105}{\frac{1}{2}(7.5)}$

$= 28 \text{ cm}$

28 cm **[4]**

15 **(a)** Sasha and Taylor are asked to find how many solutions the equation $5(x+2)^2 = 45$ has.

Here is **Sasha**'s answer.

$5(x+2)^2 = 45$

$(x+2)^2 = 9$ ÷5 √

$x+2 = 3$ √ −2

$x = 1$

There is one solution.

Here is **Taylor**'s answer.

$5(x+2)^2 = 45$

$(x+2)^2 = 9$ ÷5 √

$x + 2 = 3$ or $x + 2 = -3$ −2

$x = 1$ or $x = -5$

There are two solutions.

Decide who is correct, Sasha or Taylor, and give the reason for your decision.

Taylor is correct because $\sqrt{9}$ is +3 or −3

[1]

(b) Solve this equation algebraically.
Give your answers correct to **2** decimal places.
You must show your working.

$x^2 - 5x + 3 = 0$

$ax^2 + bx + c = 0$ $\quad a = 1 \quad b = -5 \quad c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

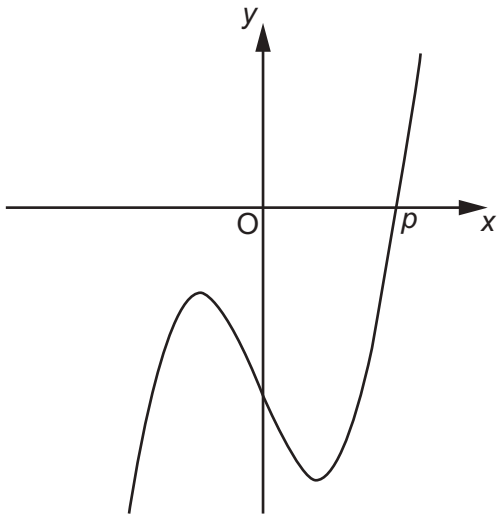
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$

$= 4.302775638$ or 0.6972243623

≈ 4.30 or 0.70

(b) $x =$ 4.30 or $x =$ 0.70 **[4]**

16 The graph of $y = x^5 - 70x - 150$ is sketched below.
The root of the equation $x^5 - 70x - 150 = 0$ is p.



(a) Show that $3 < p < 4$. **[3]**

$(3)^5 - 70(3) - 150 = -117$

$(4)^5 - 70(4) - 150 = 594$

Change of sign ∴ solution between $x = 3$ and $x = 4$

(b) Find a smaller interval that contains the value of p.
You must show calculations to support your answer.

$(3)^5 - 70(3) - 150 = -117$

$(3.4)^5 - 70(3.4) - 150 = 66.35424$

$(3.5)^5 - 70(3.5) - 150 = 130.21875$

(b) ..... 3 ..... $< p <$ ..... 3.4 ..... **[3]**

17 Hiro invests £2500 for 2 years in a bank account paying r% per year compound interest.
At the end of 2 years, the amount in the bank account is £2704.

Calculate r.

Final amount = investment $\times \left(\frac{100 + r}{100}\right)^n$ ← no. of years

$$2704 = 2500 \times \left(1 + \frac{r}{100}\right)^2$$

$\div 2500$ $\div 2500$

$$1.0816 = \left(1 + \frac{r}{100}\right)^2$$

$\sqrt{}$ $\sqrt{}$

$$1.04 = 1 + \frac{r}{100}$$

-1 -1

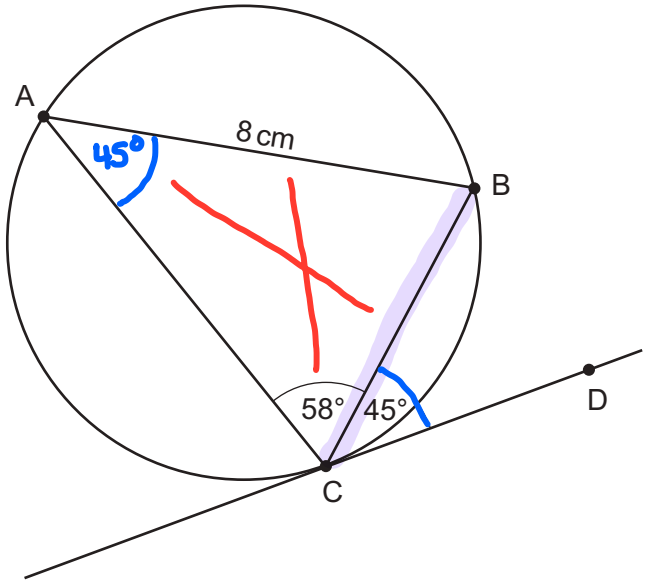
$$0.04 = \frac{r}{100}$$

$\times 100$ $\times 100$

$$4 = r$$

$r =$ 4 **[4]**

18 A, B and C are points on the circumference of a circle.



Not to scale

Points C and D lie on a tangent to the circle at C.
Angle BCD = 45°.
Angle ACB = 58°.
AB = 8 cm.

Find the length of BC.

$\angle CAB = 45°$ Alternate segment theorem

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{BC}{\sin(45)} = \frac{8}{\sin(58)}$$

$\times \sin(45)$ $\times \sin(45)$

$$BC = \frac{8}{\sin(58)} \times \sin(45)$$

$$= 6.670440362$$

$$\approx 6.67 \text{ cm}$$

.....6.67..... cm **[4]**

19 A fitness centre records how long each customer spends in the gym.
This **cumulative frequency** table summarises the results.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	6
$10 < t \leqslant 20$	24
$20 < t \leqslant 30$	35
$30 < t \leqslant 40$	48
$40 < t \leqslant 50$	60
$50 < t \leqslant 60$	74

frequency	midpoints	midpoint x frequency	
6	5	30	
18	15	270	
11	25	275	
13	35	455	
12	45	540	
14	55	770	
74		2340	

Calculate an estimate of the mean time the customers spend in the gym.
You must show your working.
You may use the table above to help present your work.

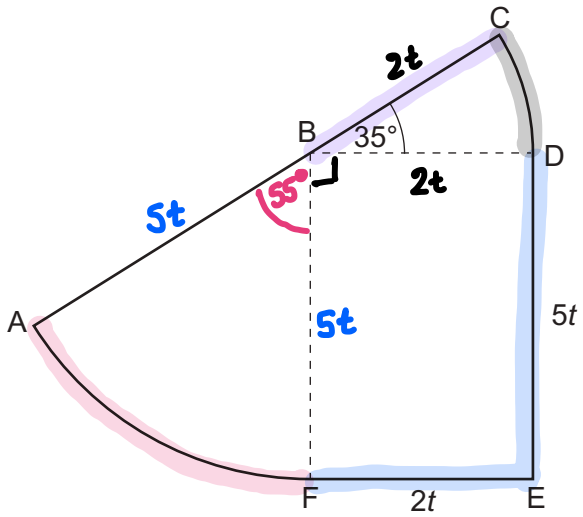
$$\text{Mean} = \frac{2340}{74}$$

$$= 31.6\dot{2}\dot{1}$$

$$\approx 31.6$$

.....31.6..... minutes **[5]**

20 This shape is formed from a rectangle and two sectors of circles.



Not to scale

Points A, B and C lie on a straight line.
Angle CBD = 35°.
DE = $5t$ and EF = $2t$.

(a) Explain why BC = $2t$.
Give a reason for each step of your explanation.

BD = 2t as opposite sides of rectangles are equal.

BC = BD = 2t they are both radii

..... **[2]**

(b) Show that the perimeter of the shape is $\frac{23}{12}\pi t + 14t$. **[5]**

$$CD = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{35}{360} \times 2\pi(2t)$$

$$= \frac{35}{360} \times 4\pi t$$

$$= \frac{7}{18}\pi t$$

$\angle ABF = 180 - 90 - 35$
$= 55^\circ$

$$AF = \frac{55}{360} \times 2\pi(5t)$$

$$= \frac{55}{360} \times 10\pi t$$

$$= \frac{55}{36}\pi t$$

$$P = \frac{7}{18}\pi t + 5t + 2t + \frac{55}{36}\pi t + 5t + 2t$$

$$= \frac{23}{12}\pi t + 14t$$

21 Simplify fully.

$$\frac{x^3 + 8x^2 + 15x}{x^3 - 9x}$$

$x^3 + 8x^2 + 15x$

$x(x^2 + 8x + 15)$

15

$+3 \quad +5$

$x(x+3)(x+5)$

$x^3 - 9x$

$x(x^2 - 9)$

$x(x+3)(x-3)$

$$\frac{x(x+3)(x+5)}{x(x+3)(x-3)}$$

$$\frac{x+5}{x-3}$$

$$\frac{x+5}{x-3}$$

..... **[5]**

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).