OCR
Oxford Cambridge and RSA

Thursday 16 May 2024 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

* J 5 6 0 0 4 *

* 1 3 9 4 9 4 4 1 4 0 *

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Work out.

$$\sqrt[3]{\frac{19.5^4 - 18^2}{1.45}}$$

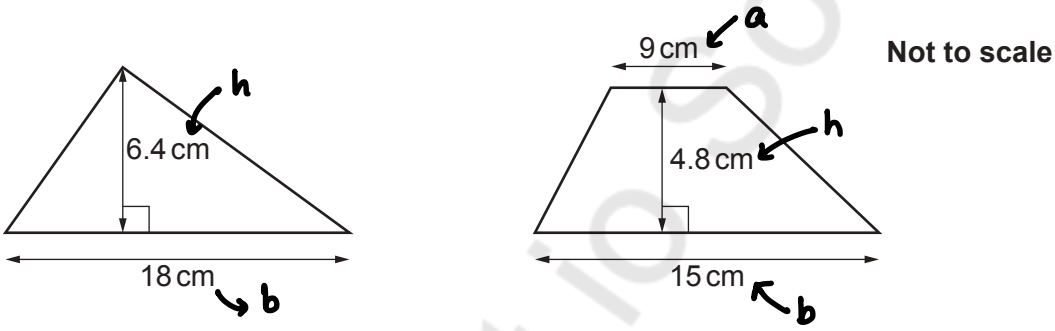
Write your answer correct to **4** significant figures.

46.337442 36

≈ 46.34

46.34 ..... **[3]**

2 The diagram shows a triangle and a trapezium.



Not to scale

Show that they have the same area. **[3]**

Area triangle $= \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 6.4$

$= 57.6\ cm^2$

Area trapezium $= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(9+15) \times 4.8$

$= 57.6\ cm^2$

3 Four numbers are written, in ascending order, as algebraic expressions.

a $\quad a+b$ $\quad a+2b$ $\quad 3a-b$

The mean of these four numbers is 27.
The range of these four numbers is 24.

Find the value of a and the value of b.
You must show your working.

mean = add all values / number of values

$$27 = \frac{a + a+b + a+2b + 3a-b}{4}$$

$$27 = \frac{6a + 2b}{4}$$

$\times 4$ $\qquad \times 4$

$$108 = 6a + 2b$$

Range = largest - smallest

$$24 = 3a - b - a$$

$$24 = 2a - b$$

$$6a + 2b = 108$$
$$2a - b = 24 \quad \times 3$$

$$6a + 2b = 108$$
$$6a - 3b = 72$$

$$5b = 36$$

$\div 5 \qquad \div 5$

$$b = 7.2$$

$$2a - b = 24$$
$$2a - 7.2 = 24$$
$$+7.2 \qquad +7.2$$
$$2a = 31.2$$

$\div 2 \qquad \div 2$

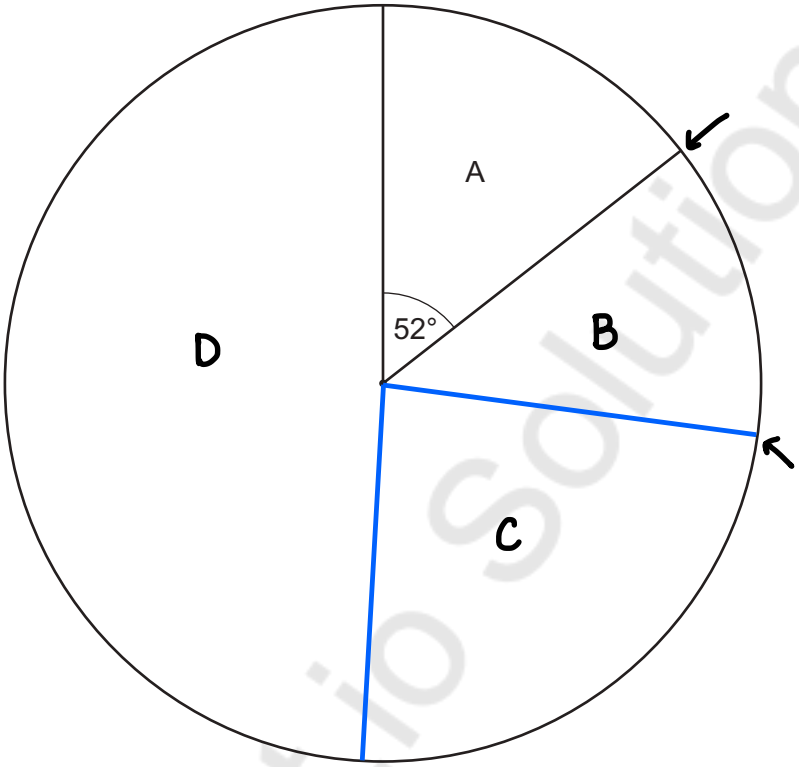
$$a = 15.6$$

$a =$ 15.6

$b =$ 7.2 **[5]**

4 A school is deciding on a charity to support.
Each student at the school votes for one of four charities, A, B, C or D.
The results are to be shown in a pie chart.

This pie chart shows the sector for charity A.
Twice as many students voted for charity C than charity B.
Twice as many students voted for charity D than charity C.



B = 44°
C = 2 × 44 = 88°
D = 4 × 44 = 176°

(a) (i) Show that the sector for charity B will have an angle of 44°. **[2]**

360° − 52° = 308°

B : C
1 : 2

C : D
1 : 2
×2 ×2
2 : 4

B : C : D
1 : 2 : 4
} 7 parts

308° ÷ 7 = 44

B = 1 × 44
= 44°

(ii) Complete the pie chart. **[3]**

(b) 39 students voted for charity A.

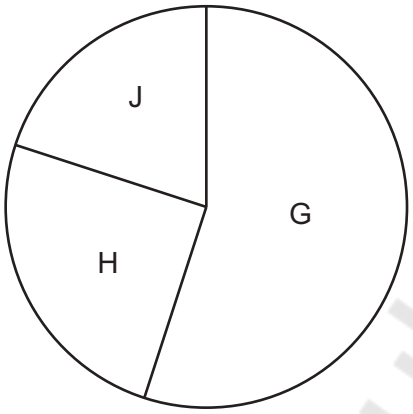
Calculate the total number of students at the school.

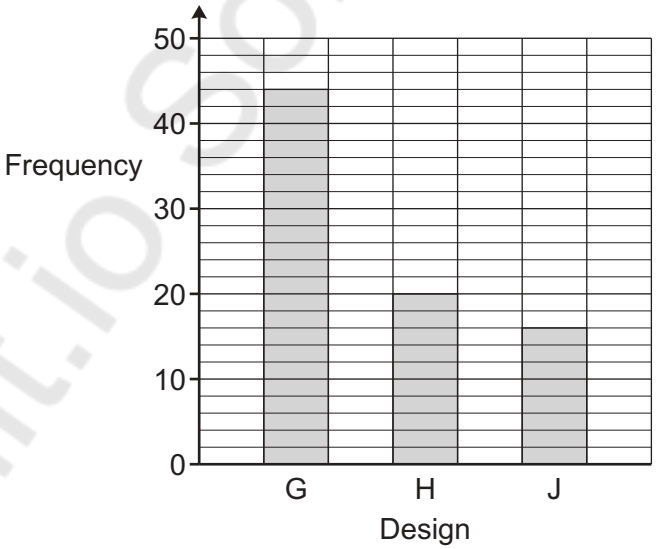
	Angle		Frequency
A	52°	×0.75 →	39
Total	360°	×0.75 →	270

$\frac{39}{52} = 0.75$

(b) 270 **[2]**

(c) The school asks 80 of the students to choose a new logo from three designs G, H and J.
The same results are shown in a pie chart and in a bar chart.





(i) Give one **advantage** of using the pie chart rather than the bar chart.

It displays data as part of a whole

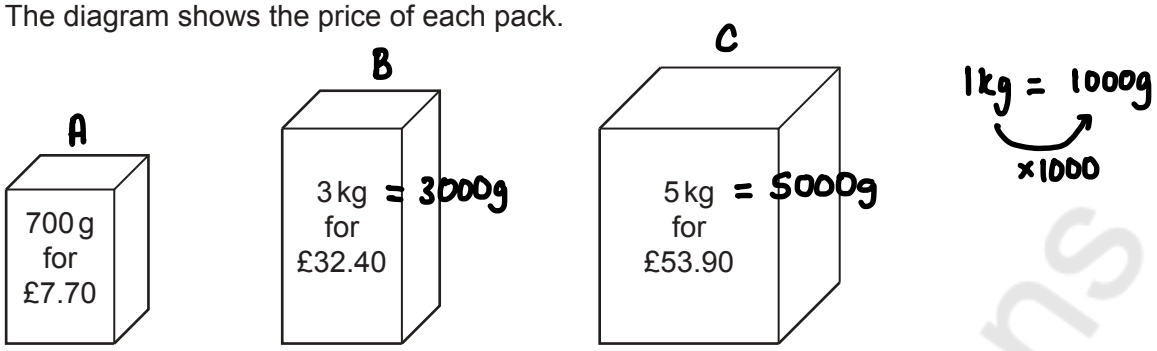
..... **[1]**

(ii) Give one **disadvantage** of using the pie chart rather than the bar chart.

It does not show exact frequencies.

..... **[1]**

5 The same dog food is sold in three different sized packs.
The diagram shows the price of each pack.



Which pack is the best value for money?
Show how you decide.

A 700g = £7.70
÷7 ÷7
100g = £1.10

B 3000g = £32.40
÷30 ÷30
100g = £1.08

C 5000g = £53.90
÷50 ÷50
100g = £1.078

The 5kg for £53.90 pack because £1.078 for 100g is less than £1.10 and £1.08 for 100g.

..... **[3]**

6 A regular polygon has n sides.
The interior angle of the polygon is 15 times its exterior angle.

Find the value of n.

$$\text{Exterior} = \frac{360}{n} \qquad \text{Interior} = 180 - \frac{360}{n}$$

$$180 - \frac{360}{n} = 15\left(\frac{360}{n}\right)$$

$$180 - \frac{360}{n} = \frac{5400}{n}$$

$$+\frac{360}{n} \qquad +\frac{360}{n}$$

$$180 = \frac{5760}{n}$$

$$\times n \qquad \times n$$

$$180n = 5760$$

$$\div 180 \qquad \div 180$$

$$n = 32$$

$n =$ 32 **[4]**

7 Write the following in order of size, smallest first.

0.2 2^{-2} 2×10^{-2}

Show how you decide.

$$x^{-a} = \frac{1}{x^a}$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

$$0.02 \times 10^{-2} = 0.02$$

2×10^{-2} , 0.2 , 2^{-2} **[3]**

smallest

8 The price of a television is increased by 35%.
In a sale, the new price of the television is decreased by r%.

The overall percentage increase in the price of the television is 16.1%.

Find the value of r.
You must show your working.

$100\% + 35\% = 135\%$

$100\% + 16.1\% = 116.1\%$

$$\%\ \text{decrease} = \frac{\text{change}}{\text{original}} \times 100$$

$$= \frac{135 - 116.1}{135} \times 100$$

$$= 14$$

$r =$ 14 **[5]**

9 Sasha and Taylor each have a stamp collection.

They organise their stamp collections according to where the stamps come from: United Kingdom (UK), European Union (EU), Other.

The table shows the number of stamps in each collection and the ratio UK : EU : Other.

	Number of stamps	**Ratio UK : EU : Other**
Sasha's collection	1638	9 : 3 : 2
Taylor's collection	660	8 : 1 : 2

When they put the two stamp collections together, Sasha and Taylor claim that at least $\frac{2}{3}$ of all the stamps come from the UK.

Are they correct?
Show how you decide.

Sasha's

UK : EU : Other
9 : 3 : 2 } 14 parts 1638 ÷ 14 = 117

×117 ↓

1053 UK stamps

Taylor's

UK : EU : Other
8 : 1 : 2 } 11 parts 660 ÷ 11 = 60

×60 ↓

480 UK stamps

Total Stamps = 1638 + 660
= 2298

$\frac{2}{3} \times 2298 = 1532$

Total UK Stamps
= 1053 + 480
= 1533

They are correct because 1533 > 1532

[5]

10 Here is a question and an incorrect solution.

Question:
You are given
$y \propto x$ and $y = 9$ when $x = 2$.

Find a formula linking x and y.

Solution:
$y \propto x$ so $y = x + c$ $y = kx$
Substituting $y = 9$ and $x = 2$ gives
$9 = 2 + c$
$c = 7$
So, $y = x + 7$

Describe the error made and write out a correct solution.

The error is the correct formula should be $y = kx$

A correct solution is $y = kx$

$9 = k \times 2$

$9 = 2k$ $\div 2$ $\div 2$

$4.5 = k$

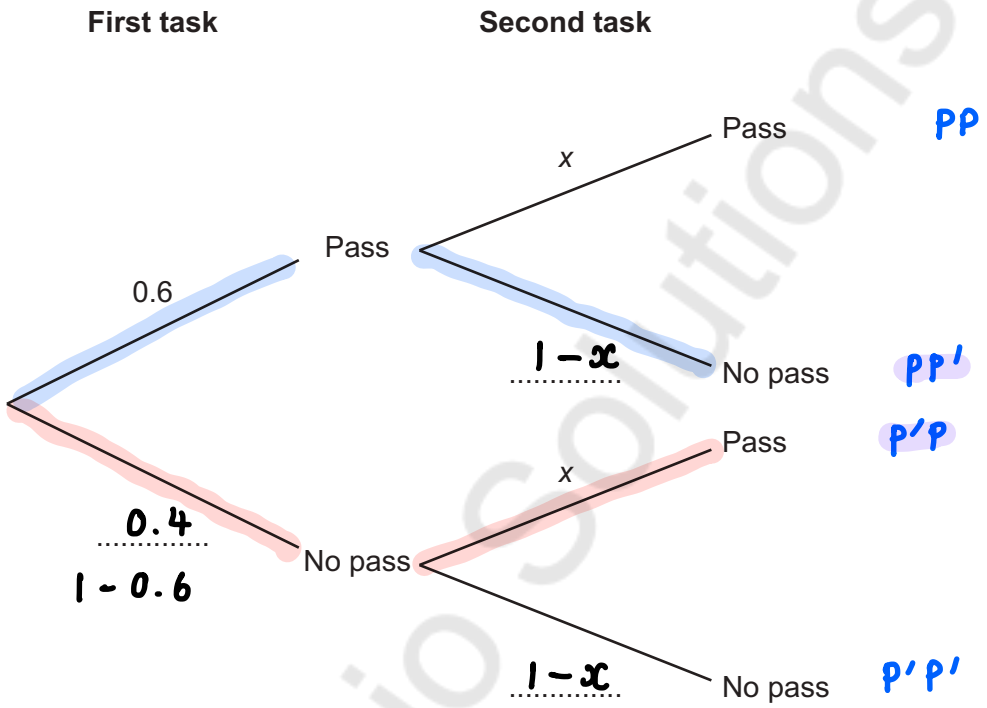
$y = 4.5x$

[3]

11 A student attempts two tasks.
The result of each task is either "Pass" or "No pass".

The probability of the student passing the first task is 0.6.
The probability of the student passing the second task is x.

(a) Complete the tree diagram.



[2]

(b) Write down the mathematical assumption that has been made about the two tasks.

They are independent ..... **[1]**

(c) The probability of the student passing just one of these two tasks is 0.528.

Work out the value of x.

$P(PP') + P(P'P)$

$0.6(1-x) + 0.4x = 0.528$

$0.6 - 0.6x + 0.4x = 0.528$

$0.6 - 0.2x = 0.528$

$-0.6 \qquad -0.6$

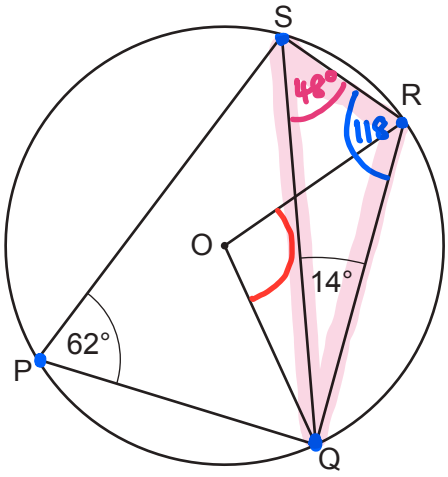
$-0.2x = -0.072$

$\div -0.2 \qquad \div -0.2$

$x = 0.36$

(c) $x =$ 0.36 ..... **[4]**

12 **(a)** P, Q, R and S are points on the circumference of a circle, centre O.
Angle SQR = 14° and angle SPQ = 62°.



Not to scale

Find the size of angle ROQ.

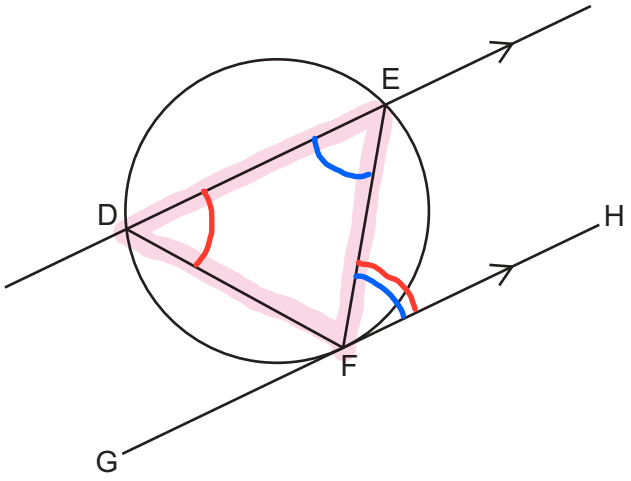
$\angle SRQ = 180 - 62$
$= 118°$

$\angle RSQ = 180 - 118 - 14$
$= 48°$

$\angle ROQ = 2 \times \angle RSQ$
$= 2 \times 48°$
$= 96°$

(a) Angle ROQ = 96 ° **[3]**

(b) D, E and F are points on the circumference of a circle.



Not to scale

Line GH is a tangent to the circle at F.
Line DE is parallel to line GH.

Complete these statements to prove that triangle DEF is isosceles.
Give reasons for your statements.
You may not need all of the lines.

Angle DEF = Angle EFH because alternate angles are equal

Angle FDE = Angle EFH because of the alternate segment theorem

Angle DEF = Angle FDE because they are both equal to angle EFH

Angle ..... = Angle ..... because .....

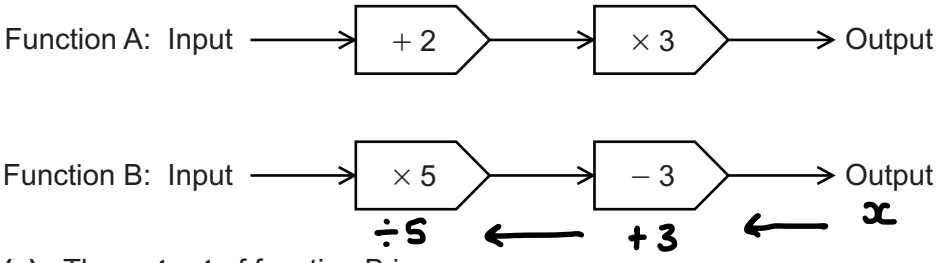
.....

Angle ..... = Angle ..... because .....

.....

Triangle DEF is isosceles because two angles are equal. **[3]**

13 Function A and function B are shown below.



(a) The **output** of function B is x.

Write an algebraic expression, in terms of x, for the inverse of function B.

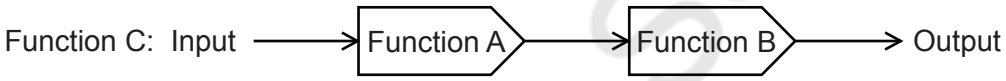
$(x + 3) \div 5$

$\frac{x+3}{5}$

$\frac{x+3}{5}$

(a) ..... **[2]**

(b) Function C is shown below as a composite function.



Complete the diagram below using two arithmetic operations to show function C as a single function.

Function A

$(x + 2) \times 3 = 3(x+2)$

Function B

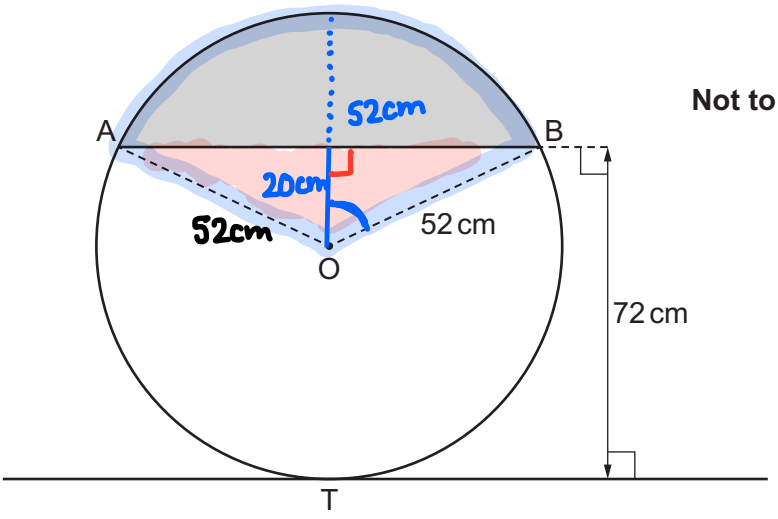
$3(x+2) = 3x + 6$

$5(3x + 6) = 15x + 30$

$15x + 30 - 3 = 15x + 27$

[4]

14 The diagram shows a circle, centre O and radius 52 cm.

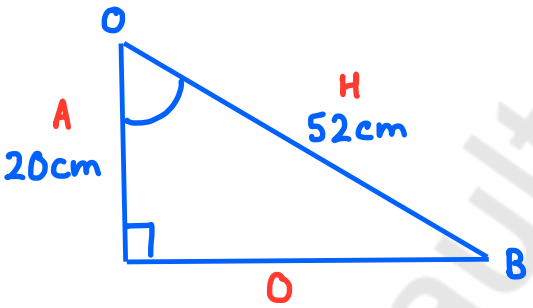


Not to scale

AB is a chord of the circle.
The line through T is a tangent to the circle.

The chord is parallel to the tangent.
The perpendicular distance between the chord and the tangent is 72 cm.

Calculate the area of the shaded segment.
You must show your working.



SOH CAH TOA

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{20}{52}$

$\theta = \cos^{-1}\left(\frac{20}{52}\right)$

$= 67.38013505$

$2 \times 67.38\ldots = 134.7602701^\circ$ [Angle of Sector]

Area sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{134.76\ldots}{360} \times \pi(52)^2$

$= 3179.91808\ \text{cm}^2$

Area triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 52 \times 52 \times \sin(134.76\ldots)$

$= 960\ \text{cm}^2$

Shaded area $= 3179.9\ldots - 960 = 2219.91808\ \text{cm}^2$

2219.9 ..... cm^2 **[6]**

15 **(a)** Solve by factorisation.

$3x^2 + 10x - 8 = 0$

$3 \times -8 = -24$

$12 \quad -2$

$3x^2 + 12x - 2x - 8$

$3x(x+4) \quad -2(x+4)$

$(3x-2)(x+4) = 0$

$3x - 2 = 0 \qquad x + 4 = 0$

$x = 2/3 \qquad x = -4$

(a) $x =$ $\frac{2}{3}$ or $x =$ -4 **[3]**

(b) Write $x^2 + 8x + 11$ in the form $(x+a)^2 - b$.

$(x+4)^2 - 4^2 + 11$

$(x+4)^2 - 16 + 11$

$(x+4)^2 - 5$

(b) $(x+4)^2 - 5$ **[3]**

(c) **(i)** Write down the coordinates of the turning point of the graph $y = (x-3)^2 + 8$.

$x - 3 = 0$

$+3 \quad +3$

$x = 3$

(c)(i) (3 , 8) **[2]**

(ii) Describe the **single** transformation which maps the graph of $y = x^2$ onto the graph of $y = (x-3)^2 + 8$. ← 8 up

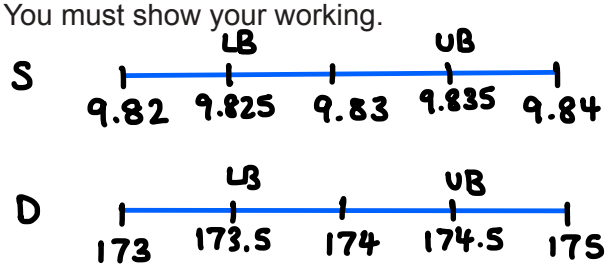
↑ 3 right

(ii) Translation $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ **[2]**

16 A lift can travel at a maximum speed of 9.83 m/s, correct to 3 significant figures.
The lift travels a distance of 174 m, correct to the nearest metre, between the ground floor and the top floor of a building.

(a) Use the above information to work out the shortest possible time for the lift to travel between the ground floor and the top floor.
You must show your working.

D
S T



$T = \frac{D}{S}$

$T_{LB} = \frac{D_{LB}}{S_{UB}}$

$= \frac{173.5}{9.835}$

$= 17.64107778$

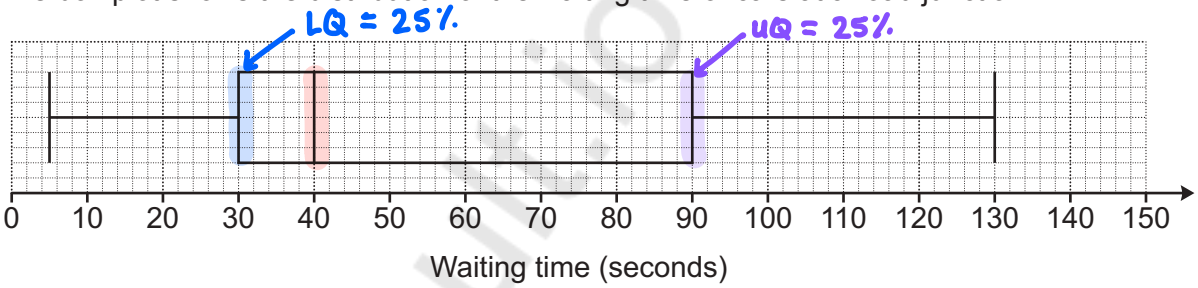
(a) ..... 17.6 ..... s **[4]**

(b) Explain why your answer to **part (a)** may not be possible to achieve.

..... The lift speeds up and slows down. .....

..... **[1]**

17 The box plot shows the distribution of the waiting time of cars at a road junction.



(a) Write down the median waiting time.

(a) ..... 40 ..... s **[1]**

(b) What percentage of the waiting times were less than 30 seconds?

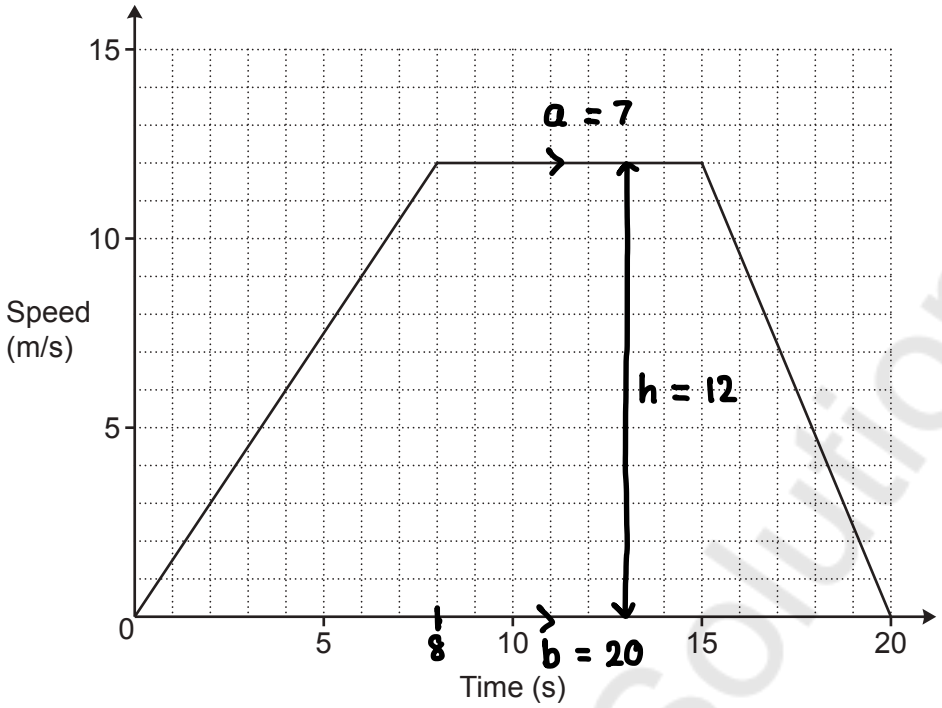
(b) ..... 25 ..... % **[1]**

(c) Given that a randomly chosen car waited for more than 30 seconds, write down the probability that the car waited for more than 90 seconds.

$\frac{25}{75} = \frac{1}{3}$

(c) ..... $\frac{1}{3}$ ..... **[1]**

18 **(a)** The graph shows the speed of an object during the first 20 seconds of its motion.



Calculate the distance travelled by the object during the 20 seconds.

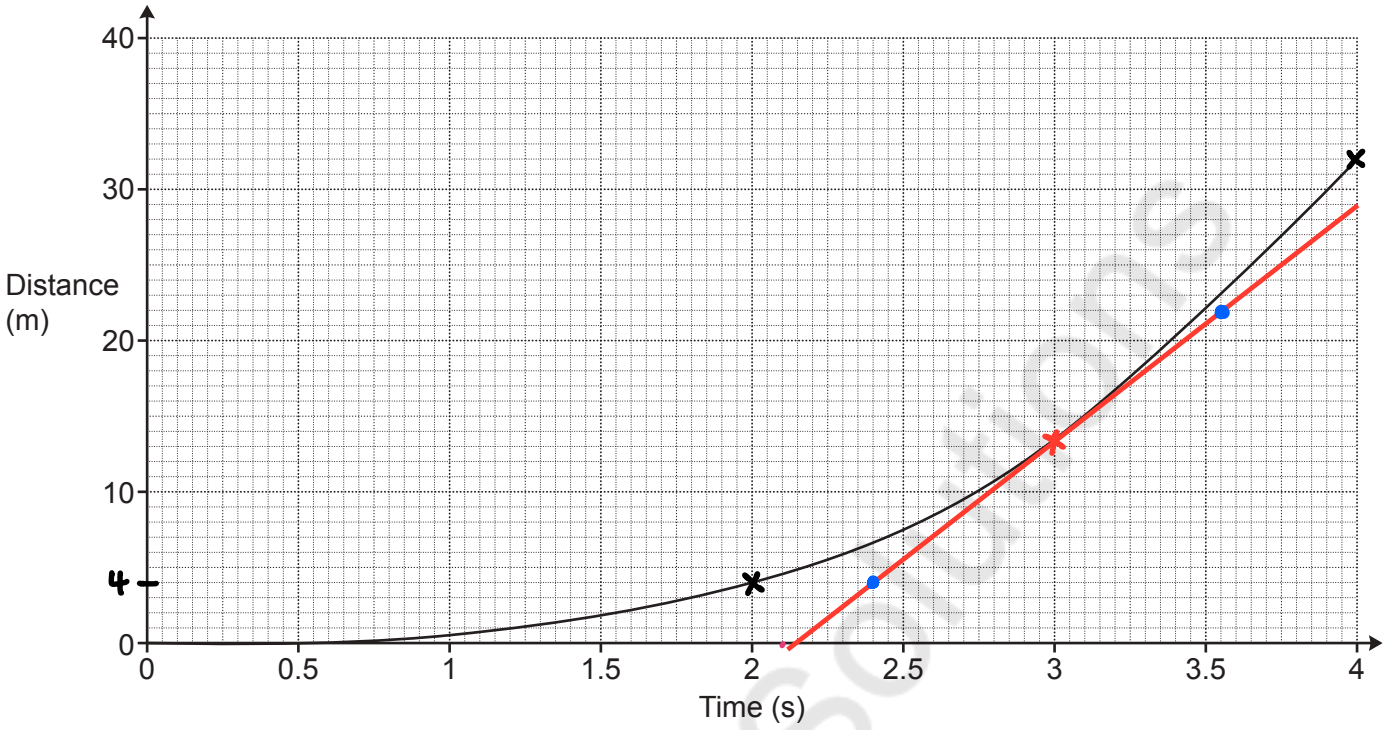
Area trapezium $= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(7+20) \times 12$

$= 162$ m

(a) ..... 162 ..... m **[3]**

(b) The graph shows the distance travelled by an object during the first 4 seconds of its motion.



(i) Work out the average speed of this object between 2 and 4 seconds.

(2, 4) (4, 32)
x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{32 - 4}{4 - 2} = \frac{28}{2} = 14$$

(b)(i) 14 m/s **[2]**

(ii) Use the graph to estimate the speed of this object at 3 seconds.
You must show working to support your estimate.

(2.4, 4) (3.55, 22)
x_1 y_1 x_2 y_2

$$m = \frac{22 - 4}{3.55 - 2.4} = \frac{18}{1.15} = 15.6521...$$

(ii) 15.6 m/s **[3]**

(iii) What happens to the speed of this object during these 4 seconds of motion.
Explain how you know.

The speed increase

I know this because the gradient is steeper.

..... **[1]**

Turn over for question 19

19 Show that $\dfrac{\sqrt{3}+2}{\sqrt{48}-6}$ can be written in the form $\dfrac{a+b\sqrt{3}}{6}$.

You must show each step in your working. **[5]**

$\sqrt{48} = \sqrt{16}\sqrt{3}$

$= 4\sqrt{3}$

$$\frac{\sqrt{3}+2}{4\sqrt{3}-6} \times \frac{4\sqrt{3}+6}{4\sqrt{3}+6}$$

$$= \frac{(\sqrt{3}+2)(4\sqrt{3}+6)}{(4\sqrt{3}-6)(4\sqrt{3}+6)}$$

$\sqrt{3} \times 4\sqrt{3} = 4 \times 3 = 12$

$4\sqrt{3} \times 4\sqrt{3} = 16 \times 3 = 48$

$$= \frac{12 + 6\sqrt{3} + 8\sqrt{3} + 12}{48 + 24\sqrt{3} - 24\sqrt{3} - 36}$$

$$= \frac{24 + 14\sqrt{3}}{12} \quad \begin{matrix} \div 2 \\ \div 2 \end{matrix}$$

$$= \frac{12 + 7\sqrt{3}}{6}$$

END OF QUESTION PAPER