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National Qualifications 2022 MODIFIED

Mark

# X847/76/01 Mathematics Paper 1 (Non-calculator) Answer booklet

FRIDAY, 6 MAY
9:00 AM – 10:15 AM

*X8477601*

**Fill in these boxes and read what is printed below.**

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth
Day Month Year

Scottish candidate number

**You must NOT use a calculator.**

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



SQA ©

*X847760101*

QUESTION NUMBER

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**1.** $5x + 2y = 7 \qquad (-1, 6)$

$2y = -5x + 7$

$y = -\frac{5}{2}x + \frac{7}{2}$

$m = -\frac{5}{2}$

$m_{\perp} = \frac{2}{5}$

$y - 6 = \frac{2}{5}(x - (-1))$

$y - 6 = \frac{2}{5}(x + 1)$

$5y - 30 = 2x + 2$

$5y - 2x = 32$

**2.** $2\log_3 6 - \log_3 4$

$\log_3 6^2 - \log_3 4$

$\log_3 36 - \log_3 4$

$\log_3 \frac{36}{4}$

$\log_3 9$

$3^2 = 9$

$\log_3 9 = 2$

QUESTION NUMBER

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**3.**

$$h(x) = 4 + \frac{1}{3}x$$

$$x = 4 + \frac{1}{3}y$$

$$x - 4 = \frac{1}{3}y$$

$$3x - 12 = y$$

$$h^{-1}(x) = 3x - 12$$

**4.**

$$y = \sqrt{x^3} - 2x^{-1}$$

$$y = (x^3)^{1/2} - 2x^{-1}$$

$$y = x^{3/2} - 2x^{-1}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 2(-1)x^{-2}$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x} + \frac{2}{x^2}$$

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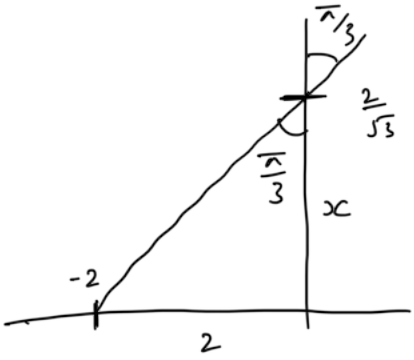
**5.**



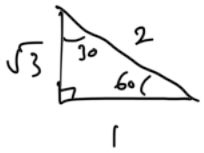

$\tan\frac{\pi}{3} = \frac{2}{x}$

$\sqrt{3} = \frac{2}{x}$

$x = \frac{2}{\sqrt{3}}$

$\pi : 180$

$\frac{\pi}{3} : 60$




$\tan 60 = \frac{\sqrt{3}}{1}$

$m = \frac{\frac{2}{\sqrt{3}} - 0}{0-(-2)} = \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{\sqrt{3}}{3}$

$y = x\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}}{3}$

**6.**

$$\int_{-5}^{2} (10-3x)^{-\frac{1}{2}}\,dx$$

$u = 10 - 3x$

$\frac{du}{dx} = -3$, $dx = \frac{du}{-3}$

$u_2 = 10 - 3(2) = 10 - 6 = 4$

$u_1 = 10 - 3(-5) = 10 + 15 = 25$

$$\int_{25}^{4} u^{-\frac{1}{2}} \times \frac{du}{-3}$$

$$-\frac{1}{3}\int_{25}^{4} u^{-1/2}\,du$$

$$-\frac{1}{3}\left[\frac{u^{1/2}}{1/2}\right]_{25}^{4}$$

$$-\frac{1}{3}\left[2\sqrt{4} - 2\sqrt{25}\right]$$

$$-\frac{1}{3}\left[4 - 10\right]$$

$$-\frac{1}{3} \times -6 = 2$$

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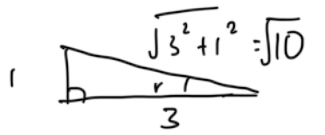
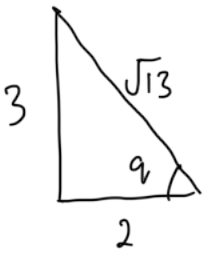
**7.(a) (i)**




$$\sin r = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

**7.(a) (ii)**




$$\sin q = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\sqrt{(\sqrt{13})^2 - 2^2}$$
$$\sqrt{13-4}$$
$$\sqrt{9} = 3$$

**7.(b)**

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\sin(q - r) = \sin q \cos r - \sin r \cos q$$

$$\cos q = \frac{2}{\sqrt{13}} \qquad \cos r = \frac{3}{\sqrt{10}}$$

$$\sin(q - r) = \left(\frac{3}{\sqrt{13}} \times \frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{13}}\right)$$

$$\frac{9}{\sqrt{130}} - \frac{2}{\sqrt{130}}$$

$$\sin(q - r) = \frac{7}{\sqrt{130}}$$

QUESTION NUMBER

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**8.**

$$\log_6 x + \log_6 (x+5) = 2$$

$$\log_6 x(x+5) = 2$$

$$6^{\log_6 x(x+5)} = 6^2$$

$$x(x+5) = 36$$

$$x^2 + 5x - 36 = 0$$

$-36$

$-1, 36$

$-2, 18$

$-3, 12$

$-6, 6$

$\boxed{-4, 9}$

$$x^2 + 9x - 4x - 36 = 0$$

$$x(x+9) - 4(x+9) = 0$$

$$(x-4)(x+9) = 0$$

$$\therefore x = 4 \quad x = -9 \text{ reject}$$

**9.**

$$\cos 2x = 5\cos x - 3$$

$$\cos^2 x - \sin^2 x = 5\cos x - 3$$

$$\cos^2 x - (1 - \cos^2 x) = 5\cos x - 3$$

$$\cos^2 x - 1 + \cos^2 x = 5\cos x - 3$$

$$2\cos^2 x - 5\cos x + 2 = 0$$

$\cos x = y \quad \cos^2 x = y^2$

$$2y^2 - 5y + 2 = 0$$

$$2y^2 - y - 4y + 2 = 0$$

$$y(2y-1) - 2(2y-1) = 0$$

$$(y-2)(2y-1) = 0$$

$$\therefore y = 2 \quad y = \tfrac{1}{2}$$

$\cos x = 2$

$x = \cos^{-1}(2)$

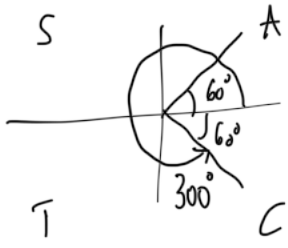
reject

no real solutions

$\cos x = \frac{1}{2}$

$x = \cos^{-1}(\frac{1}{2})$

$x = 60^\circ, 300^\circ$



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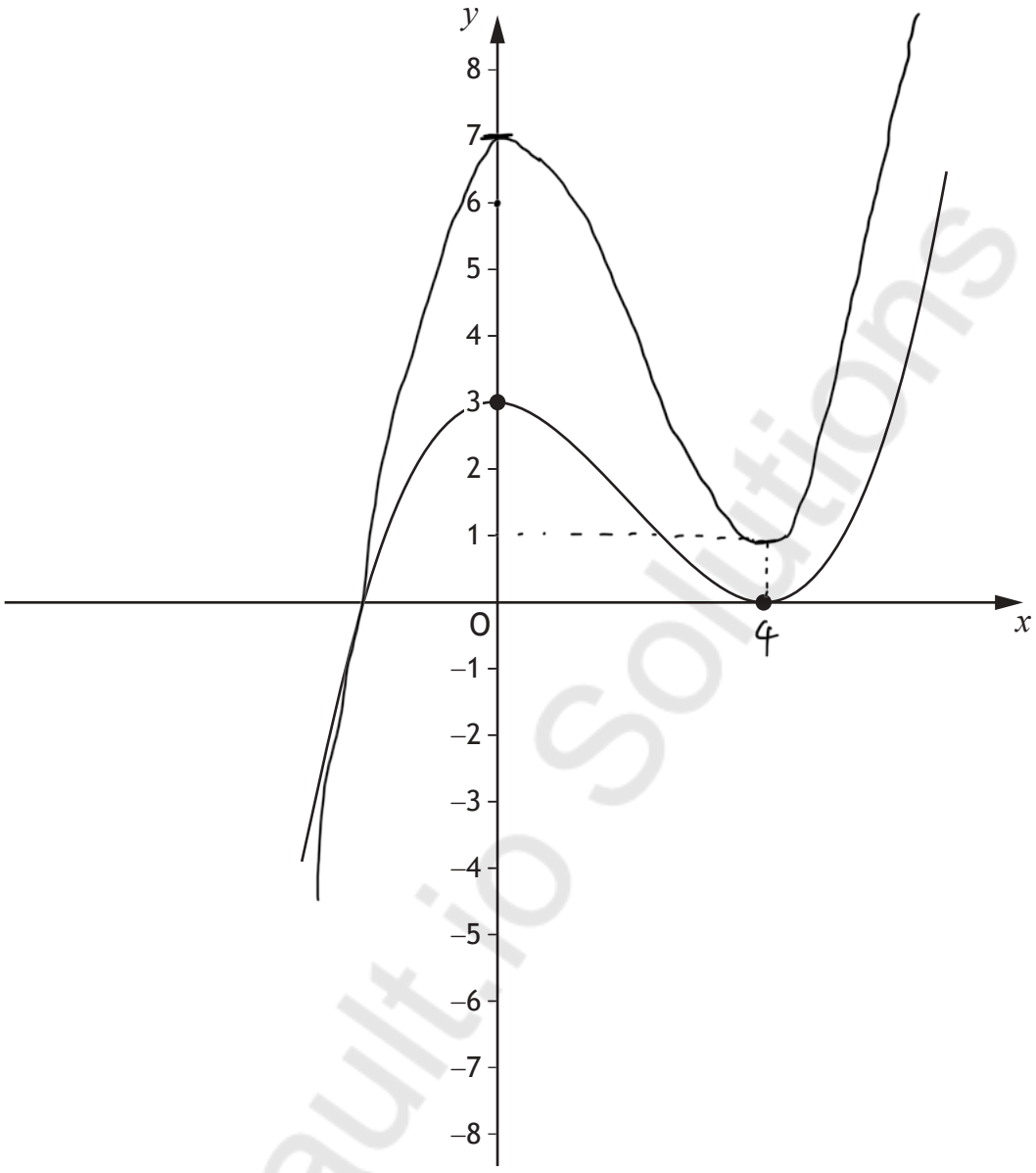
An additional diagram, if required, can be found on *page 11*.

**10.(a)**




**10.(b)** (0 × 2 , 3) (4 × 2 , 0)

(0, 3) (8, 0)

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**11.**

$2x^2 + 12x + 23$

$2(x^2 + 6x) + 23$

$2[(x+3)^2 - 9] + 23$

$2[x+3]^2 - 18 + 23$

$2(x+3)^2 + 5$

**12.**

$f(x) = 4\sin\left(3x - \frac{\pi}{3}\right)$

$\pi : 180$

$\frac{\pi}{6} : 30$

$\frac{1}{2} - \frac{1}{3}$

$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

$u = 3x - \frac{\pi}{3}$, $y = 4\sin u$

$\frac{du}{dx} = 3$ $\quad \frac{dy}{du} = 4\cos u = 4\cos\left(3x - \frac{\pi}{3}\right)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = f'(x) = 3 \times 4\cos\left(3x - \frac{\pi}{3}\right) = 12\cos\left(3x - \frac{\pi}{3}\right)$

$f'\left(\frac{\pi}{6}\right) = 12\cos\left(3\left(\frac{\pi}{6}\right) - \frac{\pi}{3}\right)$

$\frac{\pi}{2} - \frac{\pi}{3}$

$f'\left(\frac{\pi}{6}\right) = 12\cos\left(\frac{\pi}{6}\right) = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

QUESTION NUMBER

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**13.(a) (i)**

$$f(-2) = (-2)^3 - 2(-2)^2 - 20(-2) - 24$$
$$= -8 - 8 + 40 - 24$$
$$f(-2) = 0$$
$$\therefore \quad x+2 \text{ is a factor of } f(x)$$

**13.(a) (ii)**

$$x+2 \;\overline{)\; x^3 - 2x^2 - 20x - 24}$$

$x^2 - 4x - 12$

$-(x^3 + 2x^2)$

$-4x^2 - 20x$

$-(-4x^2 - 8x)$

$-12x - 24$

$-(-12x - 24)$

$0 \quad 0$

$$(x+2)(x^2 - 4x - 12) = 0$$
$$(x+2)(x^2 - 6x + 2x - 12) = 0$$
$$(x+2)[x(x-6) + 2(x-6)] = 0$$
$$(x+2)(x+2)(x-6) = 0$$

$\therefore \quad x = -2, \; x = -2, \; x = 6$

**13.(b)**

$(-2, 0) \longrightarrow (1, 0)$

$\therefore \quad y = f(x - 3)$

$k = 3$

QUESTION NUMBER

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**14.(a) (i)**

$C(7, -5)$

$r = 10$

**14.(a) (ii)**

$(-2-7)^2 + (7+5)^2$

$(-9)^2 + (12)^2$

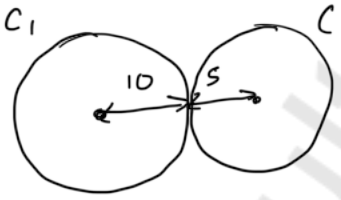
$81 + 144$

$225 \neq 100$

$\therefore P(-2, 7)$ lies outside $C_1$

$$\begin{array}{r} {}^{1}1\,{}^{1}4\,4 \\ +\ 8\,1 \\ \hline 2\,2\,5 \end{array}$$

**14.(b)**




$C_1(7, -5)$

$C_2(-2, 7)$

$D = \sqrt{[7-(-2)]^2 + [-5-7]^2}$

$D = \sqrt{9^2 + (-12)^2}$

$D = \sqrt{81 + 144}$

$D = \sqrt{225}$

$D = 15$

$r$ for $C_1$ must be 10

$r$ for $C_2$ must be 5

] Both must be true for one point of intersection between $C_1$ and $C_2$

$$\begin{array}{r} {}^{2}1\,5 \\ \times\ 1\,5 \\ \hline 7\,5 \\ 1\,5 \\ \hline 2\,2\,5 \end{array}$$

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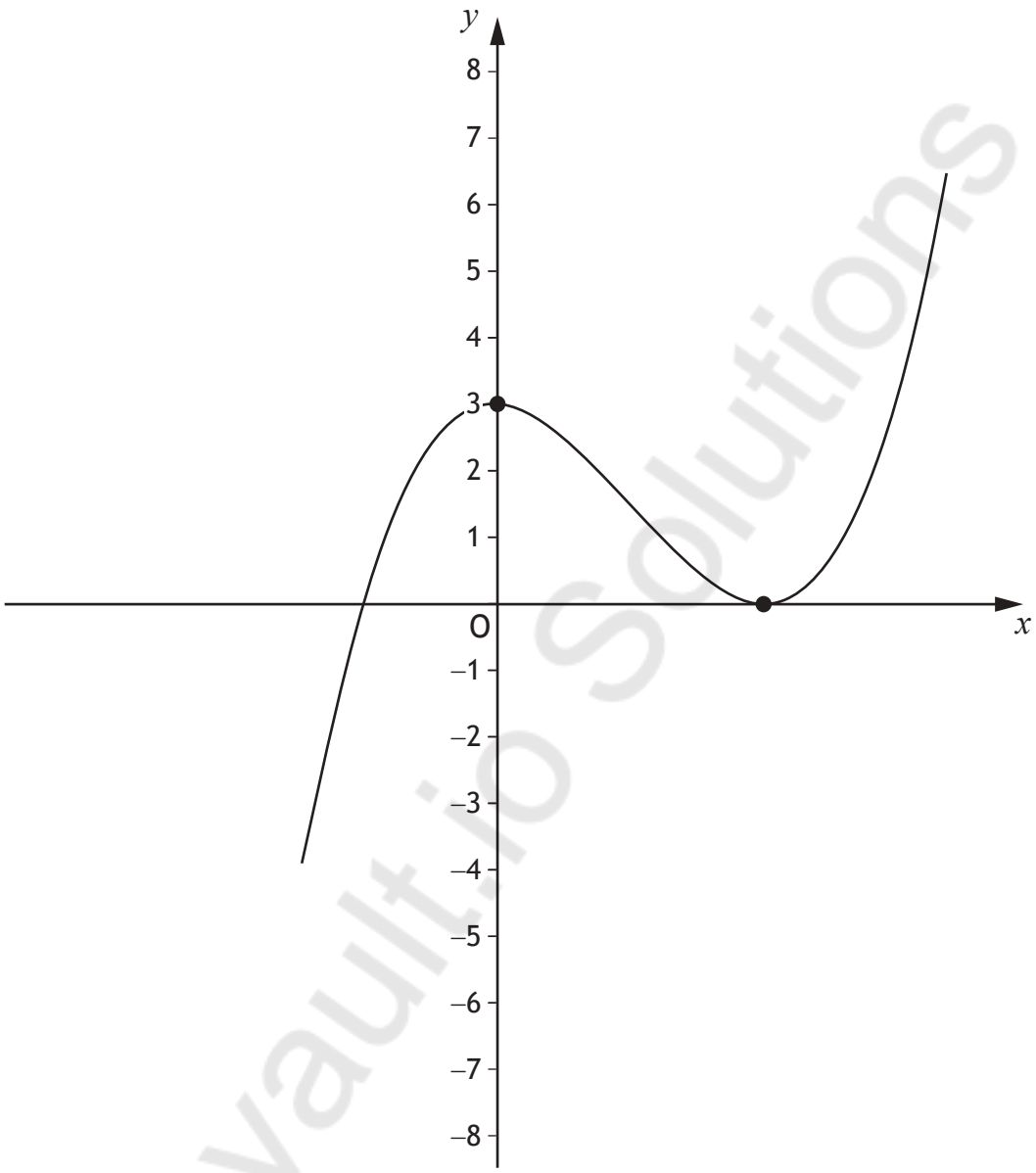
**ADDITIONAL SPACE FOR ANSWERS**

Additional diagram for question 10 (a).



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## ADDITIONAL SPACE FOR ANSWERS

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