

Monday 3 June 2024 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- geometrical instruments
- tracing paper

Do not use:

- a calculator



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.



1 Work out.

$$1.2 \div 0.03$$

$$\times 100 \quad \times 100$$

$$120 \div 3 = 40$$

..... **40** [2]

2 Kai has these four number cards.



Kai takes two of the cards at random without replacement and finds the positive difference between the two numbers.

(a) Complete the table to show all of the possible differences.

		First card			
		0	2	5	9
Second card	0		2	5	9
	2	2		3	7
	5	5	3		4
	9	9	7	4	

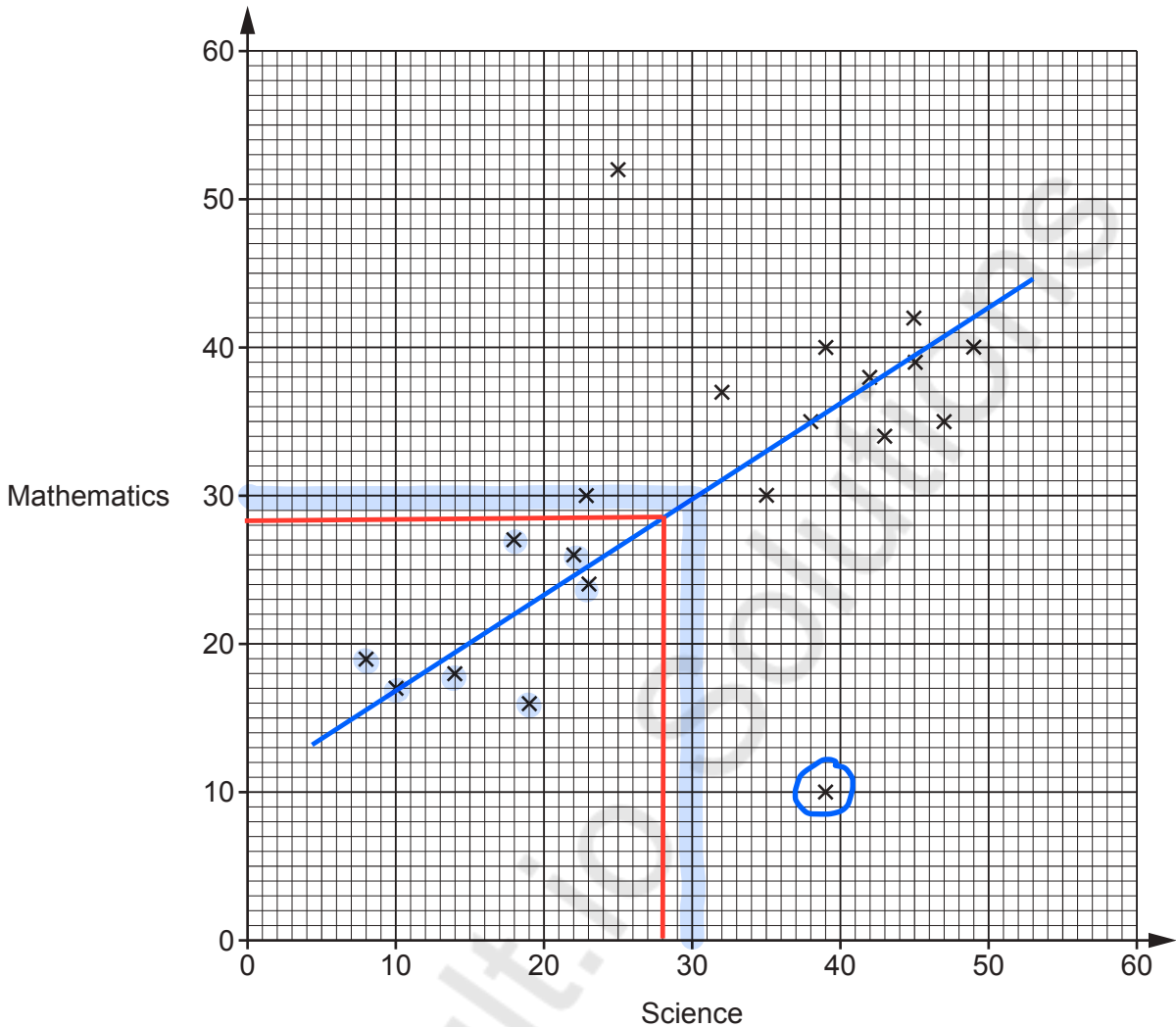
[2]

(b) Find the probability that Kai takes two cards with a difference that is an even number or a factor of 10.

1, 2, 5, 10

(b) $\frac{6}{12}$ [2]

4 The scatter diagram shows the test scores for 20 pupils in Science and Mathematics.



(a) Describe the type of correlation shown in the scatter diagram.

(a) **Positive** [1]

(b) One pupil took the Science test but was then ill during the Mathematics test and had to leave early.

On the scatter diagram, circle the point that is most likely to represent this pupil. [1]

(c) By drawing a line of best fit, estimate the test score in Mathematics for a pupil who scored 28 in the Science test.

(c) **28** [2]

(d) Explain why using the scatter diagram to estimate the test score in Mathematics for a pupil who scored 60 in Science may be unreliable.

..... **There is not enough data.**

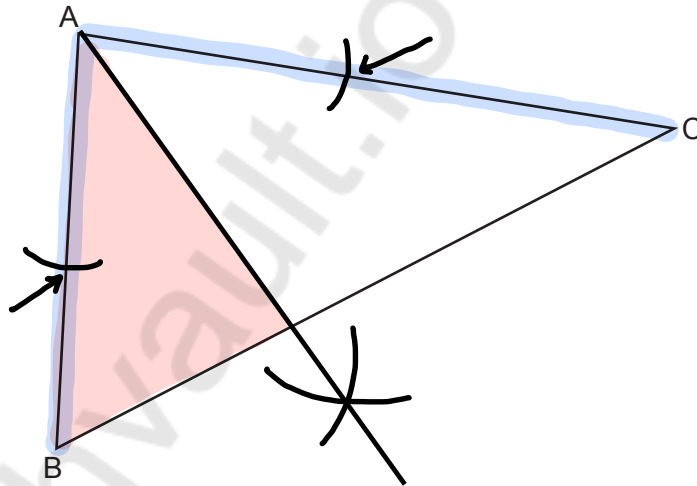
..... [1]

- (e) Find the percentage of the 20 pupils who scored less than 30 marks in **both** Science and Mathematics.

$$\frac{7}{20} \overset{\times 5}{=} \frac{35}{100} = 35\%$$

(e) **35** % [3]

- 5 Triangle ABC is drawn below.



Using a ruler and compasses only, construct and shade the region which is closer to AB than AC. [3]

- 6 A bronze ornament has a volume of 198 cm^3 .
The density of bronze is 8.9 g/cm^3 .

By rounding each value correct to one significant figure, work out an estimate for the mass of the bronze ornament.

$$V = 198 \text{ cm}^3$$

$$\approx 200 \text{ cm}^3$$

$$D = 8.9 \text{ g/cm}^3$$

$$\approx 9 \text{ g/cm}^3$$

$$m = D \times V$$

$$m = D \times V$$

$$= 9 \times 200$$

$$= 1800 \text{ g}$$

..... 1800 g [3]

- 7 A bottle contains $1\frac{3}{4}$ litres of cordial.
To make orange squash, 1 part of this cordial is mixed with 7 parts of water.
Cups that can hold $\frac{1}{6}$ of a litre are completely filled with this orange squash.

Work out the maximum number of cups that can be filled from the bottle of cordial.
You must show your working.

$$C : W$$

$$1 : 7$$

↘
x7

$$1\frac{3}{4} \times 7$$

$$\frac{7}{4} \times 7 = \frac{49}{4}$$

$$1\frac{3}{4} \text{ L cordial}$$

$$\frac{49}{4} \text{ L water}$$

$$\frac{7}{4} + \frac{49}{4} = \frac{56}{4} = 14 \text{ L}$$

Orange
Squash

Number of cups

$$14 \div \frac{1}{6}$$

$$14 \times \frac{6}{1} = \frac{84}{1} = 84 \text{ cups}$$

..... 84 [6]

- 8 (a) y is directly proportional to x .

Write down the percentage increase in y when x is increased by 100%.

(a) **100** % [1]

- (b) z is inversely proportional to x .

$\div 2$

$\times 2$

Write down the percentage decrease in z when x is increased by 100%.

(b) **50** % [1]

- 9 The following kinematics formulas may be used in this question.

$$v = u + at$$

$$v^2 = u^2 + 2as$$

A particle has an initial velocity of 0 m/s. The particle accelerates uniformly at 3 m/s² for 4 seconds.

Find the distance travelled by the particle in the 4 seconds.

$$\begin{aligned} v &= u + at \\ &= 0 + 3 \times 4 \\ &= 12 \text{ m/s} \end{aligned}$$

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

$$\frac{v^2 - u^2}{2a} = s$$

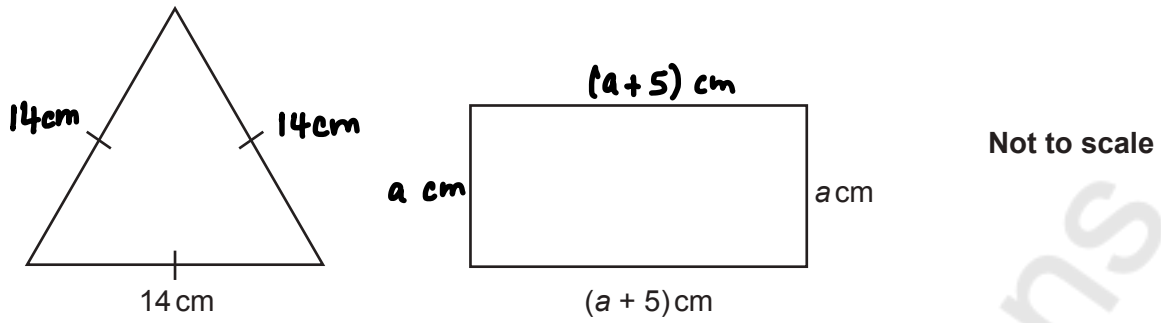
$$\frac{(12)^2 - (0)^2}{2(3)} = s$$

$$\frac{144}{6} = s$$

$$6 \overline{) 144} \begin{array}{r} 024 \\ 144 \\ \hline 000 \end{array}$$

..... **24** m [4]

- 10 The diagram shows an equilateral triangle and a rectangle.



The equilateral triangle has the same perimeter as the rectangle.

Find the value of a .

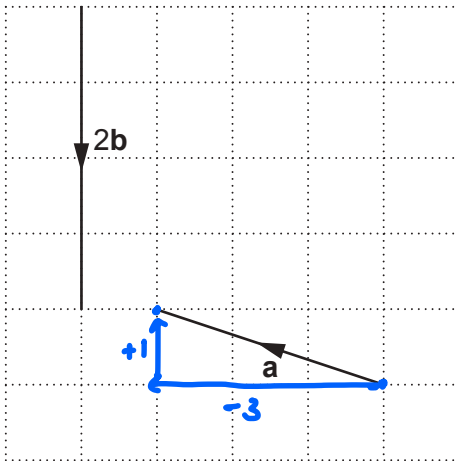
$$\begin{aligned} \text{Perimeter Triangle} &= 14 \times 3 \\ &= 42\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter Rectangle} &= a + a + a + 5 + a + 5 \\ &= 4a + 10 \end{aligned}$$

$$\begin{aligned} 4a + 10 &= 42 \\ -10 \quad -10 & \\ \hline 4a &= 32 \\ \div 4 \quad \quad \div 4 & \\ \hline a &= 8 \end{aligned}$$

$$a = \underline{8} \dots \dots \dots [4]$$

11 Vector \mathbf{a} and vector $2\mathbf{b}$ are drawn on this grid.



$$2\mathbf{b} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

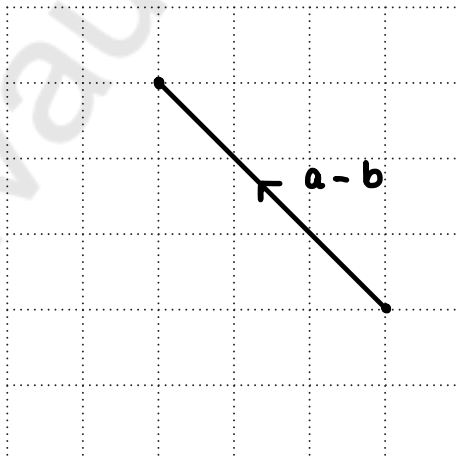
(a) Write vector \mathbf{a} as a column vector.

(a) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ [2]

(b) Write a column vector that is different in direction but has the same length as vector \mathbf{a} .

(b) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ [1]

(c) On the grid below, draw the vector $\mathbf{a} - \mathbf{b}$.



$$\begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

[3]

- 12 (a) Sasha invests £1000 at a rate of 5% per year compound interest.
Sasha says

After one year, my investment will get £50 in interest and will be worth £1050.
Therefore, after two years, my investment will get another £50 in interest and will be worth £1100.

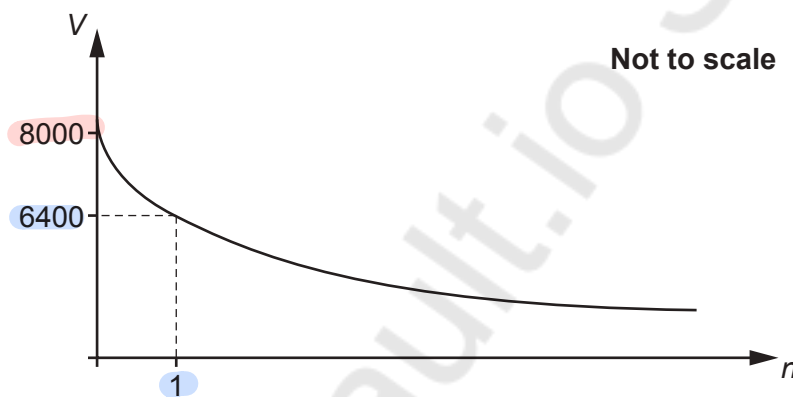
Is Sasha correct?
Give a reason for your answer.

..... **No** because **5% of £1050 is not £50 it is**
..... **£52.50** [1]

- (b) Sasha buys a car.
The value, £ V , of the car after n years is given by the formula

$$V = a \times b^n.$$

The graph shows some information about the value of the car.



Find the value of a and the value of b .

$$8000 = a \times b^0$$

$$8000 = a \times 1$$

$$8000 = a$$

$$6400 = a \times b^1$$

$$6400 = a \times b$$

$$6400 = 8000 \times b$$

$$\frac{6400}{8000} = b$$

$$\frac{8}{10} = b$$

$$(b) \quad a = 8000$$

$$0.8 = b \quad b = 0.8 \quad [4]$$

13 Solve the inequality.

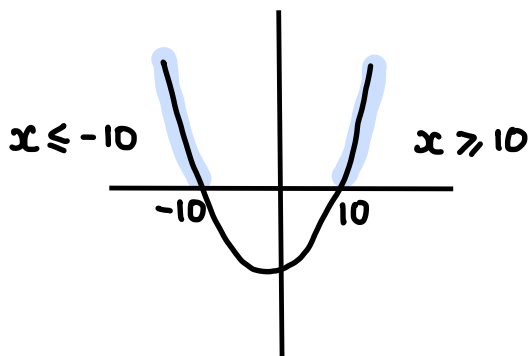
$$x^2 - 100 \geq 0.$$

$$\sqrt{\quad} \sqrt{\quad}$$

$$(x + 10)(x - 10) = 0$$

$$x + 10 = 0 \quad x - 10 = 0$$

$$x = -10 \quad x = 10$$



$$x \leq -10 \quad x \geq 10 \quad \dots [3]$$

14 Here are the first four terms of a sequence.

$$\frac{2}{5} \quad \frac{4}{9} \quad \frac{6}{13} \quad \frac{8}{17}$$

Find the n th term of the sequence.

$$\begin{array}{cccc} 2 & 4 & 6 & 8 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ +2 & +2 & +2 & \end{array} = 2n$$

$$\begin{array}{cccc} 4 & 8 & 12 & 16 \\)+1 &)+1 &)+1 &)+1 \\ 5 & 9 & 13 & 17 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ +4 & +4 & +4 & \end{array} = 4n + 1$$

$$\frac{2n}{4n + 1}$$

$$\dots [3]$$

Turn over

15 Expand and simplify.

$$(x+3)(4x+1)(x-2)$$

x	x	$+3$	
$4x$	$4x^2$	$+12x$	$4x^2 + 13x + 3$
$+1$	$+x$	$+3$	

$$(4x^2 + 13x + 3)(x-2)$$

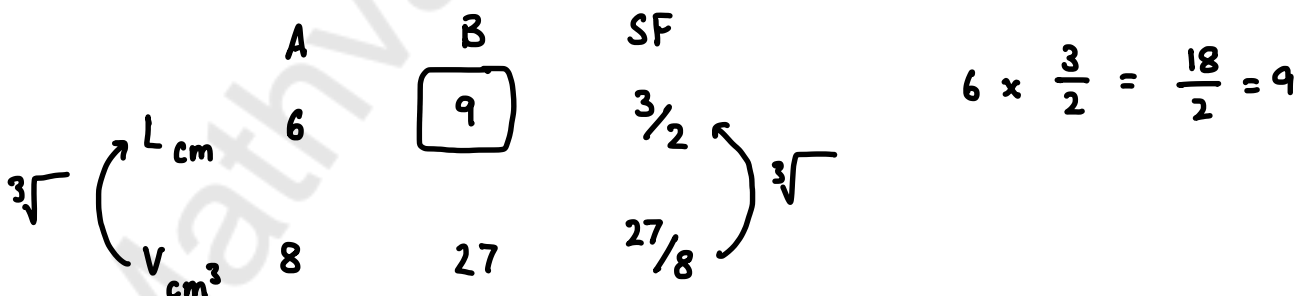
x	$4x^2$	$+13x$	$+3$	
x	$4x^3$	$+13x^2$	$+3x$	$4x^3 + 5x^2 - 23x - 6$
-2	$-8x^2$	$-26x$	-6	$4x^3 + 5x^2 - 23x - 6$ [3]

16 Two prisms, A and B, are mathematically similar.

The ratio of the volume of prism A to the volume of prism B is 8:27.

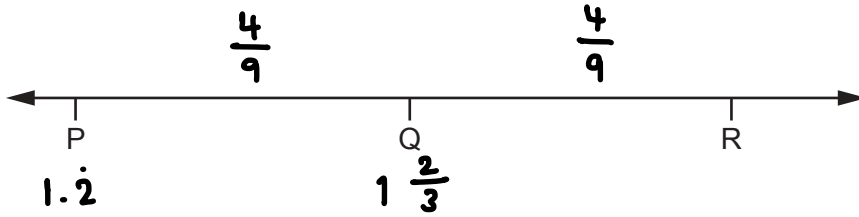
The height of prism A is 6 cm.

Work out the height of prism B.



..... 9 cm [3]

17 The diagram shows a number line.



$$P = 1.2 \text{ and } Q = 1\frac{2}{3}.$$

Q is the midpoint of PR.

Find the value of R.

Give your answer as a mixed number in its simplest form.

You must show your working.

$$x = 1.2$$

$$10x = 12.2$$

$$9x = 11$$

$$x = \frac{11}{9}$$

$$1\frac{2}{3} = \frac{5}{3}$$

$$\frac{5}{3} - \frac{11}{9}$$

$$\frac{15}{9} - \frac{11}{9} = \frac{4}{9}$$

$$R = \frac{5}{3} + \frac{4}{9}$$

$$= \frac{15}{9} + \frac{4}{9}$$

$$= \frac{19}{9}$$

$$= 2\frac{1}{9}$$

$$2\frac{1}{9}$$

[5]

- 18 A sphere has radius x cm.
A cone has radius R cm and height $2R$ cm.

The volume of the sphere is equal to the volume of the cone.

Write R in terms of x .

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Sphere

$$V = \frac{4}{3} \pi r^3 \quad r = x$$

$$V = \frac{4}{3} \pi x^3$$

Cone

$$V = \frac{1}{3} \pi r^2 h \quad r = R \quad h = 2R$$

$$V = \frac{1}{3} \pi (R)^2 (2R)$$

$$V = \frac{1}{3} \pi 2R^3$$

$$\frac{4}{3} \pi x^3 = \frac{1}{3} \pi 2R^3$$

$$4 \pi x^3 = \pi 2R^3$$

$$4x^3 = 2R^3$$

$$2x^3 = R^3$$

$$\sqrt[3]{2x^3} = R$$

$$\dots\dots\dots R = \sqrt[3]{2x^3} \dots\dots\dots [4]$$

19 (a) Describe fully the **single** transformation that is equivalent to:

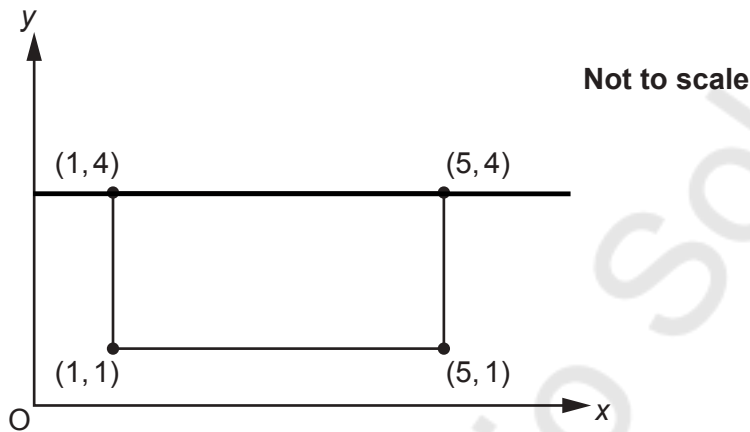
- a rotation of 20° anticlockwise about the origin, followed by
- a rotation of 70° clockwise about the origin.

$$-20 + 70 = 50$$

Rotation 50° clockwise about the origin

[2]

(b) The diagram shows the coordinates of the vertices of a rectangle.



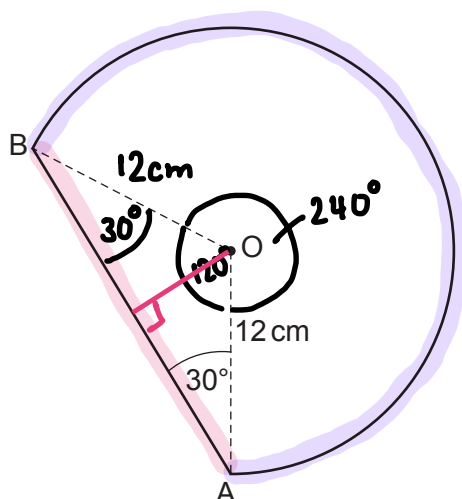
The rectangle is reflected in the line $y = 4$.

Write down the coordinates of the vertices of the rectangle that are invariant.

do not change

(b) $(1, 4)$ and $(5, 4)$ [1]

- 20 The shape below is part of a circle, centre O and radius 12 cm.
Angle OAB = 30°.



Not to scale

Arc length

$$\frac{\theta}{360} \times 2\pi r$$

Work out the perimeter of the shape.

Give your answer in its simplest terms in the form $a\sqrt{b} + k\pi$.

You must show your working.

Arc length

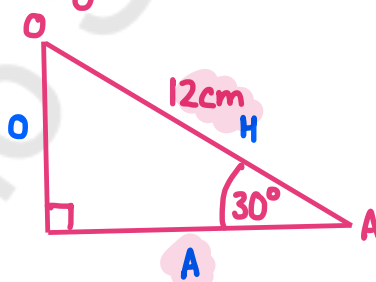
$$\frac{240}{360} \times 2\pi(12)$$

$$\frac{2}{3} \times 24\pi$$

$$\frac{48}{3} \pi$$

$$16\pi \text{ cm}$$

Triangle



$$\cos \theta = \frac{A}{H}$$

$$\cos 30 = \frac{A}{12}$$

$$A = 12 \times \cos 30$$

$$= 12 \times \frac{\sqrt{3}}{2}$$

$$= \frac{12\sqrt{3}}{2}$$

$$= 6\sqrt{3}$$

$$6\sqrt{3} \times 2 = 12\sqrt{3}$$

	0	30	60	90	
Sin	0	1	2	3	4
Cos	4	3	2	1	0
					2

$$\text{Perimeter} = (16\pi + 12\sqrt{3}) \text{ cm}$$

$$\dots\dots\dots 12\sqrt{3} + 16\pi \dots\dots\dots [7]$$

- 21 Work out the coordinates of the intersection of the graphs of $y = 5 - 2x$ and $y = 3x^2$.
You must show your working.

$$\begin{array}{r} 5 - 2x = 3x^2 \\ -5 + 2x \quad +2x - 5 \end{array}$$

$$0 = 3x^2 + 2x - 5$$

$$3x - 5 = \frac{-15}{-3 + 5}$$

$$\begin{array}{l} 3x^2 - 3x + 5x - 5 \\ 3x(x-1) + 5(x-1) \end{array}$$

$$(3x-5)(x-1) = 0$$

$$\begin{array}{l} 3x - 5 = 0 \quad x - 1 = 0 \\ +5 +5 \quad +1 +1 \end{array}$$

$$\begin{array}{l} \div 3 \quad 3x = 5 \quad \div 3 \quad x = 1 \\ x = \frac{5}{3} \end{array}$$

$$y = 3x^2$$

$$\begin{array}{l} y = 3\left(\frac{5}{3}\right)^2 \\ = 3\left(\frac{25}{9}\right) \\ = \frac{75}{9} \\ = \frac{25}{3} \end{array} \quad \begin{array}{l} y = 3(1)^2 \\ = 3(1) \\ = 3 \end{array}$$

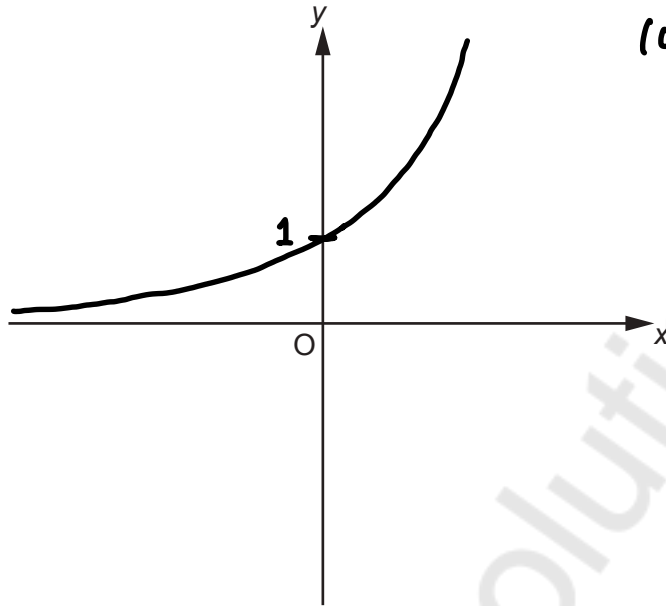
$$\left(\frac{5}{3}, \frac{25}{3}\right) \text{ and } (1, 3) \quad [6]$$

- 22 (a) Sketch the graph of $y = 8^x$.
Indicate any values where the graph crosses the axes.

$$y = 8^0$$

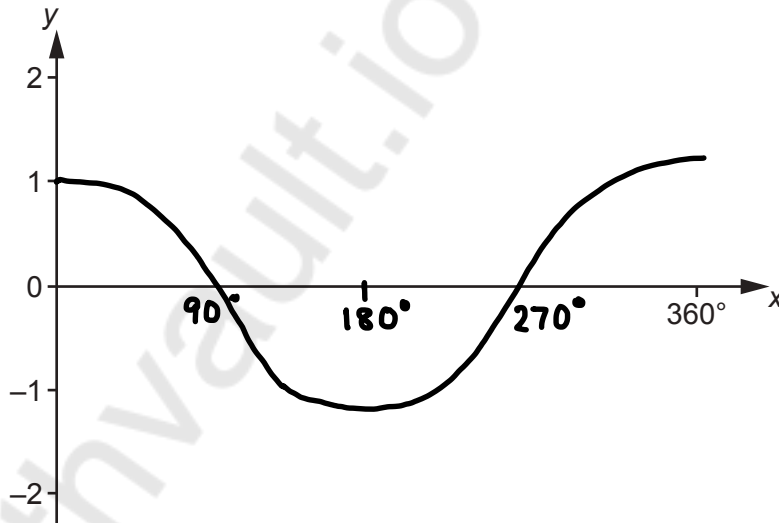
$$= 1$$

$$(0, 1)$$



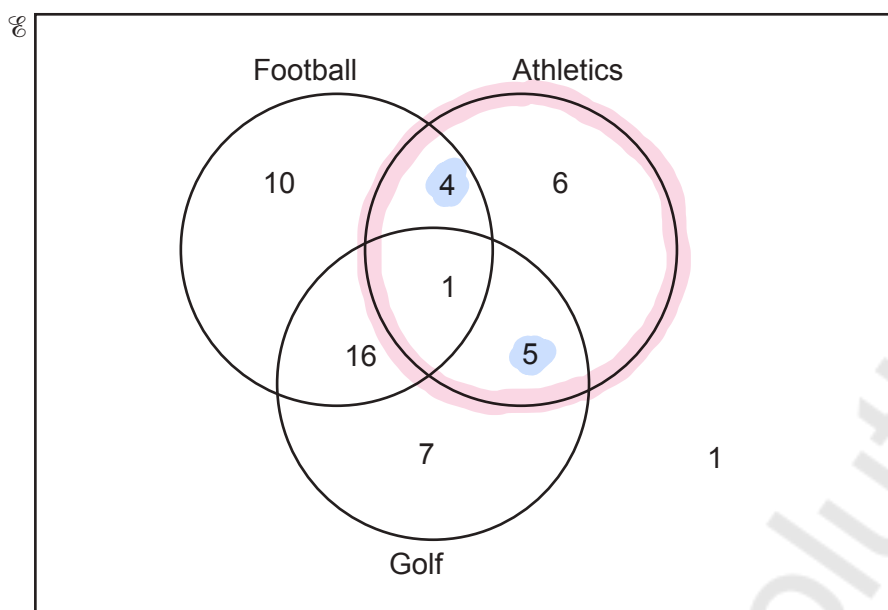
[2]

- (b) Sketch the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$.
Indicate any values where the graph crosses the x-axis.



[2]

- 23 In a survey, 50 people are asked which sports they watch. The results are shown on the Venn diagram.



- (a) One person is chosen at random from those that watch athletics.

Find the probability that this person watches **only one** other sport.

$$\frac{9}{16}$$

(a) $\frac{9}{16}$ [2]

- (b) Two of the 50 people are chosen at random.

Show that the probability that one of them watches **only** football and the other watches **only** golf is $\frac{2}{35}$. [3]

$$p(FG) + p(GF)$$

$$\left(\frac{10}{50} \times \frac{7}{49}\right) + \left(\frac{7}{50} \times \frac{10}{49}\right)$$

$$\frac{70}{2450} + \frac{70}{2450} = \frac{140}{2450} = \frac{14}{245} \div 7 = \frac{2}{35}$$

$$\begin{array}{r} 4 \ 50 \\ 49 \\ \hline 50 \\ 2400 \\ \hline 2450 \end{array}$$

END OF QUESTION PAPER

