

Monday 10 June 2024 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes



You must have:

- the Formulae Sheet for Higher Tier (inside this document)

You can use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION

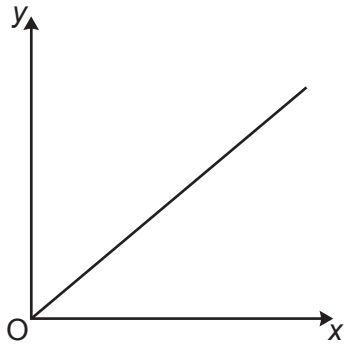
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

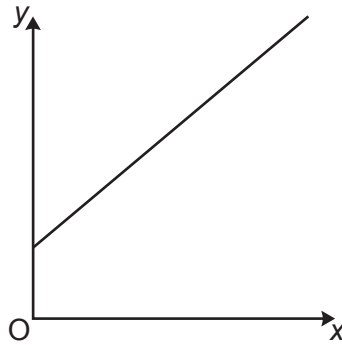
- Read each question carefully before you start your answer.

1 Below are six graphs, numbered 1 to 6, that show different relationships between x and y .

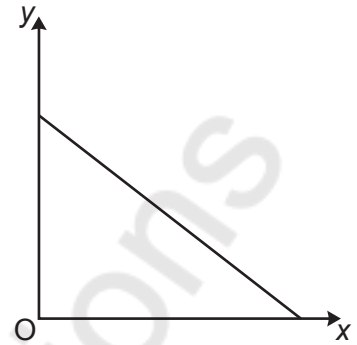
Graph 1



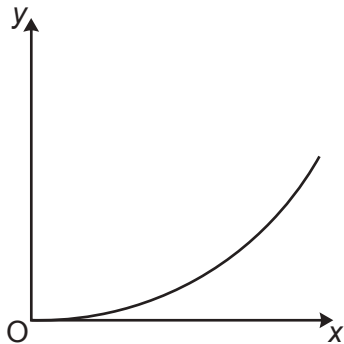
Graph 2



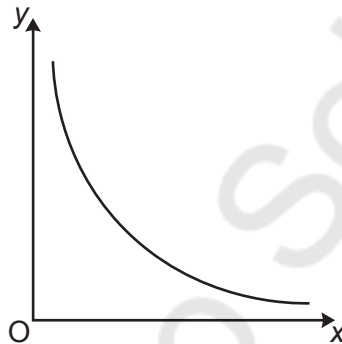
Graph 3



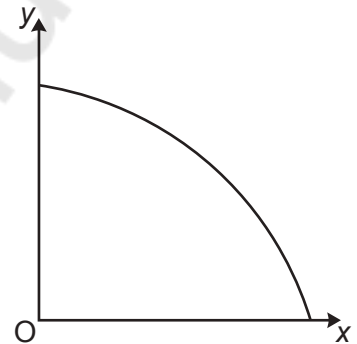
Graph 4



Graph 5



Graph 6



For each description below, write down the number of the graph that best represents the relationship between x and y .

x : temperature in $^{\circ}\text{C}$

y : temperature in $^{\circ}\text{F}$, where $y = 1.8x + 32$ **y-intercept**

Graph **2**

x : average speed when running 200 m

y : time taken to run 200 m. **inverse proportion**

Graph **5**

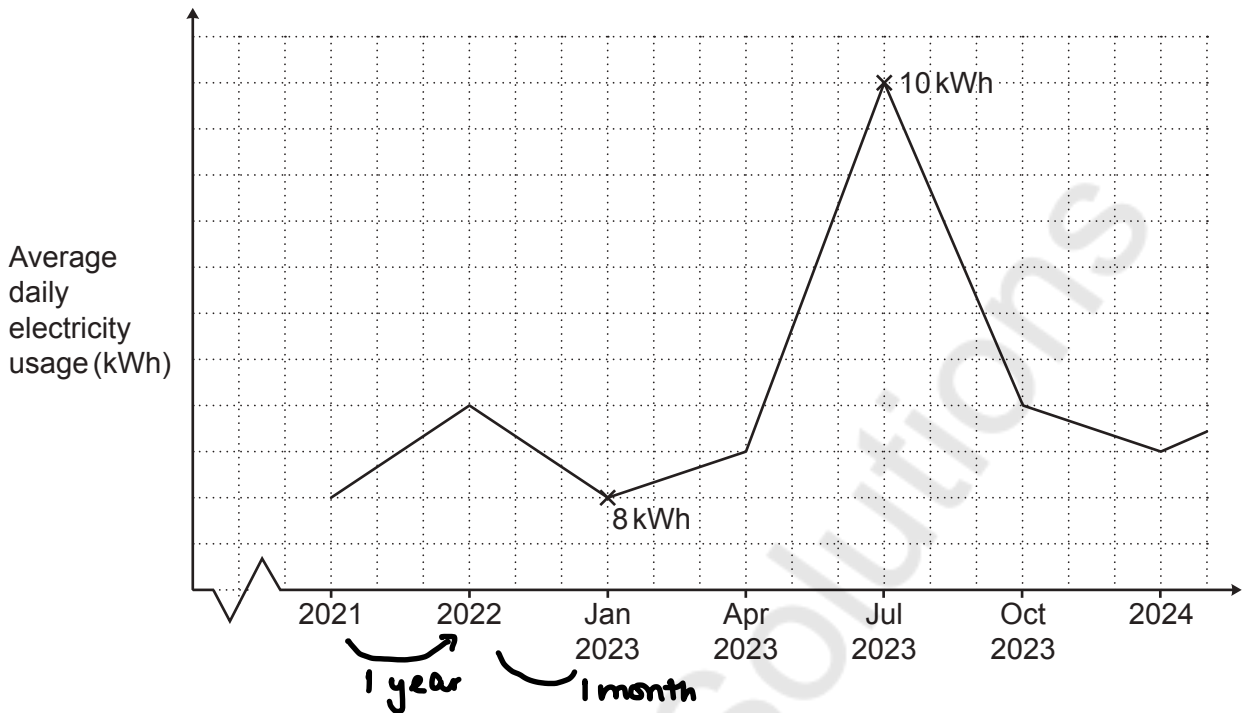
x : mass of a solid object made from clay

y : volume of the same solid object. **direct proportion**

Graph **1**

[3]

- 2 The graph shows a household's average daily electricity usage, in kilowatt hours (kWh).



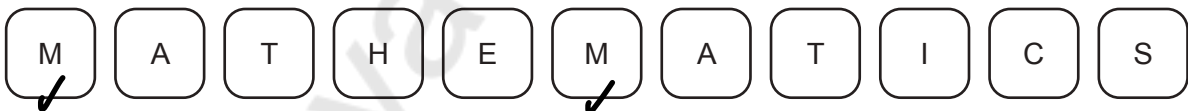
Give **two different** reasons why this graph is misleading.

Reason 1: ...The horizontal scale is not linear.

Reason 2: ...There is no scale on vertical axis.

[2]

- 3 The word MATHEMATICS is spelt using tiles.



The tiles are put into an empty bag.

One at a time, 99 children each take a tile at random from the bag. They record the letter and then put the tile back in the bag.

Work out how many times the letter M is expected to be taken from the bag.

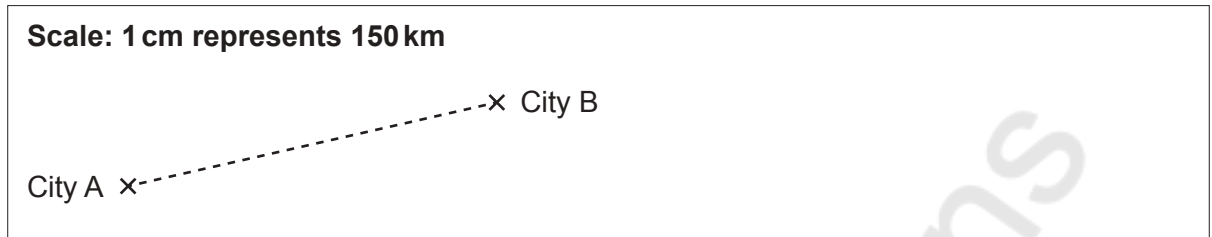
$$p(m) \times 99$$

$$\frac{2}{11} \times 99 = 18$$

18 [3]

4 1 cm on a map represents 150 km in real life.

(a) Below is part of the map showing City A and City B.



Heidi is planning to walk from City A to City B for charity.

Heidi measures the straight-line distance on the map from City A to City B accurately as 5 cm. Heidi says she will walk from City A to City B at an average speed of 2.5 km/h for 10 hours per day.

(i) Use this information to work out how many days Heidi needs to complete the walk from City A to City B.

$$D = S \times T$$

$$T = \frac{D}{S}$$

$$1 \text{ cm} = 150 \text{ km}$$

↘
x150

$$D = 750 \text{ km}$$

$$5 \text{ cm} \times 150 = 750 \text{ km}$$

$$S = 2.5 \text{ km/h}$$

$$T = 300 \text{ hours}$$

$$\begin{aligned} \text{Days} &= 300 \div 10 \\ &= 30 \end{aligned}$$

(a)(i) 30 days [4]

(ii) Explain why the information used in part (i) is likely to give an underestimate for the number of days Heidi needs to complete the walk.

..... The path may not be a straight line. [1]

(b) Yoshi writes the scale of the map, 1 cm represents 150 km, as the ratio 1 : 150.

cm km

Explain Yoshi's error and write the scale correctly in the form 1 : n.

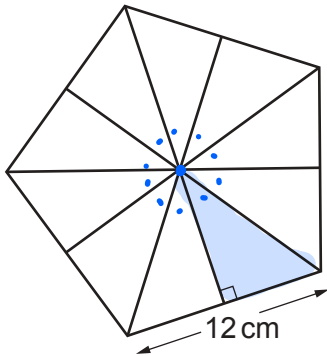
$$150 \text{ km} \times 1000 = 150,000 \text{ m}$$

$$150,000 \text{ m} \times 100 = 15,000,000 \text{ cm}$$

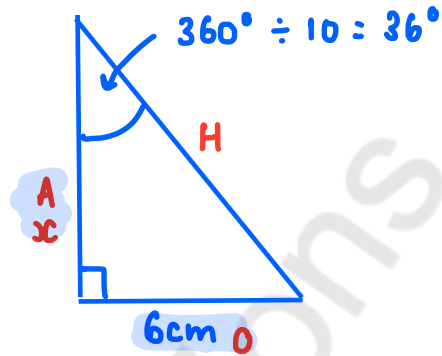
Yoshi's error is the units are different.

The correct answer is 1 : 15 000 000 [2]

- 5 The diagram shows a regular pentagon made using ten congruent right-angled triangles. The length of one side of the pentagon is 12 cm.



Not to scale



- (a) Show that the area of the pentagon is 247.75 cm^2 , correct to 2 decimal places. [6]

$$\text{Area triangle} = \frac{1}{2}bh$$

$$\tan \theta = \frac{O}{A}$$

$$\tan 36 = \frac{6}{x}$$

$$x = 6 \div \tan 36$$

$$= 8.258291523 \text{ [height triangle]}$$

$$\text{Area} = \frac{1}{2} \times 6 \times 8.25\dots$$

$$= 24.77487457 \text{ cm}^2$$

$$\text{Area pentagon} = 10 \times 24.77\dots = 247.748\dots \approx 247.75 \text{ cm}^2 \text{ (2dp)}$$

- (b) The regular pentagon is the base of a pyramid.

The pyramid has volume 450 cm^3 .

The perpendicular height of the pyramid is $h \text{ cm}$.

Calculate the value of h .

[The volume of a pyramid is $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$.]

$$450 = \frac{1}{3} \times 247.75 \times h$$

$$h = \frac{450}{\frac{1}{3} \times 247.75}$$

$$= 5.44906896$$

$$\approx 5.45 \text{ (3sf)}$$

(b) $h = \dots 5.45 \dots$ [3]

- 6 (a) Two numbers, A and B , are written as the product of their prime factors.

$$A = 2 \times 3 \times 7^2 \quad \mathbf{2 \times 3 \times 7 \times 7}$$

$$B = 2^3 \times 7 \quad \mathbf{2 \times 2 \times 2 \times 7}$$

Find the lowest common multiple (LCM) of A and B .
Give your answer as an ordinary number.

$$\mathbf{HCF = 2 \times 7 = 14}$$

$$\mathbf{LCM = 14 \times 3 \times 7 \times 2 \times 2}$$

$$\mathbf{= 1176}$$

(a) **1176** [2]

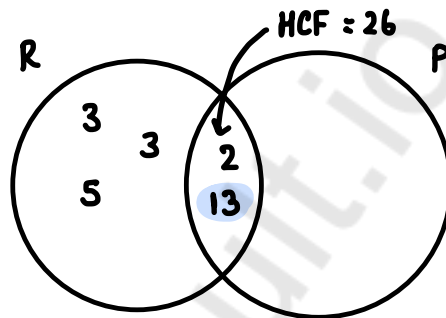
- (b) A number, R , is written as the product of its prime factors.

$$R = \cancel{7} \times 3^2 \times 5 \times k, \text{ where } k \text{ is a prime number.}$$

The highest common factor (HCF) of R and another number, P , is 26.

Find the value of k .

$$\begin{array}{c} \diagup \quad \diagdown \\ 2 \quad 13 \end{array}$$



(b) $k =$ **13** [2]

- 7 Two bags of fruit contain only apples and bananas.
In bag X, the ratio of apples to bananas is 5 : 7.
In bag Y, $\frac{5}{12}$ of the fruit are apples.

$$\text{Bag Y} \quad \begin{array}{l} A : B \\ 5 : 7 \end{array}$$

- (a) Finley says

Bag X and bag Y contain the same number of apples.

Tick the correct statement.

- Finley is definitely correct
- Finley might be correct, or might not be correct
- Finley is definitely not correct

Show how you decided.

$$24 \div 12 = 2$$

$$2 \times 5 = 10 \text{ apples}$$

$$2 \times 7 = 14 \text{ bananas}$$

$$\div 2 \left(\begin{array}{l} 10 : 14 \\ 5 : 7 \end{array} \right) \div 2$$

There could be any multiple of 12 fruits in the bags.

[3]

- (b) Finley adds 4 apples to bag X.
The ratio of apples to bananas is now 11 : 14.

How many bananas are in bag X?

$$\text{Bag X} \quad A : B = 5 : 7$$

$$\begin{array}{l} A : B \\ 5x+4 : 7x \end{array} = \begin{array}{l} A : B \\ 11 : 14 \end{array}$$

$$\frac{5x+4}{7x} = \frac{11}{14}$$

$$7x = 7(8) = 56$$

$$14(5x+4) = 11(7x)$$

$$\begin{array}{r} 70x + 56 = 77x \\ -70x \quad \quad -70x \end{array}$$

$$\begin{array}{r} 56 = 7x \\ \div 7 \quad \quad \div 7 \\ 8 = x \end{array}$$

(b) 56 [3]

- 8 (a) Complete this table for $y = x^3 - 3x^2$.

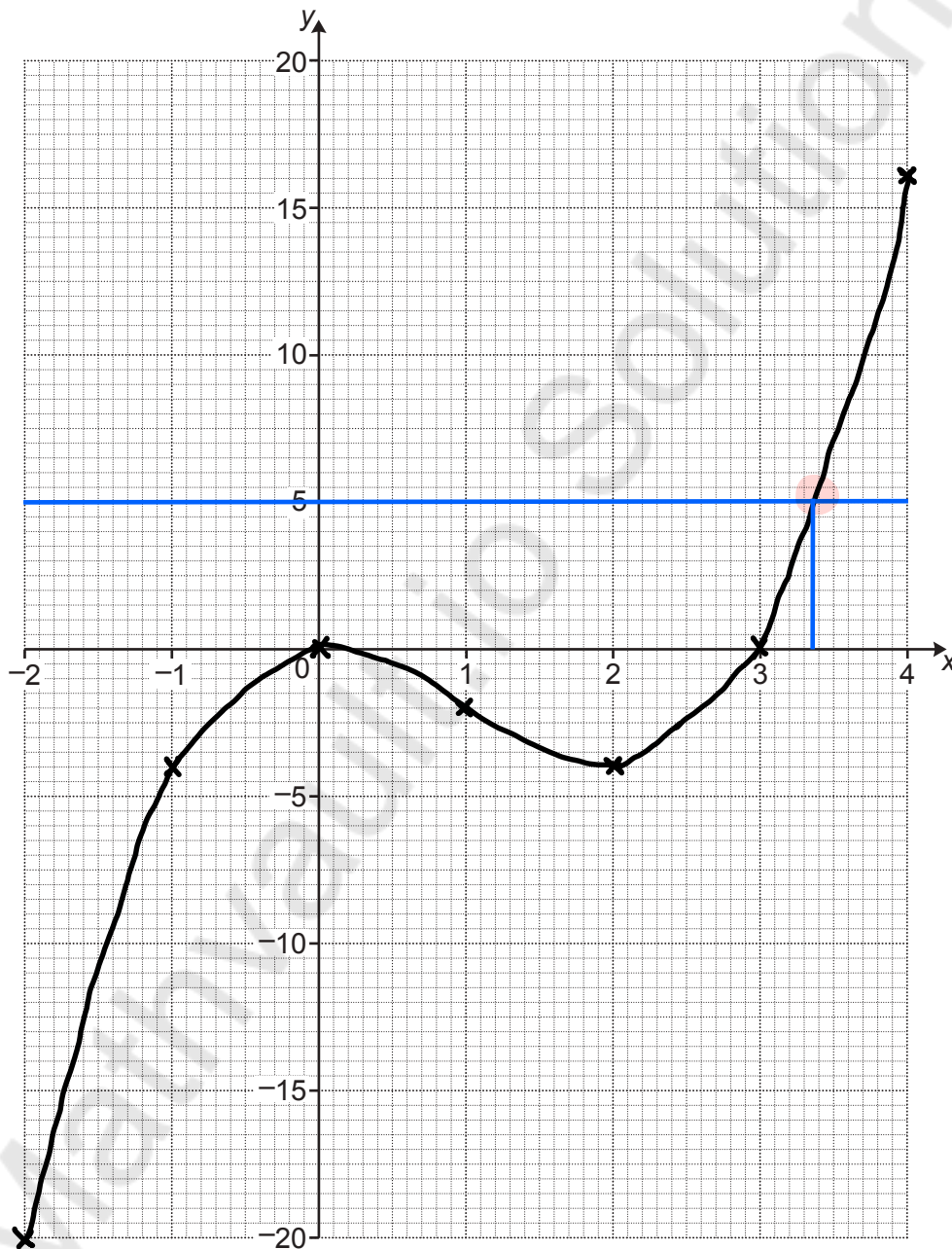
x	-2	-1	0	1	2	3	4
y	-20	-4	0	-2	-4	0	16

$$\swarrow y = (-1)^3 - 3(-1)^2$$

$$\swarrow y = (3)^3 - 3(3)^2$$

[2]

- (b) Draw the graph of $y = x^3 - 3x^2$ for values of x from -2 to 4.



[3]

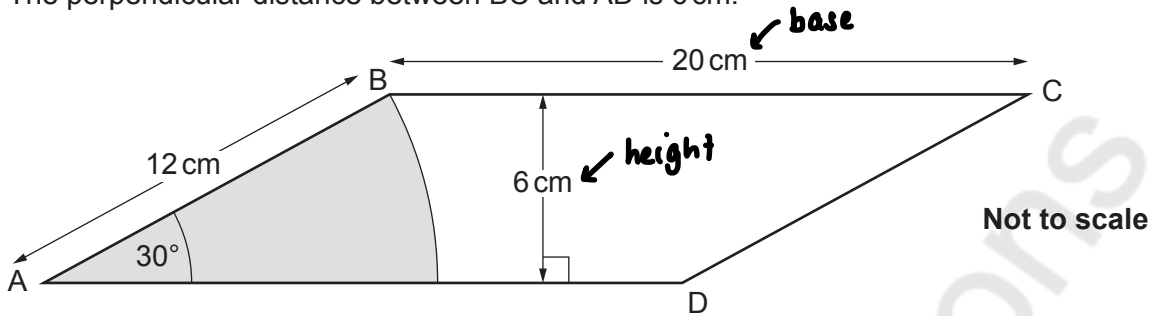
- (c) Use the graph to solve the equation $x^3 - 3x^2 = 5$.
Give your answer to 1 decimal place.

$$3.35 \approx 3.4 \text{ (1dp)}$$

Draw $y = 5$

(c) 3.4 [1]

- 9 The diagram shows a shaded sector inside a parallelogram. The sector has an angle of 30° . The parallelogram, ABCD, has length $BC = 20$ cm and $AB = 12$ cm. The perpendicular distance between BC and AD is 6 cm.



- (a) Show that the area of the sector is 37.7 cm^2 , correct to 3 significant figures. [3]

$$\begin{aligned}
 \text{Area sector} &= \frac{\theta}{360} \times \pi r^2 \quad r = 12 \\
 &= \frac{30}{360} \times \pi (12)^2 \\
 &= 12\pi \\
 &= 37.69911184 \\
 &\approx 37.7 \text{ cm}^2 \quad (3\text{sf})
 \end{aligned}$$

- (b) Work out the percentage of the parallelogram that is not shaded.

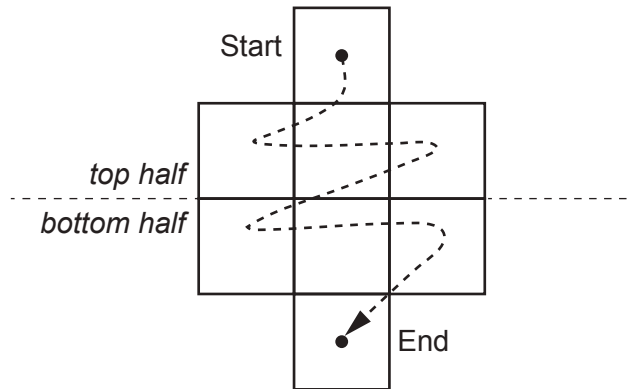
$$\begin{aligned}
 \text{Area parallelogram} &= b \times h \\
 &= 20 \times 6 \\
 &= 120 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area not shaded} &= 120 - 37.699\dots \\
 &= 82.30088816 \text{ cm}^2
 \end{aligned}$$

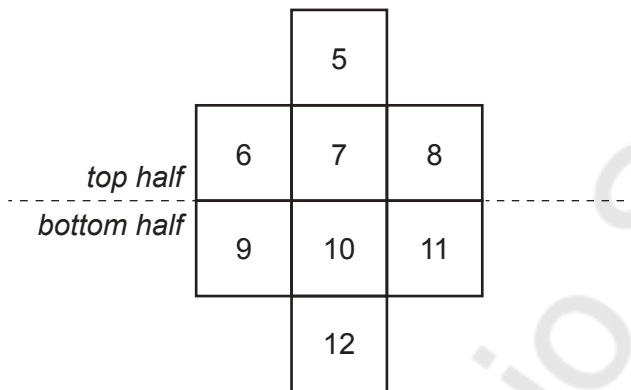
$$\begin{aligned}
 \frac{82.30\dots}{120} \times 100 &= 68.58407346 \\
 &\approx 68.6 \% \quad (3\text{sf})
 \end{aligned}$$

(b) **68.6** % [4]

- 10 Eight consecutive numbers are written in ascending order in this grid, starting from the top and working left to right.



- (a) Kareem writes the numbers 5 to 12 in the grid.



Show that for Kareem's grid, the sum of the numbers in the top half of the grid is 16 less than the sum of the numbers in the bottom half of the grid. [1]

$$5 + 6 + 7 + 8 = 26 \quad [\text{Top half}]$$

$$9 + 10 + 11 + 12 = 42 \quad [\text{bottom half}]$$

$$42 - 26 = 16$$

- (b) Use algebra to prove that for **any** set of eight consecutive numbers written in this grid in the same way, the sum of the numbers in the top half of the grid is 16 less than the sum of the numbers in the bottom half of the grid. [5]

[Top half]

$$n + n + 1 + n + 2 + n + 3 = 4n + 6$$

[Bottom half]

$$n + 4 + n + 5 + n + 6 + n + 7 = 4n + 22$$

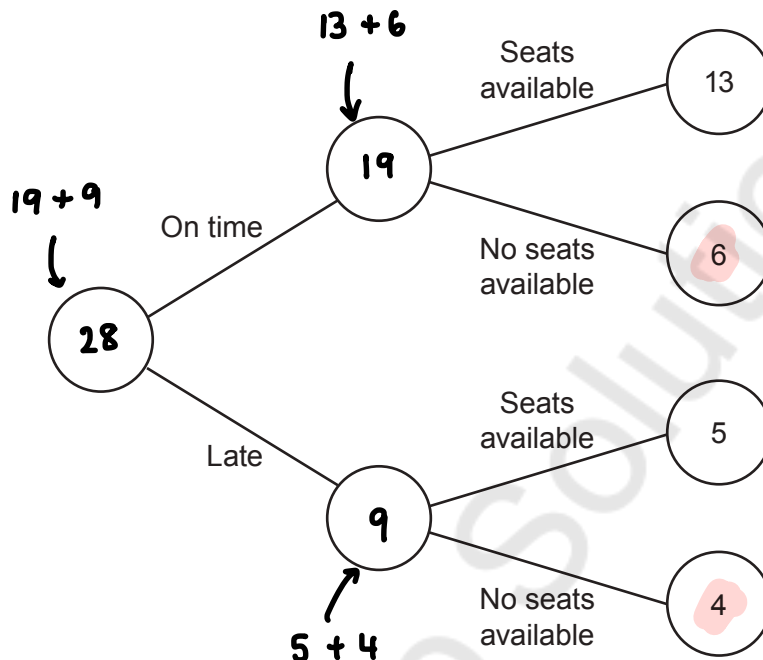
$$(4n + 22) - (4n + 6) = 16$$

11 Jack travels to work each day by train.

He records whether

- the train is on time or late
- there are seats available or no seats available.

Jack's results are shown on this partly completed frequency tree.



(a) Find the relative frequency of there being no seats available on Jack's train journey.

$$\frac{6 + 4}{28} = \frac{10}{28}$$

(a) $\frac{10}{28}$ [2]

(b) Jack says

If the train is late, travellers are less likely to find seats available than if the train was on time.

Does Jack's data suggest he is correct? Show how you decide.

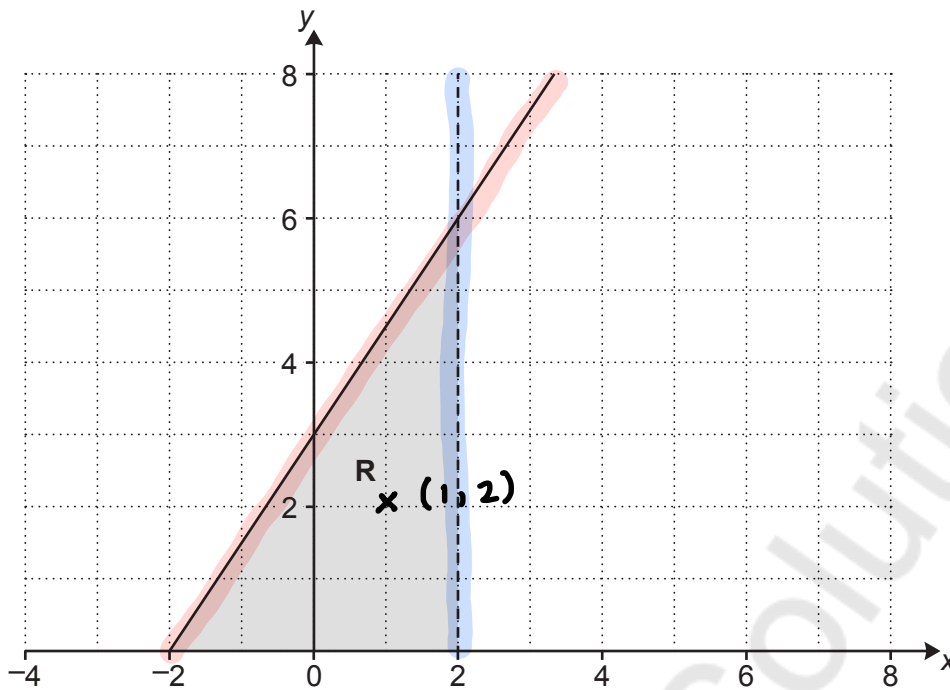
$$\text{Late, available} = \frac{5}{9} = 0.\dot{5}$$

$$\text{On time, available} = \frac{13}{19} = 0.684\dots$$

Yes because 0.5 is less than 0.684...

.....
 [3]

12 The region **R** is shown on this grid.



The region **R** is defined by three inequalities.
The first inequality is given below.

Complete the second inequality and write down the third inequality needed to define region **R**.

$$y \geq 0$$

$$2y \dots \leq 3x + 6$$

\leq \geq

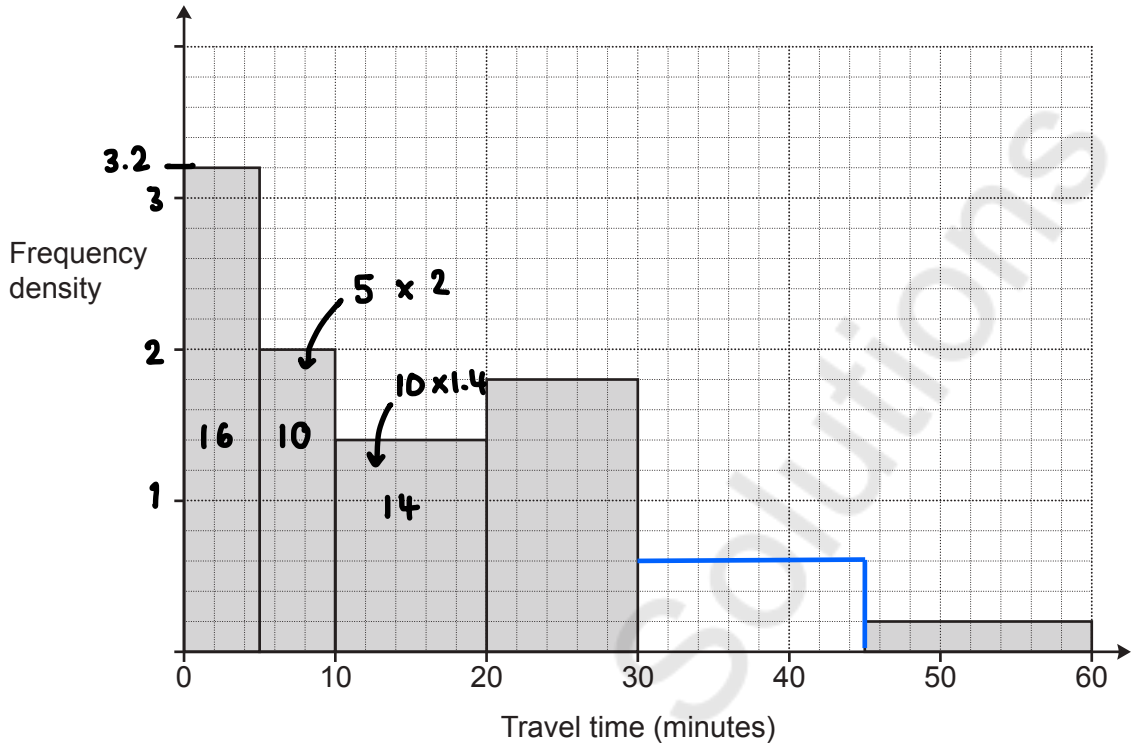
$$\frac{2(2)}{4} < \frac{3(1) + 6}{9}$$

$$x < 2$$

$<$ $>$

[3]

- 13 A group of students record the time taken to travel to school. All students in the group took less than an hour to travel to school. Some of their results are recorded on this histogram.



16 students took less than 5 minutes to travel to school.

- (a) How many students took less than 20 minutes to travel to school?

$$\begin{aligned}
 \text{FD} &= \frac{F}{\text{CW}} \\
 &= \frac{16}{5} \\
 &= 3.2
 \end{aligned}
 \qquad
 16 + 10 + 14 = 40$$

(a) **40** students [4]

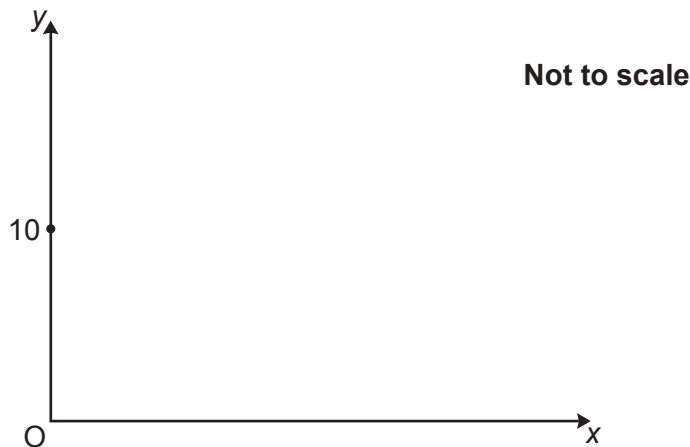
- (b) 9 students took between 30 and 45 minutes to travel to school.

Add these students' results to the histogram.

[2]

$$\begin{aligned}
 \text{FD} &= \frac{F}{\text{CW}} \\
 &= \frac{9}{15} \\
 &= 0.6
 \end{aligned}$$

- 14 The sketch shows the coordinate axes and the point (0, 10).



The distance from the point (0, 10) to a point (12, t) is 12.5 units.

x_1 y_1 x_2 y_2

Work out the **two** possible values of t .

You must show your working and you may use the sketch to help.

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$12.5 = \sqrt{(12 - 0)^2 + (t - 10)^2}$$

$$156.25 = 12^2 + (t - 10)^2$$

$$156.25 = 144 + (t - 10)^2$$

$$-144 \quad -144$$

$$12.25 = (t - 10)^2$$

$$12.25 = (t - 10)(t - 10)$$

$$12.25 = t^2 - 10t - 10t + 100$$

$$12.25 = t^2 - 20t + 100$$

$$-12.25 \quad -12.25$$

$$0 = t^2 - 20t + 87.75$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{array}{l} a = 1 \\ b = -20 \\ c = 87.75 \end{array}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(87.75)}}{2(1)}$$

$$= 13.5 \quad \text{and} \quad 6.5$$

$$t = \dots 13.5 \dots \text{and} \dots 6.5 \dots [5]$$

- 15 y is inversely proportional to x^3 .
 $y = 4$ when $x = 12$.

Find a formula linking x and y .

$$y \propto \frac{1}{x^3}$$

$$y = \frac{k}{x^3}$$

$$4 = \frac{k}{(12)^3}$$

$$4 = \frac{k}{1728}$$

$$k = 6912$$

$$y = \frac{6912}{x^3}$$

$$y = \frac{6912}{x^3}$$

[3]

- 16 Write $\frac{9x^7 \times 2\sqrt{x}}{3x^4}$ in the form kx^m .

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{9x^7 \times 2x^{\frac{1}{2}}}{3x^4} = \frac{18x^{\frac{15}{2}}}{3x^4} = 6x^{\frac{7}{2}}$$

$$6x^{\frac{7}{2}}$$

[4]

- 17 (a) Without using a calculator, show that $0.\dot{1}\dot{8}$ can be written as $\frac{2}{11}$. [3]

$$x = 0.\dot{1}\dot{8}$$

$$10x = 1.\dot{8}\dot{1}$$

$$100x = 18.\dot{1}\dot{8}$$

$$100x - x = 99x$$

$$18.\dot{1}\dot{8} - 0.\dot{1}\dot{8} = 18$$

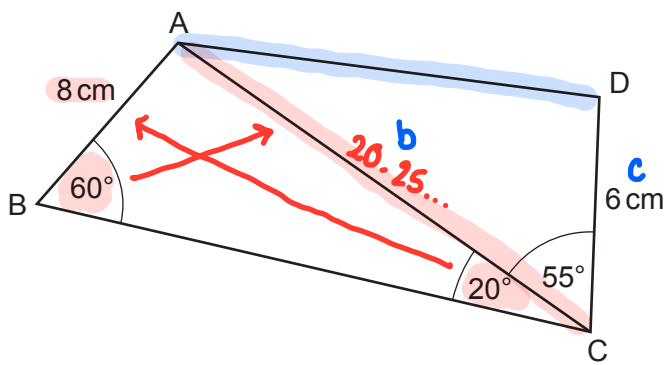
$$\begin{array}{l} 99x = 18 \\ \div 99 \qquad \qquad \div 99 \\ x = \frac{18}{99} = \frac{2}{11} \end{array}$$

- (b) Explain how $\frac{2}{11} = 0.\dot{1}\dot{8}$ can be used to find $\frac{10}{11}$ as a decimal and write down its value.

$$\frac{2}{11} \times 5 = \frac{10}{11} \quad \text{so} \quad 0.\dot{1}\dot{8} \times 5 = 0.\dot{9}\dot{0}$$

$$\frac{10}{11} = 0.\dot{9}\dot{0} \quad [2]$$

18 The diagram shows two triangles ABC and ADC.



Angle ABC = 60°, angle ACB = 20° and angle ACD = 55°.
AB = 8 cm and DC = 6 cm.

Calculate length AD.
You must show your working.

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{AC}{\sin 60} = \frac{8}{\sin 20}$$

$$AC = \frac{8}{\sin 20} \times \sin 60$$

$$= 20.25671109 \text{ cm}$$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$AD^2 = (20.25\dots)^2 + (6)^2 - 2(20.25\dots)(6) \cos 55$$

$$AD^2 = 306.9090783$$

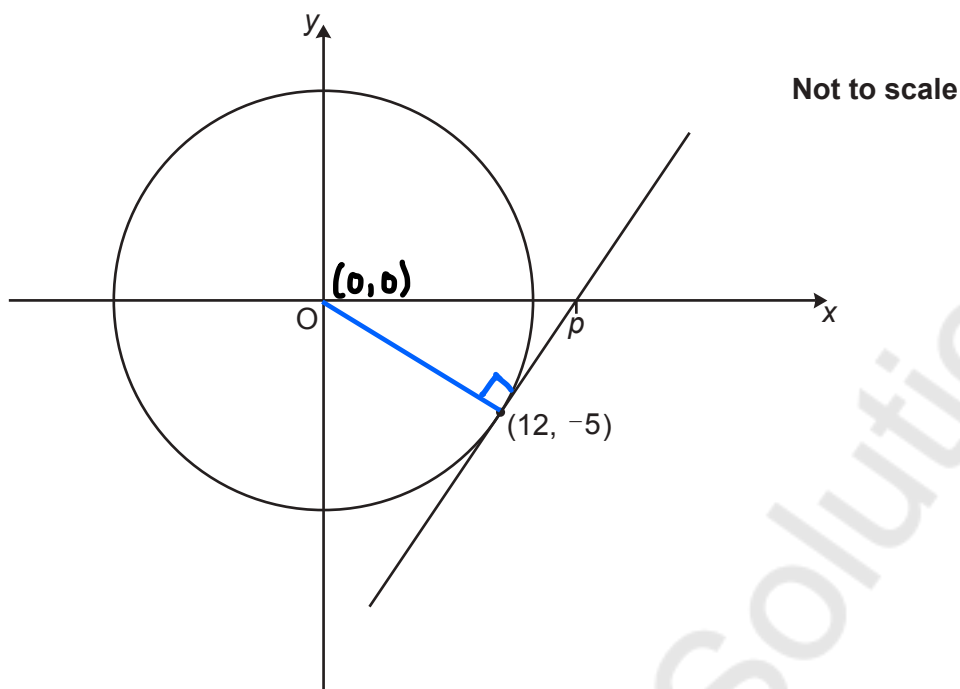
$$\sqrt{\quad} \quad \quad \sqrt{\quad}$$

$$AD = 17.51882069 \text{ cm}$$

$$\approx 17.5 \text{ cm}$$

AD = **17.5** cm [6]

- 19 The diagram shows a circle with centre $(0, 0)$ and a tangent at $(12, -5)$.
The tangent at $(12, -5)$ crosses the x -axis at $(p, 0)$.



Find the exact value of p .
You must show your working.

Gradient of radius $(0, 0)$ and $(12, -5)$
 x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 0}{12 - 0}$$

$$= \frac{-5}{12}$$

Gradient of tangent = $\frac{12}{5}$

$$y = mx + c$$

$$y = \frac{12}{5}x + c \quad \begin{matrix} (12, -5) \\ x \quad y \end{matrix}$$

$$-5 = \frac{12}{5}(12) + c$$

$$-5 = \frac{144}{5} + c$$

$$\frac{-169}{5} = c$$

Equation of tangent

$$y = \frac{12}{5}x - \frac{169}{5} \quad \begin{matrix} (p, 0) \\ x \quad y \end{matrix}$$

$$0 = \frac{12}{5}x - \frac{169}{5}$$

$$\frac{169}{5} = \frac{12}{5}x$$

$$169 = 12x$$

$$\frac{169}{12} = x$$

$$p = \frac{169}{12} \dots \dots \dots [5]$$

Turn over for Question 20

20 Write as a single fraction in its simplest form.

$$4 + \frac{x^2 - 49}{(x+7)(x-3)}$$

$$\frac{x^2 - 49}{\sqrt{\quad} \sqrt{\quad}}$$

$$(x+7)(x-7)$$

$$4 + \frac{\cancel{(x+7)}(x-7)}{\cancel{(x+7)}(x-3)}$$

$$\frac{4}{1} + \frac{x-7}{x-3}$$

$$\frac{4(x-3) + 1(x-7)}{1(x-3)}$$

$$\frac{4x - 12 + x - 7}{x - 3}$$

$$\frac{5x - 19}{x - 3}$$

$$\frac{5x - 19}{x - 3}$$

..... [5]

END OF QUESTION PAPER

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